Notes on "A Theory of Job Shopping"

Johnson, *Quarterly Journal of Economics*, Vol. 92, No. 2.

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- This is a prototype for (and more general than) the Jovanovic model.

$$Y_1 = a_1 + b_1\theta + U_1, \quad b_1 \geq 0$$
$$Y_2 = a_2 + b_2\theta + U_2, \quad b_2 \geq 0$$

  - Key feature: has no investment beyond job shopping.
  - Place more weight on θ in the earnings of occupation 2:

$$b_2 > b_1$$

  - θ receives a greater return in occupation 2.

$$\mathrm{E}(U_1) = \mathrm{E}(U_2) = \mathrm{E}(\theta) = 0 \qquad (U_1 \perp\!\!\!\perp U_2).$$

- In either job, workers can learn about θ:

$$\theta \perp\!\!\!\perp U_1 \perp\!\!\!\perp U_2$$

- Why do people leave jobs?—Two sources:
  1. Bad luck (low U_i).
  2. The more the worker learns about θ, the more likely he/she is to go to sectors that weight θ more highly if he/she has a high θ.
- Major Conclusion: Under uncertainty about productivity of matches, and own ability, its optimal to start in an occupation with high variance.
- But this is variance in true income not measurement error.

- Assume a two period model with no cost of mobility.
- An income maximizing person who starts in 1 will move to 2 if

$$E(Y_2 \mid Y_1) > Y_1 .$$

- From normal theory we have that

$$E(\theta \mid Y_1) = \beta(Y_1 - a_1)$$

$$E(Y_2 \mid Y_1) = a_2 + \beta b_2(Y_1 - a_1) ,$$

where

$$\beta = \frac{b_1 \sigma_\theta^2}{b_1^2 \sigma_\theta^2 + \sigma_{U_1}^2} .$$

- A worker starting in state 1 changes jobs if

$$Y_1 < a_2 + \beta b_2 (Y_1 - a_1).$$

- $Y_1(1 - \beta b_2) < a_2 - \beta b_2 a_1$
- To fix ideas, let $a_1 = a_2 = a$ (same mean income in all jobs).
- $\mathrm{E}(Y_2 | Y_1) > Y_1$ requires:

$$Y_1(1 - \beta b_2) < a(1 - \beta b_2)$$

- Condition for mobility out of occupation 1 for a person who starts there.
- Consider two cases: $(\beta b_2 > 1; \beta b_2 < 1)$
- With $\beta b_2 = 1$ worker indifferent.
- First let $\beta b_2 > 1$. Then worker leaves 1 having started there if $Y_1 > a$.
- Person above average in 1.
- Intuition:

$$\beta b_2 = \frac{b_2 b_1 \sigma_\theta^2}{b_1^2 \sigma_\theta^2 + \sigma_{U_1}^2}$$

- $\beta b_2 > 1 \Rightarrow b_2 > b_1$ (only way possible)
- Goes to the occupation where the reward to θ is higher.

- Second Case: If $\beta b_2 < 1$ and $Y_1 < a$, the person leaves occupation 1.
- At issue is the question of whether one would ever observe a person starting in 1 changing occupations.
- *Optimal Sequence Problem*. Which job first?
- Learning occurs about both general and specific skills:
  1. Learning about general skill (θ), and acting on it in terms of its value in each sector.
  2. Learning about occupation-specific skills (U_1 and U_2).
- Miller (1984) and Jovanovic (1979) ignore θ and focus on learning about the occupation-specific components.

Define the comparable expressions for starting in 2

- Define $\gamma = b_2\sigma_\theta^2/\sigma_2^2$.

$$b_1\gamma = \frac{b_1 b_2\sigma_\theta^2}{\sigma_2^2} = \frac{b_1 b_2\sigma_\theta^2}{b_2^2\sigma_\theta^2 + \sigma_{U_2}^2}$$

- Assume $b_2\beta > 1$. This implies:

1. $$b_2 > b_1 \text{ (as before)}$$
2. $$\therefore \quad b_1\gamma < 1$$
3. $$\sigma_1^2 < \sigma_2^2$$

since

$$\frac{\beta b_2}{\gamma b_1} = \frac{\sigma_2^2}{\sigma_1^2} > 1$$

$\therefore \sigma_2^2 > \sigma_1^2$.

$$\sigma_1^2 = \sigma_{U_1}^2 + b_1^2\sigma_\theta^2 \qquad \sigma_2^2 = \sigma_{U_2}^2 + b_2^2\sigma_\theta^2$$

$$b_1\gamma = \frac{b_1 b_2 \sigma_\theta^2}{\sigma_2^2} \qquad b_2\beta = \frac{b_1 b_2 \sigma_\theta^2}{\sigma_1^2}$$

Start in Sector 1 $(\beta b_2 > 1)$

$$E(V_1) = E(Y_1) + \Pr(Y_1 < a)E(Y_1|Y_1 \leq a) + \Pr(Y_1 > a)E(Y_2|Y_1 > a)$$

$$= \underset{\substack{\uparrow \\ 1^{\text{st}} \\ \text{period} \\ \text{mean}}}{a} + \underbrace{\Pr(Y_1 < a)}_{=1/2}E(Y_1 \mid Y_1 \leq a) + \underbrace{\Pr(Y_1 > a)}_{=1/2}E(Y_2 \mid Y_1 > a)$$

Define t as the standard normal random variable.

$$E(V_1) = a + \frac{1}{2}\left[a + \underbrace{\sigma_1 \int_{-\infty}^{0} \frac{te^{-\frac{t^2}{2}}}{\sqrt{2\pi}} \mathrm{d}t}_{= -\frac{\sigma_1}{\sqrt{2\pi}}}\right] + \frac{1}{2}\left[a + \beta b_2 \frac{\sigma_1}{\sqrt{2\pi}}\right]$$

$$= 2a + \underbrace{\frac{(\beta b_2 - 1)\sigma_1}{2\sqrt{2\pi}}}_{\substack{\text{Arises from} \\ \text{option value}}}$$

Start in Sector 2 (Recall $\beta b_2 > 1$)

$$\begin{aligned} E(V_2) = &a + \Pr(Y_2 > a)E(Y_2 \mid Y_2 \geq a) \\ &+ \Pr(Y_2 < a)E(Y_1 \mid Y_2 \leq a) \end{aligned}$$

$$\begin{aligned} E(Y_1 \mid Y_2) &= a + \frac{\text{Cov}(Y_2, Y_1)}{\text{Var}(Y_2)}(Y_2 - a) \\ &= a + \frac{(b_1 b_2)\sigma_\theta^2}{b_2^2\sigma_\theta^2 + \sigma_2^2}(Y_2 - a) \\ &= a + b_1\gamma(Y_2 - a) \end{aligned}$$

$$\gamma = \frac{b_2\sigma_\theta^2}{\sigma_2^2} \qquad \beta = \frac{b_1\sigma_\theta^2}{\sigma_1^2}$$

$$
\begin{aligned}
E(V_2) &= a + \frac{1}{2}\left[a + \frac{\sigma_2}{\sqrt{2\pi}}\right] + \frac{1}{2}\left[a - b_1\gamma\left(\frac{\sigma_2}{\sqrt{2\pi}}\right)\right] \\
&= 2a + \underbrace{\left[\frac{1 - b_1\gamma}{2}\right]\frac{\sigma_2}{\sqrt{2\pi}}}_{\text{option value}}
\end{aligned}
$$

- Therefore,

$$
E(V_1) - E(V_2) = \frac{1}{2}\left[\frac{(\beta b_2 - 1)\sigma_1}{\sqrt{2\pi}} + \frac{(b_1\gamma - 1)\sigma_2}{\sqrt{2\pi}}\right].
$$

- If $\beta b_2 > 1$ and therefore $b_1\gamma < 1$:

$$
E(V_2) - E(V_1) > 0
$$

- Why?: $\mathrm{E}(V_2) - \mathrm{E}(V_1) > 0$ requires:
  (substitute for β and γ in previous expression.)

$$0 > \left[\frac{b_1 b_2 \sigma_\theta^2}{\sigma_1^2} - 1 \right] \sigma_1 + \left[\frac{b_1 b_2 \sigma_\theta^2}{\sigma_2^2} - 1 \right] \sigma_2$$

$$0 > \frac{[b_1 b_2 \sigma_\theta^2 - \sigma_1^2]}{\sigma_1} + \frac{[b_1 b_2 \sigma_\theta^2 - \sigma_2^2]}{\sigma_2}$$

$$0 < \frac{\sigma_1^2 - b_1 b_2 \sigma_\theta^2}{\sigma_1} + \frac{\sigma_2^2 - b_1 b_2 \sigma_\theta^2}{\sigma_2}$$

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$$0 < (\sigma_1 + \sigma_2) - (b_1 b_2)\sigma_\theta^2 \left[\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right]$$

$$0 < (\sigma_1 + \sigma_2) - \frac{(b_1 b_2)\sigma_\theta^2}{\sigma_1 \sigma_2} [\sigma_2 + \sigma_1]$$

$$0 < 1 - \frac{b_1 b_2 \sigma_\theta^2}{\sigma_1 \sigma_2}$$

- Therefore, if

$$\frac{b_1 b_2 \sigma_\theta^2}{\sigma_1 \sigma_2} < 1\,,$$

always optimal to start with 2.

- Because

$$\overbrace{\frac{b_1 \sigma_\theta}{\sqrt{b_1^2 \sigma_\theta^2 + \sigma_{U_1}^2}} \frac{b_2 \sigma_\theta}{\sqrt{b_2^2 \sigma_\theta^2 + \sigma_{U_2}^2}}}^{\text{correl}(Y_1, Y_2)} < 1$$

is always true, agents start high variance occupations (Occupation 2), which are high expected value occupations.

- Therefore you go to the occupation with higher variance.
- If $b_1 \gamma > 1$, agent goes to occupation 1 because $\sigma_1^2 > \sigma_2^2$.

- Suppose next that $b_1\gamma < 1$ and $b_2\beta < 1$ (now we get the "intuitive" kind of selection).

- Then the difference in value functions is

$$
\begin{aligned}
E(V_1) - E(V_2) &= \frac{1}{\sqrt{2\pi}}\Big[(1 - b_2\beta)\sigma_1 - (1 - b_1\gamma)\sigma_2\Big] \\
&= \frac{1}{\sqrt{2\pi}}\left[\left(1 - \frac{b_1 b_2 \sigma_\theta^2}{\sigma_1^2}\right)\sigma_1 - \left(1 - \frac{b_1 b_2 \sigma_\theta^2}{\sigma_2^2}\right)\sigma_2\right] \\
&= \frac{1}{\sqrt{2\pi}}\left[\left(\frac{\sigma_1^2 - b_1 b_2 \sigma_\theta^2}{\sigma_1}\right) - \left(\frac{\sigma_2^2 - b_1 b_2 \sigma_\theta^2}{\sigma_2}\right)\right] \\
&= \frac{1}{\sqrt{2\pi}}\left[(\sigma_1 - \sigma_2) + (b_1 b_2)\left[\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right]\right].
\end{aligned}
$$

- Therefore

$$\sigma_1 > \sigma_2 \Rightarrow \mathsf{E}(V_1) > \mathsf{E}(V_2)$$

$$\sigma_1 < \sigma_2 \Rightarrow \mathsf{E}(V_2) < \mathsf{E}(V_1)\,.$$

- We have proved the following propositions:
  1. Workers start out their lives in high variance occupations.
  2. Workers *always* move from jobs with below average earnings.
     - Moving when $Y_1 > a$ requires $b_2\beta_2 > 1$, $b_1\gamma < 1$, and thus $\sigma_1 < \sigma_2$.
     - But then, a worker would never start out in occupation 1.
     - Thus the optimal sequence begins with job 2 and the mobility condition is $Y_2 < a$.
  3. Dispersion in period 2 decreases compared to period 1 (earnings variance is decreasing over time as people learn; this is a normal selection result).
  4. The mix of σ_i^2 into $\sigma_{U_i}^2$ and $b_i^2\sigma_\theta^2$ does not affect the decision to go to high variance occupation first.