TECHNICAL EXTRACT: Skills, Tasks and Technologies: Implications for Employment and Earnings

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(Handbook of Labor Economics, 2011)

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Two skills, high and low.

High and low skill workers are imperfect substitutes in production.

Let $\mathcal{L}$ denote the set of low skill workers and $\mathcal{H}$ denote the set of high skill workers.

$i \in \mathcal{L}$ has l_i efficiency units of low skill labor and each high skill worker
$i \in \mathcal{H}$ has h_i units of high skill labor.

All workers supply their efficiency unites inelastically.

$$L = \int_{i \in \mathcal{L}} l_i di \text{ and } H = \int_{i \in \mathcal{H}} h_i di.$$

Production function

$$Y = \left[(A_L L)^{\frac{\sigma-1}{\sigma}} + (A_H H)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

$\sigma \in [0, \infty)$ is the elasticity of substitution
A_L and A_H are factor-augmenting technology terms.

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The production function (1) admits three different interpretations.

1. There is only one good, and high skill and low skill workers are imperfect substitutes in the production of this good.
2. The production function (1) is also equivalent to an economy where consumers have utility function $\left[Y_l^{\frac{\sigma-1}{\sigma}} + Y_h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ defined over two goods. Good Y_h is produced using only high skill workers, and Y_l is produced using only low skill workers, with production functions $Y_h = A_H H$, and $Y_l = A_L L$.
3. A mixture of the above two whereby different sectors produce goods that are imperfect substitutes, and high and low education workers are employed in both sectors.

$$w_L = \frac{\partial Y}{\partial L} = A_L^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} (H/L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}. \quad (2)$$

earnings of worker $i \in \mathcal{L}$:

$$W_i = w_L l_i.$$

There are two important implications of Eq. (2):

1. $\partial w_L / \partial H / L > 0$
2. $\partial w_L / \partial A_L > 0$ and $\partial w_L / \partial A_H > 0$, that is, either kind of factor-augmenting technical change *increases* wages of low skill workers (except in the limit case where $\sigma = \infty$, the second inequality is weak).

High skill unit wage:

$$w_H = \frac{\partial Y}{\partial H} = A_H^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} (H/L)^{-\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}. \qquad (3)$$

Two Implications

1. $\partial w_H / \partial H / L < 0$
2. $\partial w_H / \partial A_L > 0$ and $\partial w_H / \partial A_H > 0$, so that technological progress of any kind increases high skill (as well as low skill) wages.

   the earnings of worker $i \in \mathcal{H}$ is simply

$$W_i = w_L h_i.$$

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The skill premium:

$$\omega = \frac{\omega_H}{\omega_L} = \left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}. \tag{4}$$

Taking logs:

$$\ln \omega = \frac{\sigma - 1}{\sigma} \ln \left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}. \tag{5}$$

$$\frac{\partial \ln \omega}{\partial \ln(H/L)} = -\frac{1}{\sigma} < 0. \tag{6}$$

Most estimates put σ in this context to be somewhere between 1.4 and 2 (Johnson, 1970; Freeman, 1986; Heckman et al. , 1998).

$$\ln\left(\frac{A_{H,t}}{A_{L,t}}\right) = \gamma_0 + \gamma_1 t, \tag{7}$$

$$\ln \omega_t = \frac{\sigma - 1}{\sigma}\gamma_0 + \frac{\sigma - 1}{\sigma}\gamma_1 t - \frac{1}{\sigma}\ln\left(\frac{H_t}{L_t}\right). \tag{8}$$

3.3. Changes in the US earnings distribution through the lens of the canonical model

$$\ln \omega_t = \text{constant} \quad \underset{(0.005)}{+0.027 \times t} \quad \underset{(0.128)}{-0.612 \cdot \ln\left(\frac{H_t}{L_t}\right)}. \text{ Katz-Murphy Regression}$$

$$\hat{\sigma} = 1/0.61 \approx 1.6$$

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Table 8. Regression models for the college/high school log wage gap, 1963-2008.

	1963-1987		1963-2008		
	(1)	(2)	(3)	(4)	(5)
CLG/HS relative supply	−0.612 (0.128)	−0.339 (0.043)	−0.644 (0.066)	−0.562 (0.112)	−0.556 (0.094)
Time	0.027 (0.005)	0.016 (0.001)	0.028 (0.002)	0.029 (0.006)	0.020 (0.006)
Time X post-1992			−0.010 (0.002)		
$Time^2/100$				−0.013 (0.006)	0.036 (0.012)
$Time^3/1000$					−0.007 (0.002)
Constant	−0.217 (0.134)	0.059 (0.039)	−0.254 (0.066)	−0.189 (0.122)	−0.145 (0.103)
Observations	25	46	46	46	46
R-squared	0.558	0.935	0.961	0.941	0.960

Source: March CPS data for earnings years 1963-2008. See notes to Figs 2 and 19.

Thus, taken at face value, this model suggests that relative demand for college workers decelerated in the 1990s, which "does not accord with common intuitions" regarding the nature or pace of technological changes occurring in this era. We return to this point below.

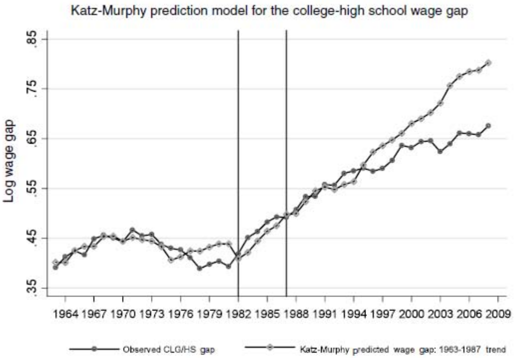

Figure 19 *Source: March CPS data for earnings years 1963-2008. Log weekly wages for full-time, full-year workers are regressed separately by sex in each year on four education dummies (high school dropout, some college, college graduate, greater than college), a quartic in experience, interactions of the education dummies and experience quartic, and two race categories (black, non-white other). The composition-adjusted mean log wage is the predicted log wage evaluated for whites at the relevant experience level (5, 15, 25, 35, 45 years) and relevant education level (high school dropout, high school graduate, some college, college graduate, greater than college). The mean log wage for college and high school is the weighted average of the relevant composition adjusted cells using a fixed set of weights equal to the average employment share of each sex by experience group. The ratio of mean log wages for college and high school graduates for each year is plotted. See the Data Appendix for more details on the treatment of March CPS data. The Katz-Murphy predicted wage gap series contains the predicted values from a regression of the college/high school wage gap on time trend term and log labor supply, as measured in efficiency units described in the note to Fig. 2, for years 1963-1987.*

These facts may better accord with a simple extension to the canonical model. To the extent that workers with similar education but different ages or experience levels are imperfect substitutes in production, one would expect age-group or cohort-specific relative skill supplies–as well as aggregate relative skill supplies–to affect the evolution of the college/high school premium by age or experience, as emphasized by Card and Lemieux (2001b).

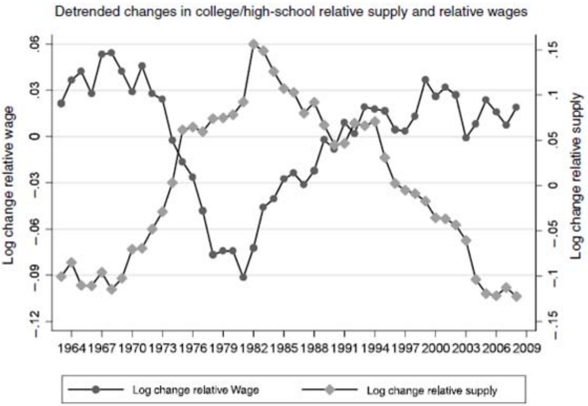

Figure 20 *Source: March CPS data for earnings years 1963-2008. See note to Fig. 19. The detrended supply and wage series are the residuals from separate OLS regressions of the relative supply and relative wage measures on a constant and a linear time trend.*

We take fuller account of these differing trends by experience group in Table 9 by estimating regression models for the college wage by experience group.

Specifically, we estimate:

$$\ln \omega_{jt} = \beta_0 + \beta_1 \left[\ln\left(\frac{H_{jt}}{L_{jt}}\right) - \ln\left(\frac{H_t}{L_t}\right) \right] + \beta_2 \ln\left(\frac{H_t}{L_t}\right) + \beta_3 \times t + \beta_4 \times t^2 + \delta_j + n_{jt},$$

j indexes experience groups, δ_j is a set of experience group main effects, and we include a quadratic time trend.

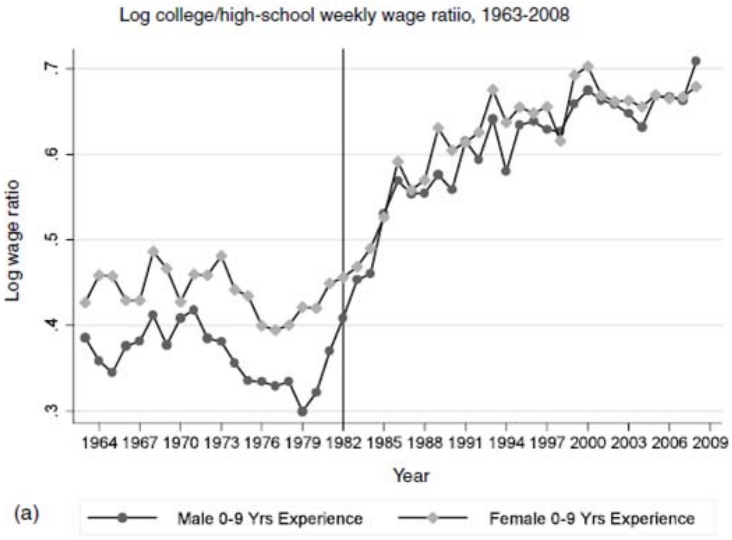
Log college/high-school weekly wage ratiio, 1963-2008
Log wage ratio
.3
.4
.5
.6
.7
1964 1967 1970 1973 1976 1979 1982 1985 1988 1991 1994 1997 2000 2003 2006 2009
Year
(a)
Male 0-9 Yrs Experience
Female 0-9 Yrs Experience

Figure 21

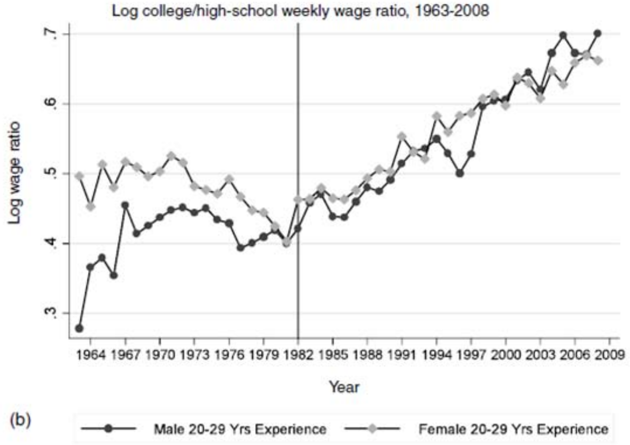

Figure 21 *Source: March CPS data for earnings years 1963-2008. See note to Fig. 19. Log college/high school weekly wage ratio for 0-9 and 20-29 years of potential experience is plotted for males and females.*

Table 9. Regression models for the college/high school log wage gap by potential experience group, 1963-2008.

	Potential experience groups (years)				
	All	*0-9*	*10-19*	*20-29*	*30-39*
Own minus aggregate supply	−0.272 (0.025)	−0.441 (0.136)	−0.349 (0.095)	0.109 (0.079)	−0.085 (0.099)
Aggregate supply	−0.553 (0.082)	−0.668 (0.209)	−0.428 (0.142)	−0.343 (0.138)	−0.407 (0.141)
Time	0.027 (0.004)	0.035 (0.011)	0.016 (0.008)	0.015 (0.007)	0.020 (0.008)
$Time^2/100$	−0.010 (0.004)	−0.023 (0.011)	0.007 (0.008)	0.001 (0.007)	−0.008 (0.009)
Constant	−0.056 (0.085)	−0.118 (0.212)	0.120 (0.169)	0.138 (0.145)	0.018 (0.144)
Observations	184	46	46	46	46
R-squared	0.885	0.885	0.959	0.929	0.771

Source: March CPS data for earnings years 1963-2008. See notes to Figs 2 and 19.

Table 9

The implied value of the partial elasticity of substitution between experience group is around 3.7.

3.4. Overall inequality in the canonical model

Within group inequality is invariant to skill prices and thus changes in overall inequality in this model will closely mimic changes in the skill premium.

$$\frac{W_i}{W_{i'}} = \frac{w_L l_i}{w_L l_{i'}} = \frac{l_i}{l_{i'}} \text{ for } i,\ i' \in \mathcal{L}.$$

Significant within group wage inequality, but inequality will be independent of the skill premium.

To make within group inequality responsive to the wage premium

Assume that the two observable groups are college and non-college

fraction of ϕ_c college graduates are high skill

fraction of $\phi_n < \phi_c$ non-college graduates are high skill

skill premium by $\omega = w_H / w_L$

college wages, w_C: non-college, w_N,

$$\omega^c = \frac{w_C}{w_N} = \frac{\phi_c w_H + (1 - \phi_c) w_L}{\phi_n w_H + (1 - \phi_n) w_L} = \frac{\phi_c \omega + (1 - \phi_c)}{\phi_n \omega + (1 - \phi_n)}.$$

Like Gorman-Lancaster Model

Because $\phi_n < \phi_c$ in ω, when the true price of skill increases, the observed college premium will also arise.

Trivially explains wage inequality within groups as a function of skill promotion.

1. Changes in the wage structure are linked to changes in factor-augmenting technologies and relative supplies.
2. Overall inequality rises in tandem with the skill premium (as within group inequality is either invariant when the skill premium changes or co-moves with the skill premium).
3. The economy-wide average wage and the real wage of each skill group should increase over time as a result of technological progress, particularly if the supply of high skill labor is increasing.
4. The rate and direction of technological change do not respond to the relative abundance or scarcity of skill groups.

Problems with the Canonical Model

1. It does not provide a natural reason for why certain groups of workers would experience real earnings declines, yet this phenomenon has been quite pronounced among less-educated workers, particularly less-educated males, during the last three decades. [AA ignore quality effects for low-skilled workers]
2. It does not provide a framework for the analysis of polarization in the earnings distribution, which they document, and relatedly, it does not easily account for differential changes in inequality in different parts of the skill distribution during different periods (decades).

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3. Because the model does not distinguish between skills and tasks (or occupations), it does not provide insights into the systematic changes observed in the composition of employment by occupation in the United States and in other advanced economies–in particular, the disproportionate growth of employment in both high education, high wage occupations and, simultaneously, low education, low wage service occupations (i.e., employment polarization).
4. The model is also silent on the question of why the allocation of skill groups across occupations has substantially shifted in the last two decades, with a rising share of middle educated workers employed in traditionally low education services, or why the importance of occupations as predictors of earnings may have increased over time.

5. Because it incorporates technical change in a factor-augmenting form, it does not provide a natural framework for the study of how new technologies, including computers and robotics, might substitute for or replace workers in certain occupations or tasks.
6. Because it treats technical change as exogenous, it is also silent on how technology might respond to changes in labor market conditions and in particular to changes in supplies.
7. Finally, the canonical model does not provide a framework for an analysis of how recent trends in offshoring and outsourcing may influence the labor market and the structure of inequality (beyond the standard results on the effect of trade on inequality through its factor content).

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Skills: a worker's *endowment* of capabilities for performing various tasks.

This endowment is a stock, which may be either exogenously given or acquired through schooling and other investments.

Workers apply their skill endowments to tasks in exchange for wages.

The distinction between skills and tasks becomes relevant, in fact central, when workers of a given skill level can potentially perform a variety of tasks and, moreover, can change the set of tasks that they perform in response to changes in supplies or technology.

1. Such a model should allow an explicit distinction between skills and tasks, and allow for general technologies in which tasks can be performed by different types of skills, by machines, or by workers in other countries ("offshored"). This will enable the model to allow for certain tasks to be become mechanized (as in Autor et al., 2003) or alternatively produced internationally.
2. To understand how different technologies may affect skill demands, earnings, and the assignment (or reassignment) of skills to tasks, it should allow for comparative advantage among workers in performing different tasks.

3. To enable a study of polarization and changes in different parts of the earnings distribution during different periods, it should incorporate at least three different skill groups.
4. As with the canonical model, the task-based approach should give rise to a well-defined set of skill demands, with downward sloping relative demand curves for skills (for a given set of technologies) and conventional substitutability and complementarity properties among skill groups.

We consider a static environment with a unique final good.

No trade in tasks (a possibility we allow for later).

The unique final good is produced by combining a continuum of tasks represented by the unit interval, $[0, 1]$.

$$Y = \exp\left[\int_0^1 \ln y(i) di\right] \tag{9}$$

Y denotes the output of a unique final good
$y(i)$ as the "service" of production level of task i.
Price of the final good is the numeraire.

There are three factors of production, high, medium and low skilled workers.

$$L, M, \text{ and } H$$

Workers homogeneous within the groups.

Each task i has the following production function:

$$y(i) = A_L \alpha_L(i) l(i) + A_M \alpha_M(i) m(i) + A_H \alpha_H(i) h(i) + A_K \alpha_K(i) k(i) \tag{10}$$

A terms represent factor-augmenting technology. (Assumed uniform across the i.)

$\alpha_L(i)$, $\alpha_M(i)$, and $\alpha_H(i)$ are the task productivity schedules, designating the productivity of low, medium, and high skill workers in different tasks.

comparative advantage of skills groups differ across tasks, as captured by the α terms.

Assumption 1. $\alpha_L(i)/\alpha_M(i)$ and $\alpha_M(i)/\alpha_H(i)$ are continuously differentiable and strictly decreasing in i.

Factor market clearing requires

$$\int_0^1 l(i)di \leq L, \ \int_0^1 m(i)di \leq M \ \text{ and } \ \int_0^1 h(i)di \leq H. \qquad (11)$$

When we introduce capital, we will assume that it is available at some constant price r.

4.2. Equilibrium without machines

Ignore capital (equivalently, $\alpha_K(\cdot) \equiv 0$).

Lemma 1. *In any equilibrium there exist* I_L *and* I_H *such that* $0 < I_L < I_H < 1$ *and for any* $i < I_L$, $m(i) = h(i) = 0$, *for any* $i \in (I_L, I_H)$, $l(i) = h(i) = 0$, *and for any* $i > I_H$, $l(i) = m(i) = 0$.

Let $p(i)$ denote the price of services of task i.

$$\exp\left[\int_0^1 \ln p(i)di\right] = 1 \text{ because final good is numeraire}$$

To determine $p(i)$:

$$w_L = p(i)A_L\alpha_L(i) \text{ for any } i < I_L.$$
$$w_M = p(i)A_M\alpha_M(i) \text{ for any } I_L < i < I_H.$$
$$w_H = p(i)A_H\alpha_H(i) \text{ for any } i > I_H.$$

Observe that w_L is set by market forces
(multipliers on aggregate constraints in social planner's problem)

$$p(i)\alpha_L(i) = p(i')\alpha_L(i') \equiv P_L, \quad (12)$$

$i, i' < I_L$

P_L is the price "index" of tasks performed by low skill workers.

This price is endogenous not only because of the usual supply-demand reasons, but also because the set of tasks performed by low skill workers is endogenously determined.

Similarly, for medium skill workers, i.e., for any $I_H > i, i' > I_L$, we have

$$p(i)\alpha_M(i) = p(i')\alpha_M(i') \equiv P_M, \quad (13)$$

and for high skill workers and any $i, i' < I_H$,

$$p(i)\alpha_H(i) = p(i')\alpha_H(i') \equiv P_H. \quad (14)$$

The Cobb-Douglas technology (the unitary elasticity of substitution between tasks) in (9) implies that expenditure across all tasks should be equalized.

The first-order conditions for cost minimization in the production of the final good imply that $p(i)y(i) = p(i')y(i')$ for any i, i'

$$p(i)y(i) = Y, \text{ for any } i \in [0,1]. \tag{15}$$

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Consider two tasks $i, i' < I_L$

$$p(i)\alpha_L(i)l(i) = p(i')\alpha_L(i')l(i').$$

but from (12) it follows that

$$l(i) = l(i')$$

$$l(i) = \frac{L}{I_L} \text{ for any } i < I_L. \tag{16}$$

$$m(i) = \frac{M}{I_H - I_L} \text{ for any } I_H > i > I_L. \tag{17}$$

$$h(i) = \frac{H}{1 - I_H} \text{ for any } i > I_H. \tag{18}$$

Compare two tasks performed by high and medium skill workers $(I_L < i < I_H < i')$

From Eq. (15) that $p(i)A_M\alpha_M(i)m(i) = p(i')A_H\alpha_H(i')h(i')$

Using (12) and (13), we have

$$\frac{P_M A_M M}{I_H - I_L} = \frac{P_H A_H H}{1 - I_H},$$

or

$$\frac{P_H}{P_M} = \left(\frac{A_H H}{1 - I_H}\right)^{-1} \left(\frac{A_M M}{I_H - I_L}\right). \tag{19}$$

Similarly, comparing two tasks performed by medium and high skill workers, we obtain

$$\frac{P_M}{P_L} = \left(\frac{A_M M}{I_H - I_L}\right)^{-1} \left(\frac{A_L L}{I_L}\right). \tag{20}$$

No arbitrage across skills

$$\frac{A_M \alpha_M(I_H) M}{I_H - I_L} = \frac{A_H \alpha_H(I_H) H}{1 - I_H}. \tag{21}$$

$$\frac{A_L \alpha_L(I_L) L}{I_L} = \frac{A_M \alpha_M(I_L) M}{I_H - I_L}. \tag{22}$$

Equilibrium wages and inequality

$$w_L = P_L A_L. \quad (23)$$

$$\frac{w_H}{w_M} = \frac{P_H A_H}{P_M A_M}.$$

$$\frac{w_H}{w_M} = \left(\frac{1 - I_H}{I_H - I_L}\right) \left(\frac{H}{M}\right)^{-1}. \tag{24}$$

$$\frac{w_M}{w_L} = \left(\frac{I_H - I_L}{I_L}\right) \left(\frac{M}{L}\right)^{-1}. \tag{25}$$

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These equations, together with the choice of the numeraire, $\int_0^1 \ln p(i)di = 0$, fully characterize the equilibrium.
We can write the last equilibrium condition as

$$\int_0^{I_L} (\ln P_L - \ln \alpha_L(i))di + \int_{I_L}^{I_H} (\ln P_M - \ln \alpha_M(i))di + \int_{I_H}^{1} (\ln P_H - \ln \alpha_H(i))di = 0. \tag{26}$$

Equations (24) and (25) give the relative wages of high to medium and medium to low skill workers. To obtain the wage level for any one of these three groups, we need to use the price normalization in (26) together with (19) and (20) to solve out for one of the price indices, for example, P_L, and then (23) will give w_L and the levels of w_M and w_H can be readily obtained from (24) and (25).

Proposition 1. *There exists a unique equilibrium summarized by* $(I_L, I_H, P_L, P_M, P_H, w_L, w_M, w_H)$ *given by Eqs (19)-(26).*

The only part of this proposition that requires proof is the claim that equilibrium is unique (the rest of it follows from the explicit construction of the equilibrium preceding the proposition). This can be seen by noting that in fact the equilibrium is considerably easier to characterize than it first appears, because it has a block recursive structure. In particular, we can first use (21) and (22) to determine I_L and I_H. Given these we can then compute relative wages from (24) and (25). Finally, to compute wage and price levels, we can use (19), (20), (23) and (26).

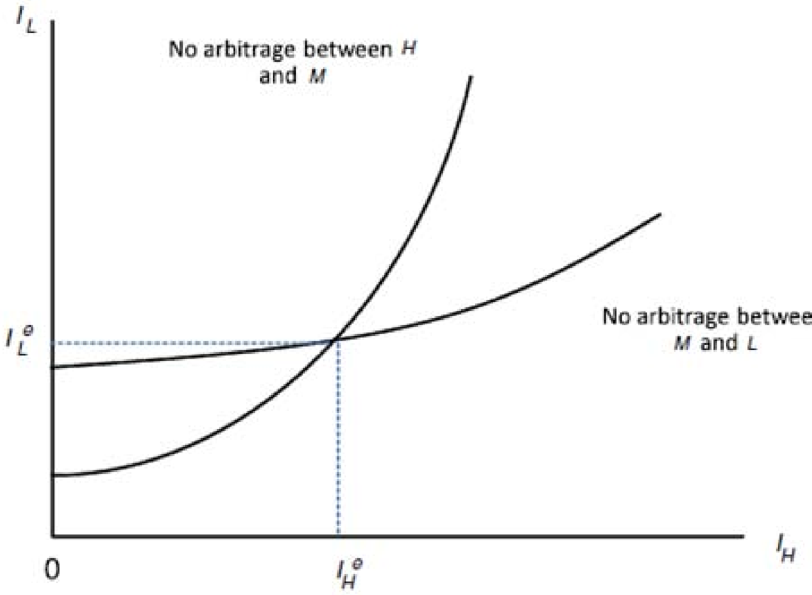

Figure 22 *Determination of equilibrium threshold tasks.*

In particular, we write (21) as follows:

$$\frac{1 - I_H}{I_H - I_L} \frac{\alpha_M(I_H)}{\alpha_H(I_H)} = \frac{A_H H}{A_M M}. \tag{27}$$

Similarly, we rewrite (22) as:

$$\frac{I_H - I_L}{I_L} \frac{\alpha_L(I_H)}{\alpha_M(I_H)} = \frac{A_M M}{A_L L}$$

for given I_H, and this expression has the same relative effective demand and supply interpretation. Since $\alpha_L(I_H)/\alpha_M(I_H)$ is strictly decreasing again from Assumption 1, the left-hand side traces a downward sloping curve as a function of I_L (for given I_H) and is shown as the inner (on the left) curve in Fig. 23.

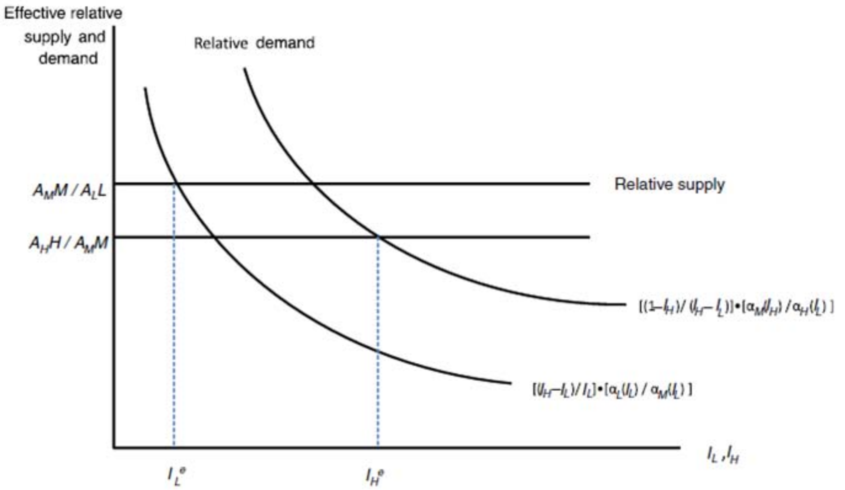

Figure 23 *Equilibrium allocation of skills to tasks.*

Take logs in Eq. (21) and (22) to obtain slightly simpler expressions:

$$\ln A_M - \ln A_H + \beta_H(I_H) + \ln M - \ln H - \ln(I_H - I_L) + \ln(1 - I_H) = 0, \tag{28}$$

and

$$\ln A_L - \ln A_M + \beta_L(I_L) + \ln L - \ln M - \ln(I_H - I_L) + \ln(I_L) = 0, \tag{29}$$

$$\beta_H(I) \equiv \ln \alpha_M(I) - \ln \alpha_H(I) \text{ and } \beta_L(I) \equiv \ln \alpha_L(I) - \ln \alpha_M(I),$$

Basic comparative statics

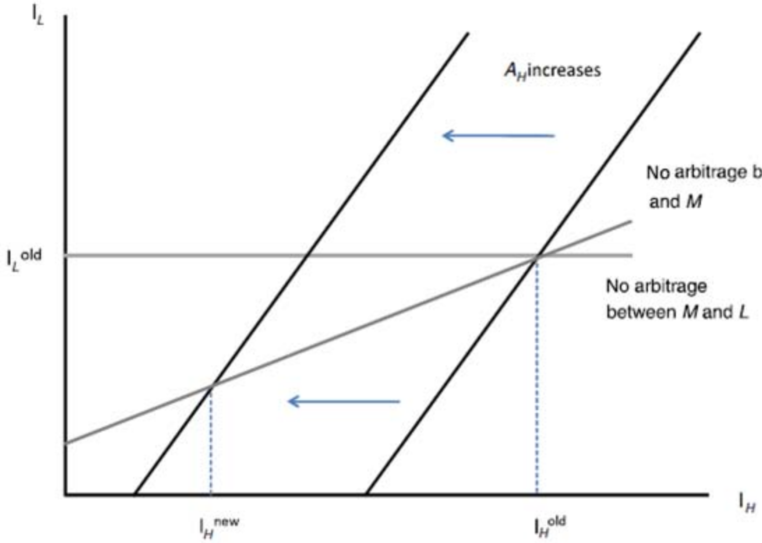

Figure 25 *Comparative statics.*

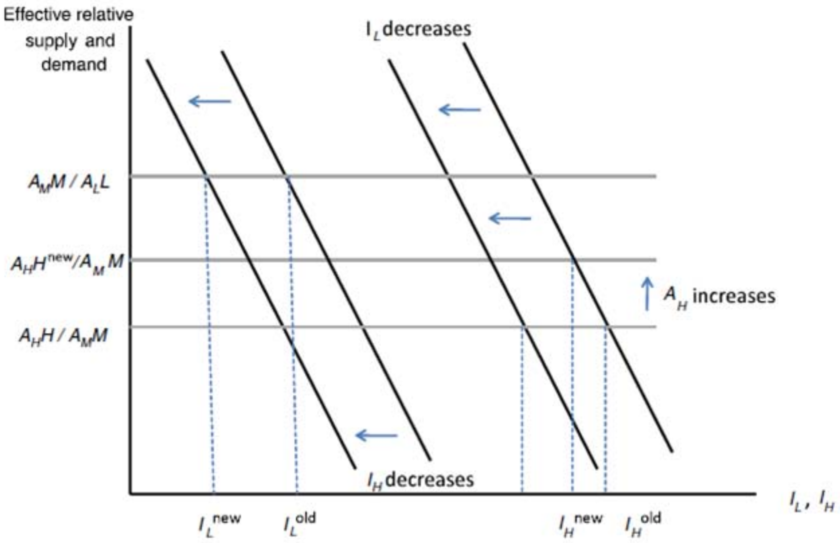

Figure 26 *Changes in equilibrium allocation.*

We obtain:

$$\begin{pmatrix} \beta_H'(I_H) - \frac{1}{I_H - I_L} - \frac{1}{1 - I_H} & \frac{1}{I_H - I_L} \\ \frac{1}{I_H - I_L} & \beta_L'(I_L) - \frac{1}{I_H - I_L} - \frac{1}{I_L} \end{pmatrix} \begin{pmatrix} dI_H \\ dI_L \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} d \ln A_H.$$

$$\Delta = \left(\beta_H'(I_H) - \frac{1}{1 - I_H} \right) \left(\beta_L'(I_L) - \frac{1}{I_L} \right) + \frac{1}{I_H - I_L} \left(\frac{1}{I_L} + \frac{1}{1 - I_H} - \beta_L'(I_L) - \beta_H'(I_H) \right).$$

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$$\frac{dI_H}{d\ln A_H} = \frac{\beta_L'(I_L) - \frac{1}{I_H - I_L} - \frac{1}{I_L}}{\Delta} < 0 \quad \text{and} \quad \frac{dI_L}{d\ln A_H} = \frac{-\frac{1}{I_H - I_L}}{\Delta} < 0,$$

$$\frac{d(I_H - I_L)}{d\ln A_H} = \frac{\beta_L'(I_L) - \frac{1}{I_L}}{\Delta} < 0$$

Using these expressions, we can obtain comparative statics for how relative wages by skill group change when there is high skill biased technical change. A similar exercise can be performed for low and medium skill biased technical change. The next proposition summarizes the main results.

Proposition 2. *The following comparative static results apply:*

1. *(The response of task allocation to technology and skill supplies):*

$$\frac{dI_H}{d\ln A_H} = \frac{dI_H}{d\ln H} < 0, \quad \frac{dI_L}{d\ln A_H} = \frac{dI_L}{d\ln A_H} < 0$$

$$\text{and } \frac{d(I_H - I_L)}{d\ln A_H} = \frac{d(I_H - I_L)}{d\ln H} < 0;$$

$$\frac{dI_H}{d\ln A_L} = \frac{dI_H}{d\ln L} > 0, \quad \frac{dI_L}{d\ln A_L} = \frac{dI_L}{d\ln L} > 0$$

$$\text{and } \frac{d(I_H - I_L)}{d\ln A_L} = \frac{d(I_H - I_L)}{d\ln L} < 0;$$

$$\frac{dI_H}{d\ln A_M} = \frac{dI_H}{d\ln M} > 0; \quad \frac{dI_L}{d\ln A_M} = \frac{dI_L}{d\ln M} < 0$$

$$\text{and } \frac{d(I_H - I_L)}{d\ln A_M} = \frac{d(I_H - I_L)}{d\ln M} > 0.$$

2. *(The response of relative wages to skill supplies):*

$$\frac{d\ln(\omega_H/\omega_L)}{d\ln H} < 0, \quad \frac{d\ln(\omega_H/\omega_M)}{d\ln H} < 0, \frac{d\ln(\omega_H/\omega_L)}{d\ln L} > 0,$$
$$\frac{d\ln(\omega_M/\omega_L)}{d\ln L} > 0, \quad \frac{d\ln(\omega_H/\omega_M)}{d\ln M} > 0, \text{ and}$$
$$\frac{d\ln(\omega_H/\omega_L)}{d\ln M} \lesseqqgtr \text{ if and only if } |\beta_L'(I_L)I_L| \lesseqqgtr |\beta_H'(I_H)(1-I_H)|.$$

3. *(The response of wages to factor-augmenting technologies):*

$$\frac{d\ln(\omega_H/\omega_L)}{d\ln A_H} > 0, \quad \frac{d\ln(\omega_M/\omega_L)}{d\ln A_H} < 0, \quad \frac{d\ln(\omega_H/\omega_M)}{d\ln A_H} > 0;$$

$$\frac{d\ln(\omega_H/\omega_L)}{d\ln A_L} < 0, \quad \frac{d\ln(\omega_M/\omega_L)}{d\ln A_L} < 0, \quad \frac{d\ln(\omega_H/\omega_M)}{d\ln A_L} > 0;$$

$$\frac{d\ln(\omega_H/\omega_L)}{d\ln A_M} \lesseqgtr 0 \text{ if and only if } |\beta_L'(I_L)I_L| \lesseqgtr |\beta_H'(I_H)(1-1_H)|.$$

The new result here concerns the impact of an increase in M on w_H / w_L. We have seen that such an increase raises I_H and reduces I_L, expanding the set of tasks performed by medium skill workers at the expense of both low and high skill workers. This will put downward pressure on the wages of both low and high skill workers, and the impact on the relative wage, w_H / w_L, is ambiguous for reasons we will encounter again below. In particular, it will depend on the form of the comparative advantage schedules in the neighborhood of I_L and I_H.

Third, the results summarized in Part 3 of the proposition, linking wages to technologies, are also intuitive.

The implications of medium skill biased technical changes are distinct from the canonical case. Medium skill biased technical changes have a direct effect on both high skill and low skill workers.

Consequently, the behavior of w_H / w_L is ambiguous. Similarly to how an increase in M affects w_H / w_L, the impact of a rise in A_M on w_H / w_L depends on the exact form of the comparative advantage schedules.

Depending on which set of tasks expands (contracts) more, wages of the relevant group increase (decrease).

To see how high skill biased technical chance, i.e.m an increase in A_H, can reduce medium skill wages more explicitly, let us work through a simple example.

Return to the special case discussed above where the task productivity schedule for the low skill workers is given by (31), implying that $I_L = \tilde{I}_L$.

Suppose also that $\beta_H(i) \equiv \ln \alpha_M(i) - \ln \alpha_H(i)$ is constant, so that the no arbitrage condition between high and medium skills in Fig. 25 (or Fig. 22) is flat.

Now consider an increase in A_H.

This will not change I_L (since $I_L = \tilde{I}_L$ in any equilibrium), but will have a large impact on I_H (in view of the fact that the no arbitrage locus between high and medium skills is flat).

Recall from the same argument leading to (23) that

$$w_M = P_M A_M .$$

This gives

$$\begin{aligned} \ln P_M = {} & I_L \left[\ln \left(\frac{A_L L}{A_M M} \right) + \ln(I_H - I_L) - \ln I_L \right] \\ & + (1 - I_H) \left[\ln \left(\frac{A_H H}{A_M M} \right) + \ln(I_H - I_L) - \ln(1 - I_H) \right] \\ & + \int_0^{I_L} \ln \alpha_L(i) di + \int_{I_L}^{I_H} \ln \alpha_M(i) di + \int_{I_H}^{1} \ln \alpha_H(i) di . \end{aligned}$$

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Now differentiating this expression, we obtain

$$\frac{\partial \ln P_M}{\partial \ln A_H} = \underbrace{\frac{1-I_H}{A_H}}_{+} + \underbrace{(\ln \alpha_M(I_H) - \ln \alpha_H(I_H))\frac{dI_H}{d\ln A_H}}_{-}$$
$$+ \left[\left(\frac{I_L}{I_H - I_L}\right) + 1 + \frac{1-I_H}{I_H - I_L} - \left(\ln\left(\frac{A_H H}{A_M M}\right)\right.\right.$$
$$\left.\left. + \ln(I_H - I_L) - \ln(1 - I_H)\right)\right] \frac{dI_H}{d\ln A_H}.$$

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This result illustrates that in our task-based framework, in which changes in technology affect the allocation of tasks across skills, a factor-augmenting increase in productivity for one group of workers can reduce the wages of another group by shrinking the set of tasks that they are performing.

For this reason, let us suppose that there now exists a range of tasks $[I', I''] \subset [I_L, I_H]$ for which $\alpha_K(i)$ increases sufficiently (with fixed cost of capital r) so that they are now more economically performed by machines than middle skill workers.

For all the remaining tasks, i.e., for all $i \notin [I', I'']$, we continue to assume that $\alpha_K(i) = 0$.

What are the implications of this type of technical change for the supply of different types of tasks and for wages?

Proposition 3. *Suppose we start with an equilibrium characterized by thresholds* $[I_L, I_H]$ *and technical change implies that the tasks in the range* $[I', I''] \subset [I_L, I_H]$ *are now performed by machines. Then after the introduction of machines, there exists new unique equilibrium characterized by new thresholds* $\hat{I}_L$ *and* $\hat{I}_H$ *such that* $0 < \hat{I}_L < I' < I'' < \hat{I}_H < 1$ *and for any* $i < \hat{I}_L$, $m(i) = h(i) = 0$ *and* $l(i) = L/\hat{I}_L$; *for any* $i \in (\hat{I}_L, I') \cup (I'', \hat{I}_H)$, $l(i) = h(i) = 0$ and $m(i) = M/(\hat{I}_H - I'' + I' - \hat{I}_L)$; *for any* $i \in (I', I'')$, $l(i) = m(i) = h(i) = 0$; *and for any* $i > \hat{I}_H, l(i) = m(i) = 0$ *and* $h(i) = H/(1 - \hat{I}_H)$.

This proposition immediately makes clear that, as a consequence of machines replacing tasks previously performed by medium skill workers, there will be a reallocation of tasks in the economy. In particular, medium skill workers will now start performing some of the tasks previously allocated to low skill workers, thus increasing the supply of these tasks (the same will happen at the top with an expansion of some of the high skill tasks). This proposition therefore gives us a way of thinking about how new technologies replacing intermediate tasks (in practice, most closely corresponding to routine, semiskilled occupations) will directly lead to the expansion of low skill tasks (corresponding to service occupations).

We next investigate the wage inequality implications of the introduction of these new tasks. For simplicity, we focus on the case where we start with $[I', I''] = \varnothing$, and then the set of tasks expands to an interval of size ε', where ε' is small. This mathematical approach is used only for expositional simplicity because it enables us to apply differential calculus as above. None of the results depend on the set of tasks performed by machines being small.

$$\ln A_M - \ln A_H + \beta_H(I_H) + \ln M - \ln H - \ln(I_H - I_L - \varepsilon) + \ln(1 - I_H) = \tag{30}$$

and

$$\ln A_L - \ln A_M + \beta_L(I_L) + \ln L - \ln M - \ln(I_H - I_L - \varepsilon) - \ln(I_L) = 0. \tag{31}$$

$$\begin{pmatrix} \beta_H'(I_H) - \frac{1}{I_H - I_L} - \frac{1}{1 - I_H} & \frac{1}{I_H - I_L} \\ \frac{1}{I_H - I_L} & \beta_L'(I_L) - \frac{1}{I_H - I_L} - \frac{1}{I_L} \end{pmatrix} \begin{pmatrix} dI_H \\ dI_L \end{pmatrix} = \begin{pmatrix} -\frac{1}{I_H - I_L} \\ \frac{1}{I_H - I_L} \end{pmatrix} d\varepsilon.$$

$$\frac{dI_H}{d\varepsilon} = \frac{1}{I_H - I_L} \frac{-\beta'_L(I_L) + \frac{1}{I_L}}{\Delta} > 0,$$

$$\frac{dI_L}{d\varepsilon} = \frac{1}{I_H - I_L} \frac{-\beta'_H(I_H) + \frac{1}{1-I_H}}{\Delta} > 0,$$

$$\frac{d(I_H - I_L)}{d\varepsilon} = \frac{1}{I_H - I_L} \frac{-\beta'_L(I_L) - \beta'_H(I_H) + \frac{1}{1-I_H} + \frac{1}{I_L}}{\Delta} > 0,$$

where recall that Δ is the determinant of the matrix on the left hand side.

Proposition 4. *Suppose we start with an equilibrium characterized by thresholds* $[I_L, I_H]$ *and technical change implies that the tasks in the range* $[I', I''] \subset [I_L, I_H]$ *are now performed by machines. Then:*

1. w_H / w_M *increases;*
2. w_M / w_L *decreases;*
3. w_H / w_L *increases if* $|\beta'_L(I_L)I_L| < |\beta'_H(I_H)(1 - I_H)|$ *and* w_H / w_L *decreases* $|\beta'_L(I_L)I_L| > |\beta'_H(I_H)(1 - I_H)|$.

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The first two parts of the proposition are intuitive. Because new machines replace the tasks previously performed by medium skill workers, their relative wages, both compared to high and low skill workers, decline. In practice, this corresponds to the wages of workers in the middle of the income distribution, previously performing relatively routine tasks, falling compared to those at the top and the bottom of the wage distribution. Thus the introduction of new machines replacing middle skilled tasks in this framework provides a possible formalization of the "routinization" hypothesis and a possible explanation for job and wage polarization discussed in Section 2.

Note that the impact of this type of technical change on the wage of high skill relative to low skill workers is ambiguous; it depends on whether medium skill workers displaced by machines are better substitutes for low or high skill workers. The condition $|\beta'_L(I_L)I_L| < |\beta'_H(I_H)(1 - I_H)|$ is the same as the condition we encountered in Proposition 3, and the intuition is similar. The inequality $|\beta'_L(I_L)| < |\beta'_H(I_H)|$ implies that medium skill workers are closer substitutes for low than high skill workers in the sense that, around I_H, there is a stronger comparative advantage of high skill relative to medium skill workers than there is comparative advantage of low relative to medium skill workers around I_L. The terms I_L and $(1 - I_H)$ have a similar intuition. If the set of tasks performed by high skill workers is larger than the set of tasks performed by low skill workers $((1 - I_H) > I_L)$, the reallocation of a small set of tasks from high to medium skill workers will have a smaller effect on high skill wages than will an equivalent reallocation of tasks from low to medium skill workers (in this case, for low skill wages).

It appears plausible that in practice, medium skill workers previously performing routine tasks are a closer substitute for low skill workers employed in manual and service occupations than they are for high skill workers in professional, managerial and technical occupations. Indeed the substantial movement of medium skill high school and some college workers out of clerical and production positions and into service occupations after 1980 (Fig. 14) may be read as *prima facie* evidence that the comparative advantage of middle skill workers (particularly middle skill males) is relatively greater in low rather than high skill tasks. If so, Part 3 of this proposition implies that we should also see an increase in w_H/w_L. Alternatively, if sufficiently many middle skill workers displaced by machines move into high skill occupations, w_H/w_L may also increase. This latter case would correspond to one in which, in relative terms, low skill workers are the main beneficiaries of the introduction of new machines into the production process.

Let us finally return to the basic comparative statics and consider a change in the task productivity schedule of high skill workers, $\alpha_H(i)$. Imagine, in particular, that this schedule is given by

$$\alpha_H(i) = \begin{cases} \theta^{\tilde{I}_{H-i}}(i) & \text{if } i \leq \tilde{I}_H \\ \tilde{\alpha_H}(i) & \text{if } i > \tilde{I}_H \end{cases} \tag{32}$$

where $\tilde{\alpha}_H(i)$ is a function that satisfies Assumption 1 and $\theta \geq 1$, and suppose that $\tilde{I}_H$ is in the neighborhood of the equilibrium threshold task for high skill workers, I_H. The presence of the term $\theta^{\tilde{I}_H - i}$ in (36) implies that an increase in θ creates a rotation of the task productivity schedule for high skill workers around $\tilde{I}_H$.

Next the implications of an increase in θ.

This will imply that high skill workers can now successfully perform tasks previously performed by medium skill workers, and hence high skills workers will replace them in tasks close to I_H (or close to the equilibrium threshold I_H).

Therefore, even absent machine-substitution for medium skill tasks, the model can generate comparative static results similar to those discussed above.

This requires that the task productivity schedule for high skill (or low skill) workers twists so as to give them comparative advantage in the tasks that were previously performed by medium skill workers.

4.6. Endogenous choice of skill supply

- This is a version of a Roy model.

Environment

To allow for substitution of workers across different types of skills, we now assume that each worker j is endowed with some amount of "low skill," "medium skill," and "high skill," respectively l_j, m_j and h_j.

Workers have one unit of time, which is subject to a skill allocation constraint

$$t_l^j + t_m^j + t_h^j \leq l.$$

The workers income is

$$w_L t_l^j l^j + w_M t_m^j m^j + w_H t_h^j h^j,$$

which captures the fact that the worker with skill vector (l_j, m_j, h_j) will have to allocate his time between jobs requiring different types of skills.

Aggregate amount of skills of different types.

Let us denote these by

$$L = \int_{j \in E_l} l^j \, dj, \quad M = \int_{j \in E_m} m^j \, dj, \quad H = \int_{j \in E_h} h^j \, dj,$$

E_l, E_m and E_h are the sets of workers choosing to supply their low, medium and high skills respectively.

Clearly, the worker will choose to be in the set Eh only if

$$\frac{l^j}{h^j} \leq \frac{w_H}{w_L} \quad \text{and} \quad \frac{m^j}{h^j} \leq \frac{w_H}{w_M}.$$

We now impose a type of *single-crossing* assumptions in supplies.

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Assumption 2. h^j / m^j and m^j / l^j are both strictly decreasing in j and $\lim_{j \to 0} h^j / m^j$ and $\lim_{j \to 1} m^j / l^j = 1$.

This assumption implies that lower index workers have a comparative advantage in high skill tasks and higher index workers have a comparative advantage in low skill tasks.

Lemma 2. *For any ratios of wages w_H/w_M and w_M/w_L, there exist $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ such that $t_h^j = 1$ for all $j < J^*(w_H/w_M)$, $t_m^j = 1$ for all $j \in (J^*(w_H/w_M), J^{**}(w_M/w_L))$ and $t_l^j = 1$ for all $j > J^{**}(w_M/w_L)$. $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are both strictly increasing in their arguments.*

Clearly, $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are defined such that

$$\frac{m^{J^*}(w_H/w_M)}{h^{J^*}(w_H/w_M)} = \frac{w_H}{w_M} \quad \text{and} \quad \frac{l^{J^*}(w_M/w_L)}{m^{J^*}(w_M/w_L)} = \frac{w_M}{w_L}. \qquad (33)$$

In light of this lemma, we can write

$$H = \int_0^{J^*(w_H/w_M)} h^j dj, \quad M = \int_{J^*(w_H/w_M)}^{J^{**}(w_M/w_L)} m^j dj \quad \text{and} \\ L = \int_{J^{**}(w_M/w_L)}^{I} l^j dj. \tag{34}$$

Assumption 2, $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are both strictly increasing in their arguments.

$$\frac{H}{M} = \frac{\int_0^{J^*(w_H/w_M)} h^j \, dj}{\int_{J^*(w_H/w_M)}^{J^{**}(w_M/w_L)}} \quad \text{and} \quad \frac{M}{L} = \frac{\int_{J^*(w_H/w_M)}^{J^{**}(w_M/w_L)}}{\int_{J^{**}(w_M/w_L)}^{1} l^j \, dj} \tag{35}$$

The first expression, together with the fact that $J^*(w_H/w_M)$ is strictly increasing, implies that holding w_M/w_L constant, an increase in w_H/w_M increases H/L. Similarly, holding w_H/w_M constant, an increase in w_M/w_L increases M/L.

Proposition 5. *In the model with endogenous supplies, there exists a unique equilibrium summarized by* $(I_L, I_H, P_L, P_M, P_H, w_L, w_M, w_H, J^*(w_H/w_M), J^{**}(w_M/w_L), L, M, H)$ *given by Eqs (19)-(26), (33) and (34).*

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The major change to the analysis introduced by allowing for the endogenous supply of skills is that when there is factor-augmenting technical change (or the introduction of capital that directly substitutes for workers in various tasks), the induced changes in wages will also affect supplies (even in the short run).

During the 1980s the US labor market experienced declining wages at the bottom of the distribution together with a relative contraction in employment in low wage occupations (though notably, a rise in employment in service occupations as underscored by Autor and Dorn, 2010), and also rising wages and employment in high skill occupations.

In terms of our model, this would be a consequence of an increase in A_H / A_M and A_M / A_L which is the analog of skill biased technical change in this three factor model.

This would result from rising penetration of information technology that replaces middle skill tasks (i.e., those with a substantial routine component).

This will depress both the wages of medium skill workers and reduce employment in tasks that were previously performed by these medium skill workers.

In the most recent decade (2000s), employment in low wage service occupations has grown even more rapidly.

In terms of our model, this could be an implication of the displacement of medium workers under the plausible assumption that the relative comparative advantage of middle skill workers is greater in low than high skill tasks.

To illustrate how offshoring of tasks affects the structure of wages, suppose that a set of tasks $[I', I''] \subset [I_L, I_H]$ can now be offshored to a foreign country, where wages are sufficiently low that such offshoring is cost minimizing for domestic final good producers.

Proposition 6. Suppose we start with an equilibrium characterized by thresholds $[I_L, I_H]$ and changes in technology allow tasks in the range $\left[I', I''\right] \subset [I_L, I_H]$ to be offshored. Then after offshoring, there exists new unique equilibrium characterized by new thresholds $\hat{I}_L < I_L$ and $\hat{I}_H > I_H$ such that $0 < \hat{I}_L < I' < I'' < \hat{I}_H < 1$ and for any $i < \hat{I}_L$, $m(i) = h(i) = 0$ and $l(i) = L/\hat{I}_L$; for any $i \in (\hat{I}_L, I') \cup (I'')$, $l(i) = m(i) = h(i) = 0$; and for any $i > \hat{I}_H$, $l(i) = m(i) = 0$ and $h(i) = H/(1 - \hat{I}_H)$. The implications of offshoring on the structure of wages are as follows:

1. ω_H/ω_M increases;
2. ω_M/ω_L decreases;
3. ω_H/ω_L increases if $|\beta_L'(I_L)I_L| < |\beta_H'(I_H)(1 - I_H)|$ and ω_H/ω_L decreases if $|\beta_L'(I_L)I_L| > |\beta_H'(I_H)(1 - I_H)|$.

While the extension of the model to offshoring is immediate, the substantive point is deeper. The task-based model offers an attractive means, in our view, to place labor supply, technological change, and trading opportunities on equal economic footing. In our model, each is viewed as offering a competing supply of tasks that, in equilibrium, are allocated to productive activities in accordance with comparative advantage and cost minimization. This approach is both quite general and, we believe, intuitively appealing.

4.8. Directed technical change

From this perspective, Tinbergen's race between supplies and technology is endogenously generated.

Autonomous changes in skill supplies–resulting from demographic trends, evolving preferences, and shifts in public and private education–induce endogenous changes in technology, which increase the demand for skills.

Formally, papers by Acemoglu (1998, 2002b) generalize the canonical model with two types of skills and two types of factor-augmenting technologies so as to endogenize the direction of technical change (and thus the relative levels of the two technologies). This work shows that an increase in the relative supply of skills will endogenously cause technology to become more skill biased.

5. Comparative Advantage and Wages: An Empirical Approach

As a stylized example of how this insight might be brought to the data, we study the evolution of wages by skill groups, where skill groups are defined according to their initial task specialization across abstract-intensive, routine-intensive, and manual intensive occupations. We take these patterns of occupational specialization as a rough proxy for comparative advantage. Consider the full set of demographic groups available in the data, indexed by gender, education, age, and region. At the start of the sample in 1959, we assume that these groups have self-selected into task specialities according to comparative advantage, taking as given overall skill supplies and task demands (reflecting also available technologies and trade opportunities). Specifically, let γ^A_{sejk}, γ^R_{sejk} and γ^S_{sejk} be the employment shares of a demographic group in abstract, routine and manual/service occupations in 1959, where s denotes gender, e denotes education group, j denotes age group, and k denotes region of the country. By construction, we have that $\gamma^A_{sejk} + \gamma^R_{sejk} + \gamma^S_{sejk} = 1$.

Let w_{sejkt} be the mean log wage of a demographic group in year t and $\Delta w_{sejk\tau}$ be the change in w during decade τ.

We then estimate the following regression model:

$$\Delta\omega_{sejk\tau} = \sum_t \beta_t^A \dot{\gamma}_{sejk}^A \mathbb{1}[\tau = t] + \sum_t \beta_t^S \dot{\gamma}_{sejk}^S \mathbb{1}[\tau = t] + \delta_\tau + \phi_e + \lambda_j + \pi_k + e_{sejk\tau}. \tag{36}$$

where δ, ϕ, λ, and π are vectors of time, education, age, and region dummies. The β_t^S and the β_t^S and B_t^A coefficients in this model estimate the decade specific slopes on the initial occupation shares in predicting wage changes by demographic group.

This hypothesis implies that we should expect the wages of workers with comparative advantage in either abstract or manual/service tasks to rise over time while the opposite should occur for skill groups with comparative advantage in routine tasks.

Formally, we anticipate that B_t^A and β_t^S will rise while the intercepts measuring the omitted routine task category (δ_τ) will decline.

Table 10. OLS stacked first-difference estimates of the relationship between demographic group occupational distributions in 1959 and subsequent changes in demographic groups' mean log wages by decade, 1959-2007.

	A. Males		B. Females	
	(1)	(2)	(1)	(2)
Abstract occupation share				
1959 share × 1959-1969 time dummy	0.021	0.033	0.146	0.159
	(0.044)	(0.104)	(0.041)	(0.081)
1959 share × 1969-1979 time dummy	−0.129	−0.123	−0.054	−0.032
	(0.044)	(0.105)	(0.036)	(0.079)
1959 share × 1979-1989 time dummy	0.409	0.407	0.143	0.174
	(0.046)	(0.106)	(0.033)	(0.079)
1959 share × 1989-1999 time dummy	0.065	0.060	0.070	0.107
	(0.049)	(0.109)	(0.033)	(0.079)
1959 share × 1999-2007 time dummy	0.198	0.194	0.075	0.113
	(0.051)	(0.11)	(0.033)	(0.08)
Service occupation share				
1959 share × 1959-1969 time dummy	−0.836	−1.014	0.359	0.404
	(0.278)	(0.303)	(0.064)	(0.09)
1959 share × 1969-1979 time dummy	−0.879	−0.991	0.304	0.363
	(0.295)	(0.316)	(0.065)	(0.091)
1959 share × 1979-1989 time dummy	1.007	0.917	−0.143	−0.060
	(0.332)	(0.349)	(0.074)	(0.096)
1959 share × 1989-1999 time dummy	0.202	0.143	0.117	0.221
	(0.378)	(0.39)	(0.086)	(0.104)
1959 share × 1999-2007 time dummy	0.229	0.212	−0.056	0.058
	(0.398)	(0.408)	(0.094)	(0.109)

Table 10. OLS stacked first-difference estimates of the relationship between demographic group occupational distributions in 1959 and subsequent changes in demographic groups' mean log wages by decade, 1959-2007.

	A. Males		B. Females	
	(1)	(2)	(1)	(2)
Abstract occupation share				
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1959 share × 1979–1989 time dummy	0.409	0.407	0.143	0.174
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	(0.051)	(0.11)	(0.033)	(0.08)
Service occupation share				
1959 share × 1959–1969 time dummy	−0.836	−1.014	0.359	0.404
	(0.278)	(0.303)	(0.064)	(0.09)
1959 share × 1969–1979 time dummy	−0.879	−0.991	0.304	0.363
	(0.295)	(0.316)	(0.065)	(0.091)
1959 share × 1979–1989 time dummy	1.007	0.917	−0.143	−0.060
	(0.332)	(0.349)	(0.074)	(0.096)
1959 share × 1989–1999 time dummy	0.202	0.143	0.117	0.221
	(0.378)	(0.39)	(0.086)	(0.104)
1959 share × 1999–2007 time dummy	0.229	0.212	−0.056	0.058
	(0.398)	(0.408)	(0.094)	(0.109)

Table 10. OLS stacked first-difference estimates of the relationship between demographic group occupational distributions in 1959 and subsequent changes in demographic groups' mean log wages by decade, 1959-2007.

Decade dummies				
1959-1969	0.274	0.274	0.120	0.046
	(0.031)	(0.037)	(0.021)	(0.032)
1969-1979	0.084	0.085	−0.083	−0.163
	(0.033)	(0.038)	(0.020)	(0.033)
1979-1989	−0.287	−0.283	−0.011	−0.099
	(0.036)	(0.041)	(0.021)	(0.034)
1989-1999	−0.002	0.002	0.061	−0.035
	(0.039)	(0.045)	(0.022)	(0.035)
1999-2007	−0.157	−0.157	−0.073	−0.171
	(0.041)	(0.046)	(0.024)	(0.036)
Education, age group, and region main effects?	No	Yes	No	Yes
R-squared	0.789	0.821	0.793	0.844
N	400	400	400	400

Source: Census IPUMS 1960, 1970, 1980, 1990 and 2000, and American Community Survey 2008. Each column presents a separate OLS regression of stacked changes in mean log real hourly wages by demographic group and year, where demographic groups are defined by sex, education group (high school dropout, high school graduate, some college, college degree, post-college degree), age group (25-34, 35-44, 45-54, 55-64), and region of residence (Northeast, South, Midwest, West). Models are weighted by the mean start and end-year share of employment of each demographic group for each decadal change. Occupation shares are calculated for each demographic group in 1959 (using the 1960 Census) and interacted with decade dummies. Occupations are grouped into three exhaustive and mutually exclusive categories: (1) abstract—professional, managerial and technical occupations; (2) service—protective service, food service and cleaning, and personal services occupations; (3) routine—clerical, sales, administrative support, production, operative and laborer occupations. The routine group is the omitted category in the regression models.