Gittins Index, Pandora's Box, and Miller's Model of Learning and Labor Market Turnover (Excerpt)

James J. Heckman
University of Chicago

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Applications: Job Search, Pandora's Box and Miller**Job Search:**

(One Spell Model)

- Jobs last forever, no learning.
- Only issue is when to stop.
- This is a degenerate, single arm bandit.
- $c =$ cost of playing machine.
- $X_t =$ reward on t^{th} trial.
- No learning implies X_t is independent and identically distributed with cdF $F(x)$ known to agent.

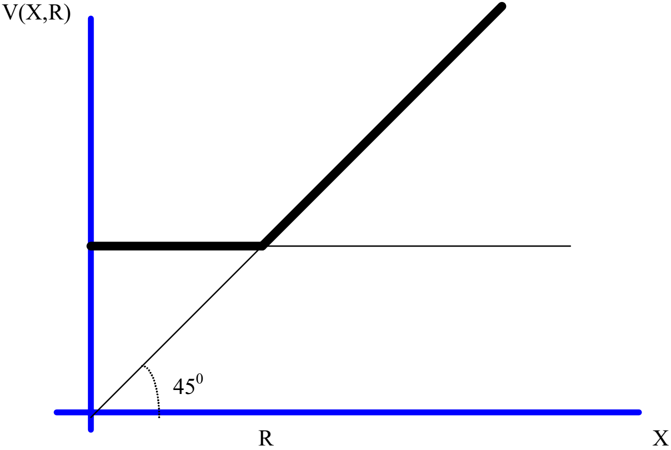
$$V(X) = \max \left[X, -c + \beta \int\limits_{0}^{\infty} V(y) dF(y) \right] \tag{1}$$

Optimal policy: Search if $X \in [0, R]$ reservation price is R :

$$V(X) = \begin{cases} R = -c + \beta \int\limits_{0}^{\infty} V(y) dF(y) & \text{if } X < R \\ X & \text{if } X \geq R \end{cases} \tag{2}$$

R is the value that makes a person indifferent between stopping and continuing.

This figure graphs the functional equation (1) and it reveals that the optimal solution is of the form of (2) (see e.g. Sargent's textbook Ch. 6):



Solving for reservation wage:

using (2) we convert (1) into an ordinary equation in the reservation wage:

$$\begin{aligned} R &= -c + \beta \left[\int_0^R R dF(x) + \int_R^\infty X \, dF(x) \right] \\ R \left(\int_0^R dF(x) + \int_R^\infty dF(x) \right) &= -c + \beta \left[\int_0^R R dF(x) + \int_R^\infty X \, dF(x) \right] \end{aligned}$$

or

$$(1-\beta) R \int_0^R dF(x) = -c + \int_R^\infty (\beta X - R) \, dF(x)$$

Solving for reservation wage:

adding $(1-\beta)R\int_R^\infty dF(x)$ to both sides we have

$$R = \frac{-c}{1-\beta} + \frac{\beta}{1-\beta}\int\limits_R^\infty (X-R)\,dF(x) \tag{3}$$

with unique solution for R. To see this, define

$$g(R) = \frac{-c}{1-\beta} + \frac{\beta}{1-\beta}\int\limits_R^\infty (X-R)\,dF(x)$$

Solving for reservation wage:

with

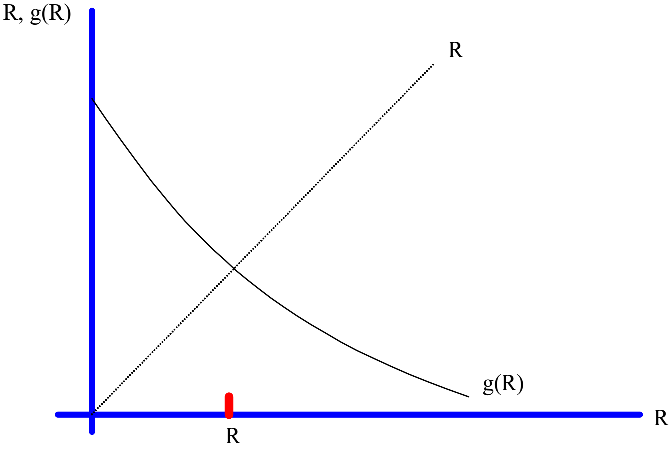
$$g'(R) = -\frac{\beta}{1-\beta}[1-F(R)] < 0$$

$$g''(R) = \frac{\beta}{1-\beta}F'(R) > 0$$

the optimal reservation wage satisfies, from (3)

$$R^* = g(R^*)$$

Solving for reservation wage:



Pandora (Weitzmann, *Econometrica*, May, 1979)

- N different occupations (or N college majors) each yielding an unknown reward
- Each occupation has its own search cost c_i and independent probability distribution F_i for the reward X_i
- Occupations are sampled sequentially, in whatever order is desired. When it has been decided to stop searching, only one occupation is accepted, the maximum sampled reward.
- Under this formulation, what sequential search strategy maximizes expected present discounted value?

- **Pandora's problem:** At each state Pandora must decide wheter or not to open a box. If she chooses to stop searching, Pandora collects at that time the maximum reward she has thus far uncovered. Should Pandora wish to continue sampling, she must select the next box to be opened and pay c_i
- **Solution:**

- Compute Reservation Price for each box
- Try jobs with highest R_i^*'s
- Keep trying until one gets $X_t(i^*) \geq \max_i\{R_i^*\}$

(*i.e.* keep going until you get a realization $\geq$ sampled values).

Proof: Compute reservation prices for each occupation $R_i(x_i)$. Consider occupation i. If occupation i tried, then X_i is known. For an i-th occupation with known trial,

$$V_i(X_i, R) = \max\left[R, X_i\right].$$

Therefore, $R_i(X_i) = X_i$. Thus, the reservation price for a sampled occupation is just the wage realized.

For an untried occupation,

$$\begin{aligned}
\widetilde{V}_i(R) &= V_i(-c_i, R) \\
&= \max\left[R, -c_i + \beta \int_0^\infty V_i(y, R)\, dF_i(y)\right] \\
&= \max\left[R, -c_i + \beta \left(F_i(R)R + \int_R^\infty y\, dF_i(y)\right)\right]
\end{aligned}$$

Reservation wage is the smallest value of R_i such that

$$
\begin{aligned}
R_i &= -c_i + \beta R_i F_i(R_i) + \beta \int_{R}^{\infty} y \, dF_i(y) \\
\therefore R_i &= -\frac{c_i}{1-\beta} + \frac{\beta}{1-\beta} \int_{R_i}^{\infty} (X - R_i) \, dF_i(y)
\end{aligned}
$$

as in the above search model.

Consider a binomial case with $\beta = 1$ (no discounting):

$$X_i = 0 \quad \text{wp.} \quad (1 - p_i)$$

$$X_i = r_i \quad \text{wp.} \quad p_i$$

and R_i satisfies

$$c_i = \int_{R_i}^{\infty} (X - R_i)\, dF_i(X)$$

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2 cases:

- If $R_i = 0$, then

$$c_i = \int_0^\infty X \, dF_i(X) = p_i r_i.$$

No reason for this to be satisfied in general. Therefore,

- We assume $R_i \in (0, r_i]$

$$c_i = \int_{R_i}^\infty (X - R_i) \, dF_i(X) = p_i r_i - R_i p_i$$

$$\therefore \quad R_i = \frac{p_i r_i - c_i}{p_i}$$

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- Thus, take two projects with equal expected value ($p_i r_i - c_i$). The project with lower expected probability of reward is the one to try (go for riskier project). Why? Suppose costs are the same. Then r_i is higher, since p_i is lower.
- Suppose that rewards are the same. But costs being lower implies that p_i is lower (trying less costly combo).
- No further learning.