Complementary Bias: A Model of Two-Sided Statistical Discrimination

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1. Introduction

2. A Brief Review of the Literature

A. Models of Discrimination

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3. The Basic Model

A. Building Blocks

- Imagine a large number of employers and a larger population of workers.
- Each employer is randomly matched with many workers from this population.
- Workers belong to one of two identifiable groups, $j \in \{A, B\}$
- Denote by λ_A the fraction of *A*s in the population and $\lambda_B = 1 - \lambda_A$ the fraction of *B*s.
- One can imagine groups being race, gender, or any other protected class.

- Nature moves first and assigns a type to each worker and a type to each employer.
- The worker's type, denoted by $c \in (0, \bar{c}), \bar{c} < \infty$, depicts her cost of investment in human capital.
- Let the fraction of workers with costs no greater than c be represented by $G^W(c)$ – a smooth and continuous cumulative distribution function – with $g^W(c)$ the associated density.
- Similarly, employers have the opportunity to invest at a cost $k_j \in \left(o, \bar{k}\right), \bar{k} < \infty$, to make their workplaces desirable and productive places to work for workers of type j.

- The fraction of employers with investment cost no greater than k_j is $G^E(k_j)$, with $g^E(k_j)$ the associated density.

- Superscripts "*W*" and "*E*" refer to workers and employers, respectively.

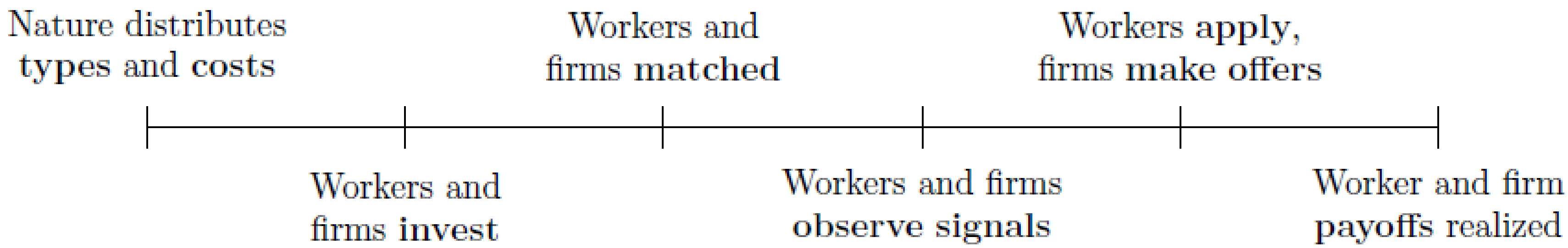


Figure 1: Sequence of Actions

- Associated smooth and continuous distribution function $F_i^W(\theta)$
- Density function $f_i^W(\theta)$ where $i \in \{q, u\}$
- $\phi(\theta) \equiv \frac{f_u^W(\theta)}{f_q^W(\theta)}$ is non-increasing in θ (i.e., $f_i^W(\theta)$ satisfies the monotone likelihood ration property)
- Noisy but informative signal $\psi \in [0,1]$ to workers
- Distribution function $F_i^E(\psi)$
- Density function $f_i^E(\psi)$ where $i \in \{q, u\}$

B. Payoffs

- If the worker is hired and works for an employer who has made a group j investment, she receives a payoff of $\omega_q - c$ if she chose to invest

- ω_q if not.

- If the worker is hired and works for an employer who has not made a group j investment, she receives $-\omega_u - c$ if she invested and $-\omega_u$ if she did not

- If she does not work for any employer, she receives $-c$ if she invested or zero otherwise

C. Strategies

- The worker's strategy consists of a pair of functions – an investment decision and an application decision

- Write as $I^W\colon\{A,B\} \times [0,\bar{c}] \rightarrow [0,1]$ and $A^W\colon\{A,B\} \times [0,1\,] \times [0,1] \times [0,\bar{c}] \rightarrow [0,1]$

- The employer's strategy also consists of a pair of functions – an investment decision and an assignment decision $-I^E\colon\{A,B\} \times \left[0,\bar{k}\right] \rightarrow [0,1], A^E\colon\{A,B\} \times [0,1] \times [0,1] \times \left[0,\bar{k}\right] \rightarrow [0,1]$

D. Expected Payoffs

- Investing in human capital increases the likelihood that a worker is accepted by an employer.
- If a worker of type j invests, she gets expected gross payoff $(1 - F_q^W(s_j))\ \overline{\omega}\ (\delta_j)$
- Conversely, if she does not invest, she gets $(1 - F_u^W(s_j))\ \overline{\omega}\ (\delta_j)$
- Thus, the net return on investment for workers can be written as:

$$\beta_W\left(s_j, \delta_j\right) \equiv \left[F_u^W\left(s_j\right) - F_q^W\left(s_j\right)\right] \overline{\omega}\left(\delta_j\right). \tag{1}$$

The net return on investment for workers can be written as:

$$\beta_E(t_j, \pi_j | \lambda_j) \equiv \lambda_j \left[F_u^E(t_j) - F_q^E(t_j)\right] \overline{\chi}(\pi_j) \,. \qquad (2)$$

E. Bayesian Nash Equilibrium

Definition. *An equilibrium of the game is a pair of beliefs* $\{\pi, \delta\}$ *satisfying:*

$$\pi_j = G^W\left(\beta_W\left(s_j, \delta_j\right)\right) \tag{3}$$

$$\delta_j = G^E\left(\beta_E\left(t_j, \pi_j | \lambda_j\right)\right) \tag{4}$$

Proposition 1. *Let* $\pi^*(\delta)$ *and* $\delta^*(\pi)$ *be the sets of solutions to equations (3) and (4) respectively. Assume that* $\phi(\theta)$ *and* $\tau(\psi)$ *are continuous, strictly decreasing and strictly positive on* $[0,1]$, *and that* $G^W(c)$ *and* $G^E(k)$ *are continuous with full support on* $[0,\bar{c}]$ *and* $[0,\bar{k}]$ *with* $G^W(0) = G^E(0) = 0$. *Further assume that for some* $\underline{\delta}$, *there exists an* s *for which* $G^W(\beta_W(s,\delta)) > \phi(s)/[\chi_q/\chi_u + \phi(s)]$. *Similarly assume that for some* $\underline{\pi}$, *there exists a* t *for which* $G^E(\beta_E(t,\pi|\lambda)) > \tau(t)/[\omega_q/\omega_u + \tau(t)]$. *Then non-zero elements of* $\pi^*(\delta)$ *and* $\delta^*(\pi)$ *exist for any* $\delta \geq \underline{\delta}$ *and* $\pi \geq \underline{\pi}$ *respectively. If there is a set of beliefs* $\{\pi,\delta\}$ *such that* $\delta \in \delta^*(\pi)$ *and* $\pi < \max\{\pi^*(\delta)\}$ *then there exist multiple solutions to the two-sided model.*

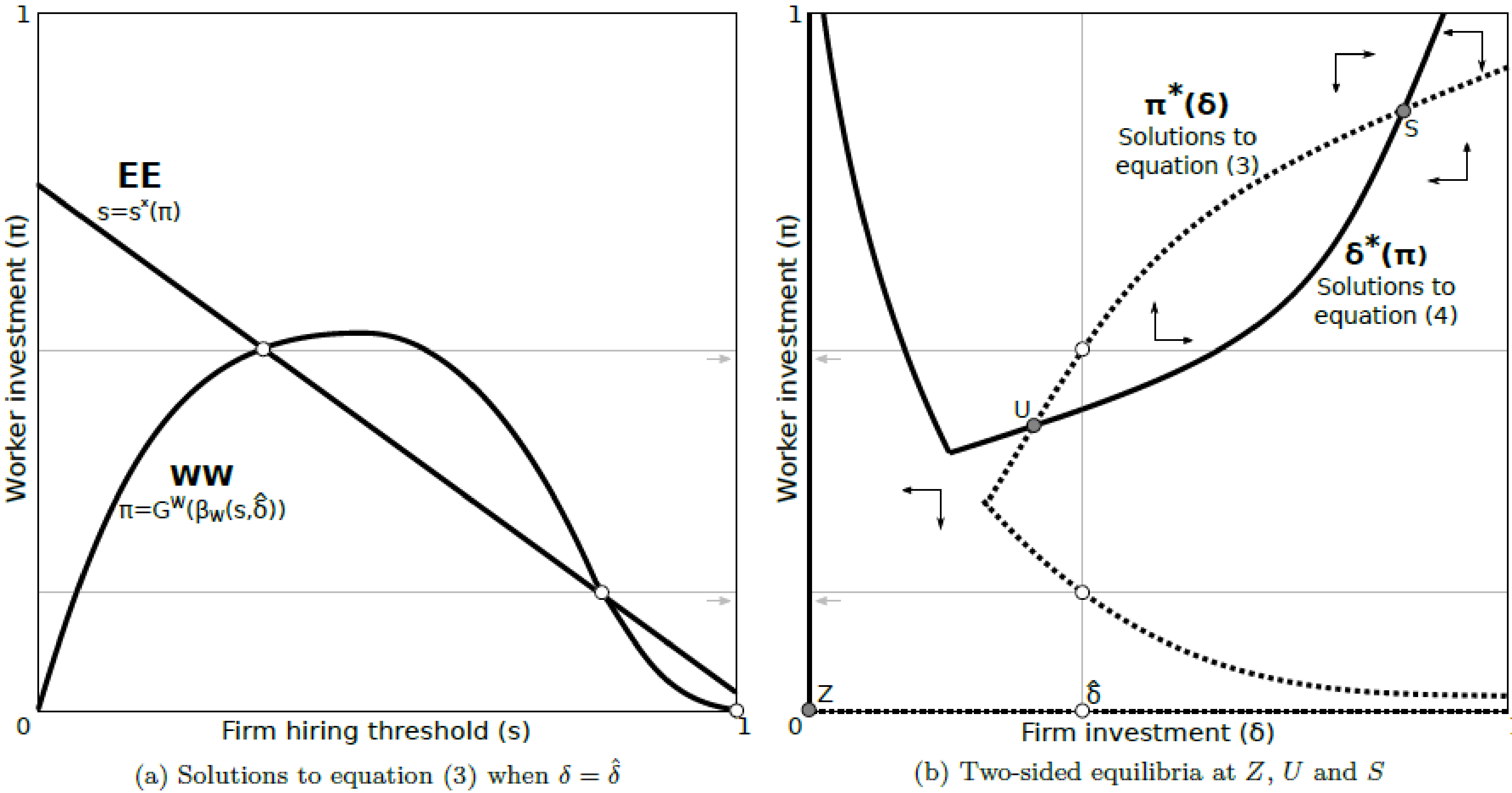


Figure 2: Equilibria in the two-sided model

F. Dynamics

To analyze stability more generally, we approximate this two-dimensional system of non-linear equations as a first-order linearized system of difference equations.

$$d_{t+1} = \begin{pmatrix} \pi_{t+1} \\ \delta_{t+1} \end{pmatrix} = \Upsilon \begin{pmatrix} \pi_t \\ \delta_t \end{pmatrix} \tag{5}$$

For ease of exposition, define the following derivatives.

$$WW_1' = G^{W\prime} \cdot \left[f_u^W\left(s^*\left(\pi\right)\right) - f_q^W\left(s^*\left(\pi\right)\right)\right] \qquad WW_2' = G^{E\prime} \cdot \left[f_u^E\left(t^*\left(\delta\right)\right) - f_q^E\left(t^*\left(\delta\right)\right)\right]$$

$$EE_1' = 1/s^{*\prime}\left(\pi\right) \qquad EE_2' = 1/t^{*\prime}\left(\delta\right)$$

$$RR_1' = \overline{\omega}'\left(\delta\right) \cdot \left[F_u^W\left(s^*\left(\pi\right)\right) - F_q^W\left(s^*\left(\pi\right)\right)\right] \cdot G^{W\prime} \qquad RR_2' = \overline{\chi}'\left(\pi\right) \cdot \lambda \cdot \left[F_u^E\left(t^*\left(\delta\right)\right) - F_q^E\left(t^*\left(\delta\right)\right)\right] \cdot G^{E\prime}$$

These definitions allow us to write the Jacobian of the system compactly.

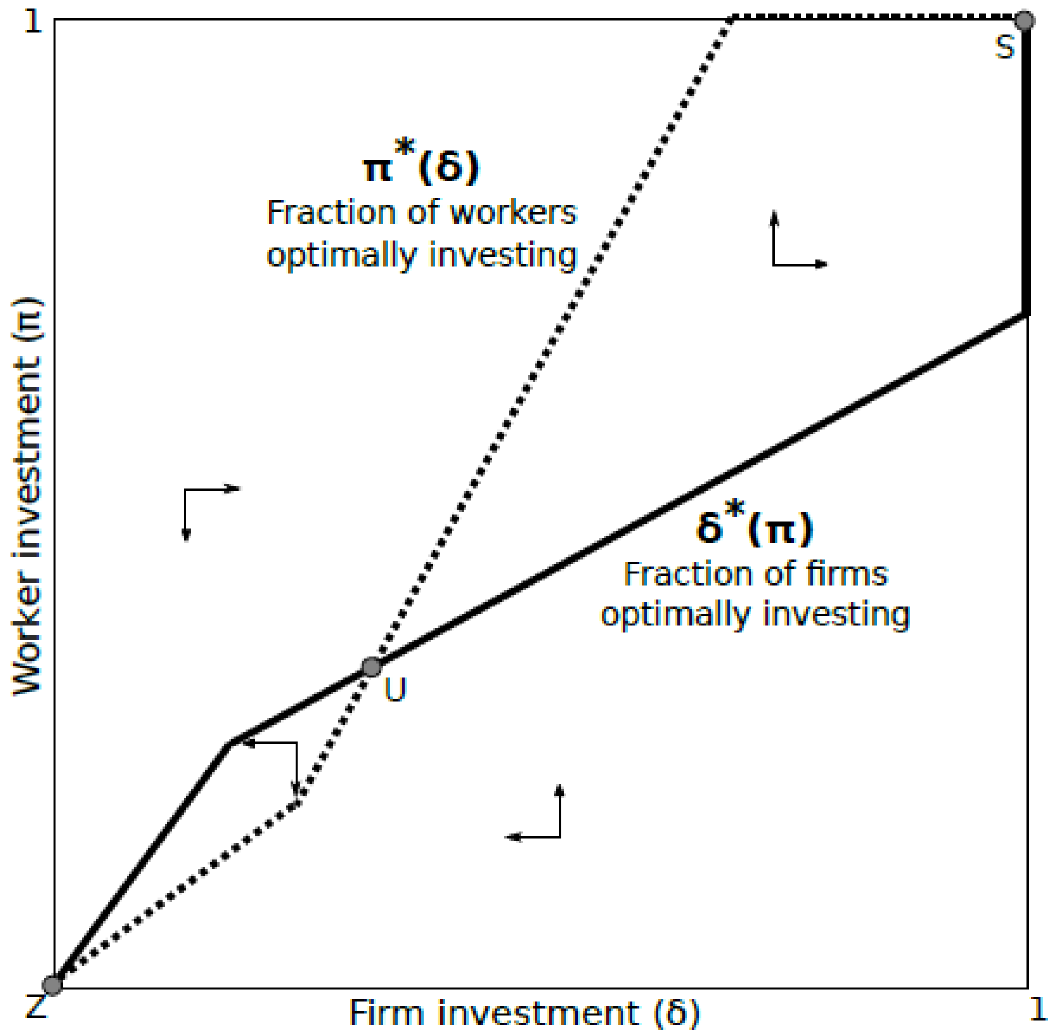
$$\begin{bmatrix} WW_1' \frac{1}{EE_1'} & RR_1' \\ RR_2' & WW_2' \frac{1}{EE_2'} \end{bmatrix}$$

The following condition is necessary and sufficient for this (Neusser 2016).

$$\left|WW_1'\frac{1}{EE_1'}+WW_2'\frac{1}{EE_2'}\right|<1+\left(WW_1'\frac{1}{EE_1'}\cdot WW_2'\frac{1}{EE_2'}\right)-(RR_1'\cdot RR_2')<2.$$

3.1 An Example with Uniform Cost and Signal Distributions

Figure 3: Equilibria in the Clear / Unclear Example



4. Extending the Basic Model: One-Sided Policies

A. Equality in Offers

- Given beliefs (π_A, π_B) and worker application standards (t_A, t_B) , it will choose hiring standards (s_A, s_B) and make an investment decision $i_j \in \{q, u\}$ for each group $j \in \{A, B\}$ to solve the following optimization problem

$$\max_{s_A, s_B, i_A, i_B} \left[\lambda_B P\left(s_B, \pi_B, i_B\right) + \lambda_A P\left(s_A, \pi_A, i_A\right)\right] \quad \text{s.t.} \quad \rho\left(s_B, \pi_B\right) = \rho\left(s_A, \pi_A\right). \tag{6}$$

Definition. *An equilibrium under affirmative action is a set of beliefs* (π_A, π_B), (δ_A, δ_B), *worker standards* (t_A, t_B) *and employer standards* (s_A, s_B) *satisfying the following conditions:*

(a) *Firm signal thresholds* (s_A, s_B) *solve problem (6), given* (π_A, π_B, t_A, t_B).[19]

(b) $t_j = t_j^*(\delta_j)$, $j \in \{A, B\}$

(c) $\pi_j = G^W(\beta_W(s_j, \delta_j))$, $j \in \{A, B\}$

(d) $\delta_j = G^E\left(\lambda_j \left[F_u^E(t_j) - F_q^E(t_j)\right]\left[\pi_j\left(1 - F_q^W(s_j)\right)\chi_q - (1 - \pi_j)\left(1 - F_u^W(s_j)\right)\chi_u\right]\right)$, $j \in \{A, B\}$

Proposition 2. *Assume that, without affirmative action, there exists an equilibrium with positive investment. Then there exists an equilibrium under affirmative action without homogeneous beliefs.*

Proposition 3. *Assume that $\phi(\theta)$ and $\tau(\psi)$ are continuous, strictly decreasing and strictly positive on $[0,1]$. Further assume that $\lambda_A \neq \lambda_B$ and that $G^E(k)$ and $G^W(c)$ are strictly increasing. Then no equilibrium with positive investment and homogeneous employer beliefs exists (with or without affirmative action).*

Proposition 4. *Assume that $\phi(\theta)$ and $\tau(\psi)$ are continuous, strictly decreasing and strictly positive on $[0,1]$. Further suppose that the A and B markets start with $\pi_A > \pi_B > 0$ and $\delta_A > \delta_B > 0$. For fixed beliefs $\{\pi_A, \pi_B, \delta_A, \delta_B\}$ and low enough δ_B and π_B, imposing affirmative action causes zero firms to invest in B amenities and zero B workers to invest.*

B. Equality in Employment

- Given beliefs (π_A, π_B) and worker application standards (t_A, t_B) , an employer will again choose hiring standards (s_A, s_B) and make investment decisions (i_A, i_B) to solve the following problem:

$$\max_{s_A, s_B, i_a, i_B} \left[\lambda_B P\left(s_B, \pi_B, i_B\right) + \lambda_A P\left(s_A, \pi_A, i_A\right)\right] \quad \text{s.t.} \quad \rho_H\left(s_B, \pi_B, i_B\right) = \rho_H\left(s_A, \pi_A, i_A\right). \tag{7}$$

Definition. *An equilibrium under an employment quota is a set of beliefs* (π_A, π_B), (δ_A, δ_B), *worker standards* (t_A, t_B) *and employer standards* $\left(s_j^{q,q}, s_j^{q,u}, s_j^{u,q}, s_j^{u,u}\right)$, $j \in \{A, B\}$ *satisfying the following conditions:*

(a) *Each firm's investment decisions* (i_A, i_B) *and thresholds* (s_A, s_B) *solve (7), given* (π_A, π_B, t_A, t_B)

(b) $t_j = t_j^* (\delta_j)$, $j \in \{A, B\}$

(c) $\pi_j = G^W \left(\overline{\beta}_W\right)$, $j \in \{A, B\}$

(d) $\delta_j = \int_0^1 G^E \left(k_j^* (k_{-j})\right) dk_{-j}$

Proposition 5. *Assume that G^E has full support on $[0, \overline{c}]$ with $\overline{c} > \omega_q$, let $\phi(\theta)$ be strictly decreasing, and define $\tilde{s}$ as the firm signal threshold such that $\phi(\tilde{s}) = 1$. If firm investment is close enough to perfectly observable, any equilibrium under an employment quota must entail homogeneous beliefs if:*

$$\eta\left(\overline{\beta}(s)\right) < \frac{\phi(s_j)}{\phi(s_j) - 1}$$

for all $s \in [0, \tilde{s})$ where $\eta(c) = \frac{d[c \cdot G(c)]}{dc}$ and $\overline{\beta}(s) = \left[F_u^W(s) - F_q^W(s)\right]\omega_q$.

Proposition 6. *Assume that* $\phi(\theta)$ *and* $\tau(\psi)$ *are continuous, strictly decreasing and strictly positive on* $[0,1]$*. Further suppose that the A and B markets start with* $\pi_A > \pi_B > 0$ *and* $\delta_A > \delta_B > 0$*. For low enough* δ_B *and* π_B*, imposing an employment quota lowers employment of A workers. Furthermore, there exists an open set of parameters such that the policy leads to zero investment by B workers.*

C. Wage and Employments Subsidies

D. An “Impossibility” Result

Proposition 7. *Suppose that we seek to move to an equilibrium* $\{s^*, t^*, \pi^*, \delta^*\}$ *from another point with* $s_0 > s^*$, $t_0 > t^*$, $\pi_0 < \pi^*$ *by independently setting some combination* C *of* s, t, π *and* δ. *There exist interventions that achieve this aim for any* $\{\pi_0, \delta_0\}$ *if and only if* $\{\delta, \pi\} \in C$, $\{t, \pi\} \in C$ *or* $\{s, \delta\} \in C$. *Targeting* $\{\delta, \pi\}$ *is faster than any alternative.*

5. Extending the Basic Model: Two-Sided Policies

A. Two-Sided Investment Insurance

Proposition 8. *Suppose that the government observes noisy but informative signals, θ^g and ψ^g, of worker and firm investment respectively. For any initial beliefs, there exist incentive payments ω^g and χ^g conditional on these signals that immediately ensure that $\pi_A = \pi_B$, $\delta_A = \delta_B$, $s_A = s_B$ and $t_A = t_B$. If and only if $\lambda_A \neq \lambda_B$, a non-zero permanent investment subsidy is required to maintain $\pi_A = \pi_B$.*

If the government sets $s_B^g = s_A$, the fraction of B workers who invest is:

$$\pi_{B,t} = G^W\left(\beta_W\left(s^*\left(\pi_{B,t-1}\right), \delta_{B,t-1}\right) + \left[\tilde{F}_u^W\left(s_A|\theta < s^*\left(\pi_{B,t-1}\right)\right) - \tilde{F}_q^W\left(s_A|\theta < s^*\left(\pi_{B,t-1}\right)\right)\right]\omega^g\right).$$

Since this is achieved immediately, the actual cost of the worker payments are as follows.

$$\delta\left[1 - \tilde{F}_q\left(s_A|\theta < s_A\right)\right]\omega^g + (1-\delta)\left[1 - \tilde{F}_u\left(s_A|\theta < s_A\right)\right]\omega^g$$

B. Affirmative Action

Proposition 9. *Assume that* $\phi(\theta)$ *and* $\tau(\psi)$ *are continuous, strictly decreasing and strictly positive on* $[0,1]$. *For any* $\pi_B \in [0,1)$ *and* $\pi_A \in (0,1)$ *with* $\pi_B < \pi_A$, *there exist cost distributions* G^W *and* G^E, *a signal distribution* $F_i^W(\theta)$ *and parameters such that: (i)* π_B *and* π_A *are part of an equilibrium; and (ii) no one-sided investment subsidy can raise* π_B *to* π_A *in any finite number of periods* T, *even if combined with affirmative action.*

6. Interpreting Group Differences in the Presence of Two-Sided Statistical Discrimination

- Two-sided statistical discrimination complicates empirical analysis, since differences between groups are generically a combination of both employer and worker decision-making.

- For example, consider a setting with employer learning as in Altonji and Pierret (2001).

- Under conditions they outline, the conditional expectation for log-wages can be written as a time-varying function of the form:

$$E\left(w_t \middle| s_i, z_i, t\right) = b_{s,t} s_i + b_{z,t} z_i + H\left(t\right)$$

- An implication of this or any other model in which investments depend on beliefs or otherwise depend on race is that empirical analysis designed to detect statistical discrimination may be misleading.
- Assuming that race is an s variable – i.e., employers statistically discriminate – the linear predictor of the wage must be modified as follows:

$$E^*\left(w_t | s_i, z_i, t\right) = \left(b_{s,t} + \rho_{s,t}\right) s_i + b_{z,t} z_i + b_t t$$

- Lang and Lehmann (2012) discuss an alternative test that is robust to differing wage profiles of black and white workers.
- Let B_i be a dummy for whether a worker is black.
- As before, z_i is correlated with productivity and initially unobserved by the employer.
- Lang and Lehmann propose comparing two regressions.

$$E^* \left(w_t | s_i, z_i, t \right) = \alpha_1 + \alpha_2 B_i + \alpha_3 t + \alpha_4 B_i t + \alpha_5 z_i$$

$$E^* \left(w_t | s_i, z_i, t \right) = \beta_1 + \beta_2 B_i + \beta_3 t + \beta_4 B_i t + \beta_5 z_i + \beta_6 z_i t$$

- If black workers are lower productivity on average and employers statistically discriminate, we would therefore expect in the following $\gamma_4 < 0$ and $\gamma_2 > 0$ in the following auxiliary regression.

$$E^*\left(z_t t \middle| s_i, z_i, t\right) = \gamma_1 + \gamma_2 B_i + \gamma_3 t + \gamma_4 B_i t + \gamma_5 z_i$$

A. Detecting Employer Discrimination

- If workers are paid their marginal product, the wage paid by a firm to a worker with ability a_i at a firm with can k_j^F be shown to be as follows

$$\ln w_i = \left(\frac{\sigma^2_{a,j}}{\sigma^2_{\varepsilon,j}+\sigma^2_{a,j}}\right)\ln a_i + \left(\frac{\sigma^2_{\varepsilon,j}}{\sigma^2_{\varepsilon,j}+\sigma^2_{a,j}}\right)\mu_{a,j} + \ln\gamma + (1-\gamma)\ln k_{j,F} + \frac{1}{2}\left(\frac{\sigma^2_{\varepsilon,j}\sigma^2_{a,j}}{\sigma^2_{\varepsilon,j}+\sigma^2_{a,j}}\right) + \left(\frac{\sigma^2_{a,j}}{\sigma^2_{\varepsilon,j}+\sigma^2_{a,j}}\right)\ln\varepsilon_i$$

Assumption. *For a worker of long enough tenure at her previous employer, her past wage exactly reflects her ability at a new firm.*

Nonetheless, this assumption allows us to write the wage offered to worker i as a particularly simple function of her wage at her previous firm, group-specific fixed effects for the source and destination firms, and an error term:

$$\ln(w_i) = \beta_j \ln\left(w_i^{OLD}\right) + \alpha_{j,f^{OLD}} + \alpha_{j,f^{NEW}} + \nu_i \qquad (8)$$

where:

$$\alpha_{j,f^{NEW}} = \left(\frac{\sigma^2_{\varepsilon,j}}{\sigma^2_{\varepsilon,j}+\sigma^2_a}\right)\mu_{a,j} + (1-\gamma)\,k_{j,F^{NEW}} + \frac{1}{2}\left(\frac{\sigma^2_{\varepsilon,j}\sigma^2_{a,j}}{\sigma^2_{\varepsilon,j}+\sigma^2_{a,j}}\right)$$

$$\alpha_{j,f^{OLD}} = -(1-\gamma)\ln k_{j,F^{OLD}}$$

$$\nu_i = \left(\frac{\sigma^2_{a,j}}{\sigma^2_{\varepsilon,j}+\sigma^2_{a,j}}\right)\ln \varepsilon_i$$

and β_j is the elasticity of the wage with respect to individual ability:

Proposition 10. *Assume that ability at the new and old firms are correlated:* $\ln a_i = c_j + \rho \ln a_i^{OLD} + \ln \eta_i$ *where* $0 < \rho \leq 1$. *Conditional on firm fixed effects, assume that* $\ln \varepsilon_i$, $\ln \varepsilon_i^{OLD}$ *and* $\ln \eta_i$ *are uncorrelated with* $\ln w_i^{OLD}$ *and that a worker's past employer has more information than her new employer:* $\sigma^2_{\varepsilon,j} \geq \sigma^2_{\varepsilon,j,OLD}$. *Then the difference in coefficients from regression (8) is* $\hat{\Gamma}$, *where*

$$\hat{\Gamma} = \rho \left[\left(\frac{\sigma^2_{\varepsilon,W,OLD} + \sigma^2_{a,W}}{\sigma^2_{\varepsilon,W} + \sigma^2_{a,W}} \right) - \left(\frac{\sigma^2_{\varepsilon,B,OLD} + \sigma^2_{a,B}}{\sigma^2_{\varepsilon,B} + \sigma^2_{a,B}} \right) \right]$$

$$= \rho \left[\underbrace{\left(\frac{\sigma^2_{a,W}}{\sigma^2_{\varepsilon,W} + \sigma^2_{a,W}} \right) - \left(\frac{\sigma^2_{a,B}}{\sigma^2_{\varepsilon,B} + \sigma^2_{a,B}} \right)}_{\Gamma \text{ (true difference in returns)}} + \left(\frac{\sigma^2_{\varepsilon,W,OLD}}{\sigma^2_{\varepsilon,W} + \sigma^2_{a,W}} \right) - \left(\frac{\sigma^2_{\varepsilon,B,OLD}}{\sigma^2_{\varepsilon,B} + \sigma^2_{a,B}} \right) \right].$$

7. Conclusion