Age, Period, and Cohort Effects

(Extract from The Mincer Equation and the Rate of Return to Schooling)

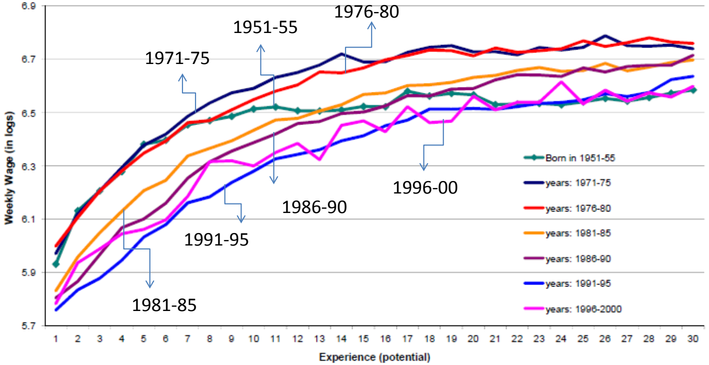
James J. Heckman
University of Chicago

Econ 350, Spring 2022

How Valid is the Synthetic Cohort Assumption? What Life Cycle Parameters Do We Identify in Cross-Sections?

Figure 1: Cohort and cross-section wage profiles for high school graduates and college graduates, white males, CPS, March supplements, 1964-–2002

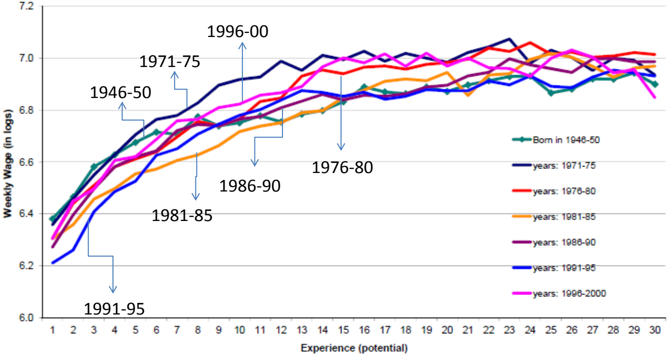
(a) High school graduates born in 1951-1955



Source: Rubinstein and Weiss (2006).
Note: The chart shows the wage-experience profiles for the cohort of high school graduates born in 1951-1955. Added to the graphs is the evolution of the cross section wage-experience profiles from 1971 to 2000 in five year intervals, where each such cross section profile shows the mean weekly wages (in log) of workers with the indicated schooling and experience in a given time interval.

Figure 1: Cohort and cross-section wage profiles for high school graduates and college graduates, white males, CPS, March supplements, 1964–2002

(b) College graduates born in 1946-1950



Source: Rubinstein and Weiss (2006).
Note: The chart shows the wage-experience profiles for the cohort of college graduates born in 1946-1950. Added to the graphs is the evolution of the cross section wage-experience profiles from 1971 to 2000 in five year intervals, where each such cross section profile shows the mean weekly wages (in log) of workers with the indicated schooling and experience in a given time interval.

Age, Period, and Cohort Effects

Add Age, Cohort, and Year Effects to Mincer Equation for Person i

$$\ln W_i = \alpha_0 + \underbrace{\alpha_1 a_i}_{\substack{\text{age} \\ \text{``maturation''}}} + \underbrace{\alpha_2 y}_{\substack{\text{year} \\ \text{aggregate} \\ \text{effects}}} + \underbrace{\alpha_3 e_i}_{\substack{\text{experience} \\ \text{(lbd or OJT)}}} + \underbrace{\alpha_4 s_i}_{\text{schooling}} + \underbrace{\alpha_5 c_i}_{\substack{\text{``vintage''} \\ \text{or cohort} \\ \text{(quality of} \\ \text{cohorts)}}} + u_i \tag{1}$$

Two Identities

$$e_i = a_i - s_i \qquad \text{“experience”} \tag{2}$$

$$y_i = a_i + c_i \qquad c_i = \text{birth year} \tag{3}$$

- Take a pedagogically convenient case where schooling and experience are initially dropped ($\alpha_3 = \alpha_4 = 0$):

$$\ln W(a, y, c) = \beta_0 + \underset{\text{(age)}}{\beta_1 a_i} + \underset{\text{(year)}}{\beta_2 y_i} + \underset{\text{(cohort)}}{\beta_3 c_i} + u_i$$

$$y_i = a_i + c_i,$$

- y_i: year wage observed.
- c_i: year of birth.
- For a given cross-section (fixing y_i) equal and exact one linear dependence: c_i and a_i move in opposite directions.
- Older people come from earlier cohorts.
- Age effects might be cohort effects, and vice versa.

THE UNIVERSITY OF CHICAGO

- Substitute $c_i = y_i - a_i$.

$$\begin{aligned} \ln W_i &= \alpha_0 + \beta_1 a_i + \beta_2 y_i + \beta_3 (y_i - a_i) + u_i \\ &= \alpha_0 + (\beta_1 - \beta_3) a_i + (\beta_2 + \beta_3) y_i + u_i \end{aligned}$$

- Can identify only combinations of coefficients.
- Cross section: y_i is the same for everyone.
- Intercept:

$$[\alpha_0 + (\beta_2 + \beta_3) y_i] .$$

- Identify $(\beta_1 - \beta_3)$: age minus cohort effect.
- If $\beta_3 > 0$, we underestimate true β_1 (e.g., schooling quality improving).
- Does longitudinal data solve the problem? Not necessarily.
- In panels, y_i generally moves over time.
- Recall that $y_i = a_i + c_i$, so we still have exact linear dependence.
- Panel data identifies model if there are no year effects.

- Supposed we substitute out for a_i?
- $y_i = a_i + c_i$.

$$\begin{aligned} \ln W_i &= \alpha_0 + \beta_1 + \beta_2(y_i - c_i) + \beta_3 c_i + u_i \\ &= \alpha_0 + (\beta_1 + \beta_2 + \beta_3) y_i + (\beta_3 - \beta_1) c_i + u_i \end{aligned}$$

- If $\beta_1 > 0$, understate cohort effects.
- **Exercise:** What are the competing stories of the labor market for each identifying assumption?

- Acquire similar problems in models with nonlinear terms:

$$y = a + c$$

- $$\begin{aligned} \ln W = \quad & \beta_0 + \beta_1 a + \beta_2 y + \beta_3 c + \beta_4 a^2 + \beta_5 ac \\ & + \beta_6 ay + \beta_7 cy + \beta_8 c^2 + \beta_9 y^2 + u, \end{aligned}$$

- $$\left. \begin{aligned} y^2 &= a^2 + 2ac + c^2 \\ ay &= a^2 + ac \\ cy &= ca + c^2 \end{aligned} \right\} \text{3 linear dependencies}$$

- Cannot identify all of the parameters (only 3 parameters associated with second order (square and interaction terms) are identified out of 6 total.

THE UNIVERSITY OF CHICAGO

Theorem. In a model with interactions of order k with j variables and one linear restriction among the j variables, then of the $\binom{j+k-1}{k}$ coefficients of order k, only $\binom{j+k-2}{k}$ are estimable. (Heckman and Robb, in S. Feinberg and W. Mason, *Age, Period and Cohort Effects: Beyond the Identification Problem*, Springer, 1986).

E.g. $k = 2$, $j = 3$; 6 coefficients and 3 are estimable, as in the preceding example.

Theorem. In a model with ℓ restrictions on the j variables, then $\binom{j+k-\ell-1}{k}$ kth order coefficients are estimable (Heckman and Robb, 1986).

- **Exercise:** Show the linear dependencies that arise if analysts use dummy variables in place of continuous variables.

General Case: Add Back Experience and Schooling

- Substitute out for c_i and a_i, using (2) and (3):

$$\ln W_i = \alpha_0 + (\alpha_2 + \alpha_5)y + (\alpha_1 + \alpha_3 - \alpha_5)e_i + (\alpha_1 + \alpha_4 - \alpha_5)s_i + u_i.$$

- In a single cross section, y is the same for everyone.
- Intercept is $\alpha_0 + (\alpha_2 + \alpha_5)y$.
- Experience "effect" $= \alpha_1 + \alpha_3 - \alpha_5 = \alpha_3 + (\alpha_1 - \alpha_5)$.
- If later vintages acquire more skills, $\alpha_5 > 0$.
- Downward bias (*e.g.* higher quality of schooling).
- If there is an aging effect ($\alpha_1 > 0$, *e.g.*, maturation) produces upward bias for α_3.

THE UNIVERSITY OF CHICAGO

Schooling Coefficient

- $\alpha_1 + \alpha_4 - \alpha_5 = \alpha_4 + (\alpha_1 - \alpha_5)$
- Vintage (cohort) effects lead to downward bias in the estimated rate of return to schooling.
- Age (maturation) effects produce upward bias.
- Experience coefficient-schooling coefficient:

$$= (\alpha_1 + \alpha_3 - \alpha_5) - (\alpha_1 + \alpha_4 - \alpha_5) = \alpha_3 - \alpha_4.$$

- Can identify difference in "returns" to experience net of schooling.

- Observe that even if $\alpha_1 = 0$ (no aging effect), still can't identify true schooling or experience effects.
- Does **longitudinal data** (observations on the same people over time) or **repeated cross section data** (observations on the same population over time but sampling different persons) help? No: because year effects activated (acquire new parameters in each wave).

- If $\alpha_2 = 0$, (no year effects), can identify α_5.
- Alternatively, for each c_i, can estimate $\alpha_1 + \alpha_3$, and hence can estimate α_5.
- We also know $\alpha_1 + \alpha_4$.
- If $\alpha_1 = 0$ (no age effect), then $\alpha_{3,}$ $\alpha_{4,}$ α_5 identified.

- If year effects are present, no gain to going to longitudinal or repeated cross section data.
- We gain a new parameter when we move to each wave of the panel or repeated cross sectional data.

Solutions in Literature

1. Redefine vintage (cohort) e.g. vintage fixed over period of years (e.g. a cohort of Depression babies).
   - Then $\ln W = (\alpha_0 + \alpha_5 c) + \alpha_1 a + \alpha_2 y + \alpha_3 e + \alpha_4 s + u$.
   - In single cross section, c and y are fixed.

- Substitute for e:

$$e = a - s$$

-

$$\ln W = [\alpha_0 + \alpha_5 c + \alpha_2 y] + (\alpha_1 + \alpha_3)a + (\alpha_4 - \alpha_3)s$$

- Can estimate $\alpha_1 + \alpha_3$ and $\alpha_4 - \alpha_3$, and thus $\alpha_1 + \alpha_4$.
- Successive time periods for the same vintage gives us α_2 directly [since c doesn't move].
- If no age effect, identify $\alpha_3, \alpha_4, \alpha_2$, and from successive vintages, we get α_5.

THE UNIVERSITY OF CHICAGO

2. If we measure actual work experience, $a \neq e + s$ (agents do not fully participate in market over their life cycles), can break the linear dependence.
    - However, better proxies may be endogenous, e.g., if experience $=$ cumulated hours of work.

Nonlinear Models

- Results carry over in an obvious way to nonlinear models.