Labor Supply and Taxes: A Survey

by Michael P. Keane
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James J. Heckman

THE UNIVERSITY OF CHICAGO

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$$\tau = \frac{1}{1 + a \cdot e}. \tag{1}$$

Here e is the labor supply elasticity (i.e., the percent increase in labor supply that accompanies a 1 percent increase in the after-tax wage rate $w(1 - \tau)$, where w is the pretax wage),[2] and a is the "Pareto parameter," an (inverse) measure of income dispersion within the top bracket.

Specifically, a is defined as $a = z_m/(z_m - z)$, where z is the income level where the top bracket starts, and z_m is the average income of people in the top bracket. For example, if the top bracket starts at \$500,000, and average income in that bracket is \$1,000,000, then $a = 1/(1–0.5) = 2$. In contrast, if average income in the top bracket is \$2,000,000 (implying more dispersion/inequality), we would have $a = 2/(2–0.5) = 1.33$. As $z_m \rightarrow \infty$, meaning inequality becomes extreme, a approaches its lower bound of 1. From (1), we see that the optimal top bracket rate *increases* if there is more inequality in the top bracket (i.e., a smaller value of a).

TABLE 1

Optimal Top Bracket Tax Rates for Different Labor Supply Elasticities

Labor supply elasticity (e)	Optimal top-bracket tax rate (τ)		
	$a = 1.50$	$a = 1.67$	$a = 2.0$
2.0	25%	23%	20%
1.0	40%	37%	33%
0.67	50%	47%	43%
0.5	57%	54%	50%
0.3	69%	67%	63%
0.2	77%	75%	71%
0.1	87%	86%	83%
0.0	100%	100%	100%

Note: These rates assume the government places essentially no value on giving extra income to the top earners.

$$\tau = 1/(1 + e). \tag{2}$$

TABLE 2
Revenue Maximizing Flat Tax Rates given different Labor Supply Elasticities

Elasticity (e)	Optimal tax rate (τ)	
	$g = 0$	$g = 0.5$
2.0	33%	20%
1.0	50%	33%
0.67	60%	43%
0.5	67%	50%
0.3	77%	63%
0.2	83%	71%
0.1	91%	83%
0.0	100%	100%

3. *Basic Models of Labor Supply*

3.1 *The Basic Static Labor Supply Model*

$$U_t = \frac{C_t^{1+\eta}}{1 + \eta} - \beta_t \frac{h_t^{1+\gamma}}{1 + \gamma}, \tag{3}$$

$$\eta \leq 0, \quad \gamma \geq 0.$$

$$(4)\quad \frac{dU_t}{dh_t} = [w_t(1-\tau)h_t + N_t]^{\eta} w_t(1+\tau)$$

$$-\ \beta_t h_t^{\gamma} = 0.$$

This can be reorganized into the familiar marginal rate of substitution (MRS) condition:

$$(5)\quad \text{MRS} = \frac{MUL(h)}{MUC(h)}$$

$$= \frac{\beta_t h_t^{\gamma}}{[w_t(1-\tau)h_t + N_t]^{\eta}}$$

$$= w_t(1-\tau).$$

$$(6) \qquad e \equiv \left.\frac{\partial \ln h_{it}}{\partial \ln w_{it}}\right|_{N_{it}} = \frac{1 + \eta \cdot S}{\gamma - \eta \cdot S},$$

$$\text{where} \quad S \equiv \frac{w_t h_t (1 - \tau)}{w_t h_t (1 - \tau) + N_t}.$$

Here S is the share of earned income in total income. If nonlabor income is a small share of total income, then, to a good approximation, the Marshallian elasticity is simply $(1 + \eta)/(\gamma - \eta)$.

Next we use the Slutsky equation to decompose the Marshallian elasticity into separate substitution and income effects. Recall that the Slutsky equation is

$$\frac{\partial h}{\partial w} = \left.\frac{\partial h}{\partial w}\right|_u + h\frac{\partial h}{\partial N}. \tag{7}$$

It is convenient to write the Slutsky equation in elasticity form, so the Marshallian elasticity appears on the left-hand side. So we manipulate (7) to obtain

$$\frac{w}{h}\frac{\partial h}{\partial w} = \frac{w}{h}\frac{\partial h}{\partial w}\bigg|_u + \frac{wh}{N}\left[\frac{N}{h}\frac{\partial h}{\partial N}\right]. \tag{8}$$

Again applying implicit differentiation to (5), we obtain that the income elasticity is

$$e_I \equiv \left.\frac{\partial \ln h_{it}}{\partial \ln N_{it}}\right|_{w_{it}} = \frac{\eta}{\gamma - \eta \cdot S}(1 - S) \tag{9}$$

and hence the income effect is

$$\begin{aligned} ie &\equiv \frac{w_t h_t (1-\tau)}{N_t} e_I \\ &= \frac{S}{1-S} \frac{\eta}{\gamma - \eta \cdot S}(1-S) \\ &= \frac{\eta \cdot S}{\gamma - \eta \cdot S} < 0. \end{aligned} \tag{10}$$

Finally, using (6), (8), and (10), the Hicks elasticity is simply given by

$$e_H \equiv \left.\frac{\partial \ln h_{it}}{\partial \ln w_{it}}\right|_U = e - ie \tag{11}$$

$$= \frac{1}{\gamma - \eta \cdot S} > 0.$$

In the special case of $N_t = 0$, we can use (5) to obtain the labor supply equation:

$$(12)\quad \ln h_t = \frac{1+\eta}{\gamma-\eta}\ln[w_t(1-\tau)] - \frac{1}{\gamma-\eta}\ln \beta_t\,.$$

From (12) we can see directly that the Marshallian labor supply elasticity is given by

$$(13)\quad e = \frac{\partial \ln h_t}{\partial \ln w_t(1-\tau)} = \frac{1+\eta}{\gamma-\eta}.$$

3.2 *The Basic Dynamic Model with Savings*

$$V = U_1 + \rho U_2, \tag{14}$$

where ρ is the discount factor. Substituting C_1 and C_2 into (3) and then (14), we obtain

$$\begin{aligned} V = {} & \frac{[w_1 h_1(1 - \tau_1) + N_1 + b]^{1+\eta}}{1 + \eta} \\ & - \beta_1 \frac{h_1^{1+\gamma}}{1 + \gamma} \\ & + \rho\left\{\frac{[w_2 h_2(1 - \tau_2) + N_2 - b(1 + r)]^{1+\eta}}{1 + \eta} \right. \\ & \left. - \beta_2 \frac{h_2^{1+\gamma}}{1 + \gamma}\right\}. \end{aligned} \tag{15}$$

$$(16)\quad \frac{\partial V}{\partial h_1} = [w_1 h_1(1-\tau_1) + N_1 + b]^{\eta}$$

$$\times\ w_1(1-\tau_1) - \beta_1 h_1^{\gamma} = 0$$

$$(17)\quad \frac{\partial V}{\partial h_2} = [w_2 h_2(1-\tau_2) + N_2 - b(1+r)]^{\eta}$$

$$\times\ w_2(1-\tau_2) - \beta_2 h_2^{\gamma} = 0$$

$$(18)\quad \frac{\partial V}{\partial b} = [w_1 h_1(1-\tau_1) + N_1 + b]^{\eta}$$

$$-\ \rho[w_2 h_2(1-\tau_2) + N_2 - b(1+r)]^{\eta}$$

$$\times\ (1 + r) = 0.$$

Utilizing the intertemporal condition, we divide (17) by (16) and take logs to obtain

$$\ln\left(\frac{h_2}{h_1}\right) = \frac{1}{\gamma}\left\{\ln\frac{w_2}{w_1} + \ln\frac{(1-\tau_2)}{(1-\tau_1)} - \ln\rho(1+r) - \ln\frac{\beta_2}{\beta_1}\right\}. \tag{19}$$

From (19) we obtain:

$$\frac{\partial \ln(h_2/h_1)}{\partial \ln(w_2/w_1)} = \frac{1}{\gamma}. \tag{20}$$

There is an important relation between the Frisch, Hicks, and Marshallian elasticities:

$$(21)\quad \frac{1}{\gamma} > \frac{1}{\gamma - \eta \cdot S} > \frac{1 + \eta \cdot S}{\gamma - \eta \cdot S}$$

$$\Rightarrow \frac{1}{\gamma} > \frac{1}{\gamma - \eta} > \frac{1 + \eta}{\gamma - \eta} \quad \text{if } S = 1.$$

$$(22)\quad [w_t h_t(1-\tau_t)+N_t+b_t]^{\eta} w_t(1-\tau_t)$$

$$= \beta_t h_t^{\gamma} \qquad \Rightarrow \qquad \frac{\beta_t h_t^{\gamma}}{C_t^{\eta}} = w_t(1-\tau_t).$$

$$(23) \quad C_t^{\eta} = E_t \rho(1 + r_{t+1}) C_{t+1}^{\eta}$$

$$\Rightarrow \quad \rho(1 + r_{t+1}) C_{t+1}^{\eta} = C_t^{\eta}(1 + \xi_{t+1}),$$

$$\text{(24)} \quad \Delta \ln C_t^{\eta} = -\ln \rho(1 + r_{t+1}) + \xi_t.$$

Taking logs and differencing (22), and using (24) to substitute out for $\Delta \ln C_{it}^{\eta}$, we obtain

$$\text{(25)} \quad \Delta \ln h_t = \frac{1}{\gamma} \Delta \ln w_t + \frac{1}{\gamma} \Delta \ln(1 - \tau_t) - \frac{1}{\gamma} \ln \rho(1 + r_t) - \frac{1}{\gamma} \Delta \ln \beta_t + \frac{1}{\gamma} \xi_t.$$

4. *Econometric Issues in Estimating Labor Supply Elasticities*

$$\ln h_{it} = \beta + e \ln w_{it}(1 - \tau_t) + \beta_I \mathbf{N}_{it} + \varepsilon_{it}, \tag{26}$$

4.1 *Endogeneity of Wages and Nonlabor Income Arising from Correlation with Tasks for Work*

$$\varepsilon_{it} = \mu_i + \eta_{it}. \tag{27}$$

4.2 *Endogeneity Arising from Simultaneity*

$$w_{it} = p_t S(X_{it}). \tag{28}$$

$$\ln h_{it} = \beta + e \ln w_{it}(1 - \tau_t) + \beta_I \mathbf{N}_{it} + \beta_T \mathbf{Z}_{it} + \varepsilon_{it}. \tag{29}$$

$$
\begin{aligned}
(30)\quad \ln h_{it} = {} & \beta \\
& + e\{\ln p(F_t) + \ln S(\mathrm{Z}_{it}) + \ln(1 - \tau_{\mathrm{t}})\} \\
& + \beta_I N_{it} + \beta_T \mathrm{Z}_{it} + \varepsilon_{it} \\
& = \beta + e\{\ln p(F_t) + \ln(1 - \tau_t)\} \\
& \qquad + \beta_I N_{it} + \beta_T^{*} \mathrm{Z}_{it} + \varepsilon_{it}.
\end{aligned}
$$

4.3 *The Treatment of Taxes*

$$
\begin{aligned}
(31)\quad \ln h_{it} \;=\; & \beta_0 + \beta_w \ln w_{it}(1 - \tau_i(w_{it}, h_{it})) \\
& +\; \beta_I\, N_{it}(w_{it}, h_{it}) \;+\; \varepsilon_{it}.
\end{aligned}
$$

4.4 *Measurement Error in Wages and Nonlabor Income*

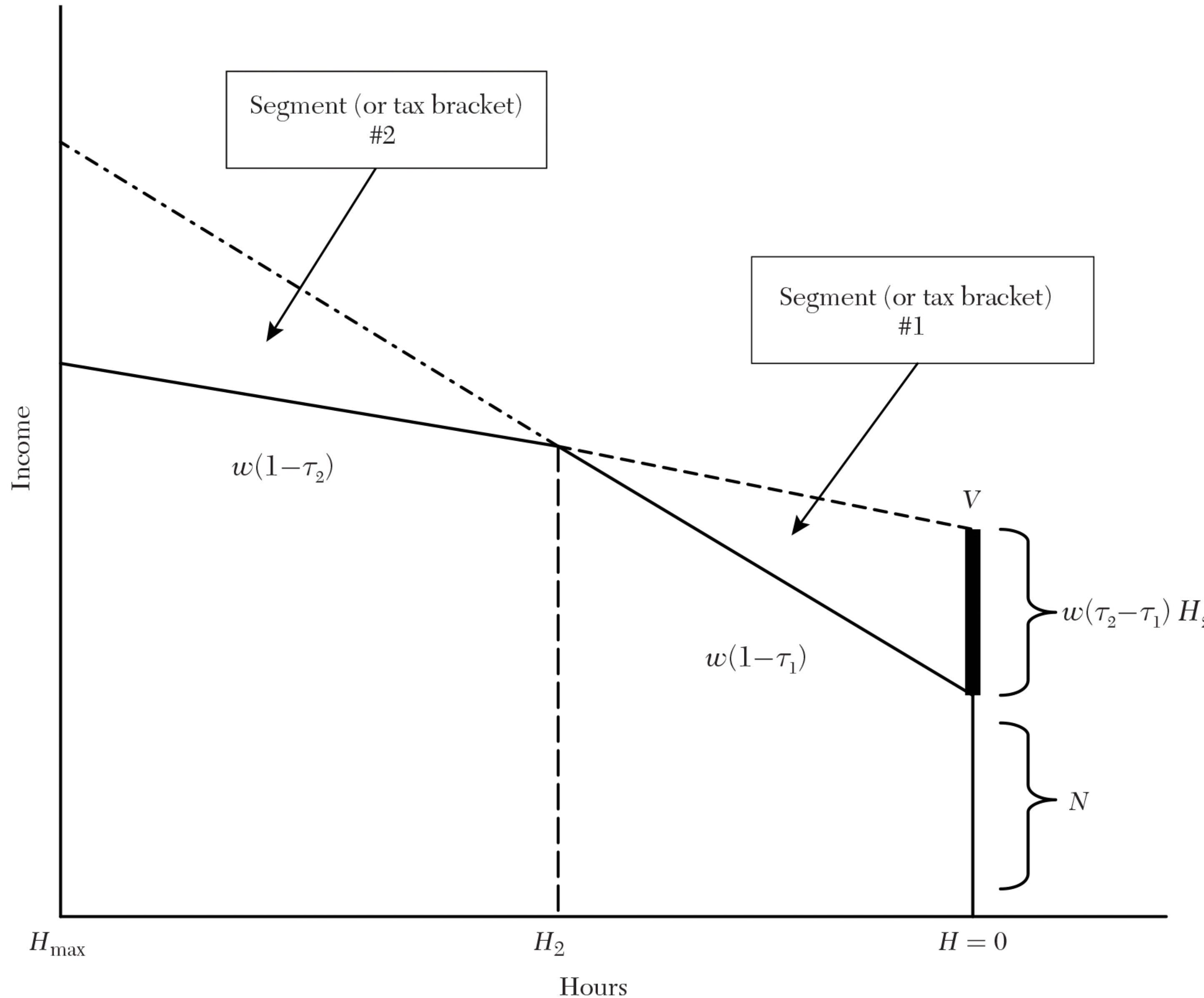

Figure 1. The Piecewise Linear Budget Constraint Created by Progressive Taxation

4.5 *The Problem that Wages Are Not Observed for Nonworkers*

4.6 *The Treatment of Nonlabor Income and Savings*

4.7 *Other Sources of Dynamics*

5. *A Roadmap to the Empirical Literature*

5.1 *The Male Labor Supply Literature*

5.2 *The Female Labor Supply Literature*

6. *A Survey of the Male Labor Supply Literature*

6.1 *A Summary of Results from Static Labor Supply Models*

6.1.1 *Attempts to Deal with Progressive Taxation (Piecewise-Linear Budget Constraints)*

6.1.1.1 *Studies Based on U.S. Data*

$$\ln h_i = \beta_0 + e \ln w_i(1 - \tau_i) + \beta_I \mathrm{V}_i + \varepsilon_i. \tag{32}$$

$$
\begin{aligned}
(33) \qquad \ln h_i^o &= \ln h_i + v_i \\
&= \ln h_i^* + \varepsilon_i + \nu_i ,
\end{aligned}
$$

$$
\begin{aligned}
(34) \quad & P(\ln h_i^o, \ln h_i > \ln H_2) \\
& = P(\varepsilon_i + v_i, \varepsilon_i + \ln h_i^* > \ln H_2) \\
& = f(\varepsilon_i + v_i | \varepsilon_i > \ln H_2 - \ln h_i^*) \\
& \quad \times P(\varepsilon_i > \ln H_2 - \ln h_i^*).
\end{aligned}
$$

$$(35)\quad h = \beta + \beta_w w(1 - \tau_2) + \beta_I V_2 + \varepsilon,$$

where τ_2 and V_2 are the tax rate and virtual income on segment #2, respectively. Plugging in the new values for the tax rate and virtual income we get

$$(35')\quad h' = \beta + \beta_w w(1 - \tau_2 - \Delta) + \beta_I(V_2 + \Delta w H_2) + \varepsilon.$$

6.1.1.2 *Studies Based on Non-U.S. Data*

6.1.2 *The NIT Experiments (Nonconvex Budget Constraints)*

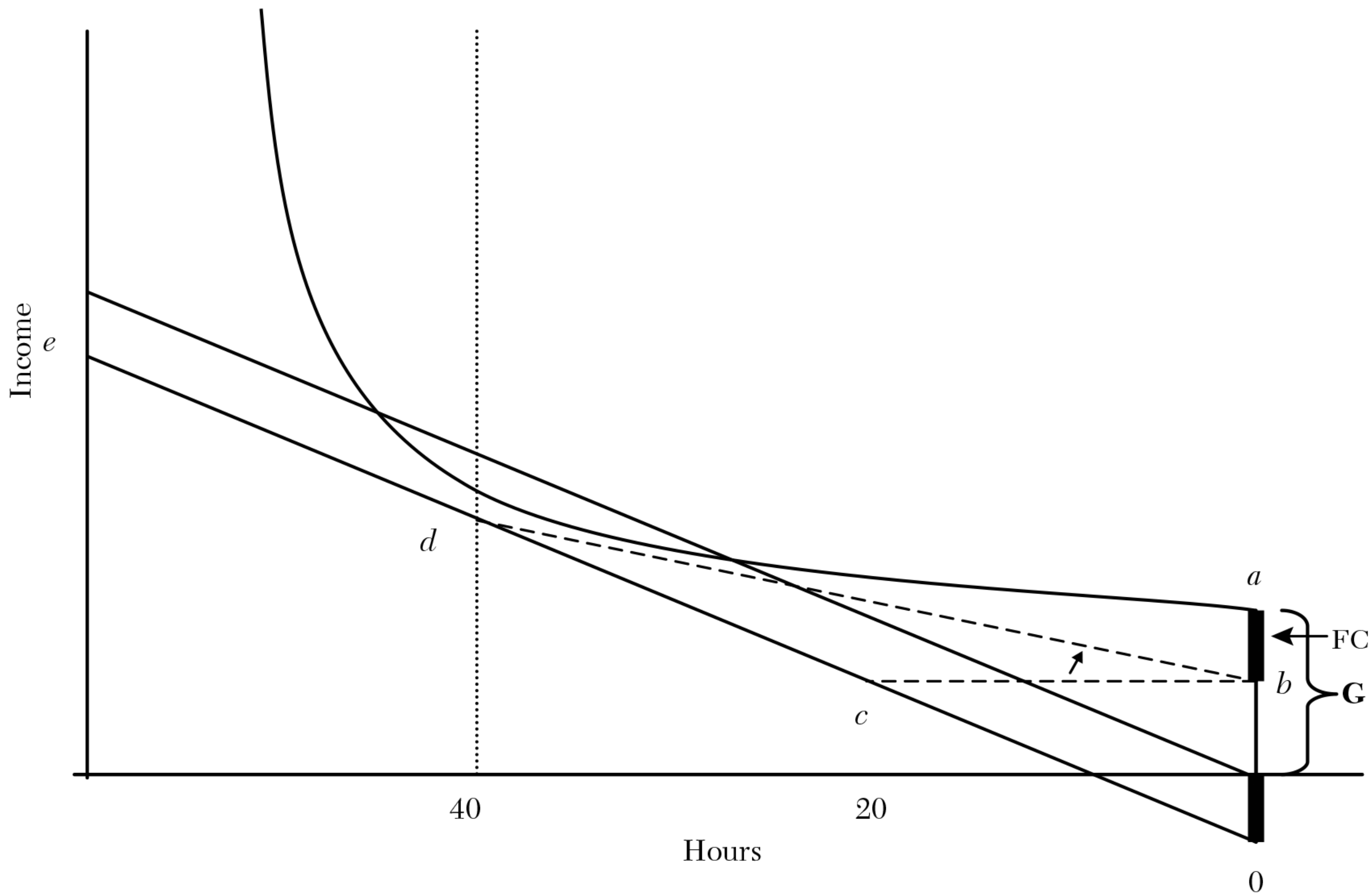

Figure 2. The Nonconvex Budget Constraint Created by NIT or AFDC type Programs

Note: The budget constraint created by the program goes through points *a*, *b*, *c*, *e*. It is generated by the program grant level (G), the fixed cost of working (FC) and the program tax rate, which render the constraint nonconvex. The straight line through the origin is the after-tax-wage line that would be the budget constraint in the hypothetical situation of a flat rate tax. The dotted line from *b* to *d* shows the shift in the budget constraint when the program tax rate on earnings is reduced to 50 percent.

$$
\begin{aligned}
(36) \quad \ln h_i \;=\; & \beta \;+\; e \ln w_i(1 - \tau_i(w_i, h_i)) \\
& +\; e_I\, N_i(w_i, h_i) \;+\; \tilde{\varepsilon}_i .
\end{aligned}
$$

Here e and e_I *would be* the Marshallian and the income elasticities in the hypothetical case of a person facing a linear budget constraint. Equation (36) implies the indirect utility function:

$$
v(w, N) \;=\; \exp(\beta_i)\, \frac{w^{1+e}}{1+e} \;+\; \frac{N^{1+e_I}}{1+e_I}.
$$

6.1.3 *A Brief Summary of the Literature up to 1990*

6.1.4 *The "Hausman–MaCurdy Controversy"*

$$(37a)\quad h = \beta + \beta_w w(1 - \tau_1) + \beta_I N + \varepsilon,$$

while, for a person located on segment #2, the labor supply equation is

$$(37b)\quad h = \beta + \beta_w w(1 - \tau_2) + \beta_I [N + w(\tau_2 - \tau_1)H_2] + \varepsilon.$$

$$\text{(38a)}\quad \beta + \beta_w w(1-\tau_2) + \beta_I[N + w(\tau_2-\tau_1)H_2] + \varepsilon < H_2$$

$$\text{(38b)}\quad \beta + \beta_w w(1-\tau_1) + \beta_I N + \varepsilon > H_2.$$

$$(38') \quad \varepsilon < H_2 - \beta - \beta_w w(1 - \tau_2) - \beta_I[N + w(\tau_2 - \tau_1)H_2] \equiv U(\varepsilon)$$

$$\varepsilon > H_2 - \beta - \beta_w w(1 - \tau_1) - \beta_I N \equiv L(\varepsilon).$$

$$-\beta_w w(1-\tau_2) - \beta_I[N + w(\tau_2-\tau_1)H_2]$$

$$> -\beta_w w(1-\tau_1) - \beta_I N,$$

which can be further simplified to

$$\beta_w[w(1-\tau_1) - w(1-\tau_2)] \quad (39)$$

$$- \beta_I w(\tau_2-\tau_1)H_2 > 0$$

or simply $\beta_w - \beta_I H_2 > 0$, which we can put in elasticity terms to obtain

$$(w/H_2)[\beta_w - \beta_I H_2] > 0. \quad (40)$$

$$(3') \quad U_t = \frac{[w_t h_t + N_t - \tau(w_t h_t)]^{1+\eta}}{1+\eta}$$

$$- \beta_t \frac{h_t^{1+\gamma}}{1+\gamma} \quad \eta \leq 0, \;\; \gamma \geq 0$$

$$(5') \quad MRS = \frac{MUL(h)}{MUC(h)}$$

$$= \frac{\beta_t \, h_t^{\gamma}}{[w_t \, h_t + N_t - \tau(w_t \, h_t)]^{\eta}}$$

$$= w_t(1 - \tau'(w_t \, h_t)).$$

6.1.4.1 *An Attempt to Resolve the Controversy—Eklöf and Sacklén (2000)*

TABLE 3
Eklöf and Sacklén (2000) Analysis of Hausman versus MaCurdy–Green–Paarsch (M–G–P)

Wage measure	Nonlabor income measure	Sample selection criteria	Hours measure	Coefficient on: Wage	Coefficient on: Nonlabor income	Marshall elasticity	Income effect	Hicks elasticity
M–G–P	M–G–P	M–G–P	M–G–P	0.0	–0.011	0.000	–0.068	0.068
M–G–P	M–G–P	Hausman	M–G–P	0.0	–0.022	0.000	–0.136	0.136
M–G–P	Hausman	M–G–P	M–G–P	0.0	–0.079	0.000	–0.488	0.488
Hausman	M–G–P	M–G–P	M–G–P	10.3	–0.004	0.030	–0.025	0.055
Hausman	Hausman	M–G–P	M–G–P	26.5	n/a	0.078	n/a	n/a
Hausman	Hausman	Hausman	M–G–P	26.9	–0.036	0.078	–0.222	0.300
Hausman	Hausman	Hausman	Hausman	16.4	–0.036	0.048	–0.222	0.270
Hausman's reported results				0.2	–0.120	0.000	–0.740	0.740

Notes: For the sake of comparability all elasticities and income effects are calculated using the mean wage of $6.18 and the mean hours 2,123 from Hausman (1981). In the authors' attempt to replicate Hausman's data set, the corresponding figures are 6.21 and 2,148. The mean values of both hours and wages are a bit higher in the MaCurdy et al. data set, but this makes little difference for the calculations. For the random nonlabor income coefficient, the table reports the median.

6.1.4.2 *A Nonparametric Approach—Blomquist, Eklöf, and Newey (2001)*

6.1.5 *A Look at the Distribution of Hours of Work*

6.1.6 *Summary of the Static Literature on Male Labor Supply*

6.2 *The Life-Cycle Labor Supply Model with Savings*

6.2.1 *Simple Methods for Estimating the Life-Cycle Labor Supply Model—MaCurdy (1983)*

$$\frac{\beta_t h_t^{\gamma}}{[w_t(1-\tau_t)h_t + N_t + b_t]^{\eta}} = \frac{\beta_t h_t^{\gamma}}{[C_t]^{\eta}} = w_t(1-\tau_t). \tag{41}$$

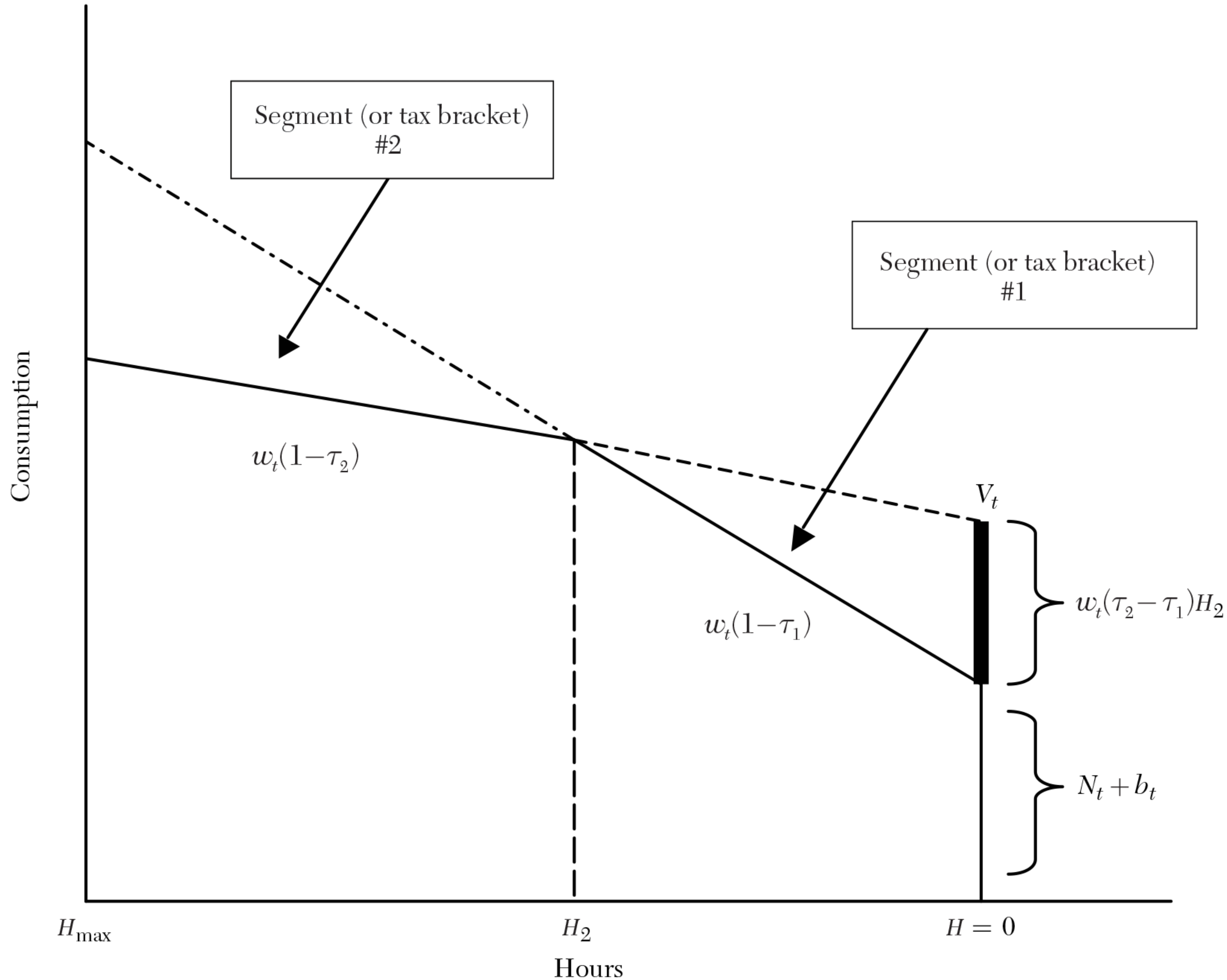

Figure 3: The Budget Constraint Created by Progressive Taxation in the Presence of Saving

$$C_t = w_t(1 - \tau_2)h_t + V_t, \tag{42a}$$

where "virtual" nonlabor income V_t is given by

$$V_t = w_t(\tau_2 - \tau_1)H_2 + N_t + b_t. \tag{42b}$$

$$\beta_{it} = \exp(X_{it}\,\alpha - \varepsilon_{it}). \tag{43}$$

Here X_{it} represents *observed* characteristics of person i that shift tastes for consumption versus leisure, and ε_{it} represents unobserved taste shifters. Now, taking logs of (41), and putting i subscripts on all variables to indicate person specific values, we get

$$\ln w_{it}(1 - \tau_{it}) = \gamma \ln h_{it} - \eta \ln C_{it} + X_{it}\,\alpha - \varepsilon_{it}. \tag{44}$$

$$V_t = C_t - w_t(1 - \tau_t)h_t. \tag{45}$$

where τ_t denotes the tax rate for the segment on which the person locates at time t. Thus, MaCurdy suggests estimating labor supply equations of the form

$$h_{it} = h(w_{it}(1 - \tau_{it}), V_{it}, X_{it}). \tag{46}$$

$$\ln h_{it} = \underset{(0.53)}{0.69} \ln w_{it}(1 - \tau_{it}) - \underset{(0.0010)}{0.0016}\, V_{it}$$

$$+ X_{it}\,\alpha + \varepsilon_{it},$$

while for the linear specification he obtains

$$h_{it} = \underset{(13.8)}{19.4}\, w_{it}(1 - \tau_{it}) - \underset{(0.07)}{0.16}\, V_{it}$$

$$+ X_{it}\,\alpha + \varepsilon_{it},$$

$$(47)\quad \ln h_{it} = \frac{1}{\gamma} \ln w_{it}(1 - \tau_{it}) + \frac{\eta}{\gamma} \ln C_{it} - X_{it}\frac{\alpha}{\gamma} + \frac{\varepsilon_{it}}{\gamma}.$$

$$(48)\quad F_t = w_t(1-\tau_t)T + N_t + b_t$$

$$= w_t(1-\tau_t)(T-h_t) + C_t,$$

where F_t is full income, T is total time in a period and $T - h_t$ is leisure.[59] One then estimates a labor supply function that conditions on the full income allocated to period t:

$$(49)\quad h_{it} = h(w_{it}(1-\tau_{it}), F_{it}, X_{it}).$$

$$U_t = G\left[\frac{F_t - a(w_t(1 - \tau_t))}{b(w_t(1 - \tau_t))}\right]. \tag{50}$$

$$(51) \qquad \frac{\partial U_t}{\partial w_t(1-\tau_t)} = h_t \cdot \frac{\partial U_t}{\partial F_t}.$$

Applying (51) to (50), we can obtain the labor supply equation

$$(52) \quad h_t = \frac{G'(\cdot)\left\{\frac{-b'(\cdot)}{b^2(\cdot)}[F_t - a(\cdot)] - \frac{a'(\cdot)}{b(\cdot)}\right\}}{G'(\cdot)\frac{1}{b(\cdot)}}$$

$$= -a'(w_t(1-\tau_t)) - \frac{b'(w_t(1-\tau_t))}{b(w_t(1-\tau_t))} \times [F_t - a(w_t(1-\tau_t))].$$

$$(53)\quad h_t = (T_m - \gamma_m) - \frac{\beta_m}{w_t^m} \times [F_t - \gamma_m w_t^m - \gamma_f w_t^f - \gamma_c p_t^c - 2\gamma_{fc}(w_t^f p_t^c)^{1/2}].$$

6.2.2 *Progressive Taxation in the Life-Cycle Model—Ziliak and Kniesner (1999)*

$$(54)\quad h_{it} = \beta + \beta_w w_{it}(1 - \tau_{it}) + \beta_{A1} A^*_{i,t-1} + \beta_{A2} A_{i,t} + X_{it}\alpha + \mu_i + \varepsilon_{it}.$$

In this equation, A^*_{t-1} is "virtual wealth," which plays a role analogous to virtual nonlabor income in static piecewise-linear budget constraint models (see figure 1). It is defined as

$$(55)\quad A^*_{t-1} = A_{t-1} + \frac{(\tau_{it} - \tau^A_{it}) I_{it}}{r_t},$$

$$
(56) \quad \Delta h_{it} = \beta_w \Delta w_{it}(1 - \tau_{it}) + \beta_{A1} \Delta A^*_{i,t-1} + \beta_{A2} \Delta A_{it} + \Delta X_{it}\alpha + \Delta\varepsilon_{it}.
$$

6.2.3 *Nonseparability between Consumption and Leisure—Ziliak and Kniesner (2005)*

$$(57) \quad U_t = G\left[\frac{C_t^{1+\eta}}{1+\eta} - \beta_t \frac{h_t^{1+\gamma}}{1+\gamma}\right]$$

$$\eta \leq 0, \;\; \gamma \geq 0$$

Assume that $G[\cdot]$ is a concave function, such as $G[X] = \log(X)$ or $G[X] = (1+\sigma)^{-1}X^{1+\sigma}$ for $\sigma \leq 0$. Notice that now the marginal utility of consumption is given by

$$(58) \quad \frac{\partial U_t}{\partial C_t} = G_t'(X_t) \cdot C_t^{\eta} \equiv \lambda_t,$$

$$\text{where} \quad X_t \equiv \frac{C_t^{1+\eta}}{1+\eta} - \beta_t \frac{h_t^{1+\gamma}}{1+\gamma}.$$

$$(59) \quad G_t'(X_{it})[C_{it}]^{\eta}$$

$$= E_t \, \rho(1 + r)\left\{\mathrm{G'}_{\mathrm{t}}(X_{i,t+1})[C_{i,t+1}]^{\eta}\right\} \quad \eta \leq 0.$$

$$(60) \qquad \frac{\partial V}{\partial h_t} = \{G'(X_t)C_t^{\eta}\}w_t(1-\tau_t)$$

$$- \{G'(X_t)\beta_t h_t^{\gamma}\} = 0$$

$$\Rightarrow C_t^{\eta} w_t(1-\tau_t) = \beta_t h_t^{\gamma}.$$

$$\text{(61)} \quad \ln h_t = \frac{1}{\gamma}\Big\{\ln w_t + \ln(1-\tau_t) + \ln \lambda_t - \ln G'(X_t) - \ln \beta_t\Big\}.$$

$$(61') \quad \ln h_t = \frac{1}{\gamma}\{\ln w_t + \ln(1 - \tau_t) + \ln \lambda_t - \sigma \ln X_t - \ln \beta_t\}.$$

$$(62)\quad e_F = \left.\frac{\partial \ln h_t}{\partial \ln w_t}\right|_{\lambda_t \textit{ fixed}}$$

$$= \frac{1}{\gamma}\left\{\frac{X_t + \left(\frac{\sigma}{\eta}\right)C_t^{1+\eta}}{X_t + \left(\frac{\sigma}{\eta}\right)C_t^{1+\eta} - \sigma\left(\frac{\beta}{\gamma}\right)h_t^{1+\gamma}}\right\}.$$

TABLE 4
How Frisch Elasticity Varies with Willingness to Substitute Utility Over Time

σ	Frisch elasticity	Changes in hours		Changes in consumption		Changes in utility	
		Hours(1)	Hours(2)	C(1)	C(2)	G(X(1))	G(X(2))
0.0	2.00	+1.03%	−0.96%	+0.97%	+0.97%	−0.05%	+1.44%
−0.5	1.40	+0.82%	−0.58%	+1.18%	+0.58%	+0.27%	+0.87%
−1.0	1.25	+0.76%	−0.48%	+1.24%	+0.48%	+0.38%	+0.72%
−2.0	1.14	+0.73%	−0.41%	+1.27%	+0.42%	+0.41%	+0.62%
−5.0	1.06	+0.70%	−0.36%	+1.30%	+0.36%	+0.45%	+0.54%
−10.0	1.03	+0.69%	−0.34%	+1.31%	+0.34%	+0.46%	+0.51%
−40.0	1.01	+0.68%	−0.33%	+1.32%	+0.33%	+0.48%	+0.49%

$$
\begin{aligned}
(63) \quad U_t = G\Big[&\alpha_1 \ln(\overline{L} - h_t) + \alpha_2 \ln C_t \\
&- \alpha_3 \ln(\overline{L} - h_t)\ln C_t - \alpha_4 \\
&\times [\ln(\overline{L} - h_t)]^2 - [\ln C_t]^2\Big],
\end{aligned}
$$

TABLE 5
Labor Supply Elasticities Based on Alternative Consumption Measures

Consumption measure	Marshall	Hicks	Income effect	Frisch
Blundell et al (2001)	−0.468	0.328	−0.796	0.535
Skinner (1987)	−0.313	0.220	−0.533	0.246
PSID unadjusted	−0.442	0.094	−0.536	0.148

6.2.4 *Methods for Estimating the Frisch Elasticity—MaCurdy (1981)*

$$\Delta \ln h_{it} = \frac{1}{\gamma}\Delta \ln w_{it}(1-\tau_{it}) - \frac{1}{\gamma}\ln \rho(1+r_t) - \frac{\alpha}{\gamma}\Delta X_{it} + \frac{1}{\gamma}\xi_{it} + \frac{1}{\gamma}\Delta\varepsilon_{it}. \tag{64}$$

6.2.5 *Attempts to Deal with Measurement Error and Weak Instruments*

$$
(65)\quad \overline{\ln h_{it}} - \overline{\ln h_{i,t-1}}
$$

$$
= \frac{1}{\gamma}\left\{\overline{\ln w_{it}(1-\tau_t)} - \overline{\ln w_{i,t-1}(1-\tau_{t-1})}\right\}
$$

$$
+ f(t) + \overline{\zeta_{it}} + \frac{1}{\gamma}\overline{(\varepsilon_{it} - \varepsilon_{i,t-1})} + e_t ,
$$

$$
\begin{aligned}
(66)\quad h_{it} - h_{i,t-1} \;=\; & \beta\left\{\ln w_{it} - \ln w_{i,t-1}\right\} \\
- \; & \beta \ln \rho(1 + r_{it}) \\
- \; & \alpha\left\{X_{it} - X_{i,t-1}\right\} \\
+ \; & \zeta_{it} + (\varepsilon_{it} - \varepsilon_{i,t-1}).
\end{aligned}
$$

$$
\begin{aligned}
(67)\quad h_{ct} - h_{c,t-1} \;=\; & \beta\left\{\ln w_{ct} - \ln w_{c,t-1}\right\} \\
& - \;\beta \ln \rho(1 + r_t) \\
& - \;\alpha\left\{X_{ct} - X_{c,t-1}\right\} \\
& + \;\zeta_{ct} \;+\; (\varepsilon_{ct} - \varepsilon_{c,t-1}).
\end{aligned}
$$

6.2.6 *The Problem of Aggregate Shocks—Altug and Miller (1990)*

$$(68)\quad \Delta \ln h_{it} = \frac{1}{\gamma}\Delta \ln w_{it}(1-\tau_t) + D_t - \frac{\alpha}{\gamma}\Delta X_{it} + (\zeta_{it} - \overline{\zeta}_{it}) + \frac{1}{\gamma}(\varepsilon_{it} - \varepsilon_{i,t-1}),$$

$$(23') \quad [C_{it}]^{\eta} = E_t\, \rho(1 + r_{t+1})[C_{i,t+1}]^{\eta} \quad \eta \leq 0.$$

$$\lambda_{it} = \mu_i \lambda_t. \tag{69}$$

$$(70) \quad \lambda_{it} = E_t\,\rho(1 + r_{t+1})\lambda_{i,t+1}$$

$$\Rightarrow \quad \mu_i\,\lambda_{\mathrm{t}} = \rho\,\mu_i\,E_t(1 + r_{t+1})\lambda_{t+1}$$

$$\Rightarrow \quad \lambda_t = \rho\,E_t(1 + r_{t+1})\lambda_{t+1}.$$

Then (23) and (24) become

$$(71) \quad \rho(1 + r_{t+1})\lambda_{t+1} = \lambda_t(1 + \xi_{t+1})$$

$$\Rightarrow \quad \Delta \ln \lambda_t \approx -\ln \rho(1 + r_t) + \xi_t.$$

$$(72)\quad \Delta\ln h_{it} = \frac{1}{\gamma}\Delta\ln w_{it}(1-\tau_{it}) - \frac{\alpha}{\gamma}\Delta X_{it} - \frac{1}{\gamma}[\ln\rho(1+r_t)] + \frac{1}{\gamma}\xi_t + \frac{1}{\gamma}(\varepsilon_{it} - \varepsilon_{i,t-1}).$$

$$
\begin{aligned}
(73)\quad \Delta \ln h_{it} \;=\; & \frac{1}{\gamma}\,\Delta \ln w_{it}(1-\tau_{it}) \\
& -\; \frac{\alpha}{\gamma}\,\Delta X_{it} \\
& -\; \frac{1}{\gamma}[\ln \lambda_t - \ln \lambda_{t-1}] \\
& +\; \frac{1}{\gamma}\,\Delta \varepsilon_{it}.
\end{aligned}
$$

$$(74)\quad \ln l_{it}^{h} = \frac{1}{\tilde{\gamma}} \ln w_{it} + \frac{\eta}{\tilde{\gamma}} \ln C_{it} - \frac{\alpha}{\tilde{\gamma}} X_{it} + \frac{\pi}{\tilde{\gamma}} \ln l_{it}^{s} + \frac{\varepsilon_{it}}{\tilde{\gamma}},$$

$$
\begin{aligned}
\frac{\partial \ln h}{\partial \ln w} &= \frac{w}{h}\,\frac{\partial h}{\partial w} = \frac{w}{h}\,\frac{\partial(1-l)}{\partial w} \\
&= -\frac{w}{h}\,\frac{\partial l}{\partial w} = -\frac{w}{h}\,\frac{l}{w}\left[\frac{w}{l}\,\frac{\partial l}{\partial w}\right] \\
&= -\frac{l}{h}(-\,0.037) \approx \frac{8760}{2300}(0.037) \\
&= 0.14.
\end{aligned}
$$

6.2.7 *A New Approach: Measuring Expectations—Pistaferri (2003)*

$$
\begin{aligned}
(75)\quad \zeta_{it} &\equiv \frac{1}{\gamma}\,\xi_{it} \\
&\equiv \frac{1}{\gamma}\{\ln \lambda_{it} - E_{t-1} \ln \lambda_{i,t-1}\} \\
&= \frac{1}{\gamma}\,\frac{d \ln \lambda_{it}}{d\,\psi_{it}}\{\Delta \ln w_{it} - E_{t-1}\Delta \ln w_{it}\},
\end{aligned}
$$

where I have defined $\psi_{it} \equiv \{\Delta \ln w_{it} - E_{t-1}\Delta \ln w_{it}\}$, the unexpected wage change from $t-1$ to t.

$$
\begin{aligned}
(76)\quad \Delta \ln h_{it} &= \frac{1}{\gamma}(\Delta \ln w_{it}) \\
&- \frac{\alpha}{\gamma}\Delta X_{it} - \frac{1}{\gamma}\ln \rho(1+r_t) \\
&+ \left[\frac{1}{\gamma}\,\frac{d\ln \lambda_{it}}{d\,\psi_{it}}\right] \\
&\times \{\Delta \ln w_{it} - E_{t-1}\Delta \ln w_{it}\} \\
&+ \frac{\Delta \varepsilon_{it}}{\gamma}.
\end{aligned}
$$

$$\ln w_{it} = \ln w_{i,t-1} + X_{i,t-1}\,\theta + \psi_{it} \tag{78}$$

$$E_{t-1}\,\psi_{it} = 0,$$

$$(79) \quad \ln \lambda_{it} = \Gamma_{at} A_{it} + \Gamma_{0t} \ln w_{it} + \sum_{\tau=t+1}^{T} \Gamma_{\tau-t,t} E_t \ln w_{i,\tau}.$$

$$
\begin{aligned}
(80)\quad \ln \lambda_{it} - E_{t-1} \ln \lambda_{it} &= \Gamma_a [A_{it} - E_{t-1} A_{it}] \\
&+ \Gamma_0 \{\ln w_{it} - E_{t-1} \ln w_{it}\} \\
&+ \sum_{\tau=t+1}^{T} \Gamma_{\tau-t} \{E_t \ln w_{i,\tau} - E_{t-1} \ln w_{i,\tau}\} \\
&= \Gamma_a [A_{it} - E_{t-1} \mathrm{A}_{it}] + \Gamma_0 \{\psi_{it}\} \\
&+ \sum_{\tau=t+1}^{T} \Gamma_{\tau-t} \{\psi_{it}\} \quad = \quad \Gamma_a \cdot 0 + \Gamma \cdot \psi_{it},
\end{aligned}
$$

where I have suppressed the time subscripts on the Γ to conserve on notation.

$$(81)\quad \Delta \ln h_{it} = \frac{1}{\gamma}(E_{t-1}\Delta \ln w_{it}) - \frac{\alpha}{\gamma}\Delta X_{it}$$

$$- \frac{1}{\gamma}\ln \rho(1 + r_t)$$

$$+ \left[\frac{1}{\gamma} + \frac{\Gamma}{\gamma}\right]$$

$$\times \{\Delta \ln w_{it} - E_{t-1}\Delta \ln w_{it}\}$$

$$+ \frac{\Delta\varepsilon_{it}}{\gamma}.$$

6.2.8 *Progressive Taxation and Tied Wage–Hours Offers—Aaronson and French (2009)*

6.3 *The Life-Cycle Model with Both Human Capital and Savings*

$$w_2 = w_1(1 + \alpha h_1), \tag{82}$$

where α is the percentage growth in the wage per unit of work. Once we introduce human capital accumulation via work experience as in (82), equation (15) is replaced by

$$\begin{aligned} V = {} & \frac{[w_1 h_1(1-\tau_1) + b]^{1+\eta}}{1+\eta} \\ & - \beta_1 \frac{h_1^{1+\gamma}}{1+\gamma} \\ + \rho \Bigg\{ & \frac{[w_1(1+\alpha h_1) h_2(1-\tau_2) - b(1+r)]^{1+\eta}}{1+\eta} \\ & - \beta_2 \frac{h_2^{1+\gamma}}{1+\gamma} \Bigg\}, \end{aligned} \tag{83}$$

$$(84)\quad \frac{\partial V}{\partial h_1} = C_1^{\eta} w_1 (1-\tau_1) - \beta_1 h_1^{\gamma}$$

$$+ \ \rho C_2^{\eta} w_1 \, \alpha \, h_2 (1-\tau_2) = 0$$

$$(85)\quad \frac{\partial V}{\partial h_2} = C_2^{\eta} w_1 (1+\alpha \, h_1)(1-\tau_2)$$

$$- \beta_2 h_2^{\gamma} = 0$$

$$(86)\quad \frac{\partial V}{\partial b} = C_1^{\eta} - \rho(1+r) C_2^{\eta} = 0.$$

$$
\begin{aligned}
MRS &= \frac{MU_L}{MU_C} = \frac{\beta h_1^{\gamma}}{C_1^{\eta}} \\
&= w_1(1-\tau_1) + \frac{\rho C_2^{\eta}}{C_1^{\eta}} \alpha w_1 h_2 (1-\tau_2.)
\end{aligned}
$$

This can be simplified by using (86) to eliminate $C_2^{\eta}/C_1^{\eta} = [\rho(1+r)]^{-1}$, giving

$$
(87) \quad \frac{\beta h_1^{\gamma}}{C_1^{\eta}} = w_1(1-\tau_1) + \frac{\alpha w_1 h_2 (1-\tau_2)}{1+r}.
$$

6.3.1 *Early Attempts to Include Human Capital—Heckman (1976), Shaw (1989)*

$$(88) \quad \frac{\beta h_t^{\gamma}}{C_t^{\eta}} = w_t(1 - \tau_t) + E_t \sum_{\tau=0}^{T-t} \frac{(\alpha w_1) h_{t+1+\tau}(1 - \tau_{t+1+\tau})}{(1+r)^{1+\tau}},$$

$$(88') \quad \frac{\beta h_t^{\gamma}}{C_t^{\eta}} = w_t(1-\tau_t) + \frac{(\alpha w_1)h_{t+1}(1-\tau_{t+1})}{(1+r)} + E_t \sum_{\tau=0}^{T-t-1} \frac{(\alpha w_1)h_{t+2+\tau}(1-\tau_{t+2+\tau})}{(1+r)^{2+\tau}}.$$

Now take (88) and date it forward one period:

$$(88'') \quad \frac{\beta h_{t+1}^{\gamma}}{C_{t+1}^{\eta}} = w_{t+1}(1-\tau_{t+1}) + E_{t+1} \sum_{\tau=0}^{T-t-1} \frac{(\alpha w_1)h_{t+2+\tau}(1-\tau_{t+2+\tau})}{(1+r)^{1+\tau}}.$$

$$(89)\quad E_t\left\{\frac{1}{1+r}\left[\frac{\beta h_{t+1}^{\gamma}}{C_{t+1}^{\eta}} - w_{t+1}(1-\tau_{t+1})\right]\right\}$$

$$= E_t \sum_{\tau=0}^{T-t-1} \frac{(\alpha w_1)h_{t+2+\tau}(1-\tau_{t+2+\tau})}{(1+r)^{2+\tau}}.$$

$$(90)\quad \frac{\beta h_t^{\gamma}}{C_t^{\eta}} = w_t(1-\tau_t) + \frac{(\alpha w_1)h_{t+1}(1-\tau_{t+1})}{(1+r)} + E_t\left\{\frac{1}{1+r}\left[\frac{\beta h_{t+1}^{\gamma}}{C_{t+1}^{\eta}} - w_{t+1}(1-\tau_{t+1})\right]\right\}.$$

$$
\begin{aligned}
(91)\quad \frac{\beta h_t^{\gamma}}{C_t^{\eta}} &= w_t(1-\tau_t) \\
&+ \frac{(\alpha w_1) h_{t+1}(1-\tau_{t+1})}{(1+r)} \\
&+ \frac{1}{1+r}\left[\frac{\beta h_{t+1}^{\gamma}}{C_{t+1}^{\eta}} - w_{t+1}(1-\tau_{t+1})\right] + \xi_{t+1}.
\end{aligned}
$$

$$K_{i,t+1} = \alpha_1 K_{it} + \alpha_2 K_{it}^2 + \alpha_3 K_{it} h_{it} + \alpha_4 h_{it} + \alpha_5 h_{it}^2 + \tau_t + \varepsilon_{it}. \tag{92}$$

$$w_{it} = R_t K_{it} \quad \Rightarrow \quad K_{it} = w_{it}/R_t. \tag{93}$$

Given (92′), the derivative of human capital with respect to hours of work is

$$\frac{\partial K_{i,t+1}}{\partial h_{it}} = \alpha_3 K_{it} + \alpha_4 + 2\alpha_5 h_{it}.$$

The estimates are $\alpha_3 = 0.30$, $\alpha_4 = –3.55$, and $\alpha_5 = 0.69$. To interpret these figures, let $R = 1$, and note that mean hours in the data is 2,160 while the mean wage rate is \$3.91. Then, noting that h_{it} is defined as hours divided by 1,000, we have, at the mean of the data

$$\frac{\partial K_{i,t+1}}{\partial h_{it}} = (0.30)(3.91) - 3.55$$

$$+ 2(0.69)(2.16) = 0.60.$$

6.3.2 *Full Solution Estimation with Human Capital and Assets—Imai and Keane (2004)*

$$K_{i,t+1} = (1 + \alpha\, h_{it})K_{it}. \tag{94}$$

$$w_{it} = RK_{it}(1 + \varepsilon_{it}). \tag{95}$$

6.3.2.1 *How to Solve the Dynamic Programming Problem—A Simple Exposition*

$$(96) \quad V_t(K_t, A_t, \varepsilon_t, \beta_t)$$

$$= \left[\frac{{C_t}^{1+\eta}}{1+\eta} - \beta_t \frac{h_t^{1+\gamma}}{1+\gamma}\right]$$

$$+ E_t\left\{\sum_{\tau=t+1}^{T} \rho^{\tau-t}\left[\frac{C_\tau^{1+\eta}}{1+\eta} - \beta_\tau \frac{h_\tau^{1+\gamma}}{1+\gamma}\right] \middle| (K_{t+1}, A_{t+1})\right\}.$$

$$
(97) \quad V_T(K_T, A_T, \varepsilon_T, \beta_\mathrm{T}) = \max_{C_T, h_T} \left\{ \frac{{C_T}^{1+\eta}}{1+\eta} - \beta_T \frac{h_T^{1+\gamma}}{1+\gamma} \right\}.
$$

$$\frac{\beta_T h_T^{\gamma}}{[w_T h_T(1-\tau_T)+A_T]^{\eta}} = w_T(1-\tau), \tag{98}$$

Denote the interpolating function that approximates $Emax_T(K_T, A_T)$ by

$$\pi_T(K_T, A_T)$$

$$\approx E_{T-1}\{V_T(K_T, A_T, \varepsilon_T, \beta_T) \,|\, (K_T, A_T)\}$$

$$where \qquad \frac{\partial \pi_T}{\partial K_T} > 0, \frac{\partial \pi_T}{\partial A_T} > 0.$$

$$\pi_T(K_T, A_T) = \pi_{T0} + \pi_{T1} \ln K_T + \pi_{T2} \ln A_T. \tag{99}$$

$$
\begin{aligned}
&V_{T-1}(K_{T-1}, A_{T-1}, \varepsilon_{T-1}, \beta_{T-1}) \\
&\quad = \max_{C_{T-1}, h_{T-1}} \left\{ \left[\frac{C_{T-1}^{1+\eta}}{1+\eta} - \beta_{T-1} \frac{h_{T-1}^{1+\gamma}}{1+\gamma} \right] \right. \\
&\quad + \rho E_{T-1} \left. \left\{ \left[\frac{C_T^{1+\eta}}{1+\eta} - \beta_T \frac{h_T^{1+\gamma}}{1+\gamma} \right] \middle| (K_T, A_T) \right\} \right\}.
\end{aligned}
$$

$$(102) \quad V_{T-2}(K_{T-2}, A_{T-2}, \varepsilon_{T-2}, \beta_{T-2}) \approx \max_{C_{T-2}, h_{T-2}} \left\{ \left[\frac{C_{T-2}^{1+\eta}}{1+\eta} - \beta_{T-2} \frac{h_{T-2}^{1+\gamma}}{1+\gamma} \right] + \rho \pi_{T-1}(K_{T-1}, A_{T-1}) \right\}.$$

6.3.2.2 *Empirical Results—Imai and Keane (2004)*

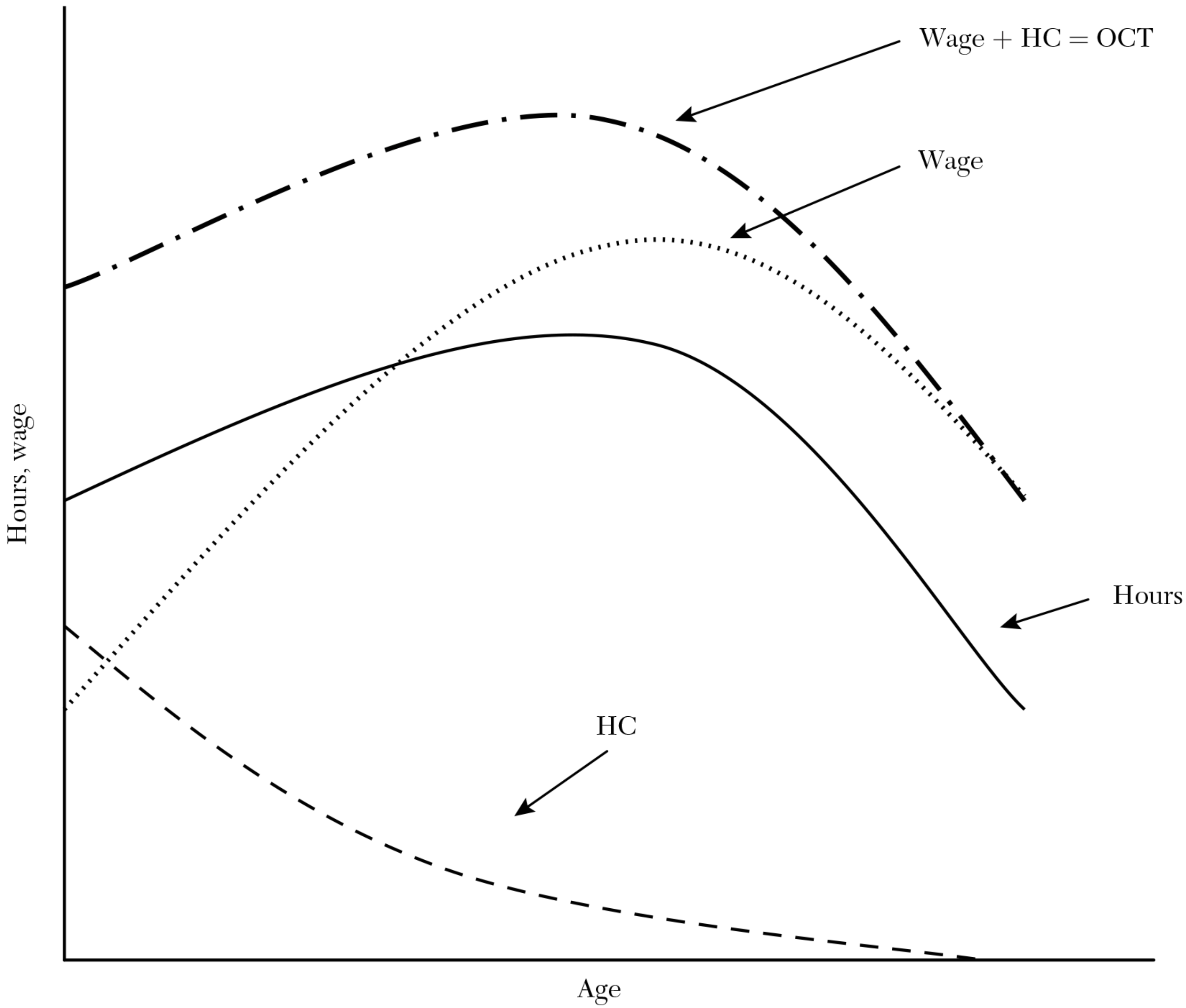

Figure 4. Hours, Wages, and Price of Time over the Life Cycle

Note: HC denotes the return to an hour of work experience, in terms of increased present value of future wages. The opportunity cost of time is Wage + HC.

6.3.2.3 *Assessing the Credibility of the Imai–Keane (2004) Results*

$$\left(\frac{h_2}{h_1}\right)^{\gamma}$$

$$= \frac{\beta_1}{\beta_2}\frac{w_1(1+\alpha h_1)(1-\tau_2)}{\rho(1+r)w_1(1-\tau_1)+\rho\alpha w_1 h_2(1-\tau_2)}.$$

To obtain a more intuitive expression, I set $w_2 = w_1(1+\alpha h_1)$, assume $\tau_1 = \tau_2 = \tau$, and take logs

$$(103) \quad \ln\left(\frac{h_2}{h_1}\right) = \left(\frac{1}{\gamma}\right)\left\{\ln\left(\frac{w_2}{w_1}\right)\right.$$

$$\left. - \ln\left(1+\frac{\alpha h_2}{1+r}\right) - \ln\rho(1+r) - \ln\left(\frac{\beta_2}{\beta_1}\right)\right\}.$$

$$
(104)\quad \frac{1}{\gamma} = \ln\left(\frac{h_2}{h_1}\right) \div \left[\ln\left(\frac{w_2}{w_1\left(1+\frac{\alpha h_2}{1+r}\right)}\right)\right].
$$

6.3.2.4 *Implications of the Imai–Keane Model for Effects of Wage and Tax Changes*

6.3.3 *Education, Experience, Saving, and Participation—Keane and Wolpin (2001)*

6.3.4 *Efficiency Costs of Taxation in a Life-Cycle Model with Human Capital*

$$
\begin{aligned}
V = \lambda f(P) &+ \frac{[w_1 h_1(1-\tau)+b]^{1+\eta}}{1+\eta} \\
&- \beta \frac{h_1^{1+\gamma}}{1+\gamma} \\
+ \rho \Bigg\{ \lambda f(P) & \\
&+ \frac{[w_2 h_2(1-\tau)-b(1+r)]^{1+\eta}}{1+\eta} \\
&- \beta \frac{h_2^{1+\gamma}}{1+\gamma} \Bigg\},
\end{aligned}
$$

6.4 *Summary of the Male Labor Supply Literature*

TABLE 6
SUMMARY OF ELASTICITY ESTIMATES FOR MALES

Authors of study	Year	Marshall	Hicks	Frisch
Static models				
Kosters	1969	–0.09	0.05	
Ashenfelter-Heckman	1973	–0.16	0.11	
Boskin	1973	–0.07	0.10	
Hall	1973	n/a	0.45	
Eight British studies[a]	1976–83	–0.16	0.13	
Eight NIT studies[a]	1977–84	0.03	0.13	
Burtless-Hausman	1978	0.00	0.07–0.13	
Wales-Woodland	1979	0.14	0.84	
Hausman	1981	0.00	0.74	
Blomquist	1983	0.08	0.11	
Blomquist-Hansson-Busewitz	1990	0.12	0.13	
MaCurdy-Green-Paarsch	1990	0.00	0.07	
Triest	1990	0.05	0.05	
Van Soest-Woittiez-Kapteyn	1990	0.19	0.28	
Ecklof-Sacklen	2000	0.05	0.27	
Blomquist-Ecklof-Newey	2001	0.08	0.09	
Dynamic models				
MaCurdy	1981	0.08[b]		0.15
MaCurdy	1983	0.70	1.22	6.25
Browning-Deaton-Irish	1985			0.09
Blundell-Walker	1986	–0.07	0.02	0.03
Altonji[c]	1986	–0.24	0.11	0.17
Altonji[d]	1986			0.31
Altug-Miller	1990			0.14
Angrist	1991			0.63
Ziliak-Kniesner	1999	0.12	0.13	0.16
Pistaferri	2003	0.51[b]		0.70
Imai-Keane	2004	0.40[e]	1.32[e]	0.30–2.75[f]
Ziliak-Kniesner	2005	–0.47	0.33	0.54
Aaronson-French	2009			0.16–0.61
Average		0.06	0.31	0.85

Notes: Where ranges are reported, mid-point is used to take means.
[a] = Average of the studies surveyed by Pencavel (1986).
[b] = Effect of surprise permanent wage increase.
[c] = Using MaCurdy Method #1.
[d] = Using first difference hours equation.
[e] = Approximation of responses to permanent wage increase based on model simulation.
[f] = Age range.

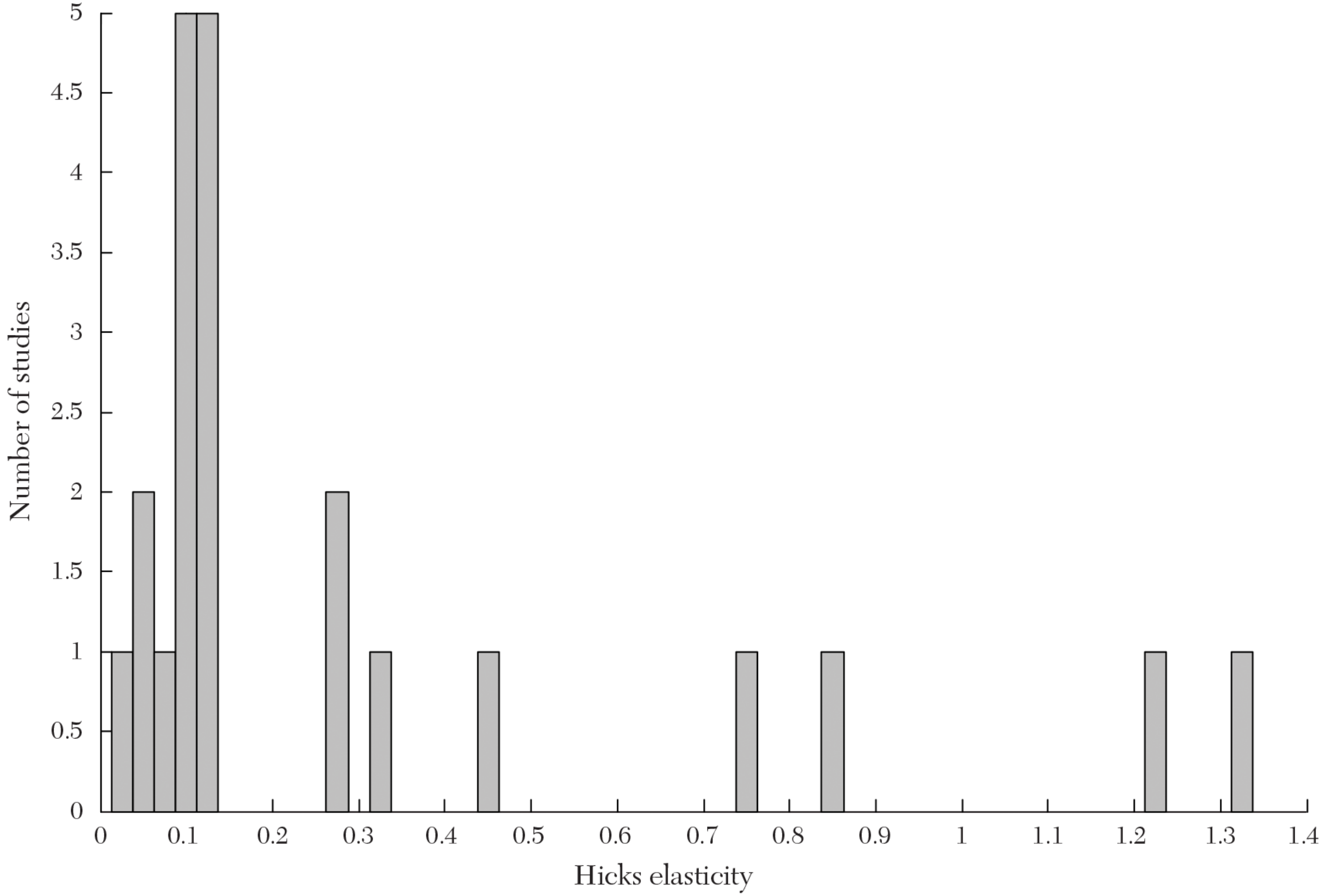

Figure 5. Distribution of Hicks Elasticity of Substitution Estimates

Note: The figure contains a frequency distribution of the twenty-two estimates of the Hicks elasticity of substitution discussed in this survey.

7. *Female Labor Supply*

7.1 *Life-Cycle Models with a Participation Margin*

$$
\begin{aligned}
(105) \quad U_{it} &= \alpha_{it}\eta^{-1}C_{it}^{\eta} \\
&\quad + \beta_{it}\gamma^{-1}(H_{\max} - h_{it})^{\gamma} \\
&\eta < 1, \ \gamma < 1.
\end{aligned}
$$

$$(106) \qquad \frac{\partial U_{it}}{\partial L_{it}} = \lambda_{it} w_{it}$$

$$\Rightarrow \quad \beta_{it} L_{it}^{\gamma-1} = [\rho(1+r)]^t \lambda_{i0} w_{it}.$$

Taking logs and rearranging, this gives the Frisch demand function for leisure:

$$(107) \quad \ln L_{it} = \frac{1}{\gamma-1} \Big\{ \ln w_{it} + \ln \lambda_{i0} + t \ln[\rho(1+r)] - \ln \beta_{it} \Big\}.$$

$$(108) \quad \left.\frac{\partial U_{it}}{\partial L_{it}}\right|_{L_{it}=H_{\max}} \geq \lambda_{it} w_{it}$$

$$\Rightarrow \quad \beta_{it} H_{\max}^{\gamma-1} \geq [\rho(1+r)]^t \lambda_{i0} w_{it}.$$

Taking logs and rearranging, we can write (108) as a reservation wage condition:

$$(109) \quad h_{it} > 0 \quad \textit{iff}$$

$$\begin{aligned} \ln w_{it} > \; & -\ln \lambda_{i0} \\ & - \; t \ln[\rho(1+r)] \\ & + \; \ln \beta_{it} \\ & - (1-\gamma) \ln H_{\max}. \end{aligned}$$

$$\text{(110a)} \quad \ln \beta_{it} = \mathrm{Z}_{it}\,\phi + \eta_{1i} + \varepsilon_{1it}$$

$$\text{(110b)} \quad \ln w_{it} = X_{it}\,\theta + \eta_{2i} + \varepsilon_{2it},$$

$$(111) \quad \ln L_{it} = f_i + X_{it}\frac{\theta}{\gamma - 1} - Z_{it}\frac{\phi}{\gamma - 1} + \frac{\ln[\rho(1+r)]}{\gamma - 1}t + \frac{\varepsilon_{2it} - \varepsilon_{1it}}{\gamma - 1}$$

$$(112) \quad h_{it} > 0 \quad \textit{iff}$$

$$\frac{\varepsilon_{2it} - \varepsilon_{1it}}{\gamma - 1} < -f_i - X_{it}\frac{\theta}{\gamma - 1} + Z_{it}\frac{\phi}{\gamma - 1} - \frac{\ln[\rho(1+r)]}{\gamma - 1}t + \ln H_{\max},$$

where

$$(113) \quad f_i \equiv \frac{1}{\gamma - 1}\{\ln \lambda_{i0} + \eta_{2i} - \eta_{1i}\}.$$

$$
\begin{aligned}
\frac{\partial \ln h_{it}}{\partial \ln w_{it}} &= \frac{\partial \ln h_{it}}{\partial \ln L_{it}} \frac{\partial \ln L_{it}}{\partial \ln w_{it}} \\
&= \frac{L_{it}}{H_{\max} - L_{it}} \frac{1}{1-\gamma} \approx \frac{L_{it}}{h_{it}}(0.41) \\
&= \frac{7460}{1300}(0.41) = 2.35.
\end{aligned}
$$

7.1.1 *Accounting for Fixed Costs of Work—Cogan (1981), Kimmel and Kniesner (1998)*

$$
(114) \quad U = C + \beta \frac{(\bar{H} - h)^{1+\gamma}}{1+\gamma}
$$

$$
= (wh + N - F) + \beta \frac{(\bar{H} - h)^{1+\gamma}}{1+\gamma},
$$

$$h^* = \bar{H} - \left(\frac{w}{\beta}\right)^{\frac{1}{\gamma}}. \tag{115}$$

In the absence of fixed costs, the reservation wage would be obtained simply as

$$\begin{aligned} h^* > 0 \quad &\Rightarrow \quad \bar{H} - \left(\frac{w}{\beta}\right)^{\frac{1}{\gamma}} > 0 \\ &\Rightarrow \quad w > \beta \bar{H}^{\gamma} \end{aligned} \tag{116}$$

$$(117)\quad U(h^*) = w\left[\overline{H} - \left(\frac{w}{\beta}\right)^{\frac{1}{\gamma}}\right] + N - F + \frac{\beta}{1+\gamma}\left[\left(\frac{w}{\beta}\right)^{\frac{1}{\gamma}}\right]^{1+\gamma}$$

$$U(0) = N + \frac{\beta}{1+\gamma}\left[\overline{H}\right]^{1+\gamma}.$$

Now the decision rule for working is $U(h^*) > U(0)$, which can be expressed as

$$
\begin{aligned}
(118) \quad h^* &= \left[\bar{H} - \left(\frac{w}{\beta}\right)^{\frac{1}{\gamma}}\right] \\
&> \frac{F}{w} + \frac{1}{w}\frac{\beta}{1+\gamma} \\
&\times \left\{\bar{H}^{1+\gamma} - \left[\left(\frac{w}{\beta}\right)^{\frac{1}{\gamma}}\right]^{1+\gamma}\right\} \\
&\equiv h_R > 0.
\end{aligned}
$$

$$(119) \quad \ln h_{it} = f_{hi} + e_F \ln w_{it} + \alpha_h \mathbf{Z}_{it} + \varepsilon_{hit}$$

$$(120) \quad P(h_{it} > 0) = F(f_{pi} + \beta \ln w_{it} + \alpha_p \mathbf{Z}_{it}).$$

$$e_P = \frac{\partial \ln P(h_{it} > 0)}{\partial \ln w_{it}} = \beta \frac{F'(\cdot)}{F(\cdot)}. \tag{121}$$

$$\frac{\partial \ln \overline{h}}{\partial \ln w} = \frac{\partial \ln P}{\partial \ln w} + \frac{\partial \ln \overline{h}_e}{\partial \ln w}$$
$$= 0.66 + 2.39 = 3.05.$$

$$(122) \quad \tilde{w}_{it} = \omega_t \nu_i \gamma(\mathrm{Z}_{it}) \exp(\varepsilon_{it})$$

$$\Rightarrow \quad \ln \tilde{w}_{it} = \ln \omega_t + \ln \nu_i + \ln \gamma(\mathrm{Z}_{it}) + \varepsilon_{it}.$$

$$(123)\quad U_{it} = \alpha_{it}\,\eta^{-1}\,C_{it}^{\eta} + d_{it}\{U_0(X_{it}) + U_1(\mathbf{Z}_{it}, h_{it}) + \varepsilon_{1it}\} + (1 - d_{it})\varepsilon_{0it}.$$

$$\alpha_{it}\, C_{it}^{\,\eta-1} \;=\; \lambda_{it} \;=\; \eta_i\, \lambda_t \tag{124}$$

$$\Rightarrow \quad \ln C_{it} \;=\; \frac{1}{\eta - 1}\{\ln \eta_i + \ln \lambda_t - \ln \alpha_{it}\}.$$

7.2 *The "Life-Cycle Consistent" Approach—Blundell, Duncan, and Meghir* (1998)

$$
\begin{aligned}
(125)\quad h_{it} \; = \; & \beta \ln w_{it}(1 - \tau_{it}) \\
& + \; \gamma[C_{it} - w_{it}(1 - \tau_{it})h_{it}] + X_{it}\phi \\
& + \; d_g + d_t + \delta_w R_{wit} + \delta_c R_{cit} \\
& + M(P_{gt}) + e_{it}.
\end{aligned}
$$

7.3 *"Approximate Reduced Form" Approach (Fertility)—Moffitt (1984)*

$$
\begin{aligned}
B_{it}^{*} = {} & a_0 + a_1 f(t) + a_2 \ln w_{1i}^{*} \\
& + a_3 Y_i + a_4 X_i + a_5 B_{i,t-1} \\
& + \mu_{Bi} + u_{it} \\
S_{it}^{*} = {} & b_0 + b_1 f(t) + b_2 \ln w_{1i}^{*} \\
& + b_3 Y_i + b_4 X_i + b_5 B_{i,t-1} \\
& + \mu_{Si} + \upsilon_{it}.
\end{aligned}
$$

$$\ln w_{it} = \ln w^{*}_{1i} + \gamma \sum_{\tau=1}^{t-1} S_{\tau} - \delta(t-1) + \varepsilon_{it}$$

$$\Rightarrow \ln w_{it} = \mathrm{Z}_i \, \eta + \gamma \sum_{\tau=1}^{t-1} S_{\tau} - \delta(t-1)$$

$$+ \mu_{wi} + \varepsilon_{it}.$$

$$
\begin{aligned}
B^*_{it} &= a_0 + a_1 f(t) + a_2(\mathrm{Z}_i \eta) \\
&\quad + a_3 Y_i + a_4 X_i + a_5 B_{i,t-1} \\
&\quad + (a_2 \mu_{wi} + \mu_{Bi} + u_{it}) \\
S^*_{it} &= b_0 + b_1 f(t) + b_2(\mathrm{Z}_i \eta) \\
&\quad + b_3 Y_i + b_4 X_i + b_5 B_{i,t-1} \\
&\quad + (b_2 \mu_{wi} + \mu_{Si} + \upsilon_{it}).
\end{aligned}
$$

7.4 *Female Labor Supply—Full Solution Structural Methods*

7.4.1 *Participation and Human Capital—Eckstein and Wolpin (1989)*

$$U_t = C_t + \alpha_1 p_t + \alpha_2 C_t p_t + \alpha_3 X_t p_t + \alpha_4 N_t p_t + \alpha_5 S_t p_t. \tag{126}$$

Here p_t is an indicator of labor force participation, X_t is work experience (a sum of lagged p_t), N_t is a vector of children in different age ranges, and S_t is schooling. The budget constraint is

$$C_t = w_t p_t + Y_t^H - cN_t - bp_t, \tag{127}$$

7.4.2 *Extensions to Make Marriage and Fertility Endogenous*

$$(128)\quad U_{pm,t} = a_{1t}\, m_t + (a_{2t} + a_{3t}\, m_t) p_t + (\beta_1 + \beta_2\, p_t + \beta_3\, m_t) \mathbf{C}_{pm,t} + \varepsilon_{pm,t}.$$

$$
\begin{aligned}
(129)\quad U_{pfn,t} = {} & C_{pfn,t} + a_{1t}\, p_t + a_{2t} f_t \\
& + (a_3 + \varepsilon_t^n)(n_t + N_{t-1}) \\
& + a_4 (n_t + N_{t-1})^2 \\
& + \{\beta_1\, p_t\, C_{pfn,t} + \beta_2 f_t\, C_{pfn,t} \\
& \quad + \beta_3\, n_t\, C_{pfn,t}\} \\
& + \{\beta_4\, p_t\, n_t + \beta_5 f_t\, n_t\}.
\end{aligned}
$$

7.4.3 *Human Capital, Marriage, Fertility, and Welfare—Keane and Wolpin (2010)*

7.5 *Assessing the Impact of Tax Reforms—Eissa (1995, 1996a)*

7.6 *Summary of the Female Labor Supply Literature*

TABLE 7
SUMMARY OF ELASTICITY ESTIMATES FOR WOMEN

Authors of study	Year	Marshall	Hicks	Frisch	Uncom-pensated (dynamic)	Tax response
Static, life-cycle and life-cycle consistent models						
Cogan	1981	0.89[a]				
Heckman-MaCurdy	1982			2.35		
Blundell-Walker	1986	−0.20	0.01	0.03		
Blundell-Duncan-Meghir	1998	0.17	0.20			
Kimmel-Kniesner	1998			3.05[b]		
Moffitt	1984				1.25	
Dynamic structural models						
Eckstein-Wolpin	1989				5.0	
Van der Klauuw	1996				3.6	
Francesconi	2002				5.6	
Keane-Wolpin	2010				2.8	
Difference-in-difference methods						
Eissa	1995, 1996a					0.77–1.60[b]

Notes:
[a] = Elasticity conditional on positive work hours.
[b] = Sum of elasticities on extensive and intensive margins.

8. *Conclusion and Suggestions for Future Work*