

# Employer Learning and Statistical Discrimination

Joseph G. Altonji & Charles R. Pierret. (2001).  
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# I. Introduction

## II. Implications of Statistical Discrimination and Employer Learning for Wages

## II.1 A Model of Employer Learning and Wages

- Our research builds on some previous work, particularly Farber and Gibbons (1996), (hereinafter FG).
- Our model is similar to FG.
- Let  $y_{it}$  be the log of labor market productivity of worker  $i$  with  $t_i$  years of experience:

$$y_{it} = rs_i + \alpha_1 q_i + \Lambda z_i + \eta_i + H(t_i). \quad (1)$$

- In (1) we separate the determinants of productivity into four categories:
- $s_i$  represents variables that are observed by both the employer and the econometrician;
- $q_i$  includes variables observed by the employer but not seen (or not used) by the econometrician;
- $z_i$  consists of correlates of productivity that are not observed directly by employers but are available to and used by the econometrician;
- and  $\eta_i$  is an index of other determinants of productivity and is not directly observed by the employers and not observed (or observed but not used) by the econometrician.

- Normalize  $z_i$  so that all the elements of the conformable coefficient vector  $\Lambda$  are positive.
- In addition,  $H(t_i)$  is the experience profile of productivity.
- For now we assume that the experience profile of productivity does not depend on  $s_i$ ,  $z_i$ ,  $q_i$ , or  $\eta_i$ .

- In the absence of knowledge of  $z$  and  $\eta$ , firms form the conditional expectations  $E(z|s, q)$  and  $E(\eta|s, q)$ , which we assume are linear in  $q$  and  $s$ .
- Consequently,

$$\begin{aligned}z &= E(z|s, q) + v = \gamma_1 q + \gamma_2 s + v \\ \eta &= E(\eta|s, q) + e = \alpha_2 s + e,\end{aligned}\tag{2}$$

- Vector  $v$  and the scalar  $e$  have mean 0 and are uncorrelated with  $q$  and  $s$  by definition of an expectation.
- Links from  $s$  to  $z$  and  $\eta$  may be due in part to a causal effect of  $s$ .



- Equations (1) and (2) imply that  $\Lambda\nu + e$  is the error in the employer's belief about the log of productivity of the worker at the time the worker enters the labor market.
- The sum  $\Lambda\nu + e$  is uncorrelated with  $q$  and  $s$ .

- $\xi_t = y + \epsilon$ , where  $y = y_t - H(t)$ .
- $\epsilon_t$  reflects transitory variation in the performance of worker  $i$  and the effects of variation in the firm environment that are hard for the firm to control for in evaluating the worker.
- Employers know  $q$  and  $s$ .

- Observing  $\xi_t$  is equivalent to observing  $d_t = \xi_t - E(y|s, q) = \Lambda\nu + e + \epsilon_t$  which is the sum of the noise  $\epsilon_t$  and the error  $\Lambda\nu + e$  in the employer's belief about initial log productivity.
- The vector  $D_t = \{d_1, d_2, \dots, d_t\}$  summarizes the worker's performance history.
- Let  $\mu_t$  be the difference between  $\Lambda\nu + e$  and  $E(\Lambda\nu + e|D_t)$ .
- $\mu_t$  is uncorrelated with  $D_t, q$ , and  $s$ .
- $\mu_t$  is distributed independently of  $D_t, q$ , and  $s$ .
- $q, s$ , and  $D_t$  are known to all employers, as in FG.

- Substituting and taking logs, we arrive at the log wage process:

$$w_t = (r + \Lambda\gamma_2 + \alpha_2)s + H^*(t) + (\alpha_1 + \Lambda\gamma_1)q + E(\Lambda\nu + e|D_t) + \zeta_t, \quad (3)$$

- $w_t = \log(W_t)$  and  $H^*(t) = H(t) + \log(E(\exp^{\mu t}))$ .
- $E(\Lambda\nu + e|D_t)$  in (3) shows that wages change over time not just because productivity changes with experience, but also because firms learn about errors in their initial assessment of worker productivity.

- Examine the parameters of the conditional expectation of  $w_t$  given  $s$ ,  $z$ ,  $t$ , and the experience profile  $H^*(t)$ .
- Begin with the case in which  $z$  and  $s$  are scalars and then turn to the more general cases.
- Consider the conditional expectation function when  $t = 0, \dots, T$ , with

$$E(w_t | s, z, t) = b_{st}s + b_{zt}z + H^*(t). \quad (4)$$

- To simplify the algebra but without any additional assumptions, we reinterpret  $s$ ,  $z$ , and  $q$  as the components of  $s$ ,  $z$ , and  $q$  that are orthogonal to  $H^*(t)$ .
- Given that the wage evolves according to (3), the omitted bias formula for least squares regression implies that

$$b_{st} = b_{s0} + \Phi_{st} = [r + \Lambda\gamma_2 + \alpha_2] + \Phi_{qs} + \Phi_{st} \quad (5)$$

$$b_{zt} = b_{z0} + \Phi_{zt} = \Phi_{qz} + \Phi_{zt},$$

- where  $\Phi_{qs}$  and  $\Phi_{qz}$  denote the coefficients of the auxiliary regressions of  $(\alpha_1 + \Lambda\gamma_1)q$  on  $s$  and  $z$ , respectively, and  $\Phi_{st}$  and  $\Phi_{zt}$  are the coefficients of the regression of  $E(\Lambda v + e|D_t)$  on  $s$  and  $z$ .

- Using the facts that  $\text{cov}(s, E(\Lambda v + e|D_t)) = 0$  and  $\text{cov}(z, E(\Lambda v + e|D_t)) = \text{cov}(v, E(\Lambda v + e|D_t))$  and the least squares regression formula, one may express  $\Phi_{st}$  and  $\Phi_{zt}$  as

$$\begin{aligned}\Phi_{st} &= \theta_t \Phi_s \\ \Phi_{zt} &= \theta_t \Phi_z,\end{aligned}\tag{6}$$

- where  $\Phi_s$  and  $\Phi_z$  are the coefficients of the regression of  $\Lambda v + e$  on  $s$  and  $z$  and

$$\theta_t = \frac{\text{cov}(E(\Lambda v + e|D_t), z)}{\text{cov}(\Lambda v + e, z)} = \frac{\text{cov}(E(\Lambda v + e|D_t), v)}{\text{cov}(\Lambda v + e, v)}.\tag{7}$$

**Proposition 1.** Under the assumptions of the above model,

- a the regression coefficient  $b_{zt}$  is nondecreasing in  $t$ , and
- b the regression coefficient  $b_{st}$  is nonincreasing in  $t$ .

**Proposition 2.** Under the assumptions of the above model,

$$\frac{\partial b_{st}}{\partial t} = -\Phi_{zs} \frac{\partial b_{zt}}{\partial t}.$$



- However, a matrix version of Proposition 2 still holds

$$\frac{\partial b_{st}}{\partial t} = -\frac{\partial b_{zt}}{\partial t} \Phi_{zs},$$

- where  $\Phi_{zs}$  is now the  $K \times J$  matrix of coefficients of the regression of  $z$  on  $s$ .

## II.2. Statistical Discrimination on the Basis of Race

## **II.3. Alternative Explanations for Variation in the Wage Coefficients with Experience**

### III. Data and Econometric Specification

## IV. Results for Education

## IV.1. AFQT as a $z$ Variable

Figure 1: The Effects of Standardized AFQT and Schooling on Wages

Dependent Variable: Log Wage; OLS estimates (standard errors)

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Panel 1 – Experience measure: potential experience

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Model:	(1)	(2)	(3)	(4)
(a) Education	0.0586 (0.0118)	0.0829 (0.0150)	0.0638 (0.0120)	0.0785 (0.0153)
(b) Black	-0.1565 (0.0256)	-0.1553 (0.0256)	0.0001 (0.0621)	-0.0565 (0.0723)
(c) Standardized AFQT	0.0834 (0.0144)	-0.0060 (0.0360)	0.0831 (0.0144)	0.0221 (0.0421)
(d) Education * experience/10	-0.0032 (0.0094)	-0.0234 (0.0123)	-0.0068 (0.0095)	-0.0193 (0.0127)
(e) Standardized AFQT * experience/10		0.0752 (0.0286)		0.0515 (0.0343)
(f) Black * experience/10			-0.1315 (0.0482)	-0.0834 (0.0581)
$R^2$	0.2861	0.2870	0.2870	0.2873

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Figure 2: The Effects of Standardized AFQT and Schooling on Wages

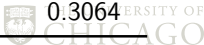
Dependent Variable: Log Wage; OLS estimates (standard errors)

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Panel 2 – Experience measure: actual experience  
instrumented by potential experience

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Model:	(1)	(2)	(3)	(4)
(a) Education	0.0836 (0.0208)	0.1218 (0.0243)	0.0969 (0.0206)	0.1170 (0.0248)
(b) Black	-0.1310 (0.0261)	-0.1306 (0.0260)	0.0972 (0.0851)	0.0178 (0.1029)
(c) Standardized AFQT	0.0925 (0.0143)	-0.0361 (0.0482)	0.0881 (0.0143)	0.0062 (0.0572)
(d) Education * experience/10	-0.0539 (0.0235)	-0.0952 (0.0276)	-0.0665 (0.0234)	-0.0889 (0.0283)
(e) Standardized AFQT * experience/10		0.1407 (0.0514)		0.0913 (0.0627)
(f) Black * experience/10			-0.2670 (0.0968)	-0.1739 (0.1184)
$R^2$	0.3056	0.3063	0.3061	0.3064





## IV.2. The Sibling Wage and Father's Education as $z$ Variables

### Figure 3: The Effects of Father's Education, Sibling Wages, and Schooling on Wages

Dependent Variable: Log Wage; Experience Measure: Potential Experience

OLS estimates (standard errors)

Model:	(1)	(2)	(3)	(4)
(a) Education	0.0511 (0.0160)	0.0630 (0.0166)	0.0568 (0.0163)	0.0659 (0.0167)
(b) Black	-0.2074 (0.0276)	-0.2076 (0.0276)	-0.0509 (0.0846)	-0.0878 (0.0871)
(c) Log of sibling's wage	0.1802 (0.0328)	-0.0260 (0.0913)	0.1817 (0.0329)	0.0010 (0.0940)
(d) Father's education/10				
(e) Education * experience/10	0.0107 (0.0131)	0.0012 (0.0136)	0.0065 (0.0133)	-0.0008 (0.0136)
(f) Log of sibling's wage * experience/10		0.1796 (0.0749)		0.1571 (0.0770)
(g) Father's education * experience/100				
(h) Black * experience/10			-0.1311 (0.0686)	-0.1004 (0.0704)
$R^2$	0.3183	0.3196	0.3191	0.3200
Observations	10746	10746	10746	10746
Individuals	1441	1441	1441	1441

## Figure 4: The Effects of Father's Education, Sibling Wages, and Schooling on Wages

Dependent Variable: Log Wage; Experience Measure: Potential Experience

OLS estimates (standard errors)

Model:	(5)	(6)	(7)	(8)
(a) Education	0.0666 (0.0129)	0.0730 (0.0140)	0.0704 (0.0130)	0.0734 (0.0140)
(b) Black	-0.2212 (0.0250)	-0.2209 (0.0250)	-0.0705 (0.0668)	-0.0793 (0.0692)
(c) Log of sibling's wage				
(d) Father's education/10	0.0826 (0.0366)	-0.0187 (0.1000)	0.0829 (0.0364)	0.0314 (0.1030)
(e) Education * experience/10	0.0023 (0.0104)	-0.0029 (0.0113)	-0.0002 (0.0105)	-0.0027 (0.0113)
(f) Log of sibling's wage * experience/10				
(g) Father's education * experience/100		0.0867 (0.0813)		0.0441 (0.0841)
(h) Black * experience/10			-0.1270 (0.0541)	-0.1194 (0.0563)
$R^2$	0.2748	0.2750	0.2755	0.2756
Observations	18523	18523	18523	18523
Individuals	2594	2594	2594	2594



Figure 5: The Effects of Standardized AFQT, Father's Education, Sibling Wage, and Schooling on Wages

Dependent Variable: Log Wage; Experience Measure: Potential Experience

OLS estimates (standard errors)

Model:	(1)	(2)	(3)	(4)
(a) Education	0.0505 (0.0118)	0.0832 (0.0151)	0.0563 (0.0120)	0.0780 (0.0155)
(b) Black	-0.1333 (0.0255)	-0.1296 (0.0257)	0.0454 (0.0609)	-0.0284 (0.0704)
(c) Standardized AFQT	0.0792 (0.0145)	-0.0206 (0.0361)	0.0789 (0.0144)	0.0065 (0.0413)
(d) Log of sibling's wage	0.1602 (0.0208)	0.0560 (0.0352)	0.1617 (0.0207)	0.0604 (0.0351)
(e) Father's education/10	0.0362 (0.0356)	0.0154 (0.0963)	0.0385 (0.0354)	0.0295 (0.0968)
(f) Education * experience/10	0.0005 (0.0093)	-0.0269 (0.0123)	-0.0035 (0.0094)	-0.0220 (0.0128)
(g) Standardized AFQT * experience/10		0.0843 (0.0285)		0.0614 (0.0333)
(h) Log of sibling wage * experience/10		0.1194 (0.0393)		0.1151 (0.0393)
(i) Father's education * experience/100		0.0176 (0.0789)		0.0055 (0.0794)
(j) Black * experience/10			-0.1500 (0.0474)	-0.0861 (0.0570)
$R^2$	0.2991	0.3014	0.3002	0.3016

## **IV.3. The Experience Profile of the Effects of AFQT and Education on Wages**

## V. Do Employers Statistically Discriminate on the Basis of Race?

## VI. Models with Training

**Figure 6: The Effects of Standardized AFQT, Father's Education, Sibling Wage, Schooling, and Training on Wages**

Dependent Variable: Log Wage; Experience Measure: Potential Experience  
 Training Measure: Predicted before 88, Actual After; OLS estimates (standard errors)

Model:	(1)	(2)	(3)	(4)
(a) Education	0.0606 (0.0119)	0.0802 (0.0151)	0.0651 (0.0121)	0.0746 (0.0155)
(b) Black	-0.1159 (0.0265)	-0.1135 (0.0267)	0.0241 (0.0616)	-0.0028 (0.0722)
(c) Standardized AFQT	0.0334 (0.0150)	-0.0199 (0.0363)	0.0338 (0.0150)	0.0102 (0.0420)
(d) Log of sibling's wage	0.1594 (0.0213)	0.0716 (0.0357)	0.1611 (0.0213)	0.0759 (0.0356)
(e) Father's education/10	0.0460 (0.0356)	0.0211 (0.0974)	0.0482 (0.0354)	0.0353 (0.0977)
(f) Education * experience/10	-0.0231 (0.0095)	-0.0392 (0.0123)	-0.0260 (0.0096)	-0.0339 (0.0128)
(g) Standardized AFQT * experience/10		0.0460 (0.0287)		0.0207 (0.0339)
(h) Log of sibling's wage * experience/10		0.1041 (0.0402)		0.1001 (0.0402)
(i) Father's education * experience/100		0.0205 (0.0803)		0.0084 (0.0805)
(j) Black * experience/10			-0.1180 (0.0476)	-0.0945 (0.0583)
(k) Training: $R_t$	-0.1143 (0.0200)	-0.1095 (0.0199)	-0.1115 (0.0199)	-0.1091 (0.0199)
(l) Cumulative training: $\Sigma R_T$	0.1881 (0.0139)	0.1830 (0.0139)	0.1854 (0.0139)	0.1827 (0.0139)
$R^2$	0.3188	0.3199	0.3195	0.3202



Figure 7: Estimates of the Effects of AFQT, Father's Education, Sibling Wage, and Schooling on Wage Growth with Controls for Training

Dependent Variable:  $\Delta \log$  Wage; Experience Measure: Potential Experience

Coefficient estimates (standard errors)				
Model:	(1)	(2)	(3)	(4)
Education *	-0.0060	-0.0694	-0.0106	-0.0729
$\Delta$ experience/10	(0.0833)	(0.0960)	(0.0832)	(0.0959)
AFQT * $\Delta$ experience/10		0.3025		0.2975
		(0.1613)		(0.1614)
Log of sibling wage *		0.2153		0.2107
$\Delta$ experience/10		(0.1477)		(0.1477)
Father's education *		-0.4306		-0.4215
$\Delta$ experience/10		(0.5034)		(0.5034)
Black * $\Delta$ experience/10	-0.0504	-0.0425	-0.0503	-0.0426
	(0.0484)	(0.0485)	(0.0483)	(0.0484)
Training: $R_t/10$			0.2468	0.2429
			(0.1024)	(0.1025)
Lag training: $R_{t-1}/10$			-0.0194	-0.0230
			(0.1108)	(0.1108)
S.E.E.	.2965	.2965	.2965	.2964

## VII. Conclusions and a Research Agenda