The Roy Model and the Generalized Roy Model

James J. Heckman
University of Chicago

Econ 312, Spring 2021
This draft, January 12, 2021 1:29pm

General Framework for Understanding Selection and Sorting in the Labor Market

Roy (1951)

- The wage W_i of individual i:

$$W_i = \pi' S_i$$

- S_i is a vector of skills
- π is a vector of prices.
- Implications for inequality and sorting?

- Models comparative advantage: Compositional effects/selection models. People respond to skill prices in different sectors (different prices of the same skill) by moving to the sector that gives the highest return. (e.g. the Generalized Roy model). The population distribution of skill is unchanged.
    - Requires that the skill prices can differ between sectors (hedonic models).
- Uncertainty.

- The Roy and the Generalized Roy models of selection under comparative advantage and can be (and have been) applied to a wide range of settings such as labor supply, search models, immigration, etc.

Basic Framework

- Agents possess advantages in tasks associated with sector j, $j \in \mathcal{J}$.
- They get an income Y_j for participating in sector j. ($Y_{j,i}$ for agent i)
- There may be a cost C_j of participating in the sector ($C_{j,i}$ for person i).
- A one-period model (will extend to multiple periods later)
- When making their choices, they are uncertain and have information set $\mathcal{I}_i$.

- For notational simplicity, drop the i subscript.
- Agents select sector $\hat{j}$ such that

$$\hat{j} = \underset{j \in \mathcal{J}}{\operatorname{argmax}} E\left\{\{Y_j - C_j\} \mid \mathcal{I}\right\}$$

- Toss a coin in the event of a tie.
- Ties are often assumed away as negligible events (i.e., absolute continuity is assumed).

- Ex post agents may regret their choices.
  E.g.,

$$Y_{\hat{j}} - C_{\hat{j}} < 0$$

or even

$$(Y_{\hat{j}} - C_{\hat{j}}) < (Y_j - C_j)_{j \in \mathcal{J} \setminus \{\hat{j}\}}$$

- The Y_j can be a variety of outcomes.
  Examples:

  1. Different labor force states (work, not work) and C_j is cost of working

     $$e.g., Y_1 = \text{ value of market time}$$
     $$Y_0 = \text{ value of home production}$$

     so if $C_1 = 0$ and $C_0 = 0$
     Y_1 is the market wage
     Y_0 is the reservation wage
     Reservation wage can come from

     1. Search Theory (see, e.g., Shimer, 2010)
     2. Value of Time in the home (see, e.g., Heckman, 1974; Mulligan and Rubinstein, 2008)

  2. Earnings in different countries (Borjas, 1987)

3. Earnings in different occupations (Miller, 1984; Jovanovic, 1979a,b; Pavan, 2008)
4. Earnings at different schooling levels (e.g., Willis and Rosen, 1979; Keane and Wolpin, 1997; Heckman, Lochner, and Taber, 1998; Johnson, 2013; Heckman, Humphries and Veramendi, 2018.)
5. Randomization bias (Kline and Walters, 2016)

- Under the earnings interpretation, let π_j be the price of skill j (the rental rate or the return)
- The quantity of skill j is S_j
- $Y_j = \pi_j' S_j$ (gross earnings)
- $Y_j - C_j = \pi_j' S_j - C_j$ (net earnings)

- Framework is also the basis for the modern treatment effect or "causal effect" literature.

Two Potential Outcome Model: Generalized Roy Model

$$Y_1 = \mu_1(X) + U_1 \qquad \mu_1(X) = E(Y_1 \mid X)$$
$$Y_0 = \mu_0(X) + U_0 \qquad \mu_0(X) = E(Y_0 \mid X)$$
$$C = \mu_C(Z) + U_C \qquad \mu_C(Z) = E(C \mid Z)$$

Net Benefit: $I = Y_1 - Y_0 - C$

$$I = \underbrace{\mu_1(X) - \mu_0(X) - \mu_C(Z)}_{\mu_D(Z)} + \underbrace{U_1 - U_0 - U_C}_{-V}$$

$$(U_0, U_1, U_C) \perp\!\!\!\perp (X, Z)$$
$$E(U_0, U_1, U_C) = (0, 0, 0)$$
$$V \perp\!\!\!\perp (X, Z)$$

"$\perp\!\!\!\perp$" denotes independence

Canonical Roy Model

- Two sectors $s \in \{1, 2\}$
- Assume (for simplicity) perfect information
- Assumes normality for $(Y_2, Y_1) \| X$
- Sets $C_2 = 0$ and $C_1 = 0$ (Roy model)
- For simplicity assume that $(U_2, U_1) \perp\!\!\!\perp X$

THE UNIVERSITY OF CHICAGO

The Roy Model

Two sector Roy model. (sectors $j \in \{1, 2\}$)

Income maximizing agents possess two skills $S_1 = s_1$ and $S_2 = s_2$ with associated positive skill prices π_1 and π_2.

Skills are scalar (for now)

Agent chooses sector 1 if his earnings are greater there

$$W_1 = Y_0 = \pi_1 S_1$$
$$W_1 = Y_1 = \pi_1 S_1$$
$$\pi_1 S_1 > \pi_2 S_2$$

Proportion of the population working in sector one,

$$P_1 = \Pr(\pi_1 S_1 > \pi_2 S_2) :$$

$$P_1 = \int_0^\infty \int_0^{\pi_1 s_1/\pi_2} f(s_1, s_2) ds_2 ds_1 \tag{2.1}$$

Density of skill employed in sector one differs from the population density of skill. (selection problem)

The latter density:

$$f_1(s_1) = \int_0^\infty f(s_1, s_2)ds_2. \tag{2.2}$$

Former density:

$$g(s_1 \,|\pi_1 s_1 > \pi_2 s_2) = \frac{1}{P_1} \int_0^{\pi_1 s_1/\pi_2} f(s_1, s_2)ds_2 \tag{2.3}$$

Density of earnings in sector 1 (using $w_1 = \pi_1 s_1$):

$$g_1(w_1) = \frac{1}{P_1 \pi_1} \int_0^{w_1/\pi_2} f(w_1/\pi_1, s_2)ds_2 \tag{2.4}$$

Similarly, the density of skill employed in sector 2 is:

$$g(s_2 \,|\pi_2 s_2 > \pi_1 s_1) = \frac{1}{P_2} \int_0^{\pi_2 s_2/\pi_1} f(s_1, s_2) ds_1 \tag{2.5}$$

The density of earnings in sector two is:

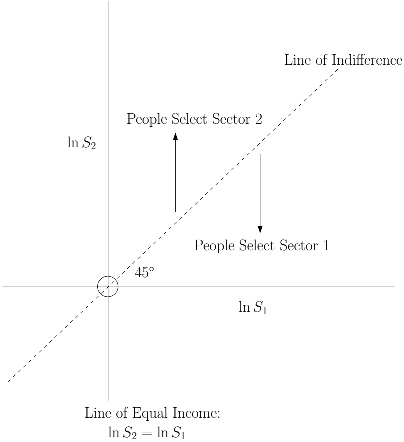
$$g_2(w_2) = \frac{1}{P_2 \pi_2} \int^{w_2/\pi_1} f(s_1, \frac{w_2}{\pi_2}) ds_1 \tag{2.6}$$

The overall density of earnings is:

$$g(w) = P_1 g_1(w) + P_2 g_2(w) \tag{2.7}$$

Set $\pi_1 = \pi_2 = 1$

Take logs: Partitions of $(\ln S_2, \ln S_1)$ space:



Question: How does sorting determine the distribution of observations on $\ln S_2$?

Normal Roy Model

$$(\ln S_1, \ln S_2) \sim N(\mu_1, \mu_2, \Sigma)$$

$$\begin{aligned} E(\ln S_j) &= \mu_j \\ \ln S_j &= \mu_j + U_j \\ \Rightarrow \ln W_j &= \ln \pi_j + \mu_j + U_j, \ j = 1, 2 \end{aligned} \tag{3.1}$$

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \right)$$

Define

$$\begin{aligned} \sigma^* &= [Var(U_1 - U_2)]^{1/2} = \sqrt{\sigma_{11} + \sigma_{22} - 2\sigma_{12}} \\ c_1 &= (\ln(\frac{\pi_1}{\pi_2}) + \mu_1 - \mu_2)/\sigma^*. \end{aligned}$$

Define:

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{\frac{-q2}{2}} dq$$

$$P_1 = P(\ln W_1 > \ln W_2) = 1 - \Phi(-c_1) = \Phi(c_1)$$

Choice equation (can be of very general functional form)

Line of Indifference:

$$\ln W_1 - \ln W_2 = \ln(\frac{\pi_1}{\pi_2}) + \mu_1 - \mu_2 + U_1 - U_2$$
$$L = U_1 - U_2$$
$$c_1^* = \ln(\pi_1/\pi_2) + \mu_1 - \mu_2.$$

$$E\left(\ln W_1 \mid \ln W_1 - \ln W_2 > 0\right) = \ln \pi_1 + \mu_1 + \underbrace{E(U_1 \mid L > -c_1^*)}_{\text{Selection Bias Term}}. \tag{3.2}$$

Selection operates through the dependence between U_1 and $(U_1 - U_2)$.

More generally through the unobservables in the $\ln W_1$ and the decision equation. $(I = Y_2 - Y_1 - (C_2 - C_1))$

Observe Y_2 if $Y_1 - Y_1 - (C_2 - C_1) > 0$
(Censoring condition and Y_2 is a censored random variable)

Observe Y_1 otherwise

- Much of the empirical literature uses normality or modest departures from it.
- Huge nonparametric and semiparametric literature. (Powell, 1994, and Matzkin, 2012)
- Under normality the dependence between U_1 and $(U_1 - U_2)$ is simple to characterize.
- We observe

$$Y_1 = \mu_1 + U_1$$

if $U_1 - U_2 > -(\ln \pi_1 - \ln \pi_2) - (\mu_1 - \mu_2)$ (*selection equation*)

- Selection equations conveys revealed preference information and what agents act on.
- U_1 generates observed Y_1.

Define:

$$
\begin{aligned}
Var(L) &= E(U_1 - U_2)^2 \\
&= \sigma_{11} + \sigma_{22} - 2\sigma_{12}, \\
&\equiv (\sigma^*)^2,
\end{aligned}
$$

$$L \sim N(0, (\sigma^*)^2).$$

- Conditional expectations of normal variables are linear.

$$U_1 = a_1 L + V_1 \tag{3.3}$$

where:

$$a_1 = \frac{Cov(L, U_1)}{Var(L)} = \frac{\sigma_{11} - \sigma_{12}}{(\sigma^*)^2}$$

- The dependence between U_1 and L the whole source of selection and sorting.
- As a consequence of normality, V_1 is *independent* of L (not generally true).
  $E(V_1) = 0$ and $Var(V_1) = \sigma_{11}(1 - \rho_1^2)$.

$$\rho_1 = \frac{Cov(L, U_1)}{(Var(L)Var(U_1))^{1/2}} = \frac{\sigma_{11} - \sigma_{12}}{(\sigma^*)\sigma_{11}^{1/2}}.$$

$$V_1 \sim N(0, \sigma_{11}(1 - \rho_1^2))$$

As a consequence of (3.3) and $V_1 \perp\!\!\!\perp L$, from (3.2):

$$\begin{aligned} E(U_1 \,|\, U_1 - U_2 > -c_1^*) &= E(U_1 \,|\, L > -c_1^*) \\ &= a_1 E(L \,|\, L > -c_1^*) \end{aligned}$$

since $E(V_1 \,|\, L > -c_1^*) = E(V_1) = 0$

because L and V_1 are independent.

Normalizing by σ^*, we define: $Z_1 = L/\sigma^*$ and $c_1 = c_1^*/\sigma^*$ where Z_1 is a unit variance mean zero normal variable. Then:

$$E(L \mid L > -c_1^*) = \sigma^* E(Z_1 \,|\, Z_1 > -c_1)$$

Collecting results:

$$
\begin{aligned}
& E\left(\ln W_1 \mid \ln W_1 > \ln W_2\right) \qquad (3.4) \\
& = \ln \pi_1 + \mu_1 + a_1 E(L \mid L > -c_1^*) \\
& = \ln \pi_1 + \mu_1 + \left(\frac{\sigma_{11} - \sigma_{12}}{(\sigma^*)^2}\right)(\sigma^*) E(Z_1 \mid Z_1 > -c_1) \\
& = \ln \pi_1 + \mu_1 + \left(\frac{\sigma_{11} - \sigma_{12}}{\sigma^*}\right) E(Z_1 \mid Z_1 > -c_1)
\end{aligned}
$$

$$
\begin{aligned}
& Var\left(U_1 \mid U_1 - U_2 > -c_1^*\right) \qquad (3.5) \\
& = Var(a_1 L + V_1 \mid U_1 - U_2 > -c_1^*) \\
& = Var(a_1 L \mid U_1 - U_2 > -c_1^*) + Var(V_1) \\
& = a_1^2 Var(L \mid L > -c_1^*) + Var(V_1) \\
& = (a_1 \sigma^*)^2 Var(Z_1 \mid Z_1 > -c_1) + Var(V_1)
\end{aligned}
$$

THE UNIVERSITY OF CHICAGO

Using $(a_1\sigma^*)^2 = \sigma_{11}\rho_1^2$:

$$\begin{aligned} & Var\left(\ln W_1 \mid \ln W_1 > \ln W_2\right) \\ & = Var(U_1 \mid U_1 - U_2 > -c_1^*) \\ & = \sigma_{11}\left\{\rho_1^2 Var(Z_1 \mid Z_1 > -c_1) + (1-\rho_1^2)\right\} \end{aligned} \tag{3.6}$$

Characterizing bias is simplified by well known properties of the truncated normal.

Link to Properties of Truncated Normal Slides

Censored Random Variables

Illustrating Properties of the Censored Normal Model

Bivariate Normal variable:

$$\begin{pmatrix} Y \\ I \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$

$$\Rightarrow Y = \rho I + v; \quad v \sim N\left(0, 1-\rho^2\right)$$

$I \perp\!\!\!\perp v \Rightarrow I \perp\!\!\!\perp v$, due to normality.

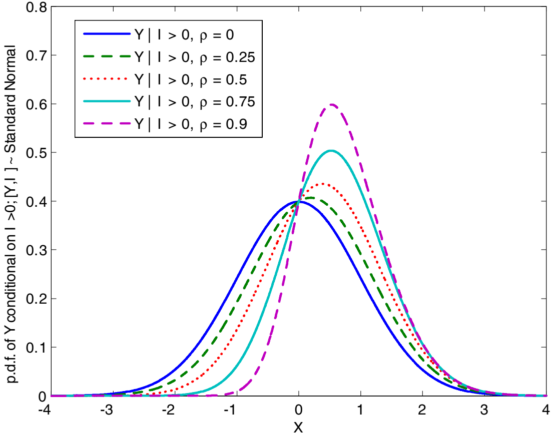
$$\begin{aligned}
(Y|I>0) &= (\rho I + v|I>0) = \rho\left(I|I>0\right) + v \\
E\left(Y|I>0\right) &= E\left(\rho I + v|I>0\right) \\
&= \rho E\left(I|I>0\right) = \rho\lambda\left(0\right) \\
Var\left(Y|I>0\right) &= Var\left(\rho I + v|I>0\right) \\
&= \rho^2 Var\left(I|I>0\right) + Var(v).
\end{aligned}$$

$$\text{Suppose: } \begin{pmatrix} Y \\ I \end{pmatrix} \sim N\left(\begin{matrix} \mu_Y \\ \mu_I \end{matrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$
$$\Rightarrow Y - \mu_Y = \rho \left(I - \mu_I \right) + v; \quad v \sim N\left(0, 1 - \rho^2\right)$$

The rest follows as above.

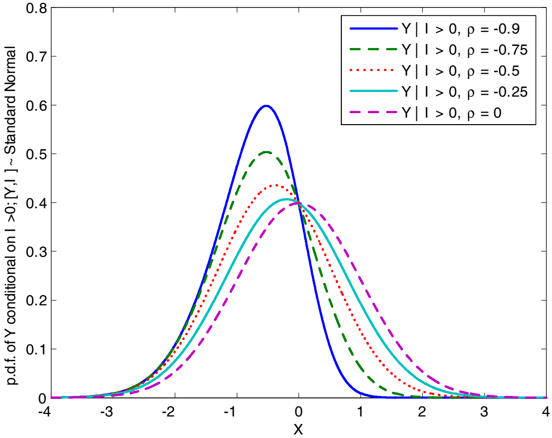
In our previous analysis, $Y = U_1$ and $I = U_1 - U_2$.

Censored Standard Normal Density Function



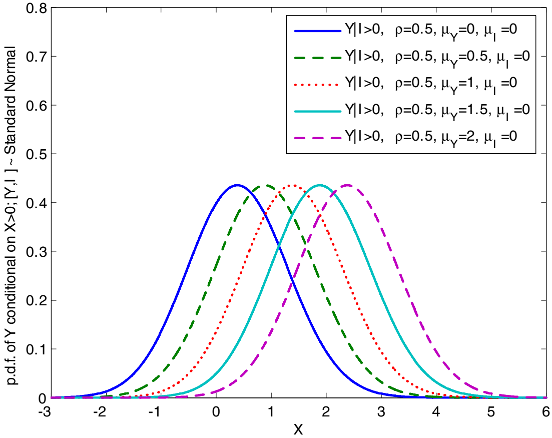
$$\begin{bmatrix} Y \\ I \end{bmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$

Censored Standard Normal Density Function



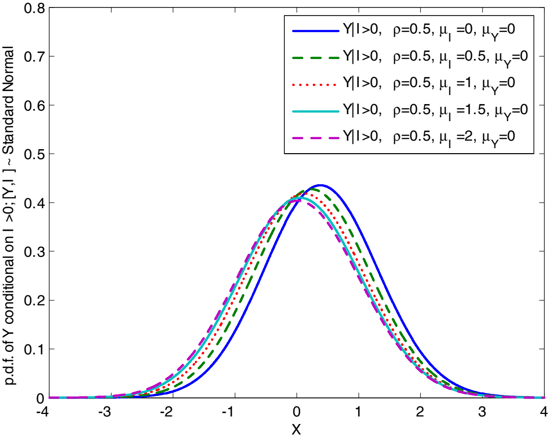
$$\begin{bmatrix} Y \\ I \end{bmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$

Censored Standard Normal Density Function



$$\begin{bmatrix} Y \\ I \end{bmatrix} \sim N\left(\begin{matrix} \mu_I \\ 0 \end{matrix}, \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}\right)$$

Censored Standard Normal Density Function



$$\begin{bmatrix} Y \\ I \end{bmatrix} \sim N\left(\begin{matrix} 0 \\ \mu_I \end{matrix}, \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \right)$$

Normal Roy Model Results on Observed Wages

Using Equation (3.4) and the results previously established:

$$\begin{aligned} & E\left(\ln W_1 \mid \ln W_1 > \ln W_2\right) \\ = & \ln \pi_1 + \mu_1 + \left(\frac{\sigma_{11} - \sigma_{12}}{\sigma^*}\right)\lambda(-c_1). \end{aligned} \tag{4.1}$$

Parallel expression for skills:

$$\begin{aligned} & E\left(\ln S_1 \mid \ln W_1 > \ln W_2\right) \\ = & \mu_1 + \left(\frac{\sigma_{11} - \sigma_{12}}{\sigma^*}\right)\lambda(-c_1). \end{aligned}$$

Now $0 \leq Correl^2(\ln S_1, \ln S_2) = \dfrac{\sigma_{12}^2}{\sigma_{11}\sigma_{22}} \leq 1$, so that:

$$0 \leq \left(\frac{\sigma_{12}}{\sigma_{11}}\right)\left(\frac{\sigma_{12}}{\sigma_{22}}\right) \leq 1$$

Both terms in parentheses cannot exceed one.

Thus if $\sigma_{11} \leq \sigma_{12}$ so that $1 \leq \dfrac{\sigma_{12}}{\sigma_{11}}$, then $\dfrac{\sigma_{12}}{\sigma_{22}} \leq 1$.

I.e. if $\sigma_{11} - \sigma_{12} \leq 0$, then $\sigma_{22} - \sigma_{12} \geq 0$.

Both sectors cannot display negative selection.

The effect of an increase in π_1 on mean skill in sector 1

Differentiate with respect to π_1.

$$E\left(\ln S_1 \mid \ln W_1 > \ln W_2\right) = \mu_1 + \frac{\sigma_{11} - \sigma_{12}}{(\sigma^*)} \lambda\left(-c_1\right)$$

with respect to π_1.

$$\frac{\partial E(\ln S_1 \mid \ln W_1 > \ln W_2)}{\partial \ln \pi_1} = -\frac{(\sigma_{11} - \sigma_{12})}{(\sigma^*)^2} \underbrace{\lambda'(-c_1)}_{+}$$

$$\text{where } c_1 = \frac{\ln \frac{\pi_1}{\pi_2} + \mu_1 - \mu_2}{\sigma^*}.$$

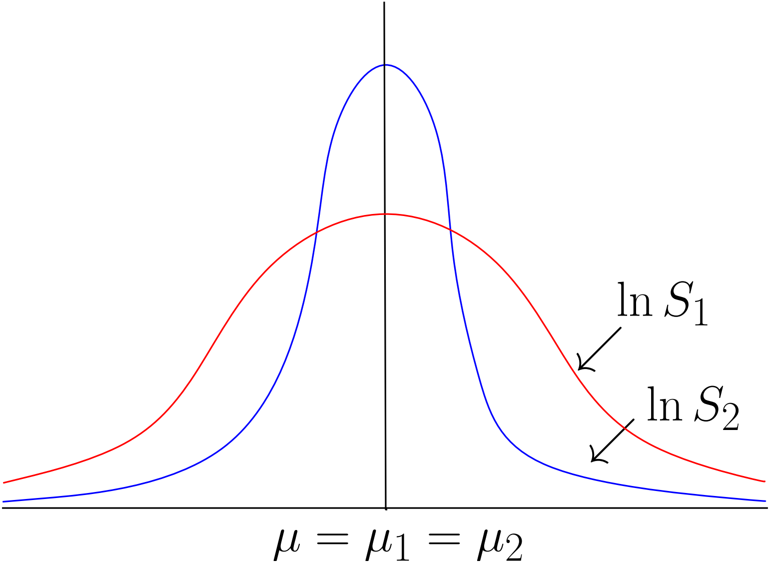
This is negative if $\sigma_{11} - \sigma_{12} > 0$, positive if $\sigma_{11} - \sigma_{12} < 0$.

When Do We Get Negative Selection in Sector 2?

Arises if $\sigma_{22} < \sigma_{12} < \sigma_{11}$

Example: Set $\pi_1 = \pi_2$

- $D = \mathbf{1}(\ln S_1 > \ln S_2)$
- $\sigma_{22} \leq \sigma_{12} \leq \sigma_{11}$
- $\mu_1 = \mu_2$



Densities of $\ln S_1$ and $\ln S_2$

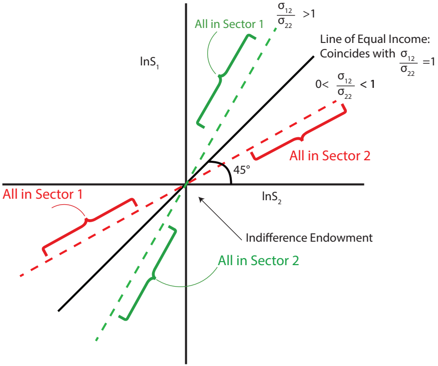
- People selected into S_2 are below average in 2.
- People selected in S_1 are above average in 1.

Intuition

Consider Case With Perfect Correlation

$$lnS_1 = \frac{\sigma_{12}}{\sigma_{22}} lnS_2$$

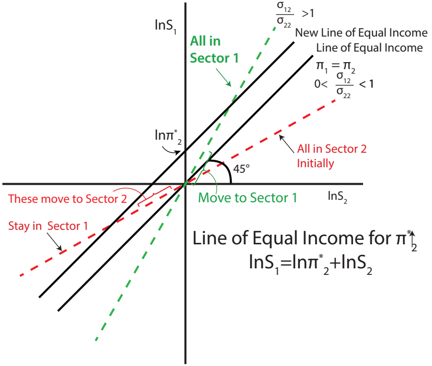
Perfectly Stratified Society



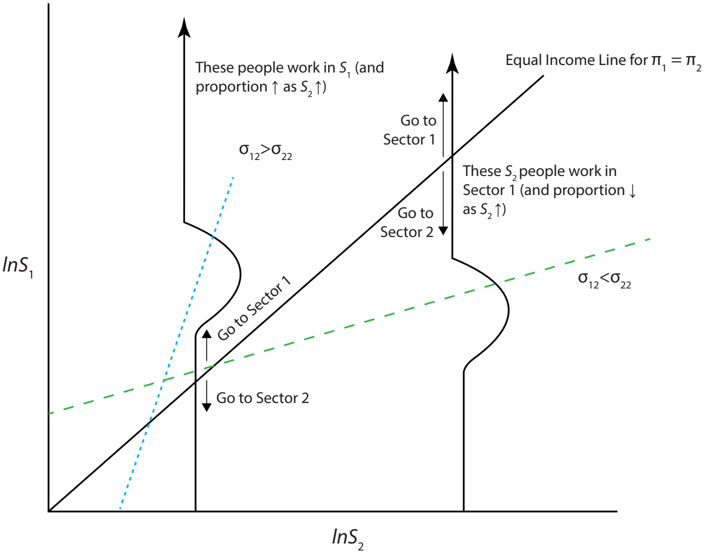
- $\mu_1 = \mu_2 = 0$ and $\pi_1 = \pi_2 = 1$

Case of Perfect Correlation With $\pi_2^* \Uparrow$ $lnS_1 = \frac{\sigma_{12}}{\sigma_{22}} lnS_2$

Perfectly Stratified Society



- $\mu_1 = \mu_2 = 0$ and $\pi_1 = 1, \pi_2^* > 1$



$$lnS_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(lnS_2 - \mu_2) + v_1$$

- $\sigma_{22} < \sigma_{12} < \sigma_{11}$
- Top S_2 tend to work in Sector 1
- If price of $S_2, \pi_2 \uparrow$, what happens?
- High S_2 now go into sector 2
- Wage growth consists of 2 components:
  a. Growth in price and growth in average skill quality (a selection effect)
  b. Increase in π_2.
- A 10% increase in π_2 produces a $>$ 10% increases in $\text{E}(\ln W_2 \mid \ln W_2 > \ln W_1)$.
- A 10% increase in π_1 produces a $<$ 10% increase in $\text{E}(\ln W_1 \mid \ln W_1 \leq \ln W_2)$
- Exercise: Prove these claims.

- Mulligan and Rubinstein (QJE, 2008) use this fact to explain rising wages of women as a selection effect
  (need variance in market sector $<$ variance in nonmarket sector)
- First evidence on this is Heckman (1980)

- Question: Under what conditions (in general) can an increase in $\pi_1 (\pi_1 \uparrow)$ reduce $\mathrm{E}(\ln W_1 \mid \ln W_1 > \ln W_2)$ and $\mathrm{E}(\ln W_2 \mid \ln W_2 \geq \ln W_1)$?

Can occur if $\sigma_{11} - \sigma_{12} > 0$ and $\sigma_{22} - \sigma_{12} < 0$ and

$$\left(\frac{\sigma_{11} - \sigma_{12}}{(\sigma^*)^2} \right) \lambda' \left(-c_1 \right) > 1, \qquad (*)$$

since $\ln W_1 = \mu_1 + \ln \pi_1$, and

$$\frac{\partial E(\ln W_1 \mid \ln W_1 > \ln W_2)}{\partial \ln \pi_1} = 1 - \frac{(\sigma_{11} - \sigma_{12})}{(\sigma^*)^2} \lambda'(-c_1)$$

$$\frac{\partial E(\ln W_2 \mid \ln W_2 > \ln W_1)}{\partial \ln \pi_1} = \frac{(\sigma_{22} - \sigma_{12})}{(\sigma^*)^2} \lambda'(c_1), \text{ because}$$

$$E\left(\ln W_2 \mid \ln W_2 > \ln W_1\right) = \ln \pi_2 + \mu_2 + \left[\frac{\sigma_{22} - \sigma_{12}}{(\sigma^*)} \right] \lambda\left(c_1\right).$$

Intuition:

- We have positive correlation between 1 and 2 and best 2 work in 1.
- The selection when π_1 increases in value is to further reduce the average quality in sector 1.
- Quality might fall enough to offset the rise in price.

Now for condition $(*)$ to arise, a necessary condition (since $\lambda'(c) \in [0,1]\ \forall c$, by fact (P-8) is that

$$\frac{\sigma_{11} - \sigma_{12}}{(\sigma_{11} - 2\sigma_{12} + \sigma_{22})^2} > 1.$$

But we can choose parameters that satisfy it and make λ big enough (vary the means) so that it is possible to have an increase in prices in one sector leading to a reduction in average wages paid in both sectors.

Results for the variance in sector wages.

From Equation (P-2) and the facts for $\lambda(c)$, we get:

$$\begin{aligned} & \mathit{Var}\left(\ln W_1 \mid \ln W_1 > \ln W_2\right) \\ = \ & \sigma_{11} \underbrace{\left\{\rho_1^2[1 - c_1\lambda(-c_1) - \lambda^2(-c_1)] + (1 - \rho_1^2)\right\}}_{\leq 1} \end{aligned} \tag{4.2}$$

As π_1 increases, $\mathit{Var} \ln\left(W_1 \mid \ln W_1 > \ln W_2\right)$ increases (to σ_{11}).

(The term in parentheses goes to 1).

Relationship between $\ln(S_1)$ and $\ln(S_2)$

$$\ln S_j = \mu_j + U_j$$

$$\ln W_j = \ln \pi_j + \mu_j + U_j, \quad j = 1, 2.$$

$$(U_1, U_2) \sim N(\mathbf{0}, \mathbf{\Sigma}) \equiv N\left(\begin{array}{c} 0 \\ 0 \end{array}, \left[\begin{array}{cc} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{array} \right] \right)$$

$$\Rightarrow \quad (\ln S_1, \ln S_2) \sim N\left(\begin{array}{c} \mu_1 \\ \mu_2 \end{array}, \left[\begin{array}{cc} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{array} \right] \right)$$

$$\Rightarrow \quad (\ln W_1, \ln W_2) \sim N\left(\begin{array}{c} \ln \pi_1 + \mu_1 \\ \ln \pi_2 + \mu_2 \end{array}, \left[\begin{array}{cc} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{array} \right] \right)$$

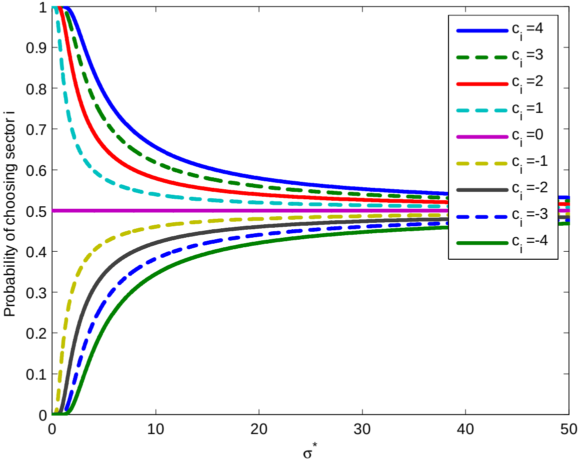
Definitions:

$$\begin{aligned} \sigma^* &\equiv \sqrt{Var(U_1 - U_2)} = (\sigma_{11} + \sigma_{22} - 2\sigma_{12})^{1/2} \\ c_1 &\equiv \frac{\ln \pi_1 - \ln \pi_2 + \mu_1 - \mu_2}{\sigma^*}. \end{aligned}$$

The probability of working in sector 1 is given by:

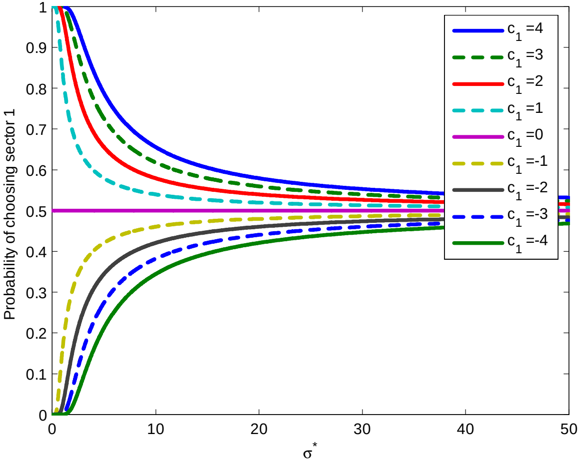
$$\begin{aligned} P_1 &= P(\ln W_1 > \ln W_2) \\ &= \Phi\left(\frac{\ln \pi_1 - \ln \pi_2 + \mu_1 - \mu_2}{\sigma^*}\right) \\ &= \Phi(c_1) \\ c_1 &= \frac{\ln \pi_1 - \ln \pi_2 + \mu_1 - \mu_2}{\sigma^*} \end{aligned}$$

Probability of Working in sector i



$$\sigma^* \equiv \sqrt{var(U_1 - U_2)} = (\sigma_{11} + \sigma_{22} - 2\sigma_{12})^{1/2}$$
$$c_i \equiv \frac{\ln \pi_i - \ln \pi_j + \mu_i - \mu_j}{\sigma^*}$$

Probability of Working in sector 1



$$\sigma^* \equiv \sqrt{var(U_1 - U_2)} = (\sigma_{11} + \sigma_{22} - 2\sigma_{12})^{1/2}$$
$$c_1 \equiv \frac{\ln \pi_1 - \ln \pi_2 + \mu_1 - \mu_2}{\sigma^*}$$

Some Examples:

Observe that for the normal case we have

$$\begin{bmatrix} Y_1 \\ I \end{bmatrix} \sim N \begin{bmatrix} \mu_{Y_1} \\ \mu_I \end{bmatrix}, \begin{pmatrix} \Sigma_{Y_1 Y_1} & \Sigma_{Y_1 I} \\ \Sigma_{I Y_1} & \Sigma_{II} \end{pmatrix} \Bigg]$$

$$[Y_1 | I = i\,] \sim N \begin{bmatrix} \mu_{Y_1} + \Sigma_{Y_1 I} \left(\Sigma_{II}\right)^{-1} \left(I - \mu_I\right), \\ \Sigma_{Y_1 Y_1} - \Sigma_{Y_1 I} \left(\Sigma_{II}\right)^{-1} \Sigma_{I Y_1} \end{bmatrix}.$$

So the conditional variables can be written as:

$$\begin{aligned}\ln S_1 &= \mathcal{P}_{\ln S_2}\ (\ln S_1) + \mathcal{P}_{\perp \ln S_2}\ (\ln S_1) \\ &= E\left[\ln S_1 | \ln S_2\right] + \left(\ln S_1 - E\left[\ln S_1 | \ln S_2\right]\right) \\ &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}\left(\ln S_2 - \mu_2\right) + v_1\end{aligned}$$

$$\begin{aligned}v_1 &\sim N(0, \sigma_{11}\left(1-\rho^2\right));\ \rho \equiv corr\left(U_1, U_2\right) \\ &\Rightarrow \ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}\left(\ln W_2 - \mu_2\right) + v_1 \\ &\Rightarrow \ln W_1 = \ln \pi_1 + \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}\left(\ln W_2 - \mu_2 - \ln \pi_2\right) + v_1\end{aligned}$$

Model:

$$\begin{aligned} \ln S_i &= \mu_i + U_i \\ \ln W_i &= \ln \pi_i + \mu_i + U_i, \quad i = 1, 2. \\ (U_1, U_2) &\sim N(\mathbf{0}, \mathbf{\Sigma}) \equiv N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix} \right) \\ \sigma_{12} &= 0.5 \quad \text{or} \quad 1.9 \\ \begin{pmatrix} \ln \pi_1 \\ \ln \pi_2 \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} -0.5 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} \end{aligned}$$

These graphs show the effect of $\sigma_{22} - \sigma_{12} < 0$ and $\sigma_{22} - \sigma_{12} > 0$ for selection of people into sector two (negative and positive selection, respectively).

Simple Model of Schooling

$$
\begin{aligned}
Y_1 &= \text{lifetime income from college} \\
Y_0 &= \text{lifetime income from high school} \\
S = 1 &: \text{go to college} \\
S = 1(Y_1 - Y_0 > 0) &: \text{ Pure Roy Model}
\end{aligned}
$$

$$
\begin{aligned}
Y_1 &= X_1\beta_1 + U_1 \\
Y_0 &= X_0\beta_0 + U_0
\end{aligned}
$$

- Can estimate β_1, β_0 and variance of U_1, U_0.
- Can we estimate $\text{Cov}(U_1, U_0)$?

Extended Model

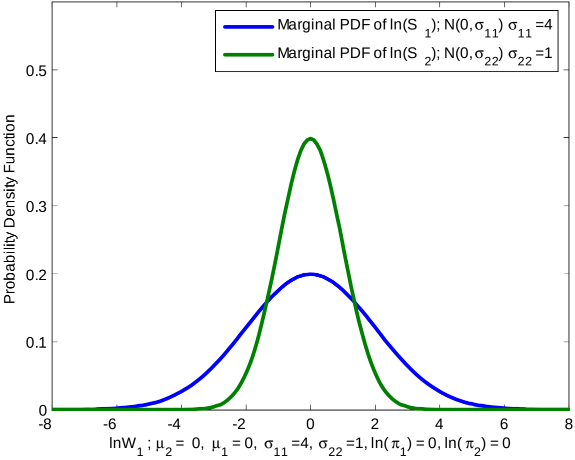
- Psychic costs C.

$$S = 1(Y_1 - Y_0 - C \geq 0)$$
$$C = Z\gamma + V$$
$$S = 1(X_1\beta_1 - X_0\beta_0 - Z\gamma \geq -U_1 + U_0 + C)$$

Roy Model

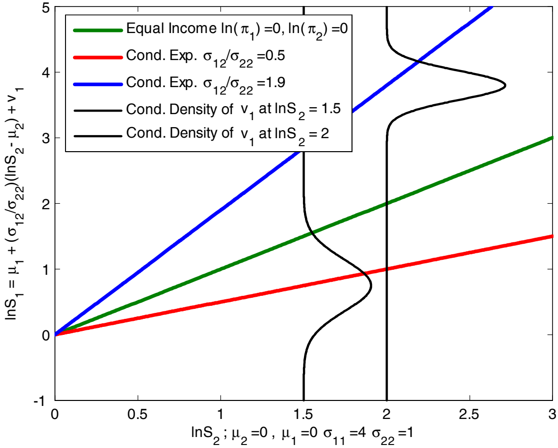
- Can we identify γ? Variance of V?
- $\mathrm{Cov}(U_1 U_0), \mathrm{Cov}(U_1, V), \mathrm{Cov}(U_0, V_1)$?

Marginal Probability Density Function (PDF) of $\ln S_1$, $\ln S_2$



$$\ln S_1 = \ln(\mu_1) + U_1; \ \ \ln S_2 = \ln(\mu_2) + U_2;$$
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix}\right); \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Graph of $\ln S_1 = f(\ln S_2)$

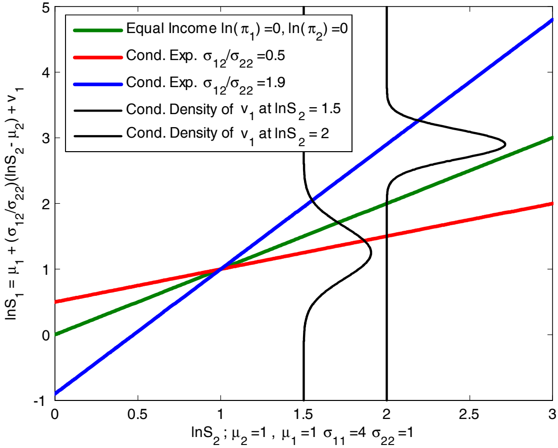


$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}\left(\ln S_2 - \mu_2\right) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \;\; \sigma_{12} = 0.5, \;\; 1.9;$$

Graph of $\ln S_1 = f(\ln S_2)$

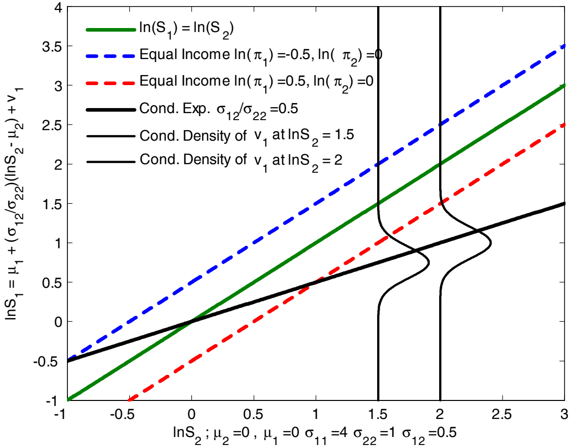


$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \ \ \sigma_{12} = 0.5, \ \ 1.9;$$

Graph of $\ln S_1 = f(\ln S_2)$



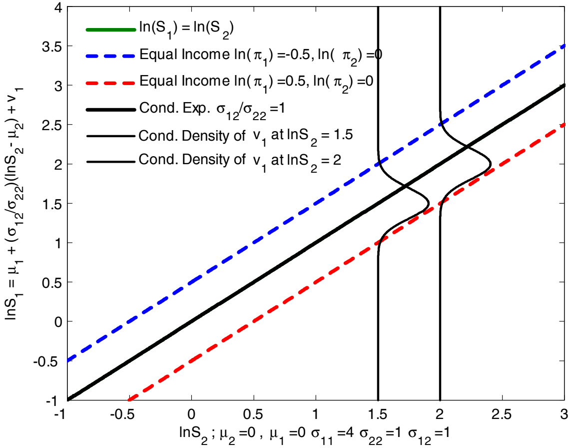
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



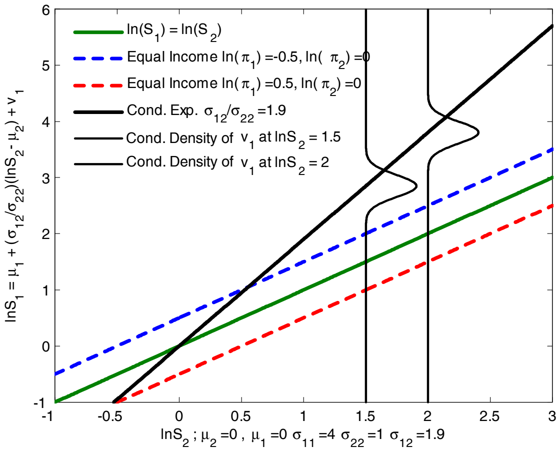
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



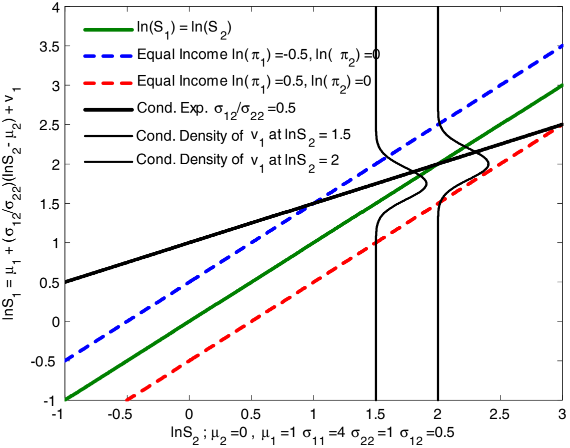
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



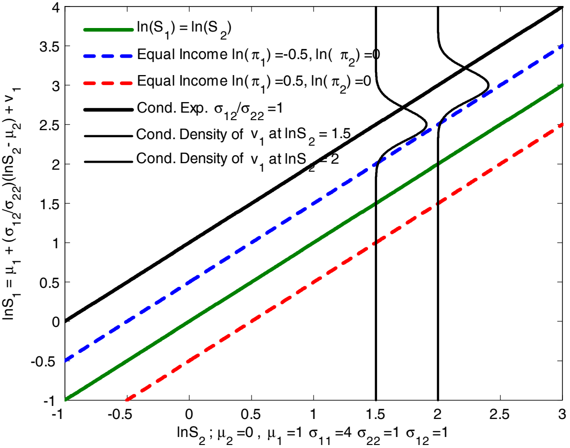
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



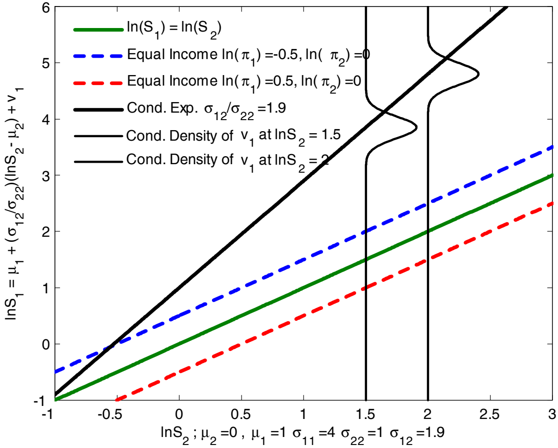
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



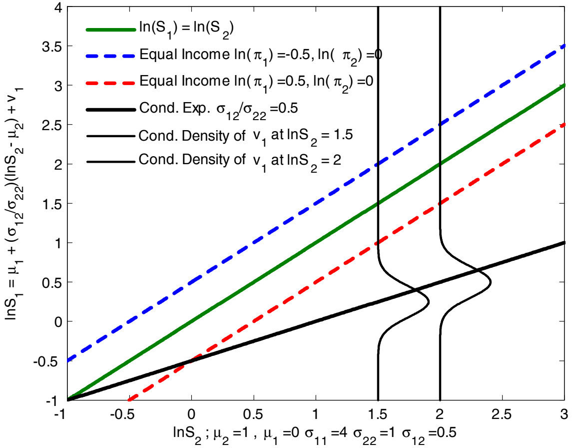
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \; 1.9 \\ 1 \; 1.9 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



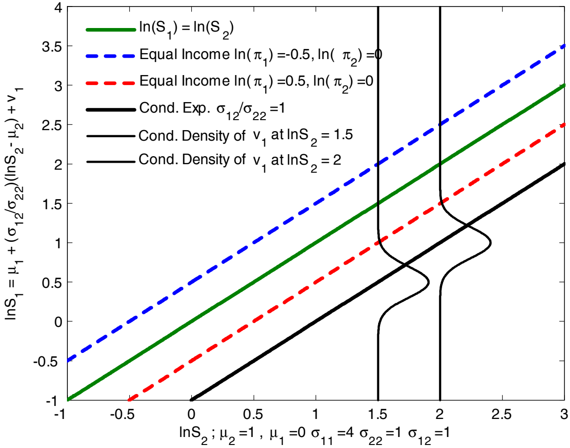
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



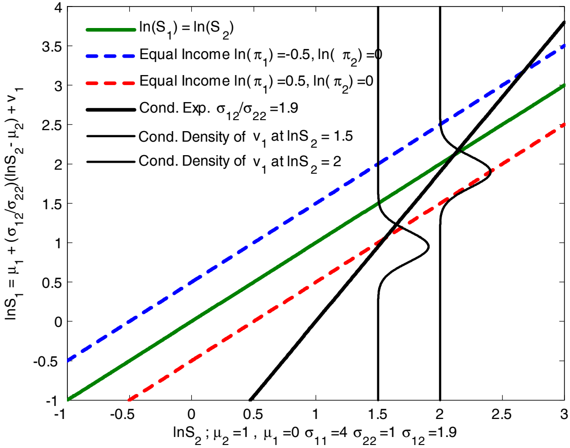
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



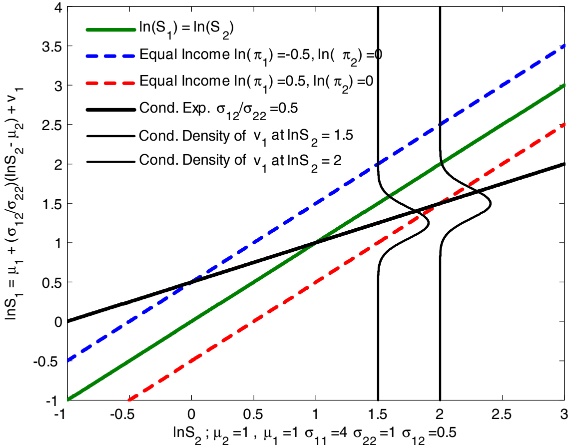
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1.9 \\ 1 \, 1.9 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



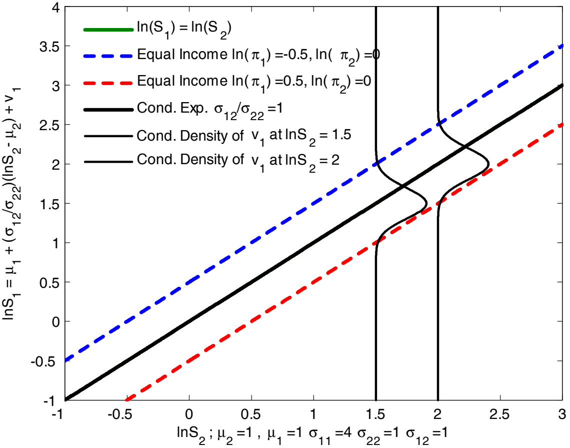
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



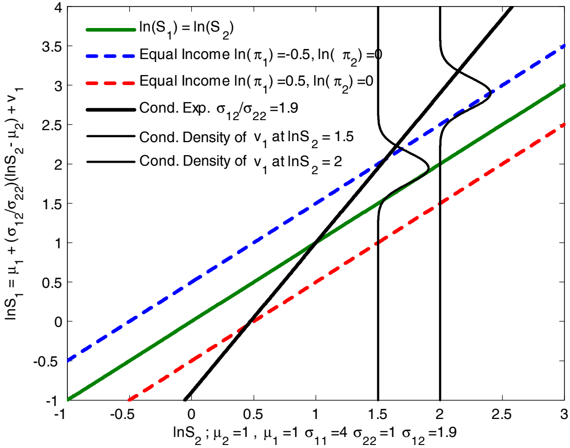
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



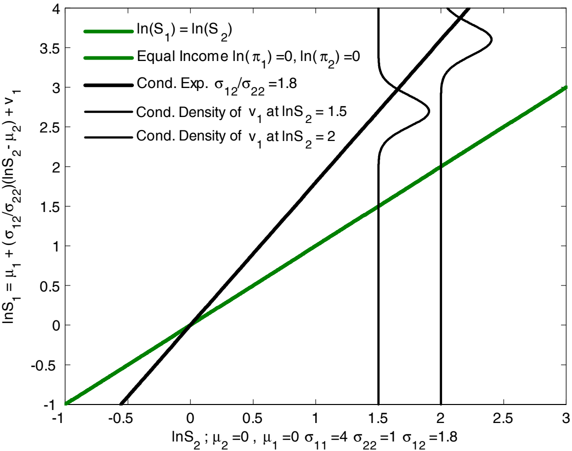
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1.9 \\ 1 \, 1.9 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



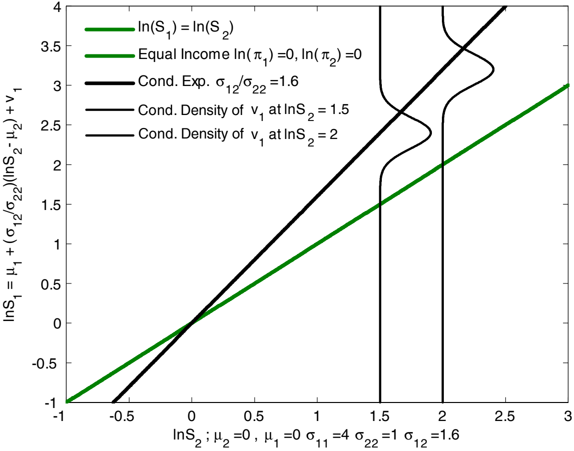
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.8 \\ 1.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



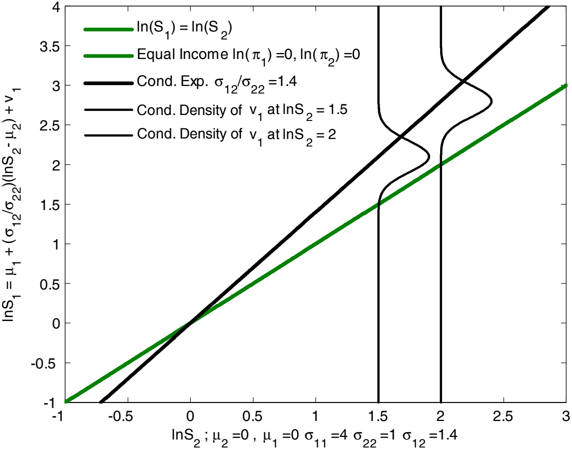
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.6 \\ 1.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



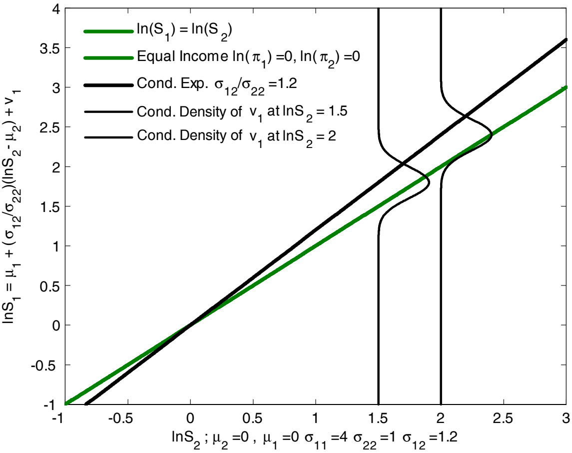
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.4 \\ 1.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



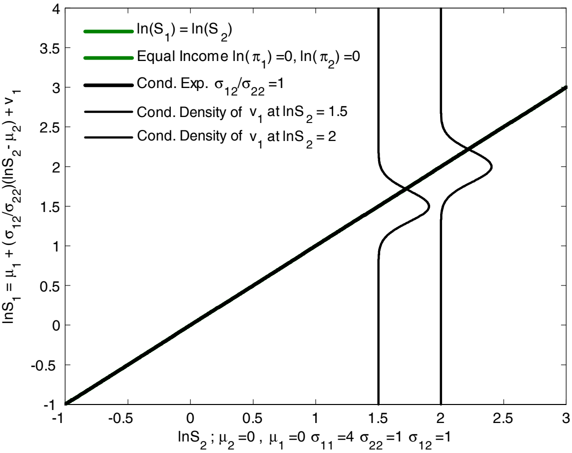
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.2 \\ 1.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



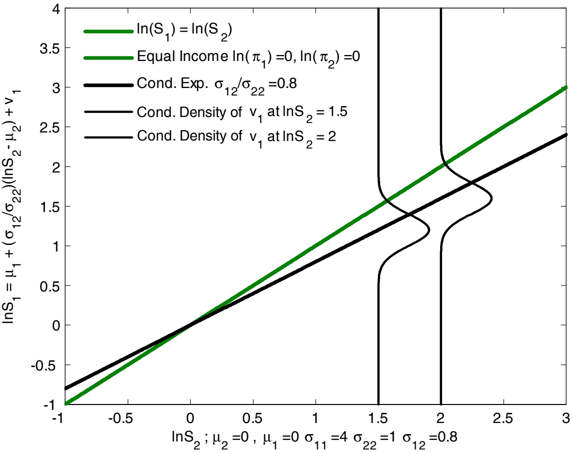
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.0 \\ 1.0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



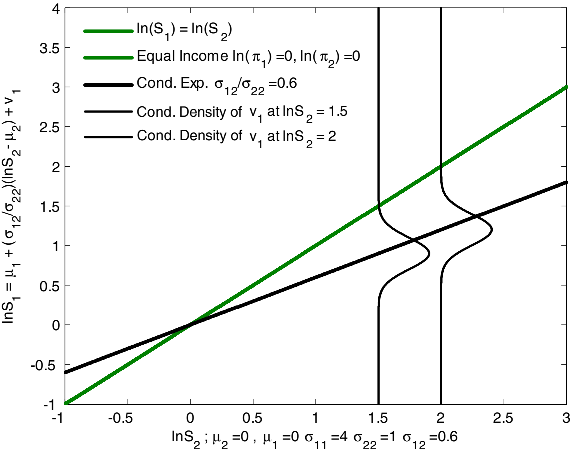
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



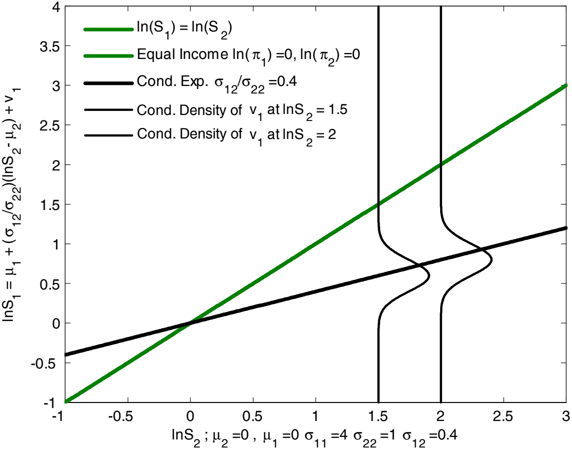
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.6 \\ 0.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



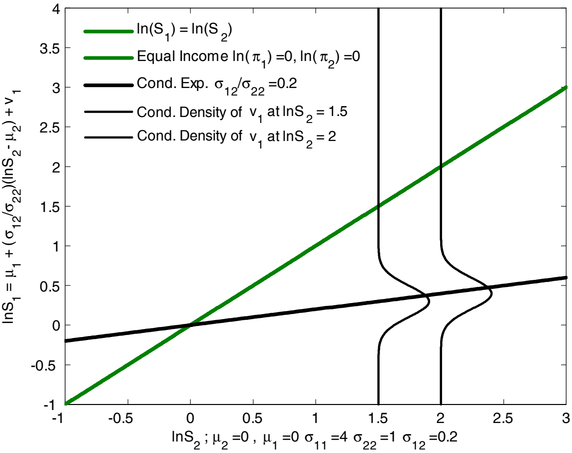
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.4 \\ 0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



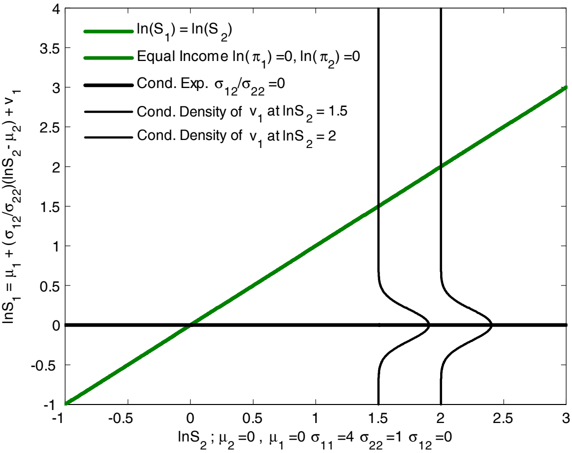
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.2 \\ 0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



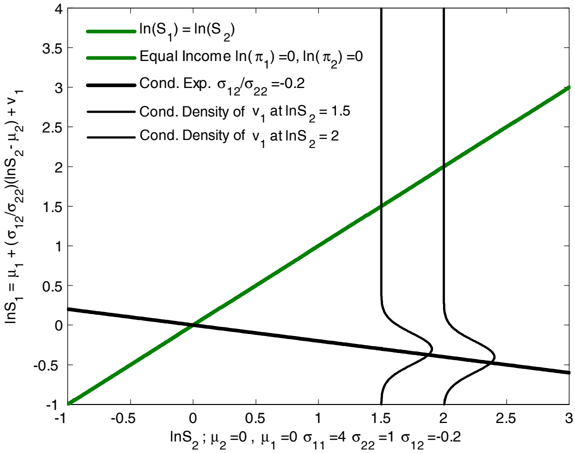
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 0 \\ 0 \, 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



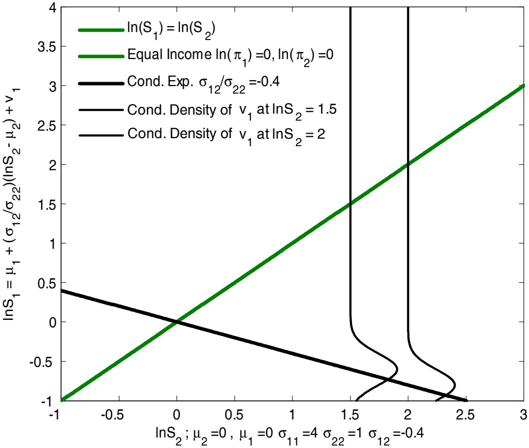
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.2 \\ -0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



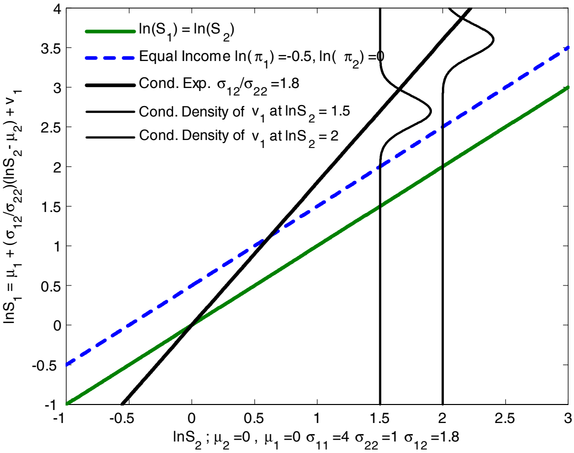
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.4 \\ -0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



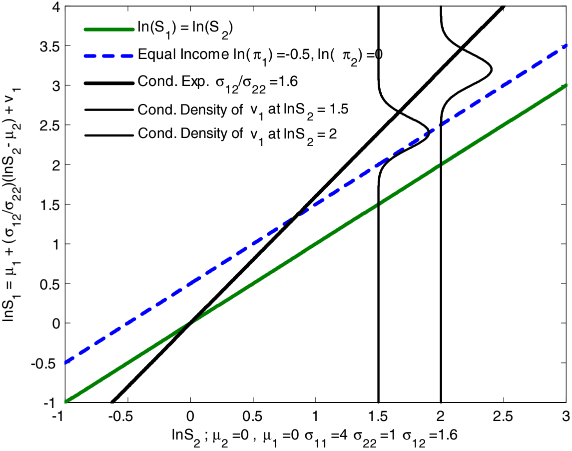
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.8 \\ 1.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



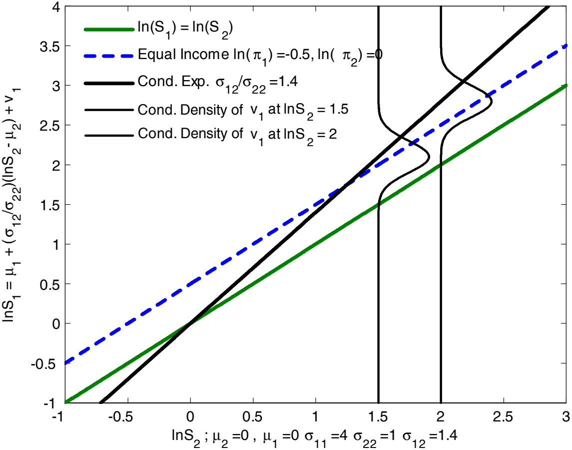
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.6 \\ 1.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



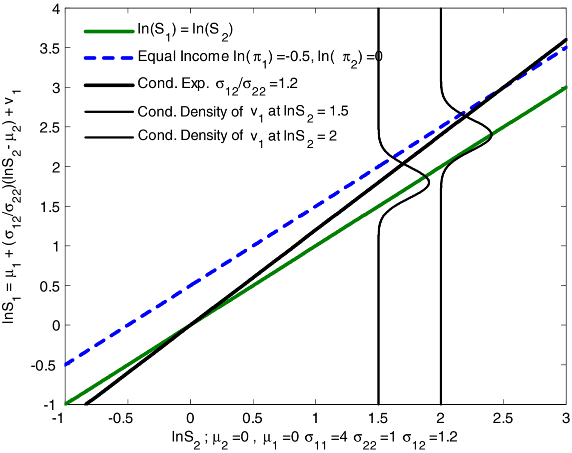
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.4 \\ 1.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



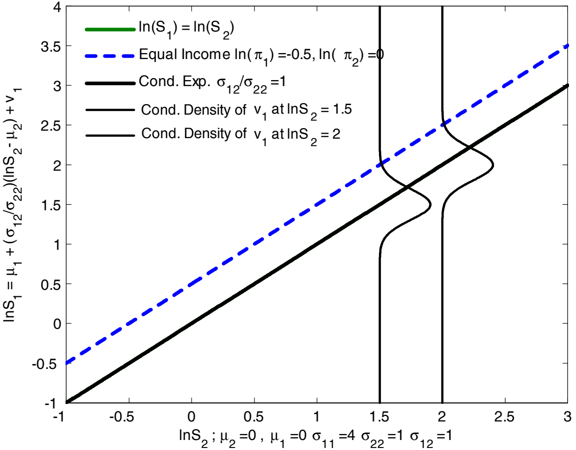
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.2 \\ 1.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



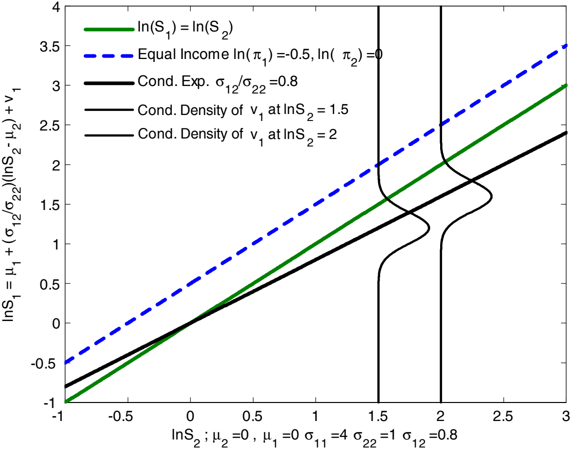
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.0 \\ 1.0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



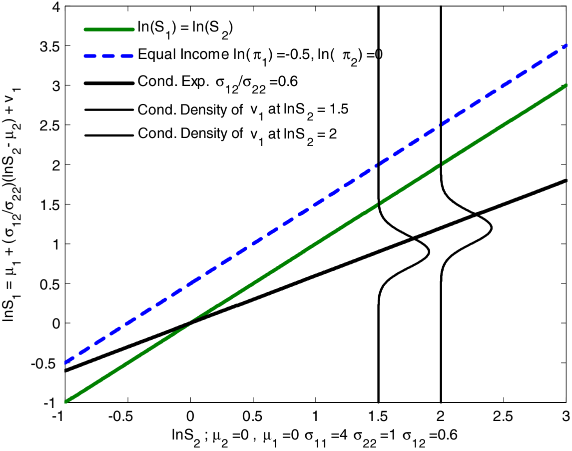
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



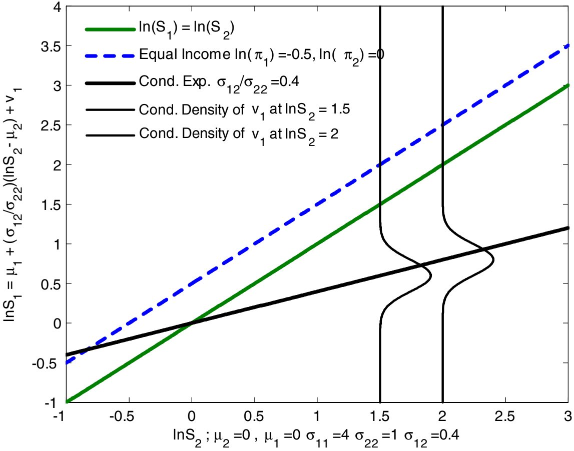
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.6 \\ 0.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



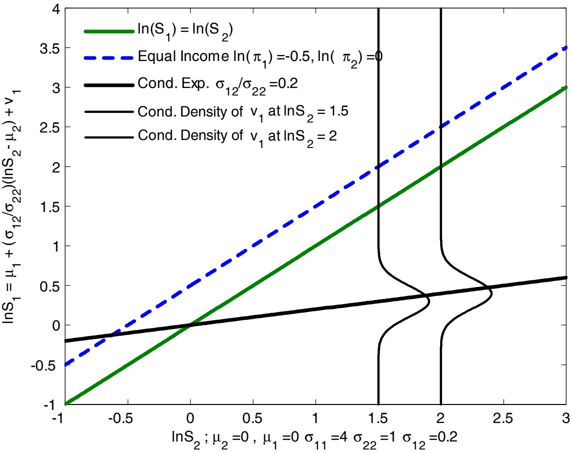
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.4 \\ 0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



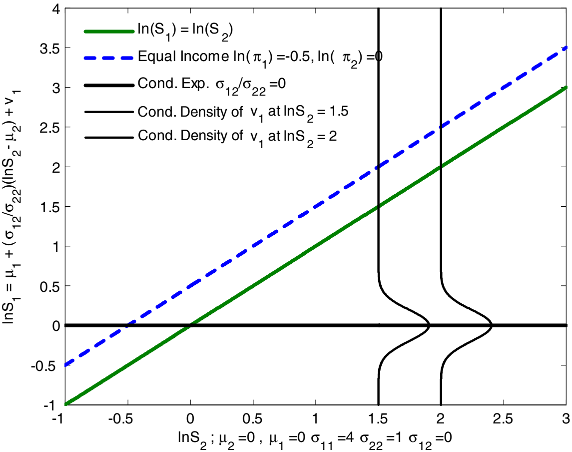
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.2 \\ 0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



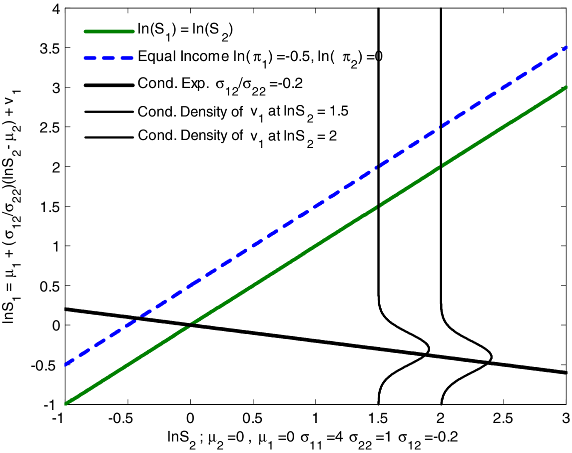
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 0 \\ 0 \, 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



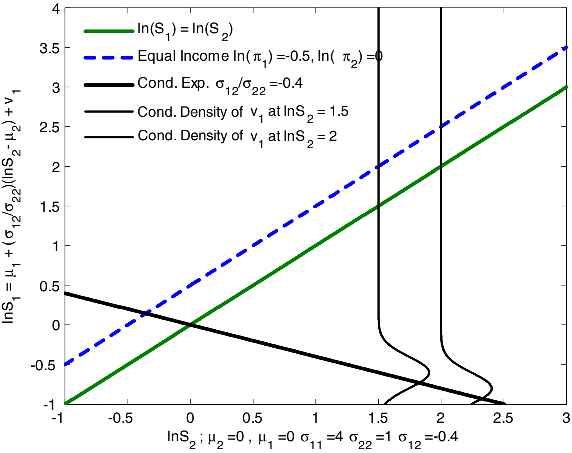
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.2 \\ -0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



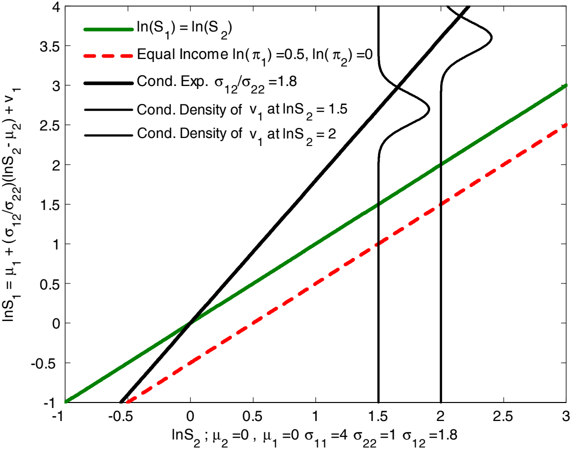
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.4 \\ -0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



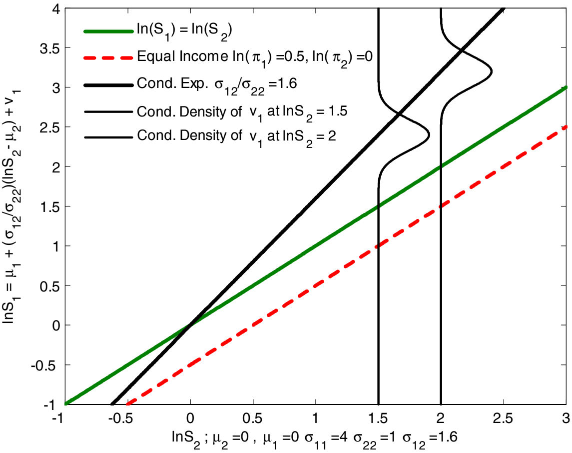
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.8 \\ 1.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



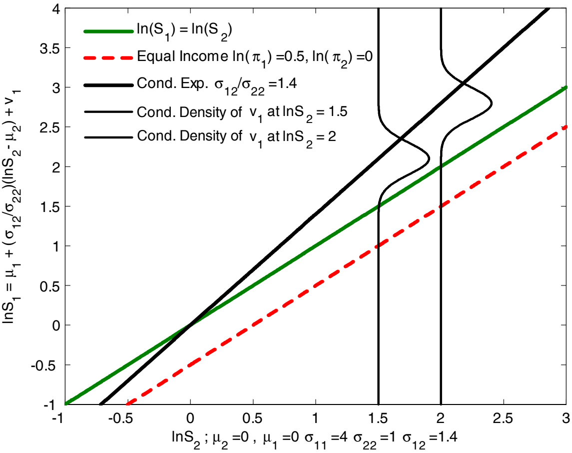
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.6 \\ 1.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



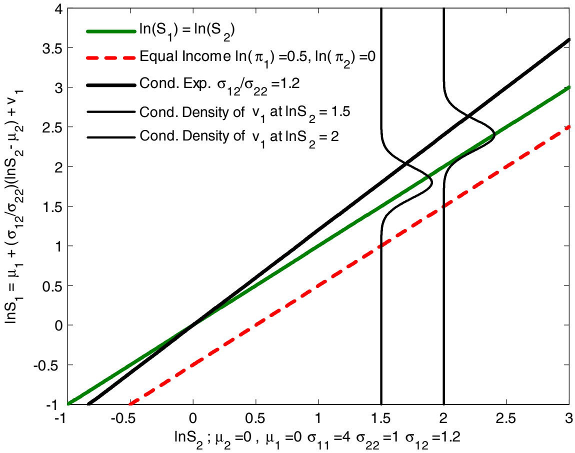
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.4 \\ 1.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



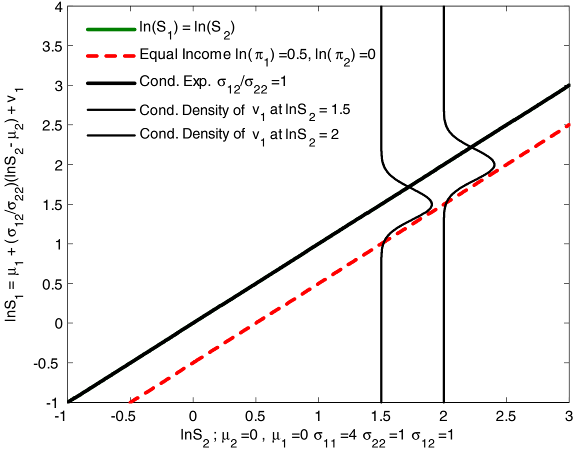
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.2 \\ 1.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



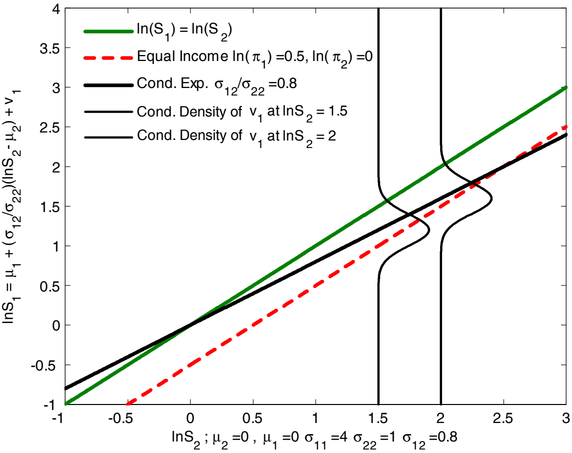
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.0 \\ 1.0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



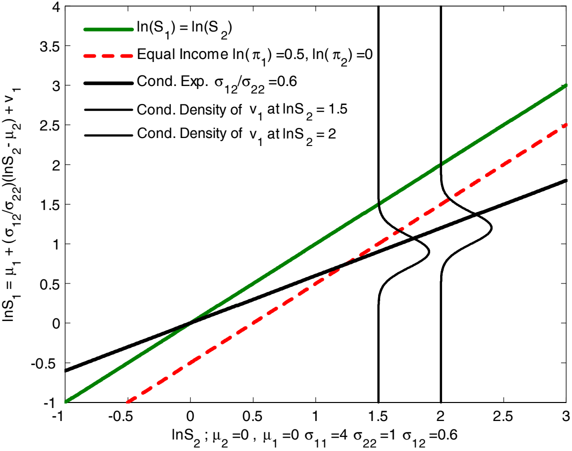
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



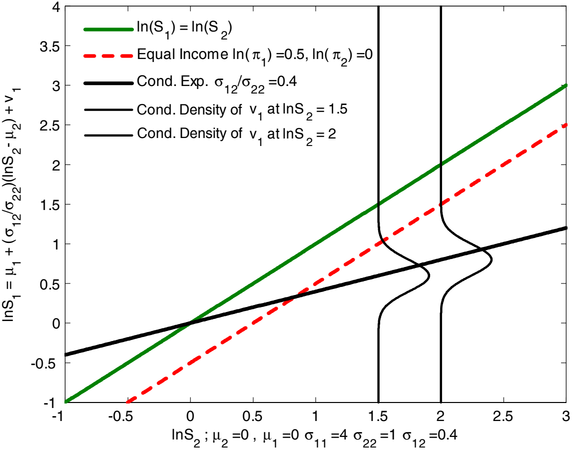
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.6 \\ 0.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



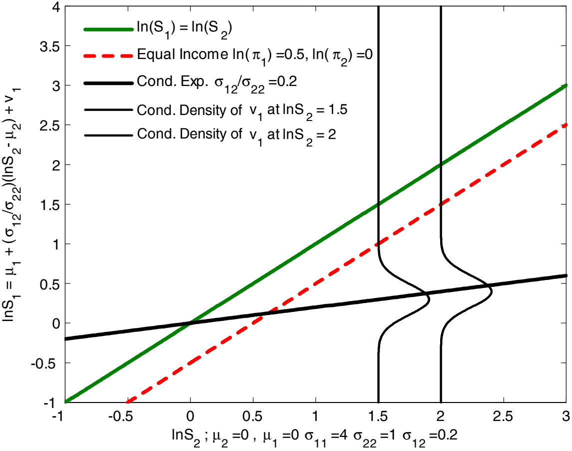
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.4 \\ 0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



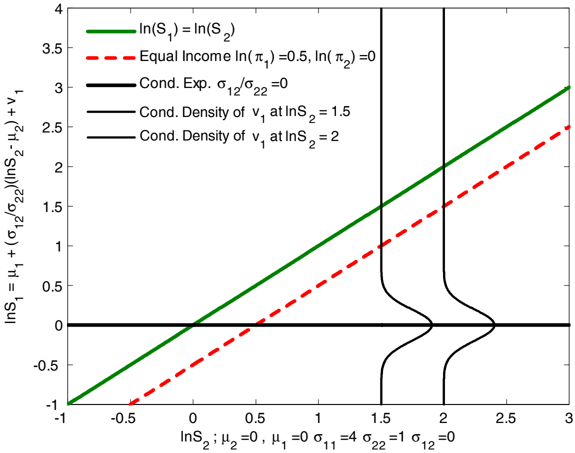
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.2 \\ 0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



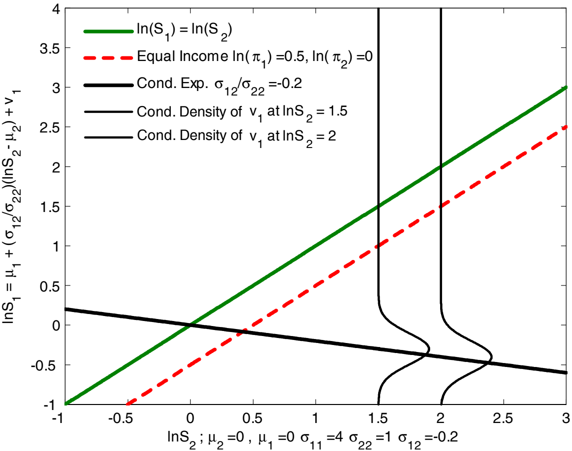
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 0 \\ 0 \, 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



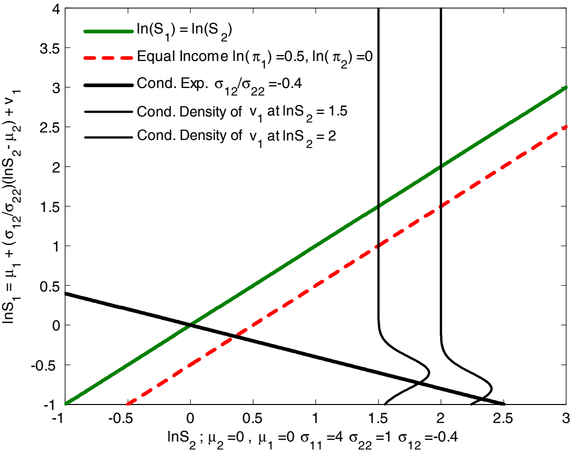
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.2 \\ -0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



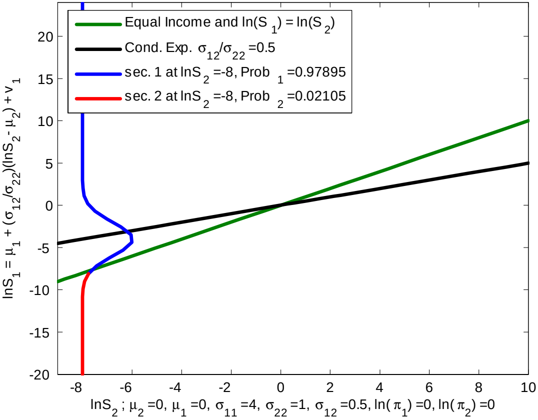
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.4 \\ -0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

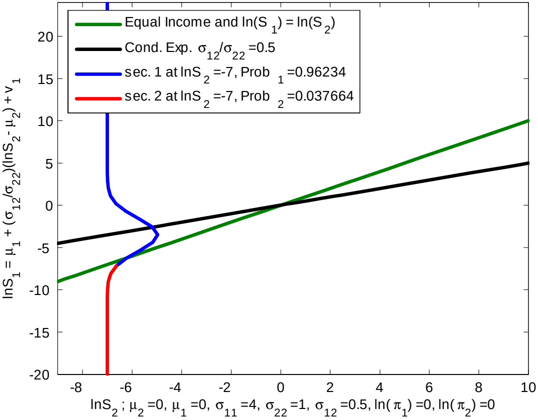
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

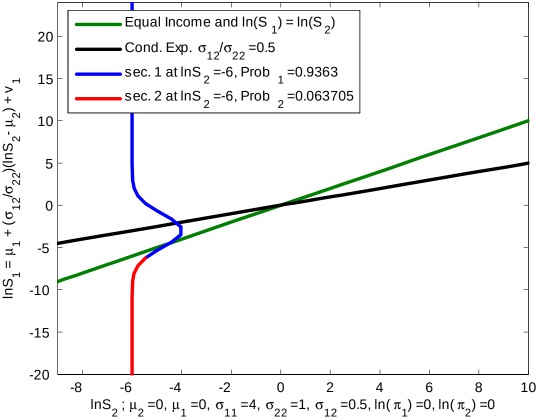
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -7) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -7) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

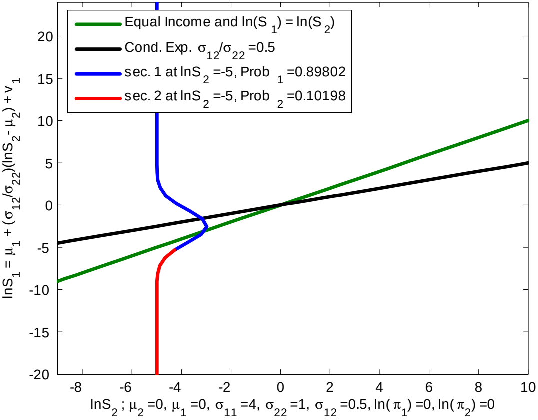
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 1}$$

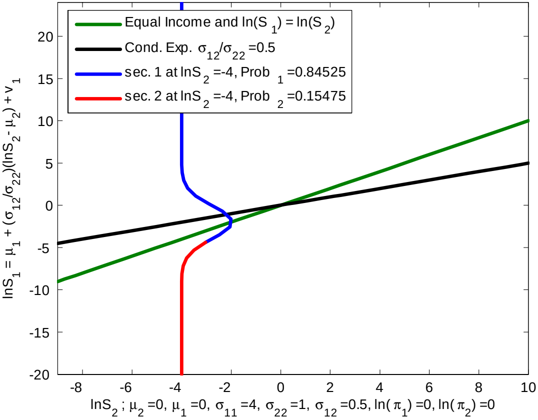
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

THE UNIVERSITY OF CHICAGO

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

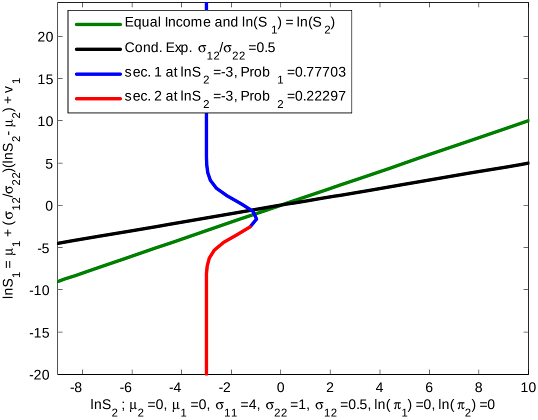
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

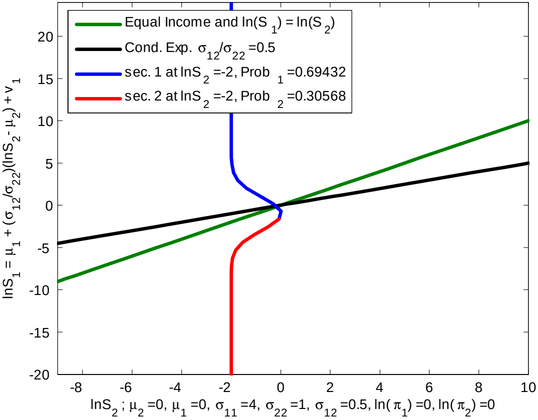
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

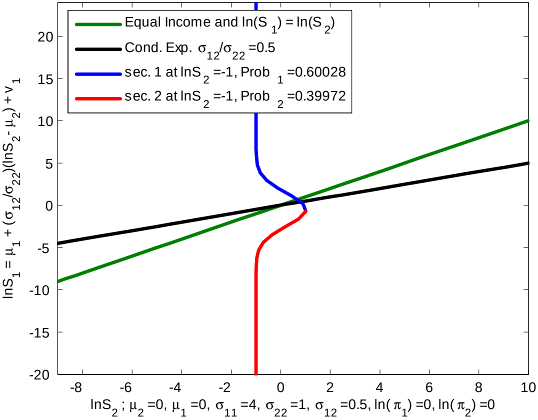
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

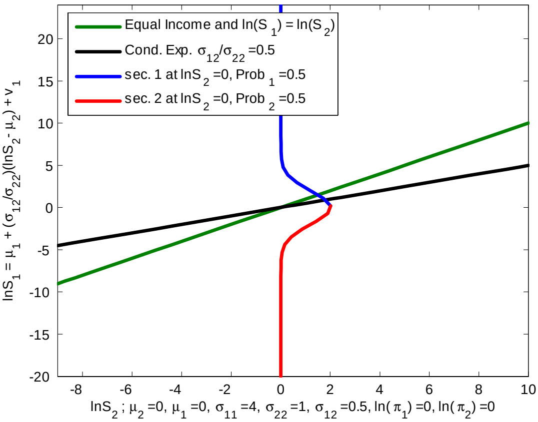
$$\text{Prob}_1 = \Pr \left(W_1 > W_2 | \ln S_2 = -1 \right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr \left(W_1 < W_2 | \ln S_2 = -1 \right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 1}$$

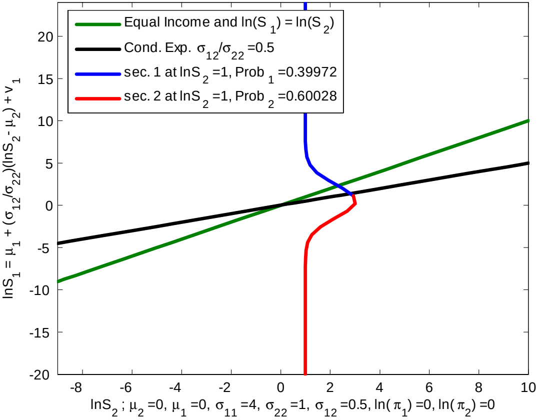
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

THE UNIVERSITY OF CHICAGO

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

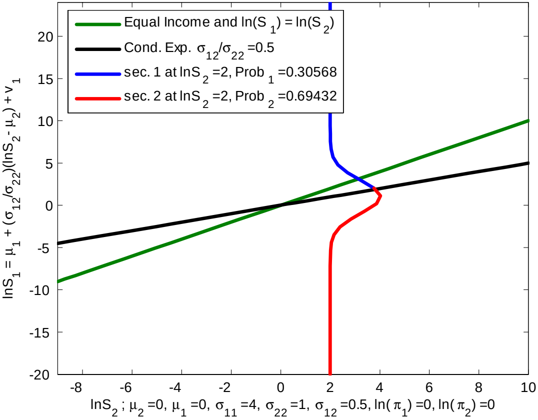
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

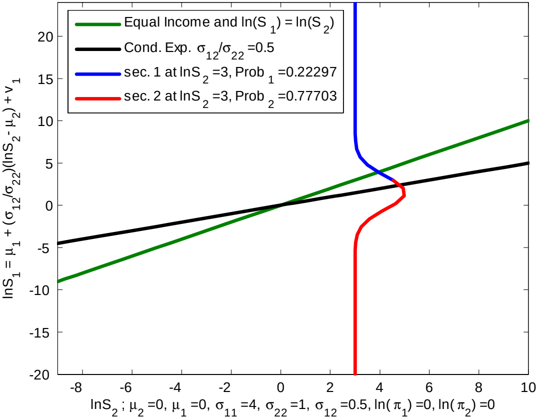
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 2) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 2) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

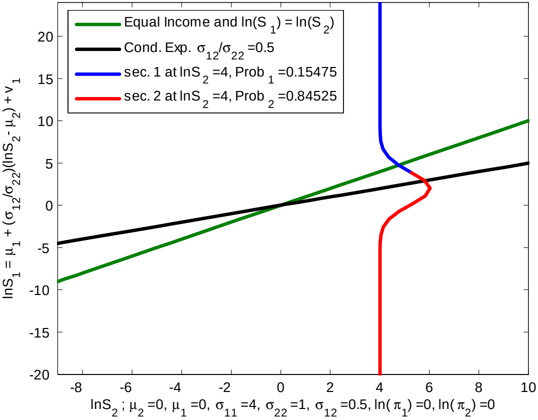
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

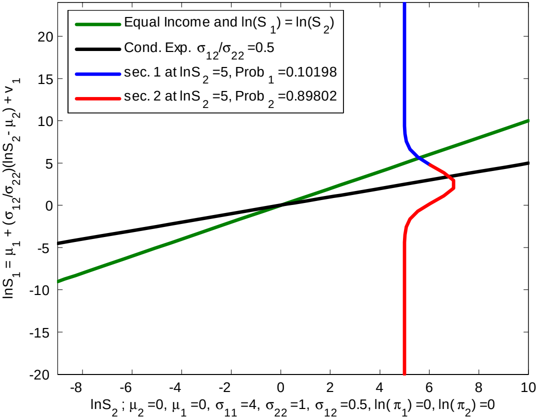
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

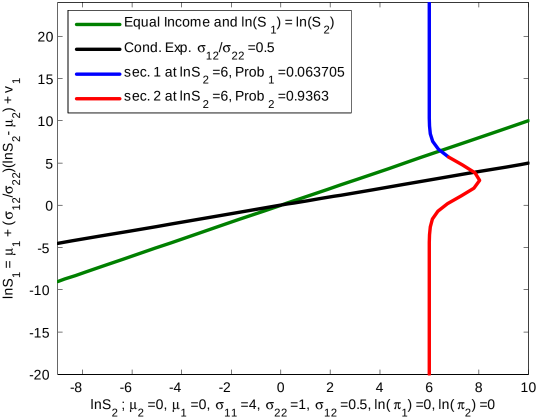
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 5) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 5) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

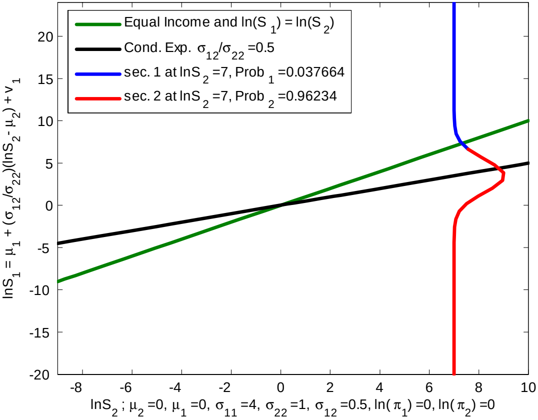
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = 6\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = 6\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$

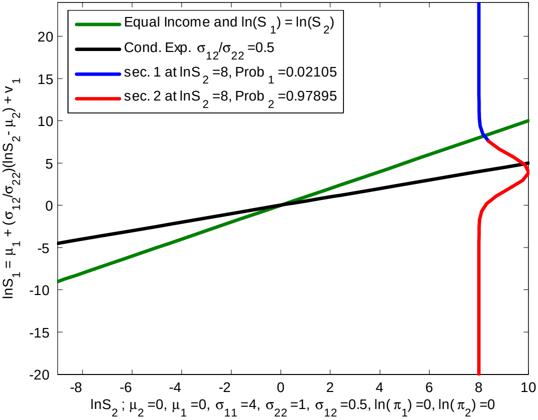


$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

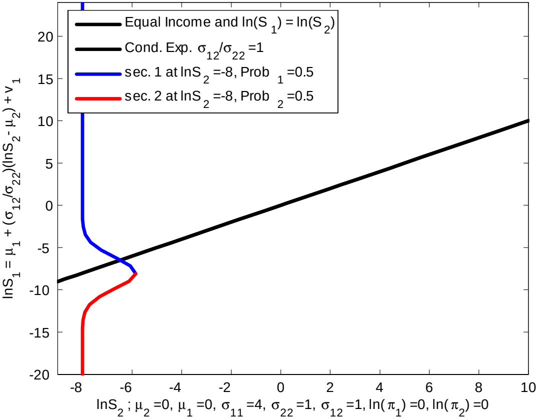
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 8) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 8) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

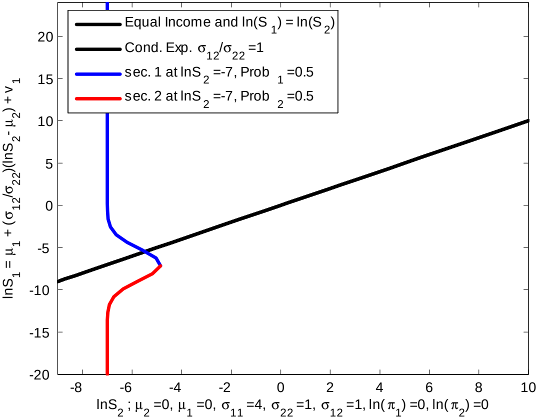
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

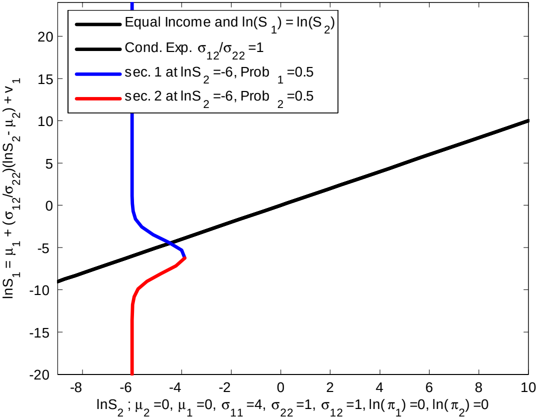
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -7\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -7\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

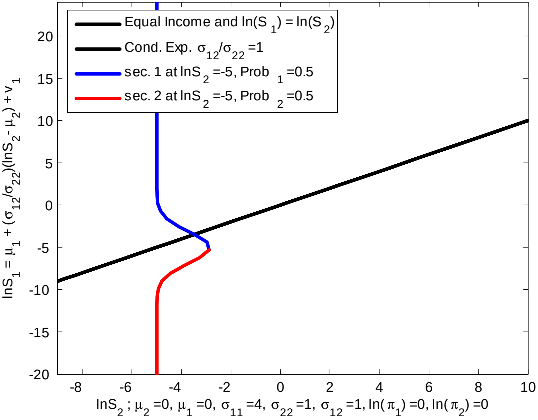
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -6\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -6\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

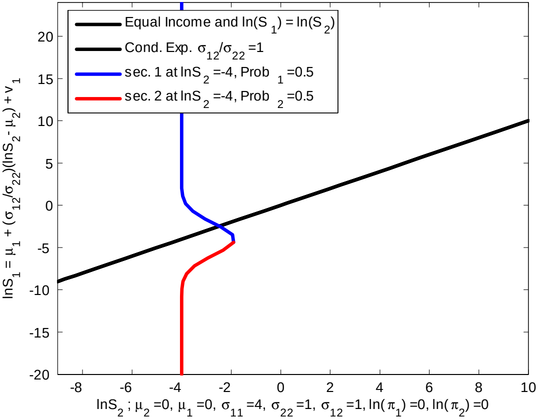
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

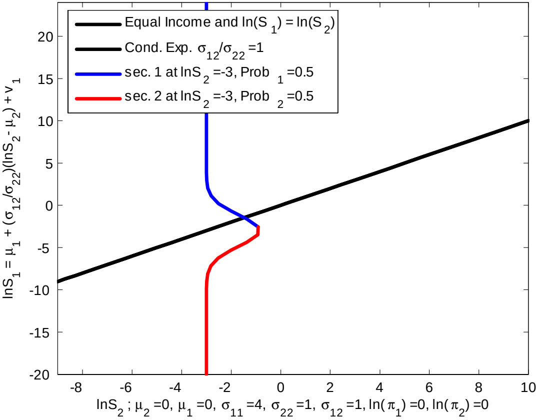
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

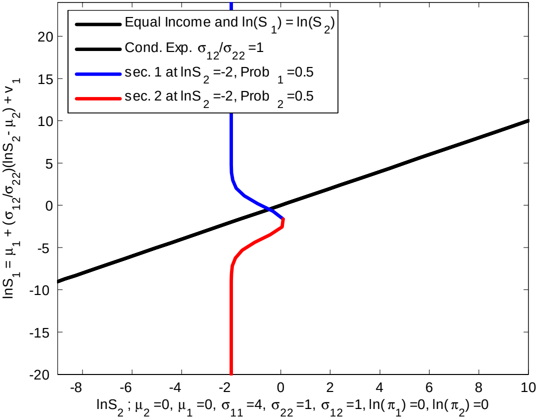
$$\text{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 1}$$

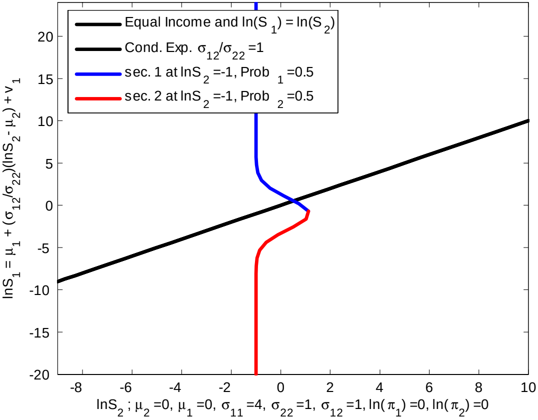
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

THE UNIVERSITY OF CHICAGO

Graph of $\ln S_1 = f(\ln S_2)$



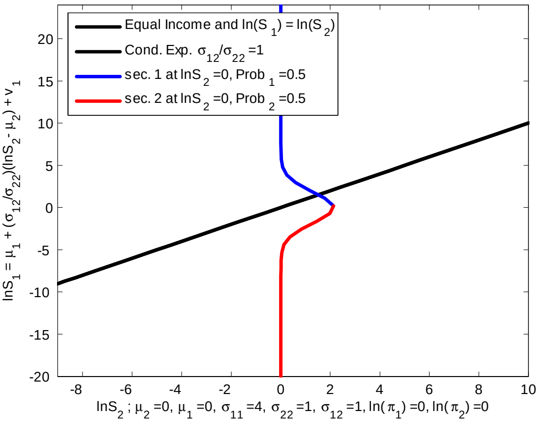
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -1) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -1) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

THE UNIVERSITY OF CHICAGO

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

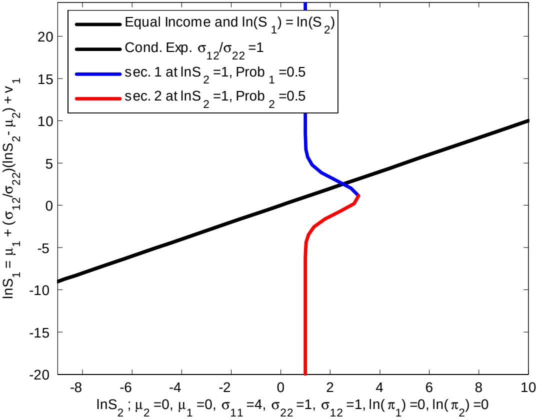
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 1}$$

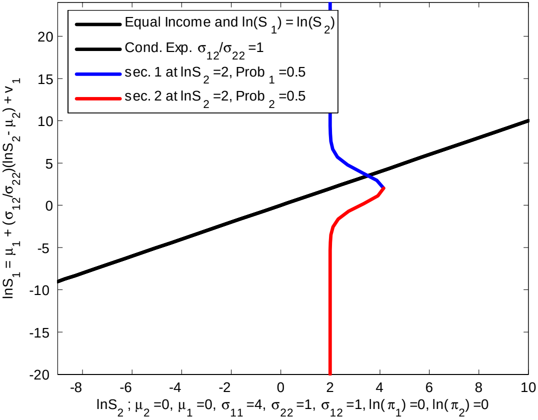
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

THE UNIVERSITY OF CHICAGO

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

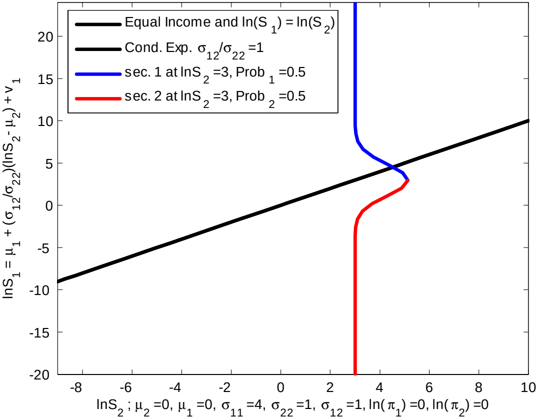
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

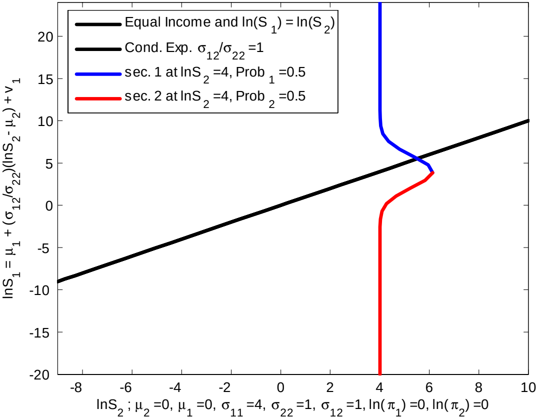
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

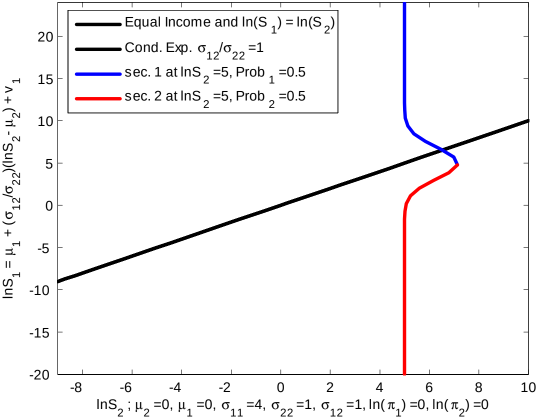
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 1}$$

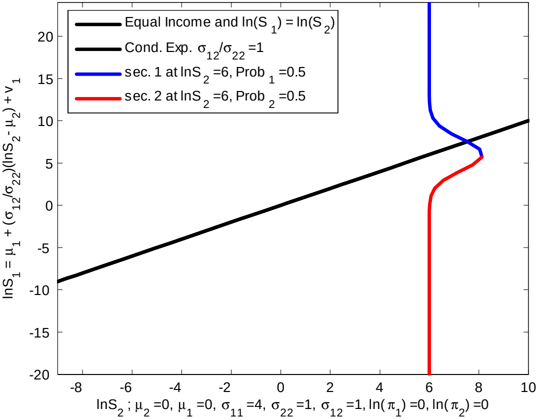
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

THE UNIVERSITY OF CHICAGO

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

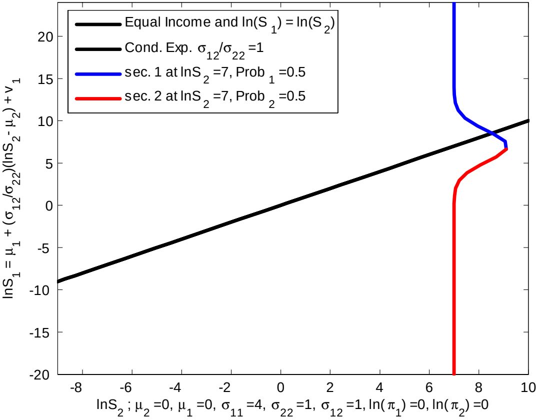
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 6) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 6) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

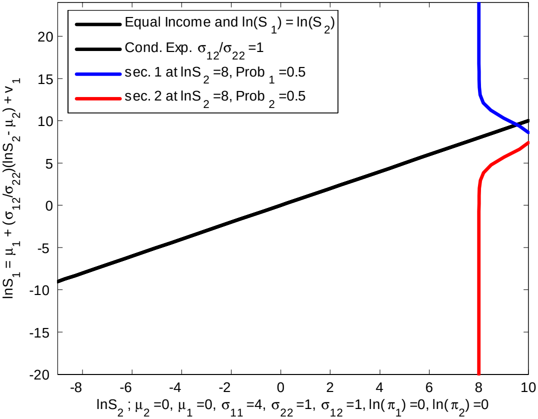
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 7) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 7) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

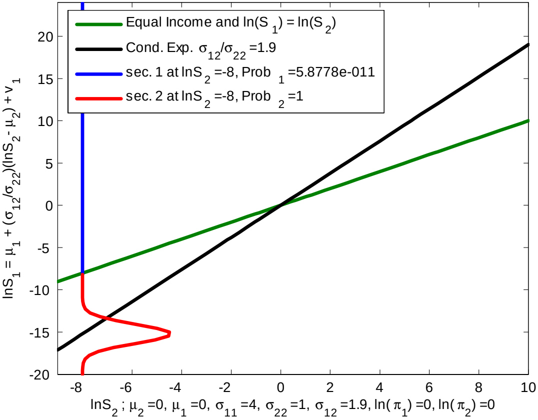
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 8) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 8) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

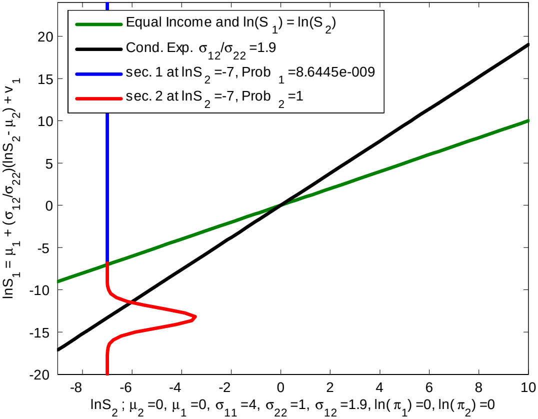
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

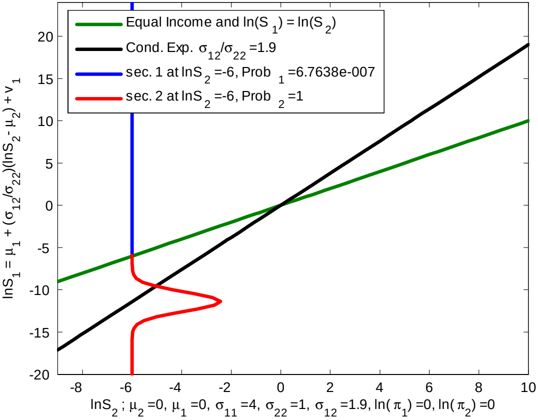
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -7) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -7) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

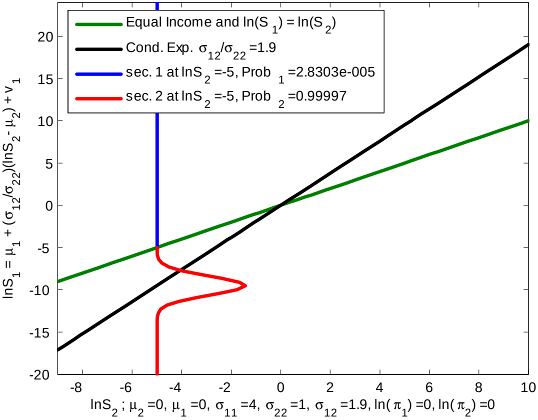
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

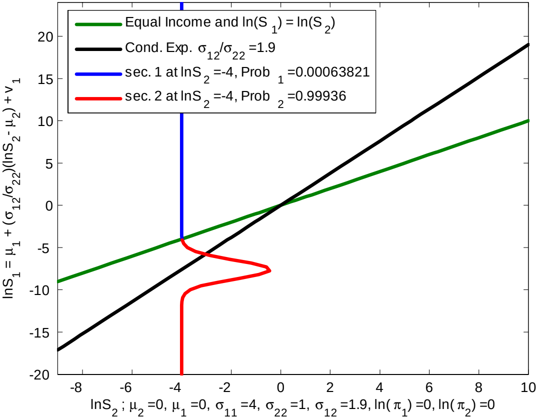
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -5\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -5\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$

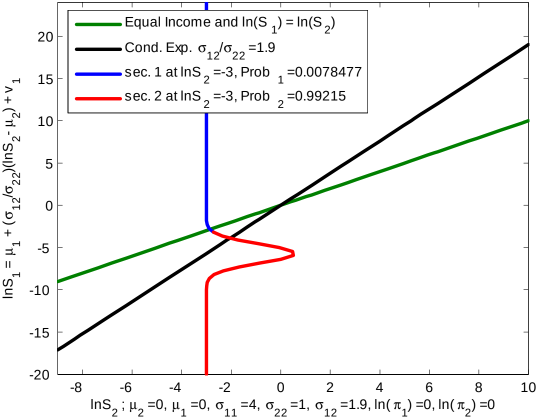


$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

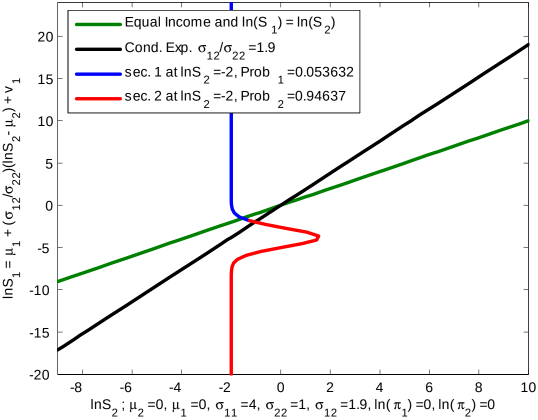
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -3\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -3\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

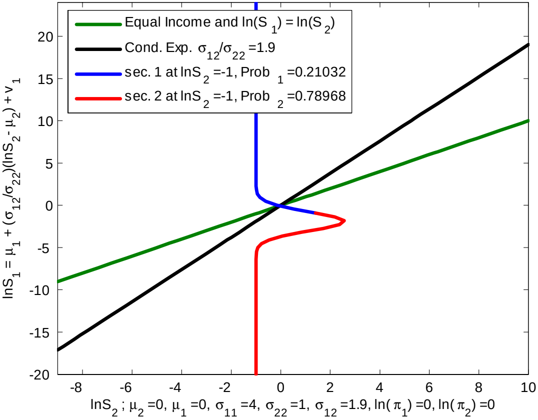
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

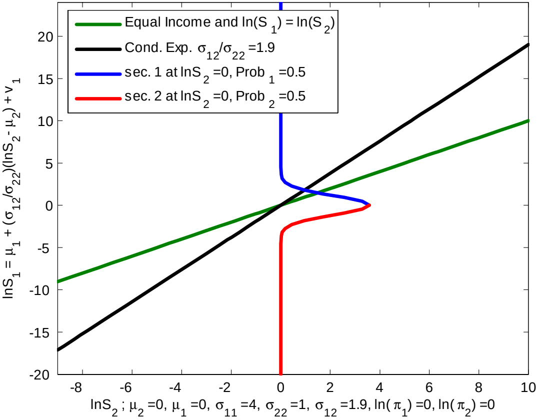
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -1) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -1) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

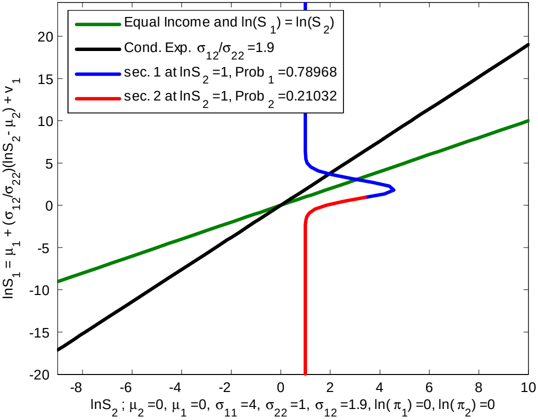
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

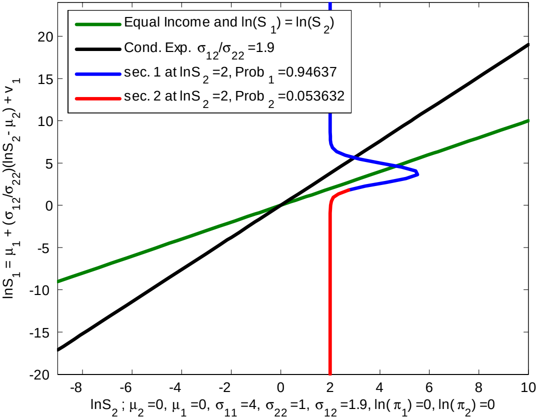
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

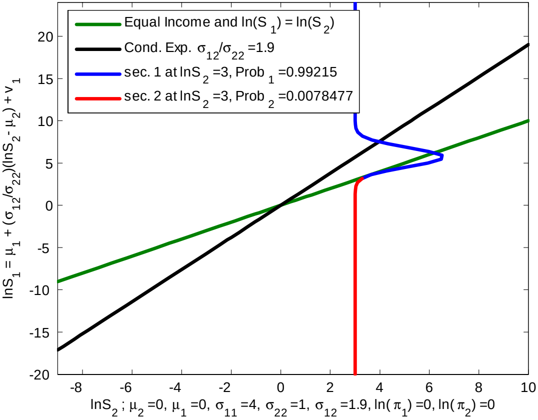
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

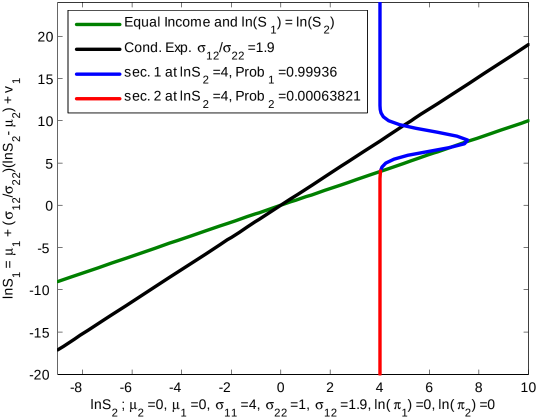
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = 3\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = 3\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

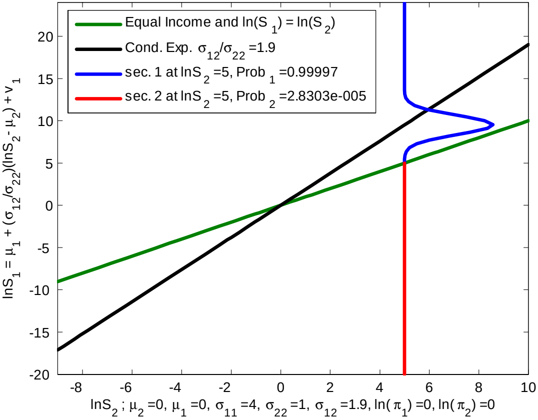
$$\text{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

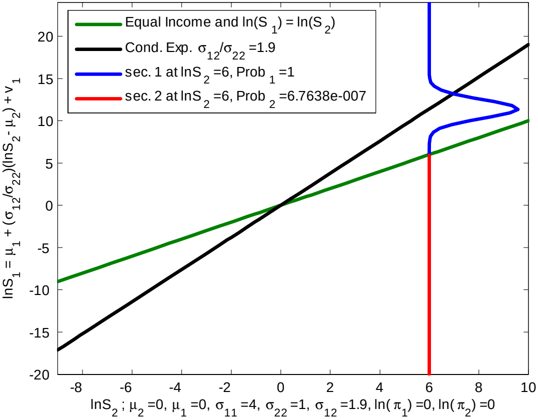
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 6) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 6) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Theorem 1 For any random assignment of persons to sectors, a Roy economy with the same proportion of persons in each sector as the random assignment economy has lower variance in log earnings provided some individuals are assigned to each sector.

Proof: Heckman and Honoré (1990).■

Skewness

We also look at one result for the skewness of the sectorial wages. We have expression for skewness of wage in sector 1 as:

$$E\left\{[\ln W_1 - E(\ln W_1 \mid \ln W_1 > \ln W_2)]^3 \mid \ln W_1 > \ln W_2\right\}$$

$$\begin{aligned} &= E\left\{(a_1)^3[L - E[L \,\middle|\, L > c_1^*)]^3 \,|\, L > -c_1^*\right\} \\ &= (\sigma_{11})^{3/2}(\rho_1)^3 E\left\{[Z - \lambda(-c_1)]^3 \,|\, Z > -c_1\right\} \end{aligned}$$

Theorem 2 : In a Roy economy, log earnings distributions are right skewed as long as some positive fraction of the population works in each sector.■

Theorem 3 : The right tail of the Roy density of earnings $f(w)$ within sectors or in the overall economy are thinner than Pareto tails from density $g(w)$ in the sense that

$$\lim_{w \to \infty} \frac{f(w)}{g(w)} \to 0.$$

Summary of Normal Case

(i) Self-selection raises the mean of employed log skill 1 above the population mean μ_1 if $\sigma_{11} > \sigma_{12}$.

(ii) Self-selection reduces the mean of employed log skill 1 below the population mean μ_1 if $\sigma_{11} < \sigma_{12}$ (the "unusual case").

(iii) The "unusual" case can arise in at most one sector.

(iv) In response to a 10% increase in π_1, mean log wages in 1 rise by more than 10%, 10% or less than 10% depending on whether or not $\sigma_{11} \lesseqgtr \sigma_{12}$. If $\sigma_{11} > \sigma_{12}$, a 10% increase in π_1 may result in a decrease in sector 1 earnings.

(v) In response to an increase in π_1, mean log wages in 2 rise unless the unusual case occurs in sector 2.

(vi) Self-selection reduces sectoral and aggregate earnings inequality (measured by the variance of log earnings) compared to a random assignment economy.

(vii) As π_1 increases, the variance of log earnings in sector 1 increases and the variance of log earnings in sector 2 decreases.

(viii) Self-selection produces right skewness in sector 1 log earnings distributions if $\sigma_{11} > \sigma_{12}$.
If $\sigma_{11} < \sigma_{12}$ (the "unusual" case), sector 1 earnings distributions are left skewed. Self-selection produces right skewness in the aggregate log earnings distribution with the exception of cases where all agents work in one sector.

How General Are These Results?

- What depends on normality and what does not?
- $E(Z \mid Z > c) \uparrow c$ is general (as long as means exist).
- $E(Z - c \mid Z > c) \downarrow c$ not general.
- $Var(Z - c \mid Z > c) \downarrow c$ not general.
- But motivates widely used estimation strategies.
- Log concave random variables preserve this property.

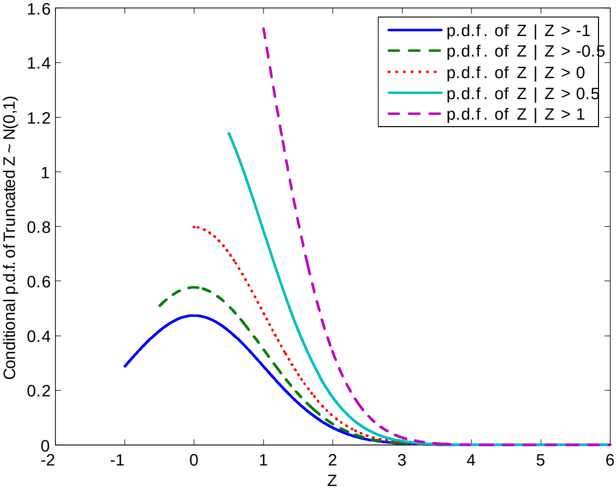
Properties of Truncated Normal Random Variable

- $E(Z\,|Z > t) = \dfrac{\frac{1}{\sqrt{2\pi}}\exp -\frac{t^2}{2}}{\Phi(-t)} = \frac{\phi(t)}{\Phi(-t)} \equiv \lambda(t)$; $\lambda(t) \geq 0$ and $\lambda(t) \geq t$.
- $Var(Z \mid Z > t) = 1 + \lambda(t)t - (\lambda(t))^2$
  $= 1 + \lambda(t)\,(t - \lambda(t))$
  $= 1 - E(Z\,|Z > t)E(Z - t\,|Z > t)$
- $\lim\limits_{t\to\infty} \lambda(t) = \infty$
- $\lim\limits_{t\to-\infty} \lambda(t) = 0$
- $\dfrac{\partial\lambda(t)}{\partial t} > 0 \qquad -\infty < t < \infty$

- $\lim_{t \to -\infty} \frac{\partial \lambda(t)}{\partial t} = 0$
- $\lim_{t \to \infty} \frac{\partial \lambda(t)}{\partial (t)} = 1$
- $0 < \frac{\partial \lambda(t)}{\partial t} = \lambda'(t) = \lambda(t)(\lambda(t) - t) < 1, -\infty < t < \infty$
- $\partial^2 \lambda(t) / \partial t^2 > 0, \qquad t < \infty$
- $\frac{\partial Var(Z \,|Z > t)}{\partial t} < 0, \quad t < \infty$
- $\lim_{t \to \infty} Var(Z \,|Z > t) = 0$

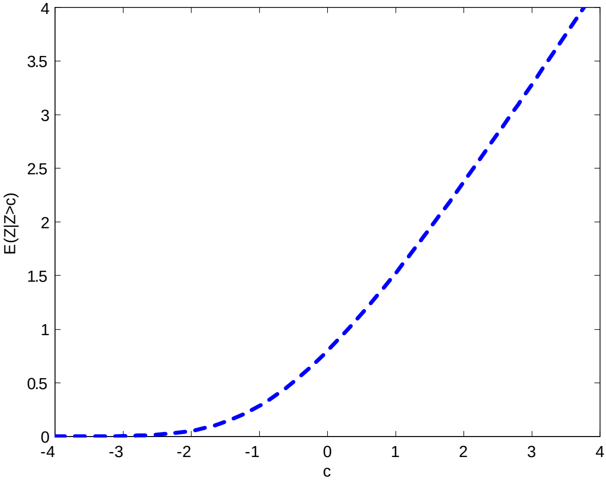
- $\lim_{t \to -\infty} Var(Z \,|Z > t) = 1$
- $E([Z - \lambda(t)]^3 \,|Z > t) > 0, \quad t < \infty$
  and $\lim_{t \to \infty} E[[Z - \lambda(t)]^3 \,|Z > t] = 0$
  so Z is positively skewed if $t < \infty$
- Mean of $Z \geq$ mode of Z.

Truncated Standard Normal Density Function



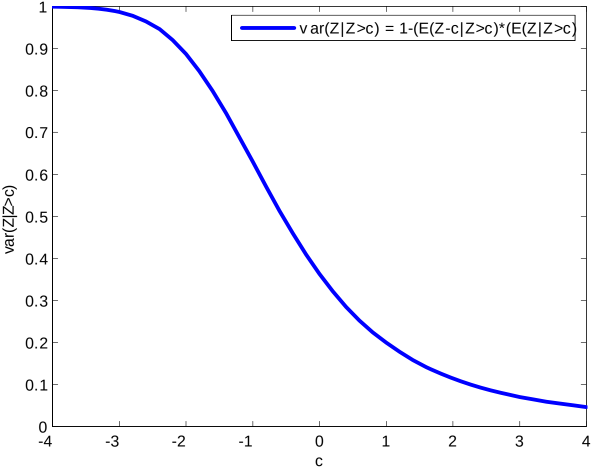
$$Z \sim N(0,1)$$

Truncated Standard Normal Expectation



$$E\left[Z|Z>c\right]; \quad Z\sim N(0,1)$$

Truncated Standard Normal Variance



$$var(Z|Z > c); \quad Z \sim N(0,1)$$

The Variance of a Truncated Standard Normal:

$$Var(Z \mid Z > t) = 1 - E(Z \,|Z > t) E(Z - t \,|Z > t)$$

but:

$$\frac{E(Z \,|Z > t)}{t} \rightarrow 1 \text{ as } t \rightarrow \infty$$

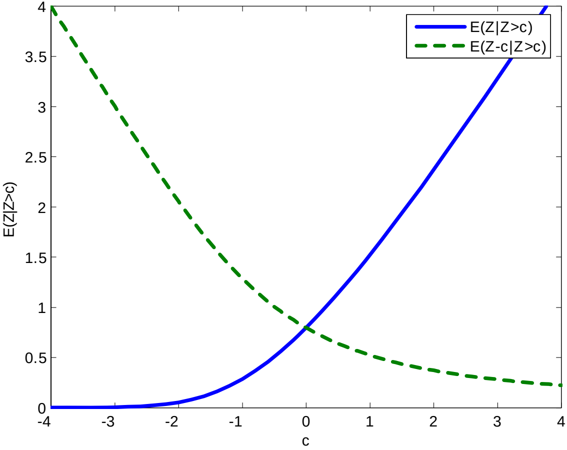
$$\Rightarrow \; E(Z \,|Z > t) \rightarrow \infty \;\; \text{as } t \rightarrow \infty$$

$$E(Z - t \,|Z > t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

$$
\begin{aligned}
0 &\leq Var(Z \mid Z > t) \leqslant 1 \\
&\Rightarrow 0 \leq 1 - E(Z \,|Z > t) E(Z - t \,|Z > t) \leqslant 1 \\
&\Rightarrow -1 \leq -E(Z \,|Z > t) E(Z - t \,|Z > t) \leqslant 0 \\
&\Rightarrow 0 \leq E(Z \,|Z > t) E(Z - t \,|Z > t) \leqslant 1
\end{aligned}
$$

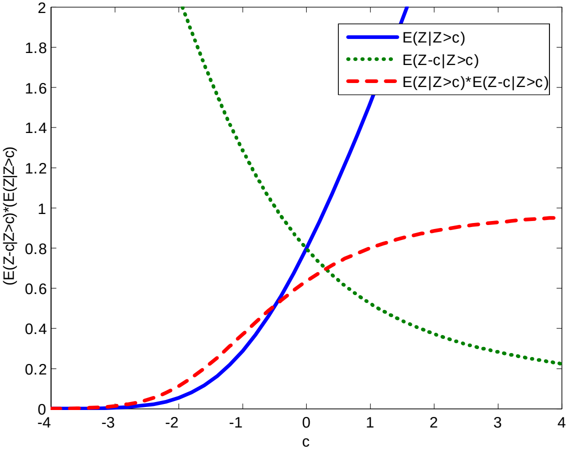
The following figures illustrate these properties.

Truncated Standard Normal Expectations



$$E[Z|Z>c] \text{ and } E[Z-c|Z>c]; \quad Z \sim N(0,1)$$

Truncated Standard Normal Expectations



$$E[Z|Z > c] \text{ and } E[Z - c|Z > c]; \quad Z \sim N(0, 1)$$

Return to main text