Rate of Return Continuation Values and Option Values in a Simple Dynamic Model

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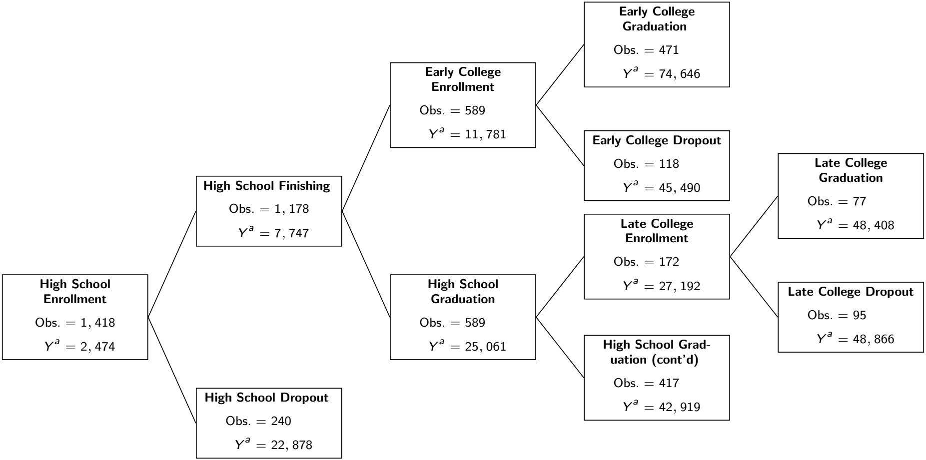
Structural Dynamic Discrete Choice Model of Schooling

Sequential Model

Figure 1: Atemporal Stage-wise Decision Tree

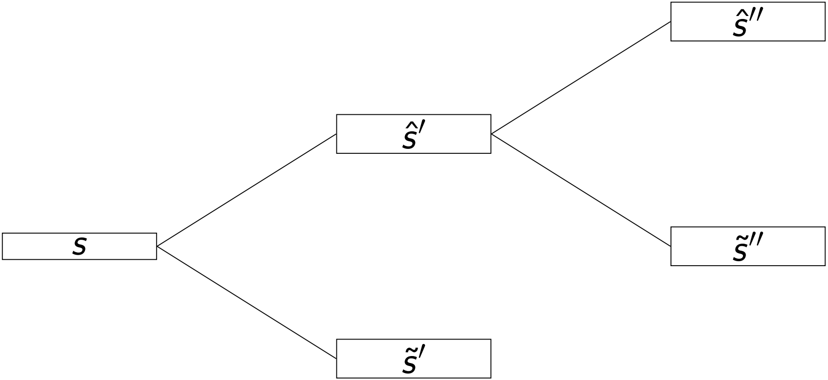


Notes: Y^a refers to average annual earnings in the state in 2005 dollars. Obs. refers to the number of observations in the state.

Setup

- Current state $s \in \mathcal{S} = \{s_1, \dots, s_N\}$.
- $\mathcal{S}^v(s) \subseteq \mathcal{S}$: set of visited states (in agent history).
- $\mathcal{S}^f(s) \subseteq \mathcal{S}$ the set of feasible states that can be reached from s.
- Choice set of the agent in state s: $\Omega(s) = \{s' \mid s' \in \mathcal{S}^f(s)\}$.
- Consider binary choices only, so $\Omega(s)$ has at most two elements at each stage.
- *Ex post*, the agent receives per period rewards $R(s') = Y(s') - C(s', s)$.
- Costs $C(s', s)$ associated with moving from state s to state s' (monetary and psychic).
- $R(s') = Y(s') - C(s', s)$.

Figure 2: Generic Decision Problem



- ˜ denotes absorbing state
- ˆ denotes a transition state

Payoffs and Costs: Example of Person at s

$$Y(s) = \mu_s(X(s)) + \theta'\alpha_s + \epsilon(s) \text{ (outcome at } s) \quad (1)$$

$$\text{Person at state } s \text{ has two options: } s' = \hat{s}' \text{ or } s' = \tilde{s}' \quad (2)$$

$$\underset{\text{transition cost}}{C(s', s)} = \begin{cases} K_{s',s}(Q(\hat{s}', s)) + \theta'\varphi_{\hat{s}',s} + \eta(\hat{s}', s) & \text{if} \quad s' = \hat{s}' \\ 0 & \text{if} \quad s' = \tilde{s}' \end{cases}$$

- In advance, $\eta(s', s)$ is known; so is θ
- $Q(\hat{s}', s)$ is the vector of variables observed before making the transition
- $\epsilon(s')$ not known

System of Measurement Equations: State Space Model

$$M(j) \quad = \quad B(j)'\kappa_j + \theta'\gamma_j + \nu(j) \qquad \forall \quad j \in J \tag{3}$$

- θ is unobserved (by agent) ability vector (cognitive and noncognitive)
- $B(j)$ may be observed by agent as well as $\theta, \nu_j, \forall j \in J$
- $B(j)$ is a device used to aid the econometrician; It's an indicator; agent already knows the information
- Econometrician does not know $\theta, \nu_j, j \in J$

Table 1: Who Knows What and When?

	Agent Studied	Observing Economist
$\epsilon(s)$:	known at s	unknown at s
$\epsilon(s')$:	unknown at s	unknown at s
$C(s', s)$:	known at s, but realized at s'	components may be known, i.e., $Q(\hat{s}', s)$
$\eta(\hat{s}', s)$:	known at s	unknown
$B(j)$:	known	known
$M(j), j = 1, \ldots, J$:	known (but irrelevant)	known
θ:	known	unknown
$\nu(j), j = 1, \ldots, J$:	may or may not be unknown j	unknown
$X(s)$:	known $\forall s \in \mathcal{S}$ (but irrelevant)	known all $s \in \mathcal{S}$; clearly some components may not be known; created an omitted variable problem
Model Parameters:	known	unknown

- $Y_t =$ decision made at t
- $I_t =$ relevant information known and acted on at t
- $W_t =$ not known and/or acted on at t
-

$$Y_t = I_t\beta + W_t\Gamma + U_t$$
$$U_t \perp\!\!\!\perp (I_t, W_t)$$

- Test: I_t properly specified if estimated $\beta \neq 0, \Gamma = 0$

- All "error terms" are mutually independent

$$
\begin{aligned}
&\epsilon(s) \perp\!\!\!\perp \epsilon(l)\ \forall l \neq s,\ \forall s \\
&\eta(s', s) \perp\!\!\!\perp \eta(j', j)\ \forall (j', j) \neq (s', s), \forall j, s \\
&\theta \perp\!\!\!\perp [\epsilon(l), \eta(s', s)]\ \forall l, s, s' \in \mathcal{S} \\
&\nu_j \perp\!\!\!\perp [\theta, \epsilon(l), \eta(s', s)]\ \forall l, s, s' \in \mathcal{S}
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{I}(s) = \{\epsilon(s), C(s', s), \eta(\hat{s}, s), \theta, X(s), Q(\hat{s}', s)\} \\
&\tilde{\mathcal{I}}(s) :\ \text{information set of the econometrician}
\end{aligned}
$$

Value Function

$$V(s \,|\, \mathcal{I}(s)) = Y(s) + \max_{s' \in \Omega(s)} \left\{ \frac{1}{1+r} \Big(\underbrace{- C(s', s) + \mathbb{E}[V(s' \,|\, \mathcal{I}(s')) \,\big|\, \mathcal{I}(s)\,]}_{\text{Continuation value}} \Big) \right\}$$

Decision Rule

$$s' = \begin{cases} \hat{s}' & \text{if} \quad \mathbb{E}\left[V(\hat{s}') \,\middle|\, \mathcal{I}(s)\right] - C(\hat{s}', s) > \mathbb{E}\left[V(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right] \\ \tilde{s}' & \text{otherwise} \end{cases}$$

Ex Ante Net Return

$$NR(\hat{s}', \tilde{s}', s) = \frac{\mathbb{E}\left[V(\hat{s}') - V(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right] - C(\hat{s}', s)}{\mathbb{E}\left[V(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right]}$$

Ex Ante Gross Return

$$GR(\hat{s}', \tilde{s}', s) = \frac{\mathbb{E}\left[\tilde{V}(\hat{s}') - \tilde{V}(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right]}{\mathbb{E}\left[\tilde{V}(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right]}$$

Option Value

$$OV(s', s) =$$

$$\frac{1}{1+r}\,\mathbb{E}\Bigg[\underbrace{\max_{s''\in\Omega(s')}\Big\{-C(s'',s') + \mathbb{E}\Big(V(s'')\Big)\Big\}}_{\text{value of options arising from } s'} - \underbrace{\Big(V(\tilde{s}'')\Big)}_{\text{fallback value}}\ \Bigg|\ \mathcal{I}(s)\Bigg]$$

Choice Probabilities: An Example

- Define

$$\Pr(s' = \hat{s}' | \mathcal{I}(s)) =$$

$$F_{\eta(\hat{s}',s)}\left(\mathbb{E}\left[V(\hat{s}') - V(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right] - (K_{\hat{s}',s}(Q(\hat{s}', s)) + \theta' \varphi_{\hat{s}',s})\right)$$

- **Problem:** Justify this expression
- **Problem:** Suppose $\eta(\hat{s}', s) \sim N(0, \sigma_\eta^2)$
- **Under the information set given, what is the functional form of:** $\Pr(s' = \hat{s}' | \mathcal{I}(s))$?
- **Problem:** Relate this model to the standard two sector Roy model

Likelihood:

- Let $D(s) = 1$ if s is the terminal state
- $D(s) = 0$ otherwise
- $D(s', s) = 1$ if transit $s \to s' = 0$; otherwise

$$\mathcal{L} = \int_{\underline{\Theta}} \left\{ \left[\prod_{j \in J} f\left(M(j) \middle| B(j), \gamma, \theta\right) \right] \times \right. \tag{4}$$

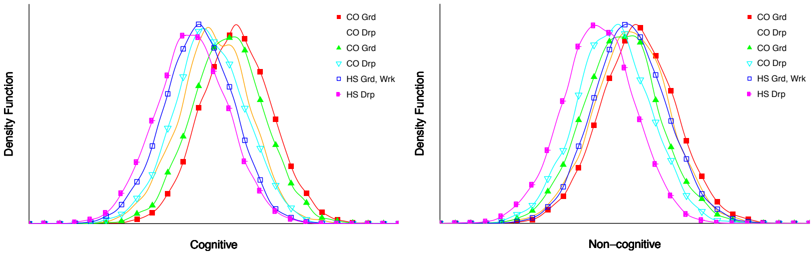
$$\left. \prod_{s \in \mathcal{S}} \left[f\left(Y(s) \middle| X(s), \theta, D(s) = 1\right) \Pr(D(s = 1) | \tilde{\mathcal{I}}(s)) \right] \right\}^{D(s)} dF(\theta) \tag{5}$$

where $\tilde{\mathcal{I}}(s)$ is the information set of the econometrician

- $D(s) = \prod_{j=0}^{s-1} D(j', j)(1 - D(s, s-1))$
- Problem: Justify this expression (derive it)

Empirical Results

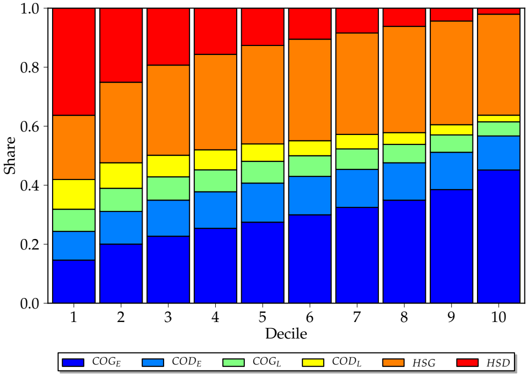
Figure 3: Ability Distributions by Terminal States



Simulate a sample of 50,000 agents based on the estimates of the model.

Ability Distributions by Final Education

Figure 4: Non-Cognitive Skills



Ability Distributions by Final Education

Figure 5: Cognitive Skills

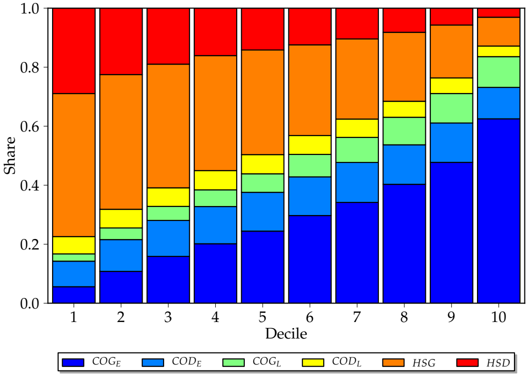
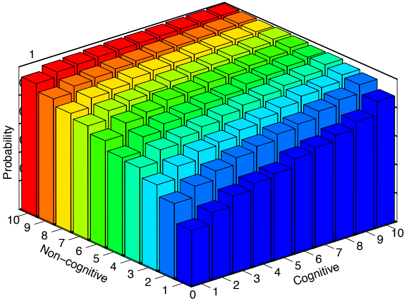
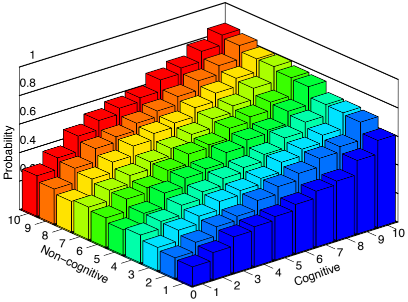


Figure 6: Transition Probabilities by Abilities



(a) High School Completion



(b) Early College Enrollment

Notes: We simulate a sample of 50,000 agents based on the estimates of the model. In each subfigure, we condition on the agents that actually visit the relevant decision state.

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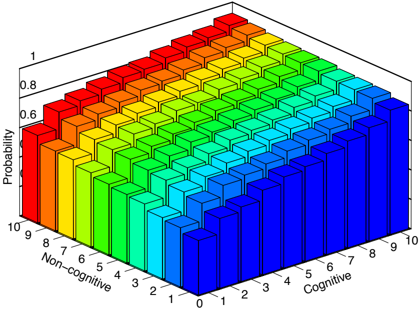
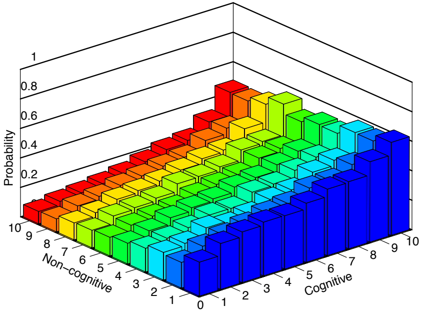
Figure 6: Transition Probabilities by Abilities (continued)



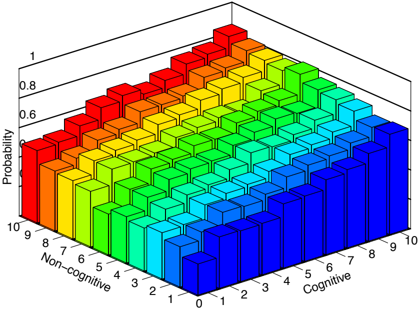
(a) Early College Graduation



(b) Late College Enrollment

Notes: We simulate a sample of 50,000 agents based on the estimates of the model. In each subfigure, we condition on the agents that actually visit the relevant decision state.

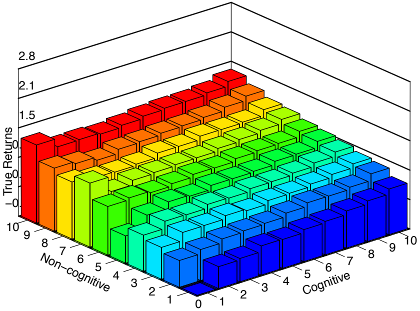
Figure 6: Transition Probabilities by Abilities (continued)



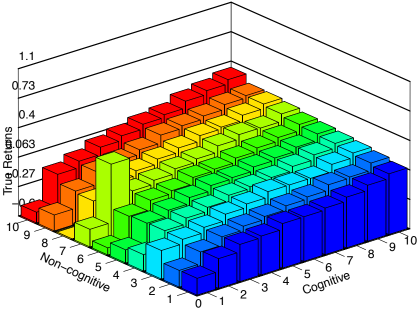
(a) Late College Graduation

Notes: We simulate a sample of 50,000 agents based on the estimates of the model. In each subfigure, we condition on the agents that actually visit the relevant decision state.

Figure 7: Ex Ante Net Returns by Abilities



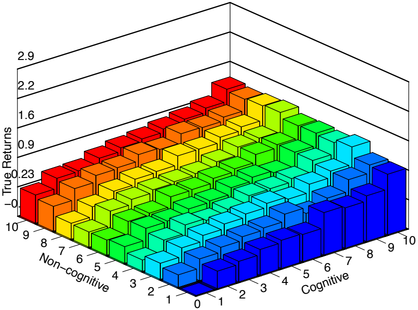
(a) High School Completion
$NR^a = 0.64$
$GR^a = 0.30$



(b) Early College Enrl.
$NR^a =$ -0.06
$GR^a = 0.17$

Notes: We simulate a sample of 50,000 agents based on the estimates of the model. In each subfigure, we condition on the agents that actually visit the relevant decision state. Enrl. = Enrollment, Grad. = Graduation.

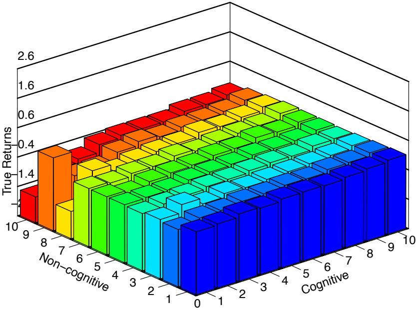
Figure 7: Ex Ante Net Returns by Abilities (continued)



(a) Early College Grad.

$NR^a = 0.57$

$GR^a = 0.89$



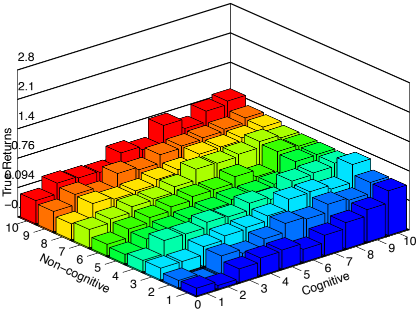
(b) Late College Enrl.

$NR^a =$ -0.23

$GR^a = 0.34$

Notes: We simulate a sample of 50,000 agents based on the estimates of the model. In each subfigure, we condition on the agents that actually visit the relevant decision state. Enrl. = Enrollment, Grad. = Graduation.

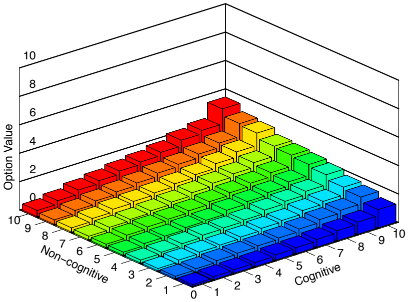
Figure 7: Ex Ante Net Returns by Abilities (continued)



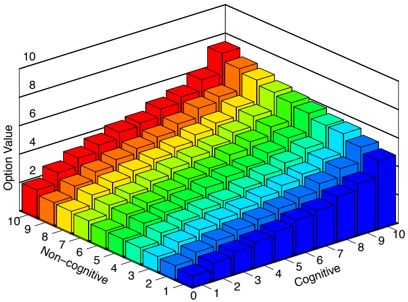
(a) Late College Grad.
$NR^a = 0.15$
$GR^a = 0.33$

Notes: We simulate a sample of 50,000 agents based on the estimates of the model. In each subfigure, we condition on the agents that actually visit the relevant decision state. Enrl. = Enrollment, Grad. = Graduation.

Figure 8: Option Values by Abilities



(a) High School Completion
$OV \quad = 0.99$
$OVC = 0.10$



(b) Early College Enrollment
$OV \quad = 3.33$
$OVC = 0.30$

Notes: We simulate a sample of 50,000 agents based on the estimates of the model. In each subfigure, we condition on the agents that actually visit the relevant decision state. In units of $100,000.

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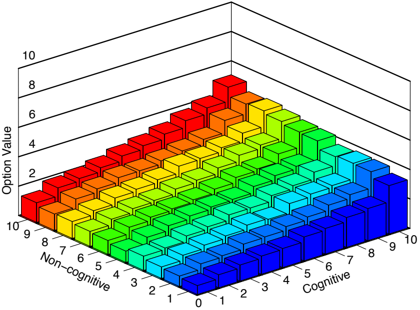
Figure 8: Option Values by Abilities (continued)



(a) Late College Enrollment

$OV = 2.19$

$\text{v}OVC = 0.19$

Notes: We simulate a sample of 50,000 agents based on the estimates of the model. In each subfigure, we condition on the agents that actually visit the relevant decision state. In units of $100,000.

Figure 9: Choice Probability, Early College Enrollment

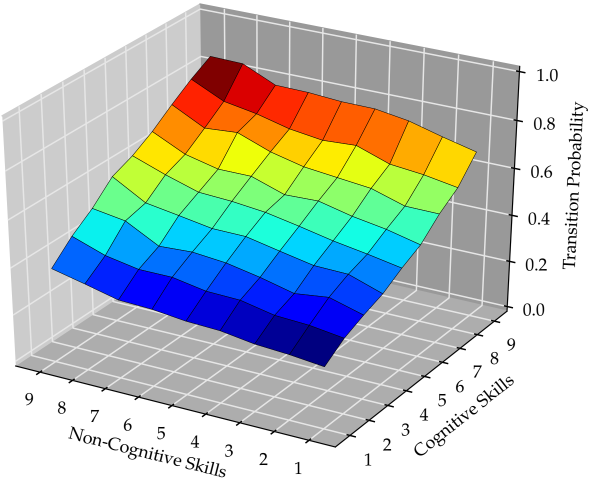


Figure 10: Gross Return, Early College Enrollment

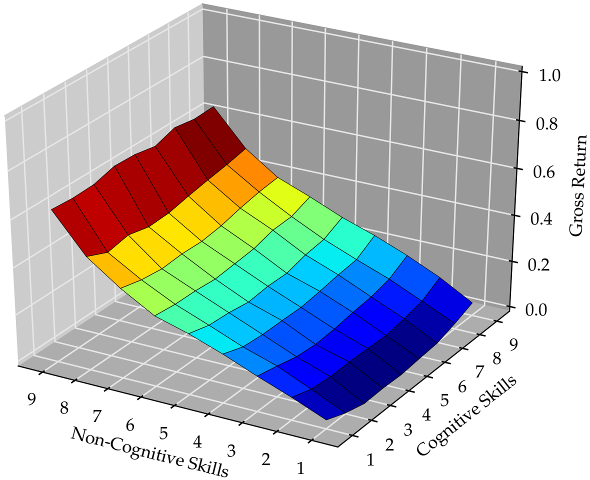


Figure 11: Net Return, Early College Enrollment

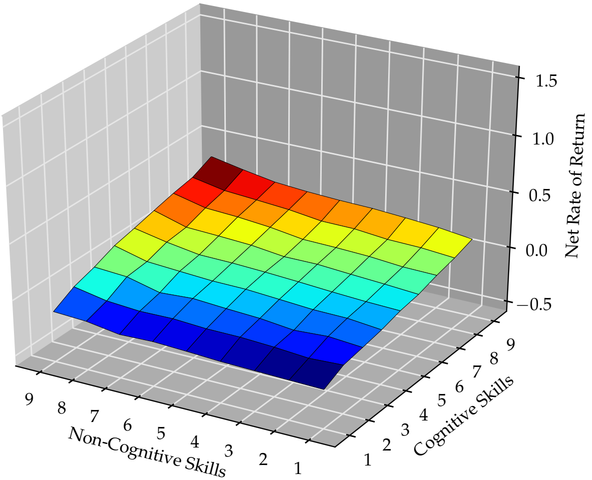


Figure 12: Schooling Attainment by Cognitive Skills

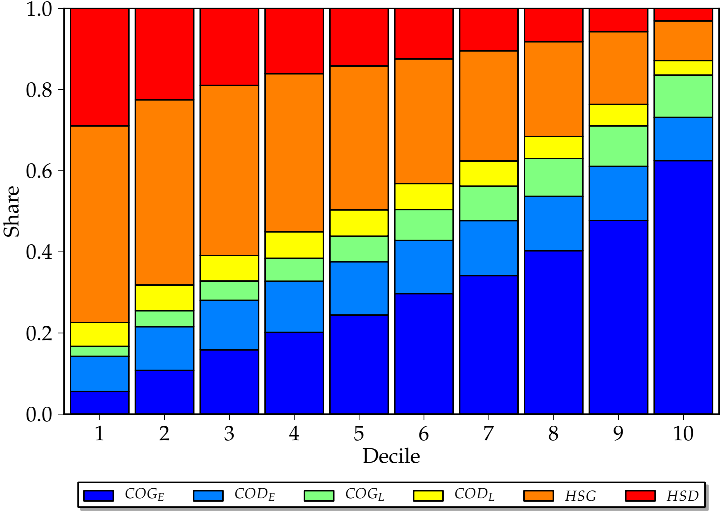


Figure 13: Schooling Attainment by Non-Cognitive Skills

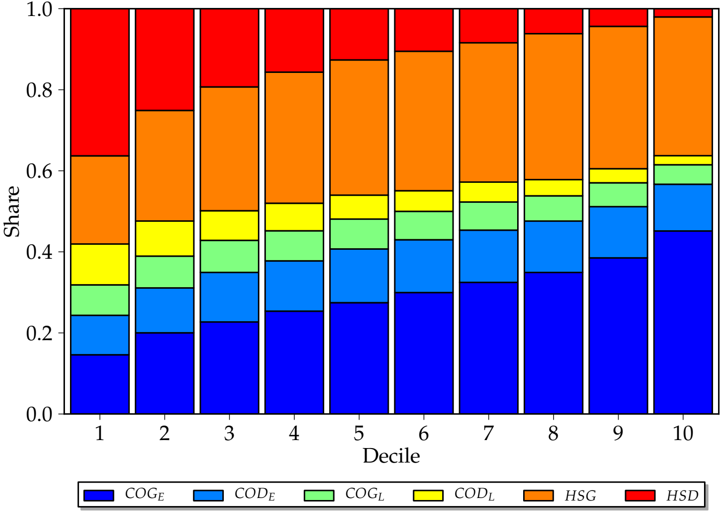


Figure 14: Net Returns (ex ante), High School Graduation

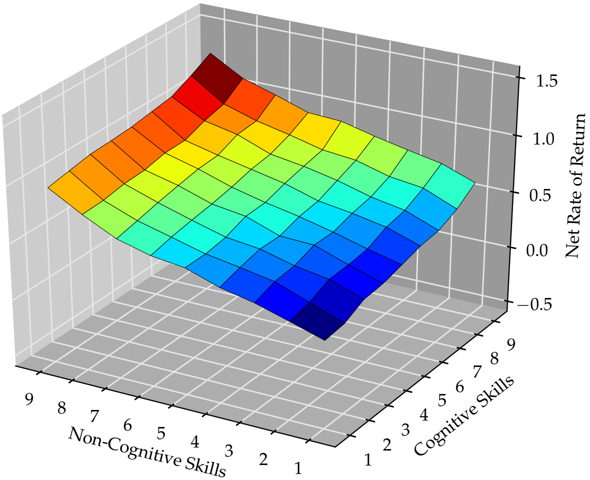


Figure 15: Net Returns (ex ante), Early College Enrollment

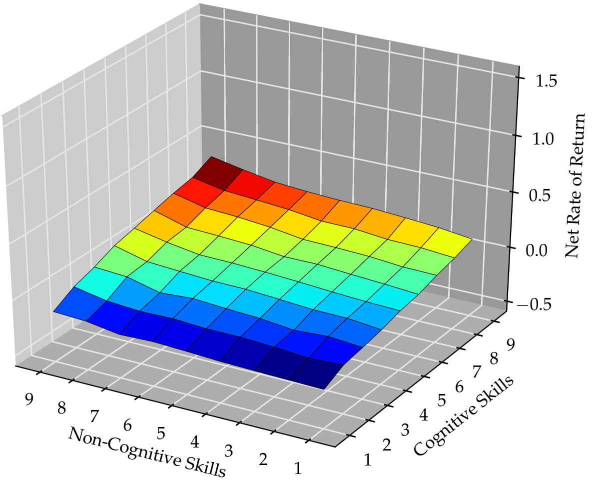


Figure 16: Net Returns (ex ante), Early College Graduation

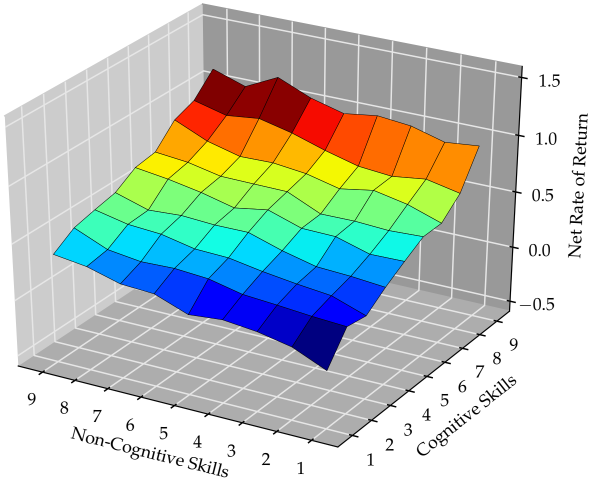


Figure 17: Net Returns (ex ante), Late College Enrollment

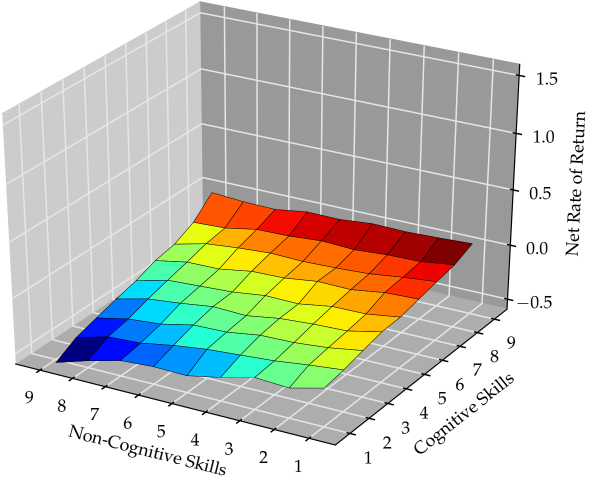


Figure 18: Net Returns (ex ante), Late College Graduation

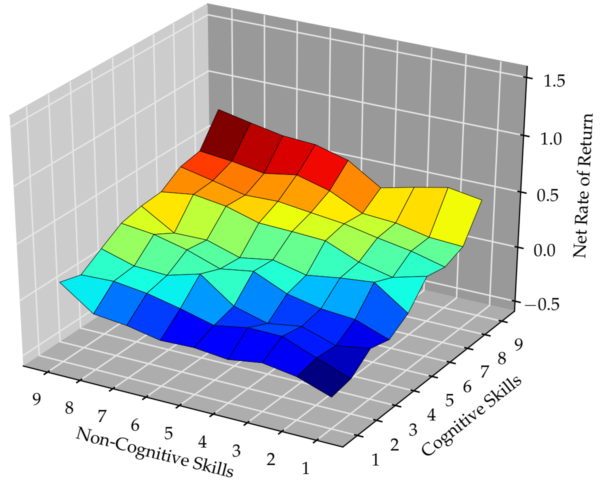


Figure 19: Option Values, High School Graduation

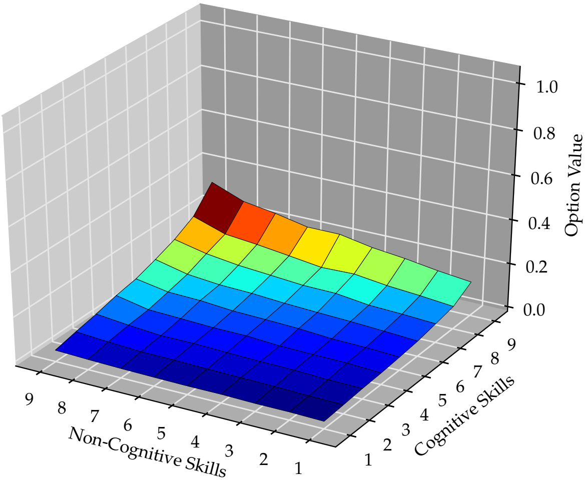


Figure 20: Option Values, Early College Enrollment

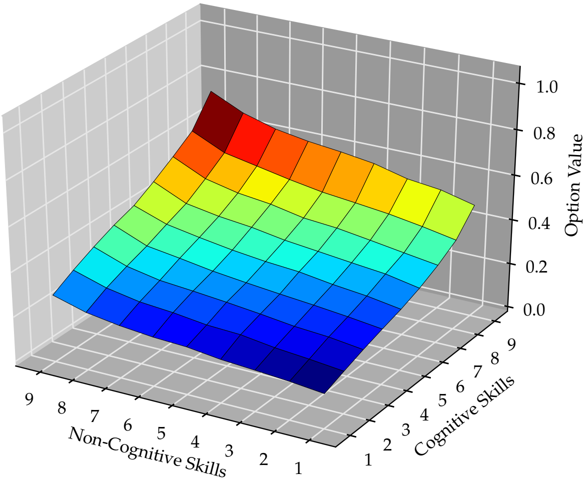


Figure 21: Option Values, Late College Graduation

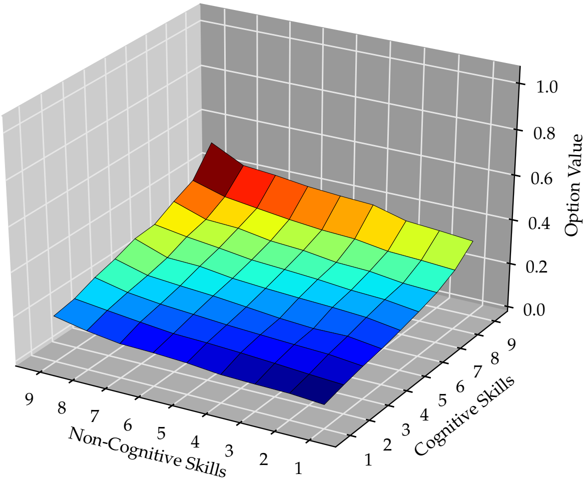


Figure 22: Choice Probability, High School Graduation

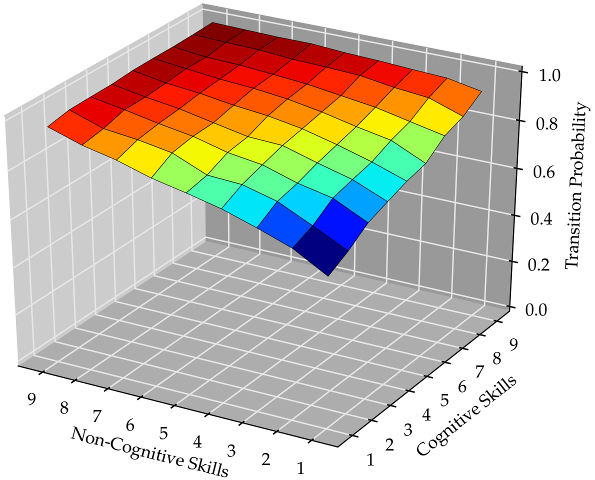


Figure 23: Choice Probability, Early College Enrollment

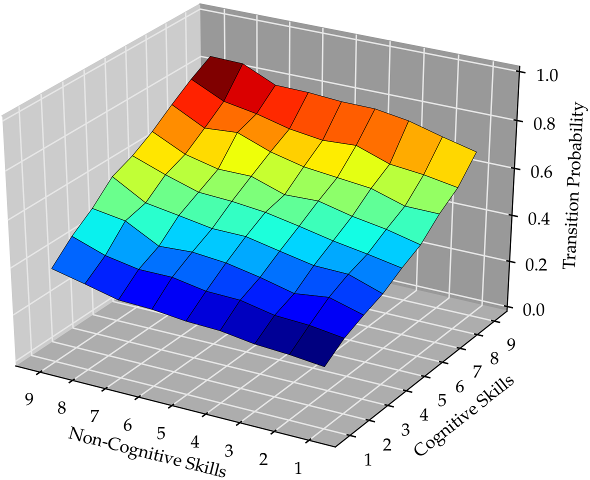


Figure 24: Choice Probability, Early College Graduation

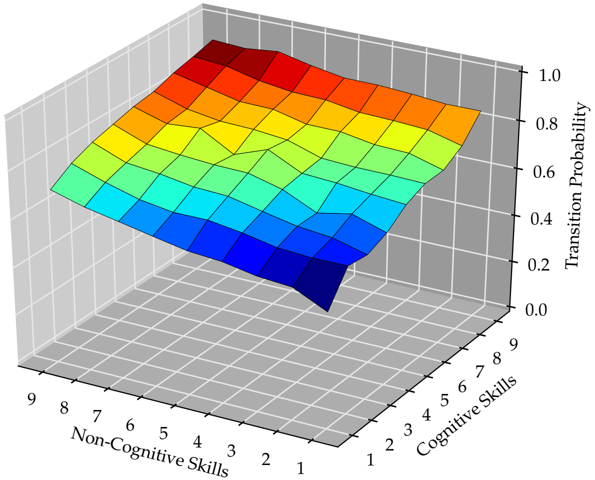


Figure 25: Choice Probability, Late College Enrollment

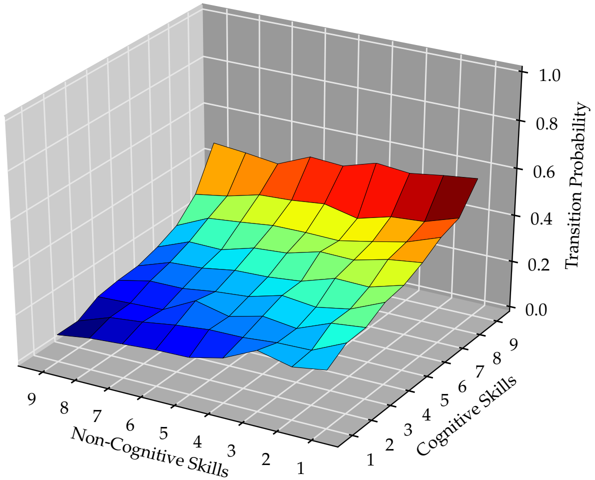


Figure 26: Choice Probability, Late College Graduation

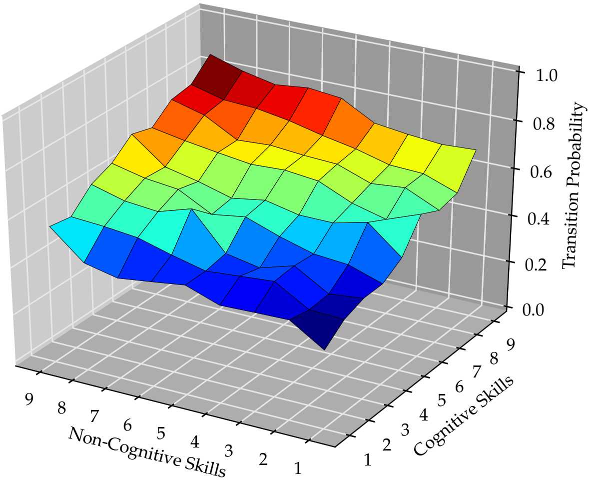


Table 2: Cross Section Model Fit

State	Average Earnings		State Frequencies	
	Observed	ML	Observed	ML
High School Graduates	4.29	3.84	0.30	0.32
High School Dropouts	2.29	2.59	0.17	0.14
Early College Graduates	6.73	7.46	0.29	0.29
Early College Dropouts	4.55	3.87	0.12	0.12
Late College Graduates	4.84	6.22	0.06	0.07
Late College Dropouts	4.89	4.88	0.06	0.06

Table 3: Conditional Model Fit

State	Number of Children	Baby in Household	Parental Education	Broken Home
High School Dropout	0.77	0.26	0.37	0.03
High School Finishing	0.88	0.73	0.55	0.35
High School Graduation	0.91	0.94	0.65	0.91
High School Graduation (cont'd)	0.95	0.33	0.40	0.85
Early College Enrollment	0.46	0.54	0.01	0.15
Early College Graduation	0.06	0.86	0.00	0.14
Early College Dropout	0.33	0.27	0.54	0.75
Late College Enrollment	0.80	0.23	0.90	0.60
Late College Graduation	0.90	0.39	0.90	0.60
Late College Dropout	0.89	0.42	0.91	0.76

Table 4: Internal Rates of Return (Calculated as in Mincer Handout)

		All	
High School Graduation	vs.	High School Dropout	215%
Early College Graduation	vs.	Early College Dropout	24%
Early College Graduation	vs.	High School Graduation (cont'd)	19%
Late College Dropout	vs.	High School Graduation (cont'd)	10%
Late College Graduation	vs.	High School Graduation (cont'd)	17%
Late College Dropout	vs.	High School Graduation (cont'd)	16%

Notes: The calculation is based on 1,407 individuals in the observed data.

Table 5: Net Returns

State	All	Treated	Untreated
High School Finishing	64%	80%	-39%
Early College Enrollment	-6%	30%	-38%
Early College Graduation	57%	103%	-59%
Late College Enrollment	-23%	31%	-45%
Late College Graduation	15%	79%	-61%

Notes: We simulate a sample of 50,000 agents based on the estimates of the model.

- Treated returns: Net return of treated comparing outcome if graduated versus not for those who graduated
- Net return untreated is the net return for people who didn't visit state compared to what they would have experienced

Table 6: Gross Returns

State	All	Treated	Untreated
High School Finishing	30%	32%	16%
Early College Enrollment	17%	23%	13%
Early College Graduation	89%	102%	57%
Late College Enrollment	34%	43%	30%
Late College Graduation	33%	48%	15%

Notes: We simulate a sample of 50,000 agents based on the estimates of the model.

Table 7: Regret: Ex Ante and Ex Post Returns Disagree

State	All	Treated	Untreated
High School Finishing	7%	4%	24%
Early College Enrollment	15%	28%	2%
Early College Graduation	29%	33%	19%
Late College Enrollment	21%	27%	19%
Late College Graduation	27%	34%	18%

Notes: We simulate a sample of 50,000 agents based on the estimates of the model.

Table 8: Option Value Contribution: Relative Share of the Option Value in the Overall Value of Each State

State	All	Treated	Untreated
High School Finishing	10%	11%	5%
Early College Enrollment	30%	37%	24%
Late College Enrollment	19%	25%	16%

Notes: We simulate a sample of 50,000 agents based on the estimates of the model.

Table 9: Psychic Costs

State	Mean
High School Finishing	—
Early College Enrollment	23%
Early College Graduation	12%
Late College Enrollment	47%
Late College Graduation	10%

Notes: We simulate a sample of 50,000 individuals based on the estimates of the model. We condition on the agents who actually visit the relevant decision state.

- **Problem:** How can you identify psychic costs?