Skills, Tasks, and Complexity

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1. Introduction

2. The Model

2.1 Macroeconomic Environment

- Households derive utility from the consumption of products.
- Each product (i, j) is a variety j that belongs to industry i.
- The utility of household r is described by a nested CES-function

$$U^r\left(\left\{c_{i,j}^r\right\}_{(i,j)\in\mathcal{I}\times[0,n_i]}\right) = C^r, \tag{1}$$

where

$$C^r := \left[\sum_{i\in\mathcal{I}}\left[\psi_i^{\frac{1}{\sigma_I-1}}\left[\int_0^{n_i} c_{i,j}^{r\ \frac{\sigma_v-1}{\sigma_v}}\,dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I-1}}.$$

- The wage household r receives is denoted by w^r and the profits he obtains from ownership are denoted by $\prod r$.
- We do not make any assumption about the distribution of firm ownership across households since equilibria can be determined independently of the distribution of ownership, and thus of the profit income distribution.
- The budget constraint of household r is therefore

$$\sum_{i \in \mathcal{I}} \int_0^{n_i} p_{i,j} c_{i,j}^r dj \leq L^r w^r + \Pi^r \ , \tag{2}$$

where $p_{i,j}$ denotes the price of product (i, j).

- The demand of household r for a product (i,j) is

$$c_{i,j}^r = \psi_i \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C^r \ ,$$

- Where $P_i := \left[\int_{ni} p_{i,j}^{1-\sigma_v} dj\right]^{\frac{1}{1-\sigma_v}}$ and $P := [\sum i \in \mathcal{I} \psi_i P_i^{1-\sigma_v}]$ are the price indices for industry i and the aggregate price index, respectively.

- The derivation of household r's demand is presented in Appendix A.1.

- Total demand for the product (i,j) is

$$c_{i,j} = \psi_i \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \qquad (3)$$

where $C := \int_{\mathcal{R}} C^r f(r) dr$ is the total consumption.

- Aggregation of budget constraints yields

$$PC := \int_{\mathcal{R}} L^r w^r f(r) dr + \int_{\mathcal{R}} \Pi^r f(r) dr \ ,$$

where PC denotes the total nominal consumption expenditures.

- We assume that every firm holds a patent for its product.
- A representative firm i e.g., holds a patent for a product (i, j).
- Specifically, if it hires an amount $li(r)$ labor of skill level r its output—denoted by xi – is given by

$$x_i = \int_{r \in \mathcal{R}_i} \kappa_1(r)\kappa_2(i)l_i(r)dr \ , \tag{4}$$

where $k_1(r)k_2(i)$ are the skill-dependent and complexity-dependent productivities of the employed labor, respectively.

- The profit maximization problem of a representative firm i is

$$\begin{aligned}
\max_{\mathcal{R}_i, p_i, \{x_i(r)\}_{r\in\mathcal{R}_i}, \{l_i(r)\}_{r\in\mathcal{R}_i}} \quad & \int_{r\in\mathcal{R}_i} \left[p_i x_i(r) - l_i(r) w^r\right] dr \ , \qquad (5) \\
s.t. \quad & x_i(r) = \kappa_1(r)\kappa_2(i) l_i(r) \ , \\
& x_i = \int_{r\in\mathcal{R}_i} x_i(r) dr = \psi_i \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \\
& r \geq \tilde{r}(i) \quad \forall r \in \mathcal{R}_i \ , \\
& \mathcal{R}_i \subseteq \mathcal{R} \ .
\end{aligned}$$

i. *Cost Minimization:* The firm chooses skill levels $\mathcal{R}_i$ suitable for production that minimize the cost per unit of output.

ii. *Profit Maximization:* Given the minimal cost per unit of output, the firm chooses a price p_i to maximize its profits.

The price, in turn, determines the output and the amount of labor input. Note that the cost per unit of output might be minimized for different skill levels. Then a firm is indifferent between these skill levels since they are perfect substitutes. We next study each of these two sub-problems of firm i in detail.

i. Cost Minimization

Firm i minimizes the cost per unit of output by choosing a subset of skills, $\mathcal{R}_i \subseteq \mathcal{R}$, that fulfills the minimization problem

$$\min_{r} \quad \frac{w^r}{\kappa_1(r)\kappa_2(i)} \qquad s.t. \quad r \geq \bar{r}(i) \ .$$

Note that the firm takes Assumption 1 into account.

ii. Profit Maximization

- Given the cost-minimizing set of skill levels in production, R_i, firm i chooses a price to solve its profit maximization problem given in (5).

- Without loss of generality, we can assume that all of firm i's production is performed by a single skill level, i.e., $R_i = \{r\}$.

- Firm i's profit maximization problem then is

$$\max_{p_i} \quad p_i x_i - x_i \frac{w^r}{\kappa_1(r)\kappa_2(i)} ,$$

$$s.t. \quad x_i = \psi_i \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C .$$

- This yields

$$p_i = \frac{\sigma_v}{\sigma_v - 1} \frac{w^r}{\kappa_1(r)\kappa_2(i)} . \tag{6}$$

2.2 Equilibrium

- Before we derive the equilibrium, we relate skill level r to the productivity of the highest skill level in the economy,

$$\tilde{l}(r) = l(r)\frac{\kappa_1(r)}{\kappa_1(\bar{r})} \ . \tag{7}$$

- We note that $\tilde{l}(r)$ expresses labor input, normalized by productivity across skill levels. We call $\tilde{l}(r)$ "effective" labor. Furthermore, we denote effective labor demand of a representative firm i by

$$\tilde{l}_i = \int_{\mathcal{R}_i} l_i(r)\frac{\kappa_1(r)}{\kappa_1(\bar{r})}dr \ .$$

LMCC (Labor Market Clearing Condition)

$$\int_{\tilde{r}(\hat{i})}^{\bar{r}} \frac{\kappa_1(r)}{\kappa_1(\bar{r})} L^r f(r) dr \geq \sum_{i \in \mathcal{I}: i \geq \hat{i}} n_i \tilde{l}_i(\mathcal{W}) \quad \forall\, \hat{i} \in \mathcal{I} \quad \text{(LMCC)}\,, \tag{8}$$

where $\mathcal{W} = \{w^r\}_{r \in \mathcal{R}}$ denotes the "wage scheme" that describes the wages workers of different skill levels will receive in an equilibrium.

- From (6) we know that a firm i is indifferent between producing a product with skill levels r' or r —under the condition that both are greater or equal to $\tilde{r}(i)$— if and only if the wages reflect their relative productivity differences, i.e., if

$$w^r = \frac{\kappa_1(r)}{\kappa_1(r')} w^{r'}\,. \tag{9}$$

- For both cases we assume the following wage scheme of *potential wages*:

$$\mathbf{W} = \left\{ \omega_i \frac{\kappa_1(r)}{\kappa_1(\bar{r})} \right\}_{(i,r)\in\mathcal{I}\times\mathcal{R}} , \tag{10}$$

where ω_i denotes the "scaling factor" in industry i and $\frac{k_1(r)}{k_1(r)}$ is the "productivity factor" of skill r. The wage scheme $\mathbf{W}$ is a menu of wages and describes the wage of a worker of skill r when he is working in industry i.

- We denote the households' demand for representative product i by c_i and we use firms' optimal price choice (6) and wage scheme (10) to obtain

$$p_i = \frac{\sigma_v}{\sigma_v - 1} \frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} \quad \forall i \in \mathcal{I} ,$$

$$P_i = \frac{\sigma_v}{\sigma_v - 1} \frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} n_i^{\frac{1}{1-\sigma_v}} \quad \forall i \in \mathcal{I} ,$$

$$P = \frac{\sigma_v}{\sigma_v - 1} \left[\sum_{i\in\mathcal{I}} \psi_i \left[\frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} n_i^{\frac{1}{1-\sigma_v}} \right]^{1-\sigma_I} \right]^{\frac{1}{1-\sigma_I}} .$$

- Using (3) and the price indices from above, the households' demand for representative product i is

$$c_i = \psi_i \left[\frac{\kappa_1(\bar{r})\kappa_2(i)}{\omega_i}\right]^{\sigma_I} n_i^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \left[\frac{\omega_{\hat{i}}}{\kappa_1(\bar{r})\kappa_2(\hat{i})} n_{\hat{i}}^{\frac{1}{1-\sigma_v}}\right]^{1-\sigma_I}\right]^{\frac{\sigma_I}{1-\sigma_I}} C \ .$$

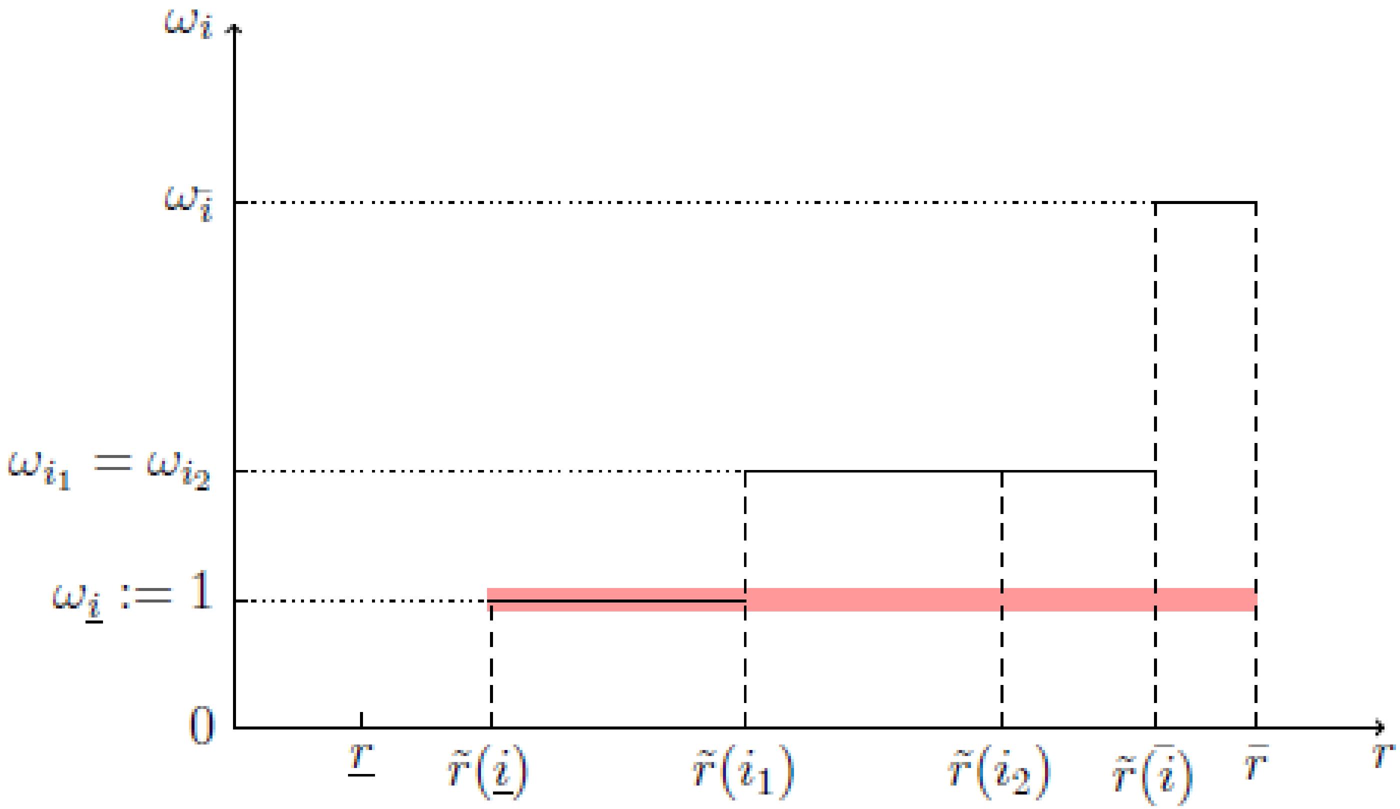
- Goods market clearing implies that $c_i = x_i$. Then, effective labor demand of firm i is

$$\tilde{l}_i = \psi_i \omega_i^{-\sigma_I} \left[\kappa_1(\bar{r})\kappa_2(i)\right]^{\sigma_I-1} n_i^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \left[\frac{\omega_{\hat{i}}}{\kappa_1(\bar{r})\kappa_2(\hat{i})} n_{\hat{i}}^{\frac{1}{1-\sigma_v}}\right]^{1-\sigma_I}\right]^{\frac{\sigma_I}{1-\sigma_I}} C \quad \forall i \in \mathcal{I} \ . \tag{11}$$

- Labor market clearing implies that total wages paid by firms must equal total wages earned by households, i.e.,

$$\sum_{i\in\mathcal{I}} n_i \omega_i \tilde{l}_i = \sum_{i\in\mathcal{I}} \int_{\bar{r}(i)}^{\bar{r}(i_+)} \omega_i \frac{\kappa_1(r)}{\kappa_1(\bar{r})} L^r f(r) dr \ ,$$

Figure 1: Two wage schemes with different scaling factors



- In Figure 1, we show two possible wage schemes in an economy with $I = \{i, i_1, i_2, \bar{\imath}\}$, where $i < i_1 < i_2 < \bar{\imath}$.

- We order groups according to skill levels and $g+$ denotes the next higher group to g.

- We call a particular set of groups a group structure denoted by $\mathcal{G}$.

- Then, $|\mathcal{G}|$ equals the number of labor market separations plus 1 in such a group structure, i.e., the number of jumps in the scaling factor plus 1.

- The scaling factor within a group is denoted by w_g and by construction,

$$\omega_{g_+} > \omega_g \quad \forall\, g \in \mathcal{G} \,, \tag{12}$$

3. Numerical Examples

Table 1: Skill Categories

Unemployed	: $F(\tilde{r}(\underline{i}))$
Low	: $F(\tilde{r}(i_1)) - F(\tilde{r}(\underline{i}))$
Low-Mid	: $F(\tilde{r}(i_2)) - F(\tilde{r}(i_1))$
Mid-High	: $F(\tilde{r}(\bar{i})) - F(\tilde{r}(i_2))$
High	: $1 - F(\tilde{r}(\bar{i}))$

- The set of skills employed in an industry i that cannot be employed in industry $i+$ is called a "category"—or, equivalently, a skill category.

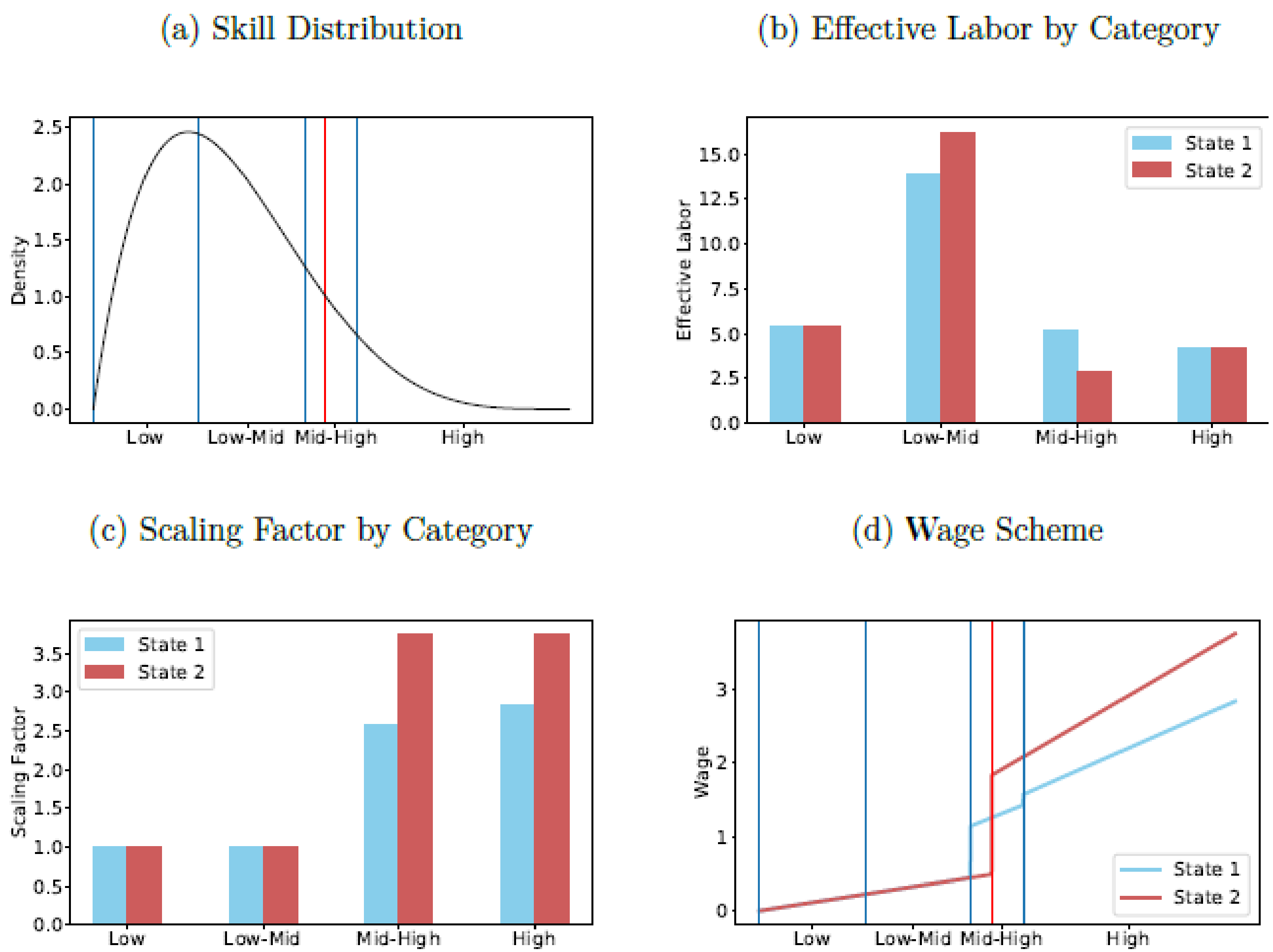
- There are four different task complexities which transform the skill distribution into five categories which we will call: "Unemployed", "Low," "Low-Mid," "Mid-High," and "High."

- Table 1 presents the skill categories and their respective relative labor masses.

- In all examples, the threshold skill levels are chosen such that there are no unemployed workers.

- Thus, we focus on the remaining four categories.

Figure 2: Equilibrium DLM - Increase in i_2 from 2 to 2.1



Notes: i_2 is the upper (lower) bound of the Low-Mid (Mid-High) skill group.

- In Figure 2, we display the consequences of an increase in the task-complexity in industry i_2 from 2 to 2.1.

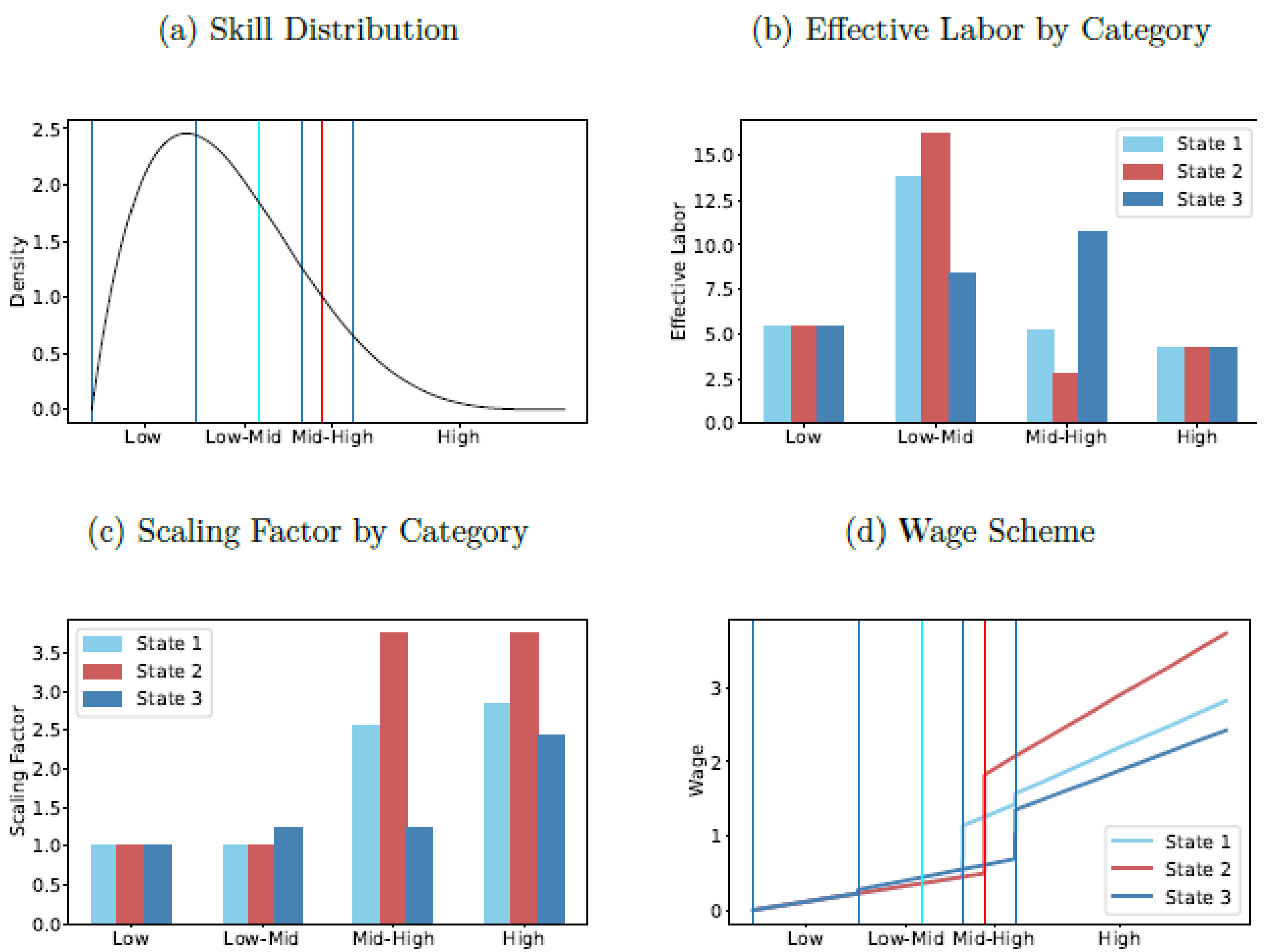
- We show the resulting re-categorization of labor, the changes for the distribution of effective labor across categories, the scaling factors, and the wage scheme.

- The sky-blue graphs and bars describe State 1, where $i_2 = 2$, whereas the red graphs and bars describe State 2, where $i_2 = 2.1$.

- The vertical red lines mark $\tilde{r}(2.1)$.

- Figure 2a presents the skill distribution and the skill categories.

Figure 3: Equilibrium DLM - Decrease in i2 from 2 to 1.8



Notes: i_2 is the upper (lower) bound of the Low-Mid (Mid-High) skill group.

- In Figure 3, we add to Figure 2 the impact of a decrease in i_2 from 2 to 1.8, which we call State 3 (displayed in dark blue). The vertical lines in cyan mark $\tilde{r}(1.8)$.

- The highest-skilled Low-Mid workers are re-categorized as the lowest-skilled Mid-High workers.

- Now, the increase in effective labor supply in category Mid-High drives down wages in this category. This effect is sufficiently strong such that the workers in categories

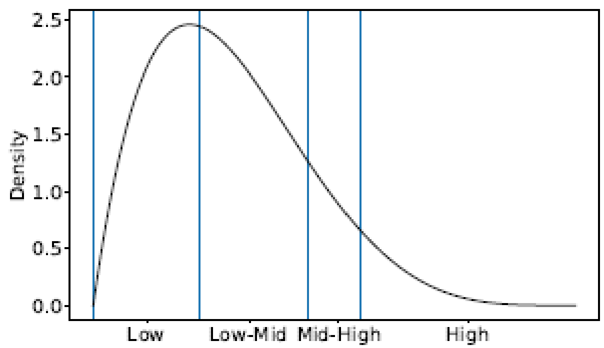
- Low-Mid and Mid-High form one group.

- This can be seen in Figure 3c, where the scaling factors of the two skill categories are the same.

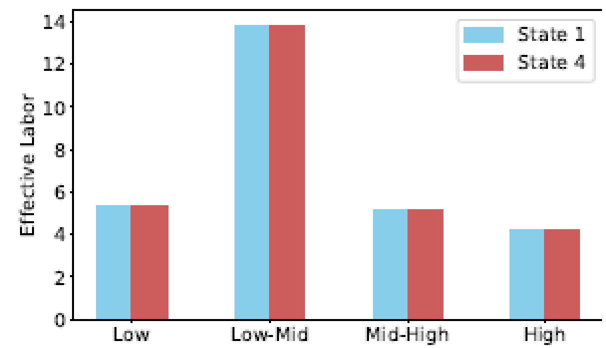
- Equivalently, the wage scheme in Figure 3d displays no jump at $\tilde{r}(1.8)$ (the vertical line in cyan).

Figure 4: Equilibrium DLM - Increase in ni2 from 5 to 9

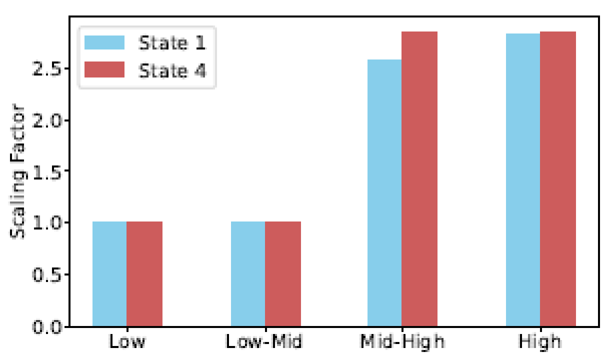
(a) Skill Distribution



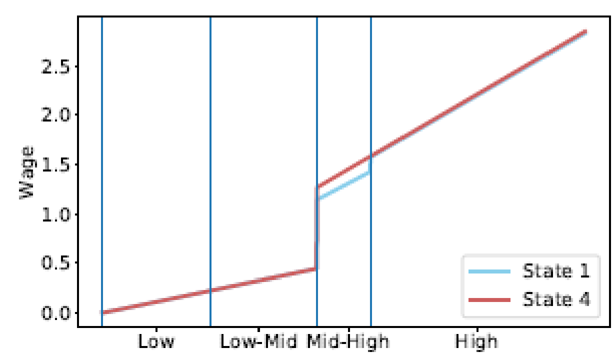
(b) Effective Labor by Category



(c) Scaling Factor by Category



(d) Wage Scheme



Notes: An increase in n_{i_2} increases the demand for the Mid-High skill group.

- In Figure 4, we display the effects of an increase in the number of firms in industry i_2, ni_2 , from 5 to 9.
- Thus, more varieties are offered in industry i_2 which impacts the number of groups and the wage jumps.
- The sky-blue graphs and bars display State 1 from the previous figures, while State 4 (red graphs and bars) depicts the situation in which the firm number is increased.
- As a consequence of this increase, the demand for labor of category Mid-High increases, thereby pushing the scaling factor upwards.
- Another consequence of the increased demand for the Mid-High skill category is that the Mid-High category and the High category now form one group.

3. Applications

Automation

- Automation can be understood in the context of our model as the ability of machines to execute tasks so far performed by human beings.
- We examine what happens if automation takes place.
- We distinguish several cases. First, automation could address the high-complexity tasks—when for instance software tools are developed by software machines or trading algorithms replace traders.
- Second, automation could also eliminate low-complexity tasks—when for instance driving is automated by self driving cars.
- In both cases we encounter subtle general equilibrium effects which go in very different directions for both cases.

Task-facilitating Innovations and the Task Life-cycle

- Another interesting aspect of technological progress are innovations that facilitate the performance of tasks and thus help workers to perform them (henceforth "task facilitating innovations").
- Together with automation of low-complexity tasks and the emergence of new production methods with high task-complexities, we could examine a task life-cycle.
- Such a task life-cycle thus describes the emergence of a task with relatively high task-complexity, the following descend of this task-complexity due to task-facilitating innovations, and the final automation of the task.

Jobs and Wages

- Typically, a set of tasks can be divided among a set of workers which facilitates to match task-complexity to worker skills.
- However, this division has technological limits and subsets of tasks cannot be further divided. Such limits give rise to the concept of jobs.
- A job encompasses a set of tasks which cannot be split further into subsets and thus has to be performed by one person.
- For instance, a person that creates a homepage must be able to use the underlying program tools, must be able to structure the activities for the homepage in a collectively exhaustive and mutually exclusive way and must be able to write the text in the languages required.