Multidimensional Skills, Sorting, and Human Capital Accumulation

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October (2018)

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Econ 350, Winter 2021

1 Introduction

- The traditional approach to studying wage and employment inequality, as emphasized by Acemoglu and Autor (2011), relies on a view of labor markets where each worker is endowed with a certain level of “human capital” that rigidly dictates the type of job they are able to hold.

- This view has gradually evolved into one of labor markets as institutions mediating the endogenous allocation of workers with heterogeneous skills into tasks with heterogeneous skill requirements: any worker can now potentially perform any job, with their skills determining how good they are at any given job, while the market determines the assignment of skills to tasks.

- This more general view of labor markets has afforded great progress in our understanding of wage and employment inequality.

2 Related Literature

- This paper is related to the vast empirical literature on the returns to firm and occupation tenure.

- Recent work on task-specific human capital (see, for example, Poletaev and Robinson, 2008; Gathmann and Schonberg, 2010, among others).

- As a preamble to their review of the empirical literature, Sanders and Taber (2012) offer an elegant theoretical model of job search and investment in multi-dimensional skills which, on many aspects, can be seen as a special case of the model in this paper.

- However, they only use their model to provide intuition and highlight key qualitative predictions of the theory, and do not bring it to the data.

2 Related Literature

3 Job Search with Multi-dimensional Job and Worker Attributes

3.1 The Model

The Environment

- Workers are characterized by general and specialized skills.
- The market productivity of specialized skills depends on the technology of a particular firm, while general skills have a common effect on output, independent of the particular firm technology a worker is currently matched with.
- Match output is $f(x, y)$, where $x \in X \subset RK$ describes the worker's set of skills, and $y \in Y \subset RL$ describes the firm's technology, with $L \leq K$.
- The first L worker skills are specialized with the remaining $K - L$ being general skills. Time is continuous.

$$\dot{x} = g(x, y),$$

- The firm's technology is fixed, but the worker's skills gradually adjust to the firm's technology as follows:

where $g : RK \times RL \to RK$ is a continuous function.

Firm, worker, and match values

- We denote the total private value (i.e. the value to the firm-worker pair) of a match between a type-x worker and a type-y firm by $P(x, y)$.
- Under linear preferences over wages, this value is independent of the way in which it is shared between the two parties, and only depends on match attributes (x, y).
- We further denote the value of unemployment by $U(x)$, and the worker's value of her/his current wage contract by W, which we discuss in detail below.
- Admissible worker values imply $W \geq U(x)$ (otherwise the worker would quit into unemployment), and $W \leq P(x, y)$ (otherwise the firm would fire the worker).
- Assuming that the employer's value of a job vacancy is zero (which would arise under free entry and exit of vacancies on the search market), the total surplus generated by a type-(x, y) match is $P(x, y) - U(x)$, and the worker's share of that surplus is $(W - U(x)) / (P(x, y) - U(x))$.

Rent sharing and wages

- Wage contracts are renegotiated sequentially by mutual agreement, as in the sequential auction model of Postel-Vinay and Robin (2002).
- Workers have the possibility of playing off their current employer against any firm from which they receive an outside offer.
- If they do so, the current and outside employers Bertrand-compete over the worker's services.
- Consider a type-x worker employed at a type-y firm and assume that the worker receives an outside offer from a firm of type y′.
- Bertrand competition between the type-y and type-y′ employers implies that the worker ends up in the match that has higher total value — that is, he stays in his initial job if $P(x, y) \geq P(x, y')$ and moves to the type-y′ job otherwise — with a new wage contract worth $W' = \min\{P(x, y), P(x, y')\}$.

Value functions and wage equation

- The total private value of a match between a type-x worker and a type-y firm, P(x, y), solves:

$$(r + \mu + \delta)P(\mathbf{x}, \mathbf{y}) = f(\mathbf{x}, \mathbf{y}) - c(\mathbf{x}, \mathbf{y}) + \delta U(\mathbf{x}) + \mathbf{g}(\mathbf{x}, \mathbf{y}) \cdot \nabla_{\mathbf{x}} P(\mathbf{x}, \mathbf{y}). \qquad (2)$$

- Note that the frequency at which the worker collects offers, λ1, does not affect P(x, y).

- Upon receiving an outside offer, the worker either stays in his initial match, in which case the continuation value for that match is P(x, y), or he accepts the offer, in which case he extracts a value of P(x, y) from the poacher (as a result of Bertrand competition) and leaves his initial employer with a vacant job worth 0.

- The value of unemployment, U(x), solves:

$$(r + \mu)U(\mathbf{x}) = b(\mathbf{x}) + \mathbf{g}(\mathbf{x}, 0) \cdot \nabla U(\mathbf{x}), \tag{3}$$

where the employer type is set to y = 0L for an unemployed worker.

- For reasons similar to those just discussed about P(x, y), the worker fails to internalize the gain in surplus associated with accepting a job offer, and the private value of unemployment is independent of the frequency at which those offers arrive.

- The worker receives an endogenous share σ of the match surplus P(x, y) − U(x), which s/he values at W(x, y, σ) = (1−σ)U(x)+σP(x, y). The wage w(x, y, σ) implementing that value solves:

$$(r+\delta+\mu)W(\mathbf{x},\mathbf{y},\sigma) = w(\mathbf{x},\mathbf{y},\sigma) - c(\mathbf{x},\mathbf{y}) + \delta U(\mathbf{x}) \\ + \lambda_1 \mathbf{E} \max\{0, \min\{P(\mathbf{x},\mathbf{y}), P(\mathbf{x},\mathbf{y}')\} - W(\mathbf{x},\mathbf{y},\sigma)\} + \mathbf{g}(\mathbf{x},\mathbf{y}) \cdot \nabla_{\mathbf{x}} W(\mathbf{x},\mathbf{y},\sigma), \quad (4)$$

where the expectation is taken over the sampling distribution, y′ ∼ .

- Combining (2), (3) and (4) (using W(x, y, σ) = (1 − σ)U(x) + σP(x, y)) further yields the following wage equation:

$$w(\mathbf{x},\mathbf{y},\sigma) = \sigma f(\mathbf{x},\mathbf{y}) + (1-\sigma)\left[b(\mathbf{x}) + c(\mathbf{x},\mathbf{y})\right] \\ - \lambda_1 \mathbf{E}\Big[\max\Big\{0, \min\Big\{P(\mathbf{x},\mathbf{y}') - P(\mathbf{x},\mathbf{y}), 0\Big\} + (1-\sigma)\left(P(\mathbf{x},\mathbf{y}) - U(\mathbf{x})\right)\Big\}\Big] \\ - (1-\sigma)\left(\mathbf{g}(\mathbf{x},\mathbf{y}) - \mathbf{g}(\mathbf{x},0)\right) \cdot \nabla U(\mathbf{x}). \quad (5)$$

3.2 Model Analysis

A fully closed-form case

- Full closed-form solutions can be obtained under specific functional form assumptions. We now give an example, which we will use in our empirical specification below.

- We first restrict the dimensionality of worker and job attributes, both for simplicity of exposition and because those restrictions are relevant to the empirical application below (nothing in the theory depends on those particular restrictions).

- We think of a typical worker's skill bundle x = (xC, xM, xI , xT) as capturing (i) the worker's cognitive skills xC, (ii) the worker's manual skills xM, (ii) the worker's interpersonal skills xI , and (iv) the worker's "general efficiency" xT.

- Jobs are likewise characterized by a three-dimensional bundle y = (yC, yM, yI) capturing measures of the job's requirements in cognitive, manual, and interpersonal skills.

- The key functional form assumption: linear adjustment for skills.

- In particular, we assume that a worker's specialized (i.e. cognitive, manual, and interpersonal) skills adjust linearly to his/her job's skill requirements:

$$\mathrm{g}(\mathrm{x},\mathrm{y}) = \begin{pmatrix} \dot{x}_C \\ \dot{x}_M \\ \dot{x}_I \\ \dot{x}_T \end{pmatrix} = \begin{pmatrix} \gamma_C^u \max\{y_C - x_C, 0\} + \gamma_C^o \min\{y_C - x_C, 0\} \\ \gamma_M^u \max\{y_M - x_M, 0\} + \gamma_M^o \min\{y_M - x_M, 0\} \\ \gamma_I^u \max\{y_I - x_I, 0\} + \gamma_I^o \min\{y_I - x_I, 0\} \\ g x_T \end{pmatrix}, \qquad (6)$$

- The $\gamma_k^{u/0}$'s are all positive constants governing the speed at which worker skills adjust to a job's requirements.

- Note that we allow that speed to differ between upward and downward adjustments (γ_k^u vs γ_k^0 for $k = C, M, I$, where "u" stands for "under-qualified" and "o" stands for "over-qualified"), and between skill types ($\gamma_C^{u/0}$ vs $\gamma_M^{u/0}$ vs $\gamma_I^{u/0}$).

- In this case a worker's skills relate to job tenure as follows:

$$x_k(s) = y_k - e^{-\gamma_k^{u/o}(s-t)}\left(y_k - x_k(t)\right), \qquad (7)$$

- The production function:

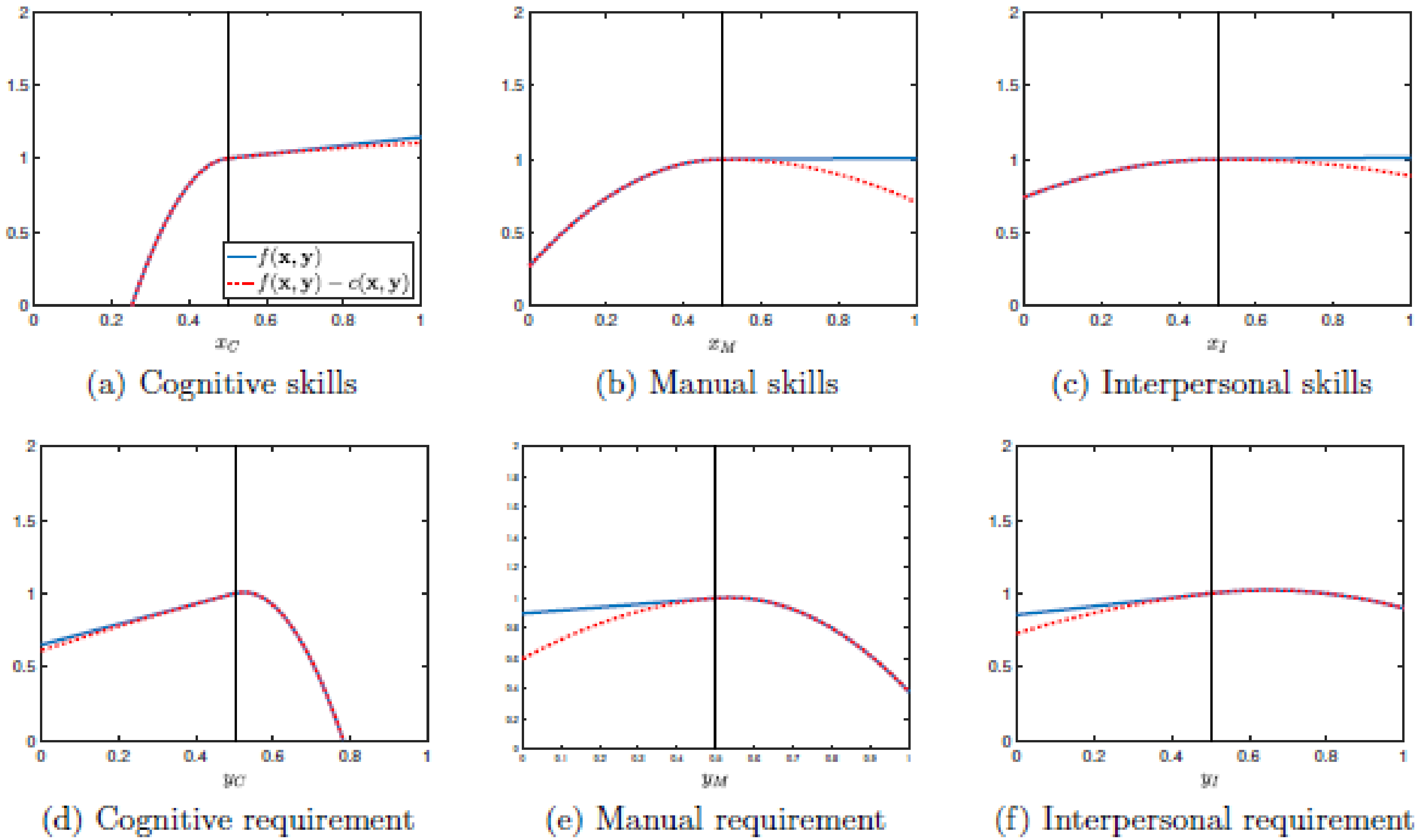
$$f(\mathbf{x},\mathbf{y}) = x_T \times \left[\alpha_T + \sum_{k=C,M,I} \left(\alpha_k y_k - \kappa_k^u \min\{x_k - y_k, 0\}^2 + \alpha_{kk} x_k y_k\right)\right]. \qquad (8)$$

- Flow disutility of work:

$$c(\mathbf{x},\mathbf{y}) = x_T \times \sum_{k=C,M,I} \kappa_k^o \max\{x_k - y_k, 0\}^2. \qquad (9)$$

- The utility cost of being under-matched further allows for an excess of skills to cause a loss of match value, albeit without causing a loss of output.
- An appealing implication of this specification over-qualified workers will have to be compensated for that utility cost, and will therefore have to be paid more in a given job than workers whose skills exactly match the job's requirements.
- A visual impression of the flow surplus from a match, $f(x, y) - c(x, y)$, is is given by the dotted lines on Figure 1, which are constructed in the same way as the corresponding production function lines discussed above.
- The relatively small vertical distance between the solid and dotted lines in regions of the graphs where the worker is overqualified suggests that the utility cost of over-qualification is quantitatively relatively small - a point to which we will return when we discuss estimation results.

Figure 1: The production function



(a) Cognitive skills (b) Manual skills (c) Interpersonal skills

(d) Cognitive requirement (e) Manual requirement (f) Interpersonal requirement

With those specifications, equations (2) and (3) imply (see Appendix A.1):

$$P(\mathbf{x}(t), \mathbf{y}) - U(\mathbf{x}(t)) = x_T(t) \times \left\{ \frac{\alpha_T + \sum_{k=C,M,I} (\alpha_k y_k + \alpha_{kk} y_k^2) - b}{r + \delta + \mu - g} \right.$$

$$- \sum_{k=C,M,I} \left(\frac{\kappa_k^u \min\{x_k(t) - y_k, 0\}^2}{r + \delta + \mu - g + 2\gamma_k^u} + \frac{\kappa_k^o \max\{x_k(t) - y_k, 0\}^2}{r + \delta + \mu - g + 2\gamma_k^o} \right)$$

$$\left. + \sum_{k=C,M,I} \alpha_{kk} y_k \times \left(\frac{\min\{x_k(t) - y_k, 0\}}{r + \delta + \mu - g + \gamma_k^u} + \frac{\max\{x_k(t) - y_k, 0\}}{r + \delta + \mu - g + \gamma_k^o} \right) \right\}. \quad (10)$$

4 Data

4.1 Construction of the Estimation Sample

4.2 Empirical content of skill and skill requirement bundles

- This can be seen in Table 1, which presents the results from regressions of log wages on various sets of skill and skill requirement measures.
- The R2 including our skill requirement measures is higher than when including the AD task measures (0.38 compared to 0.34).
- When both sets of measures are included in the regression, the R2 barely increases relative to the specification using only our measures, and many of the coefficient estimates on the AD task measures become small and statistically insignificant, while our measures continue to have the same statistically significant coefficients as when entered alone.
- These results hold both when we include only level effects and interactions between initial worker skills and job skill requirement (columns 1–3 compared to 4–6).

Table 1: Empirical content of skill and task measures

log wage	(1)	(2)	(3)	(4)	(5)	(6)
x_{C0}	0.339	0.483	0.316	-0.054	0.318	-0.076
	(0.115)	(0.117)	(0.115)	(0.152)	(0.129)	(0.153)
x_{M0}	-0.088	-0.093	-0.070	0.064	-0.122	0.024
	(0.087)	(0.089)	(0.087)	(0.168)	(0.099)	(0.169)
x_{I0}	0.268	0.311	0.262	0.234	0.394	0.305
	(0.053)	(0.054)	(0.053)	(0.101)	(0.095)	(0.141)
$\tilde{y}_C$	0.657		0.704	0.048		0.197
	(0.071)		(0.082)	(0.165)		(0.185)
$\tilde{y}_M$	0.259		0.170	0.418		0.402
	(0.058)		(0.066)	(0.165)		(0.178)
$\tilde{y}_I$	0.389		0.285	0.411		0.361
	(0.063)		(0.066)	(0.142)		(0.147)
$\tau^{AD}_{abstract}$		0.483	0.121		0.191	-0.052
		(0.032)	(0.034)		(0.104)	(0.115)
τ^{AD}_{manual}		0.251	0.194		-0.017	-0.291
		(0.048)	(0.050)		(0.177)	(0.179)
$\tau^{AD}_{routine}$		0.171	-0.012		0.273	0.071
		(0.028)	(0.030)		(0.088)	(0.089)
$x_{C0} \times \tilde{y}_C$				0.902		0.691
				(0.221)		(0.250)

Table 1: Empirical content of skill and task measures

log wage	(1)	(2)	(3)	(4)	(5)	(6)
$x_{M0} \times \tilde{y}_M$				-0.172		-0.267
				(0.249)		(0.266)
$x_{I0} \times \tilde{y}_I$				0.084		0.026
				(0.233)		(0.239)
$x_{C0} \times \tau^{AD}_{abstract}$					0.466	0.264
					(0.158)	(0.173)
$x_{M0} \times \tau^{AD}_{manual}$					0.419	0.730
					(0.282)	(0.285)
$x_{I0} \times \tau^{AD}_{routine}$					-0.199	-0.130
					(0.162)	(0.163)
tenure	0.238	0.239	0.234	0.235	0.240	0.233
	(0.025)	(0.026)	(0.025)	(0.025)	(0.026)	(0.025)
experience	0.268	0.299	0.266	0.269	0.300	0.267
	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)
years of schooling	0.270	0.336	0.274	0.271	0.329	0.266
	(0.082)	(0.084)	(0.081)	(0.081)	(0.083)	(0.080)
constant	4.436	4.524	4.448	4.552	4.600	4.562
	(0.094)	(0.095)	(0.094)	(0.145)	(0.110)	(0.149)
N	224,417	224,417	224,417	224,417	224,417	224,417
R^2	0.373	0.339	0.375	0.376	0.340	0.378

Standard errors clustered at the individual level.

τ^{AD} are the task measures from Autor and Dorn (2013).

- In Table 2 we consider the empirical content of our skill measures relative to occupation fixed effects, again in terms of the descriptive wage regression.
- Column 1 regresses log wages on our vector of skill measures for workers, the skill requirements of their occupation and the interactions.
- These coefficients are all used as moments in our estimation.
- In column 2 we drop our occupation skill demand measures and replace them with occupation fixed effects at the one-digit level23.
- In column 3 we include the occupation fixed effects and the interactions between our worker skill measures and our occupation skill requirement measures.
- There are several things to note.

Table 2: Occupation and Individual Fixed Effects

log wage	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
x_{C0}	-0.036	0.567	-0.130	0.449	-0.144					
	(0.153)	(0.116)	(0.127)	(0.105)	(0.121)					
x_{M0}	0.014	-0.153	-0.150	-0.124	-0.065					
	(0.169)	(0.090)	(0.107)	(0.082)	(0.110)					
x_{I0}	0.232	0.311	0.033	0.276	0.105					
	(0.101)	(0.055)	(0.067)	(0.049)	(0.069)					
$\tilde{y}_C$	0.041					-0.532				
	(0.164)					(0.154)				
$\tilde{y}_M$	0.365					0.561				
	(0.171)					(0.154)				
$\tilde{y}_I$	0.395					0.388				
	(0.143)					(0.148)				
$x_{C0} \times \tilde{y}_C$	0.921		1.161		1.114	1.356		0.731		0.752
	(0.221)		(0.102)		(0.123)	(0.228)		(0.117)		(0.116)
$x_{M0} \times \tilde{y}_M$	-0.109		0.202		0.076	-0.279		0.279		0.170
	(0.254)		(0.091)		(0.110)	(0.237)		(0.085)		(0.088)
$x_{I0} \times \tilde{y}_I$	0.095		0.556		0.350	-0.144		0.304		0.183
	(0.233)		(0.101)		(0.124)	(0.257)		(0.109)		(0.112)
tenure	0.234	0.261	0.242	0.232	0.232	0.142	0.121	0.138	0.115	0.134
	(0.025)	(0.025)	(0.024)	(0.023)	(0.023)	(0.019)	(0.019)	(0.019)	(0.018)	(0.018)
experience	0.269	0.289	0.264	0.257	0.244	0.335	0.363	0.334	0.343	0.322
	(0.014)	(0.014)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)
years of education	0.256	0.321	0.294	0.306	0.289					
	(0.081)	(0.085)	(0.080)	(0.075)	(0.073)					
constant	4.603	4.237	4.440	4.579	4.751	5.297	5.303	4.991	5.548	5.332
	(0.148)	(0.194)	(0.200)	(0.221)	(0.248)	(0.058)	(0.173)	(0.185)	(0.130)	(0.151)
occupation FE 1 digit		✓	✓				✓	✓		
occupation FE 3 digit				✓	✓				✓	✓
worker FE						✓	✓	✓	✓	✓
N	232,303	232,303	232,303	232,303	232,303	232,303	232,303	232,303	232,303	232,303
R^2	0.374	0.347	0.388	0.431	0.449	0.684	0.679	0.687	0.700	0.704

Standard errors clustered at the individual level

Table 3: Effect of quality and duration of first job on quality of second job

	(1)	(2)	(3)
	$\tilde{y}_C^+$	$\tilde{y}_M^+$	$\tilde{y}_I^+$
x_{C0}	0.650	-0.300	0.460
	(0.062)	(0.074)	(0.061)
x_{M0}	-0.117	0.687	-0.409
	(0.062)	(0.074)	(0.061)
x_{I0}	0.054	0.013	0.395
	(0.065)	(0.077)	(0.064)
$\max\{\tilde{y}_C - x_{C0}, 0\}^2$	3.044	0.998	1.102
	(0.694)	(0.827)	(0.686)
$\min\{\tilde{y}_C - x_{C0}, 0\}^2$	-0.677	-0.164	-0.096
	(0.106)	(0.126)	(0.104)
$\max\{\tilde{y}_M - x_{M0}, 0\}^2$	-0.171	0.682	-0.450
	(0.227)	(0.270)	(0.224)
$\min\{\tilde{y}_M - x_{M0}, 0\}^2$	0.226	-0.420	0.190
	(0.123)	(0.146)	(0.121)
$\max\{\tilde{y}_I - x_{I0}, 0\}^2$	-0.049	0.011	0.980
	(0.312)	(0.371)	(0.308)
$\min\{\tilde{y}_I - x_{I0}, 0\}^2$	0.104	0.026	-0.399
	(0.109)	(0.129)	(0.107)
duration	0.001	-0.000	0.001
	(0.000)	(0.000)	(0.000)
duration$\times(\tilde{y}_C - x_{C0})$	0.004	-0.003	0.003
	(0.002)	(0.002)	(0.001)
duration$\times(\tilde{y}_M - x_{M0})$	-0.000	0.006	-0.002
	(0.001)	(0.001)	(0.001)
duration$\times(\tilde{y}_I - x_{I0})$	0.000	-0.000	0.002
	(0.001)	(0.001)	(0.001)
constant	0.091	0.327	0.161
	(0.040)	(0.047)	(0.039)
N	528	528	528
R^2	0.392	0.294	0.510

Standard errors in parentheses

5 Estimation

5.1 Simulation

5.2 Targeted Moments

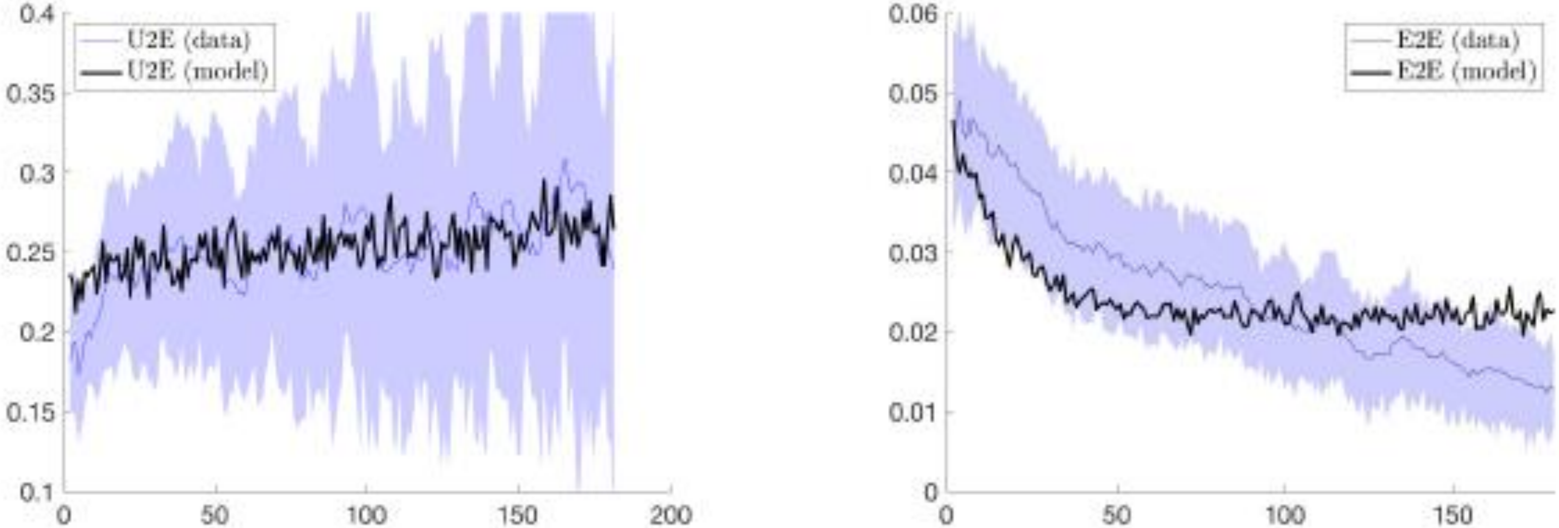
5.3 Identification

6 Results

6.1 Model Fit

- Figure 2 illustrates various aspects of the fit. All time series in Figure 2 are plotted over a period of 15 years (180 months, i.e. the sample window used for estimation), together with 95% confidence bands (based on 1,500 bootstrap replications) around the data series. Figure 2h further shows the fit in terms of the descriptive wage regression discussed in Section 4.

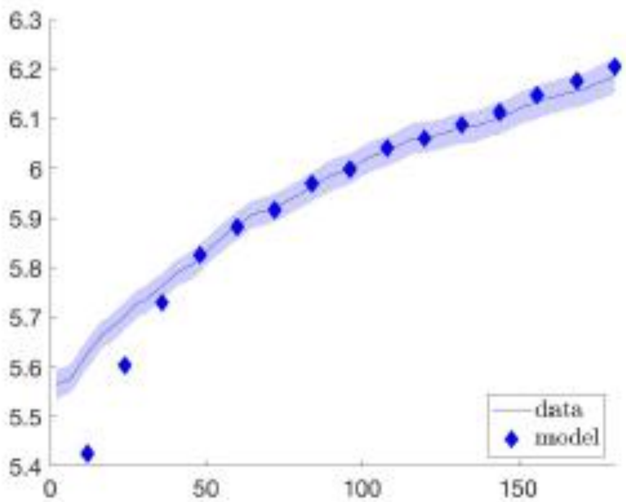
Figure 2: Model fit



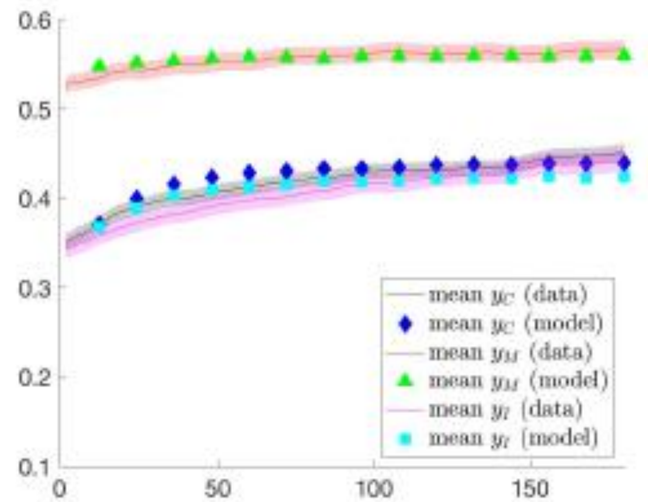
(a) U2E rate

(b) E2E rate

Figure 2: Model fit

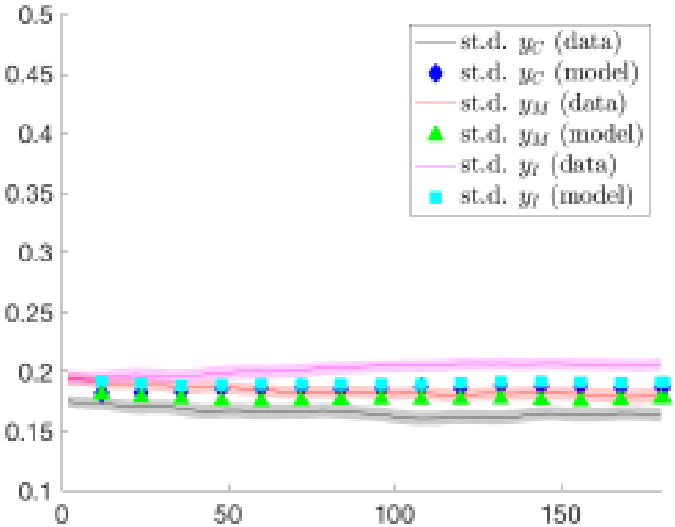


(c) Log wage/experience profile

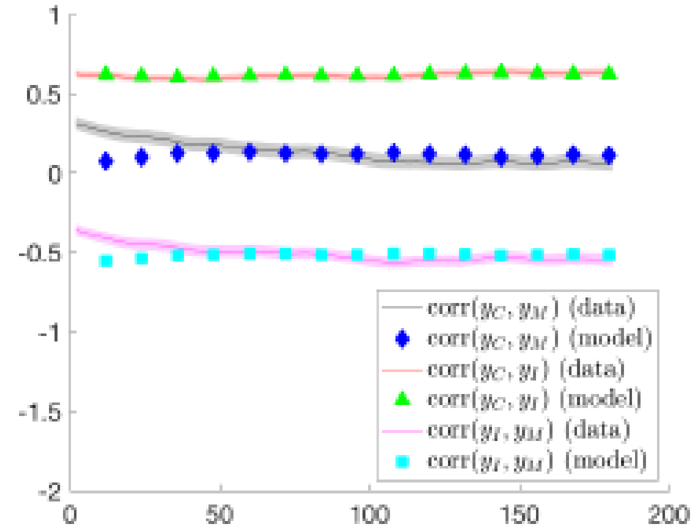


(d) Cross-sectional mean job attributes

Figure 2: Model fit

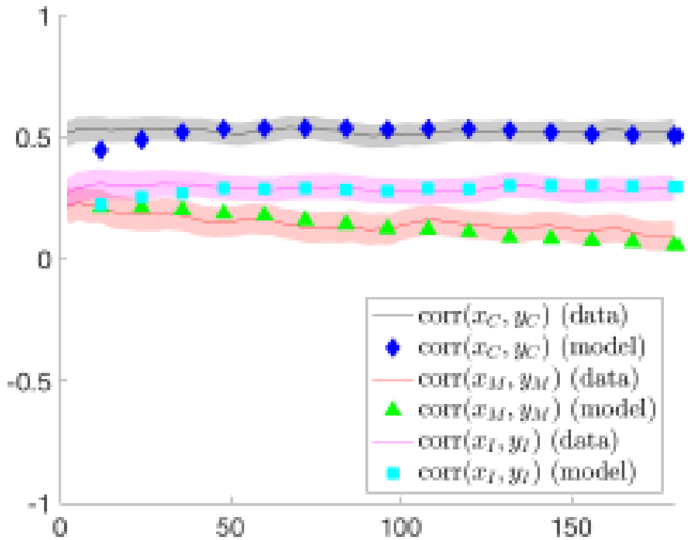


(e) Cross-sectional st.d. of job attributes

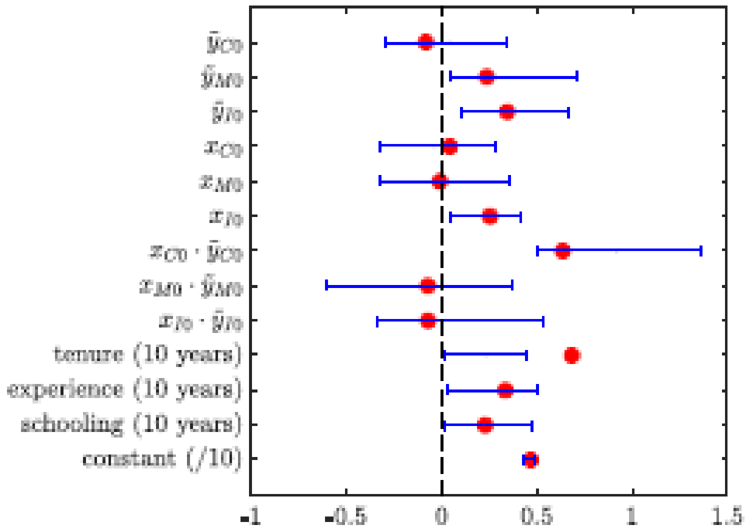


(f) Correlation of job attributes

Figure 2: Model fit



(g) Corr. of job and worker attributes



(h) Descriptive (log) wage regression

6.2 Parameter Estimates

- Table 4 shows point estimates of the model parameters with asymptotic standard errors in parentheses below each estimate.

- There is little to say about the offer arrival and job destruction rates, which are within the range of standard estimates on US data even though the ratio $\lambda 1/\lambda 0 \simeq 0.42$ is on the high end of that range.

- Overall job productivity is increasing in all cognitive, manual and interpersonal skill requirements, with the loading on cognitive skills between 1.5 and two times as large as the ones on manual and interpersonal skills.

- Job skill requirements are complementary to the corresponding worker skills (αCC, αMM and αII are all positive), although complementarity is an order of magnitude stronger in the cognitive than in the other two skill dimensions.

Table 4: Parameter estimates

production function*										disutility of work*			un. inc.
α_T	α_C	α_M	α_I	α_{CC}	α_{MM}	α_{II}	κ_C^u	κ_M^u	κ_I^u	κ_C^o	κ_M^o	κ_I^o	b
137.5	140.3	64.4	92.4	195.6	10.7	15.4	5,165.3	984.5	337.6	54.1	409.6	171.9	137.5
(17.0)	(24.8)	(10.1)	(11.1)	(24.2)	(2.40)	(4.29)	(533.5)	(265.4)	(97.4)	(7.14)	(71.9)	(23.9)	(17.0)
							(45.7)	*(3.9)*	*(4.1)*	*(20.9)*	*(8.8)*	*(5.1)*	

skill accumulation function**							general efficiency			
γ_C^u	γ_C^o	γ_M^u	γ_M^o	γ_I^u	γ_I^o	g	ζ_S	ζ_C	ζ_M	ζ_I
7.7e − 3	2.1e − 3	3.4e − 2	7.7e − 3	1.0e − 3	5.8e − 7	2.3e − 3	2.4e − 2	0.18	−0.17	0.20
(8.4e−4)	(5.3e−4)	(3.1e−3)	(6.4e−4)	(3.8e−3)	(1.4e−3)	(5.0e−4)	(.031)	(.501)	(.521)	(.261)
(7.54)	*(27.3)*	*(1.70)*	*(7.51)*	*(55.8)*	*(99,407)*					

sampling distribution***												transition rates		
ξ_C	ξ_M	ξ_I	ρ_{CM}	ρ_{CI}	ρ_{IM}	η_C^1	η_C^2	η_M^1	η_M^2	η_I^1	η_I^2	λ_0	λ_1	δ^{****}
1.21	0.79	0.88	0.14	0.73	−0.44	1.22	2.86	2.15	2.76	0.93	2.96	0.39	0.16	2.1e − 2
(.038)	(.038)	(.040)	(.019)	(.011)	(.019)	(.066)	(.095)	(.143)	(.117)	(.085)	(.124)	(.011)	(1.2e−3)	(3.3e−7)
			(0.13)	*(0.71)*	*(−0.42)*	*(0.30, 0.20)*		*(0.44, 0.20)*		*(0.24, 0.19)*				

*percent surplus loss caused by deviating from output-maximizing match by 1 SD of Υ at mean $\mathbf{x}$ in italics;

** half-life in years in italics ; *** implied correlations and (means, standard deviations) in italics ; **** estimated in first step

6.3 Skill Mismatch, Skill Changes, and Sorting

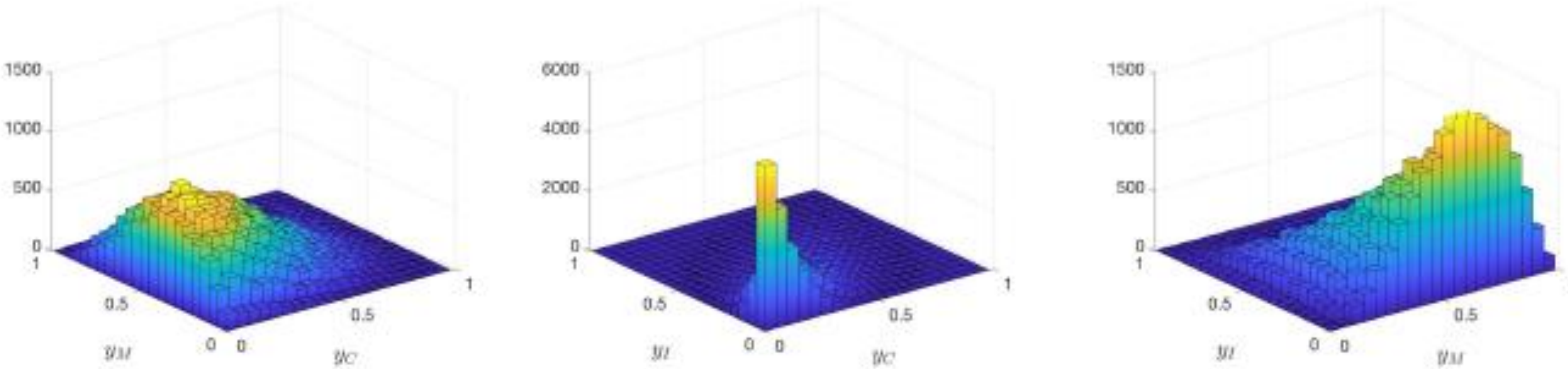
- The top row of Figure 3 (Panels a, b and c) show the marginal sampling distributions of pairs of job attributes, integrating out one skill dimension at a time.

- The second row of Figure 3 (Panels d, e and f) do the same for the distribution of initial skills among labor market entrants, N(·).

- Plots of the sampling distribution suggest that labor demand is concentrated around jobs with intermediate to high manual skill requirements (y_M around 0.5 to 0.6), and modest levels of cognitive and interpersonal skill requirements (y_C and y_I round 0.2).

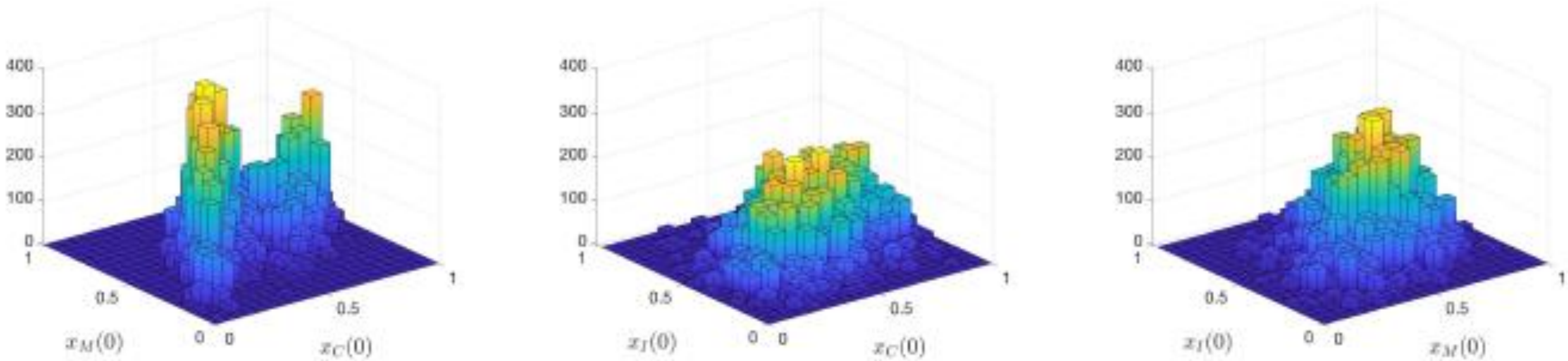
- A visual comparison of the top two rows of Figure 3 further suggests that labor market entrants are, on average, endowed with levels of manual skills that roughly coincide with what the sampling distribution suggests employers are looking for, but also seem to have much higher levels of cognitive and interpersonal skills than is required in most jobs.

Figure 3: Distribution of skill requirements and evolution of worker skills with experience



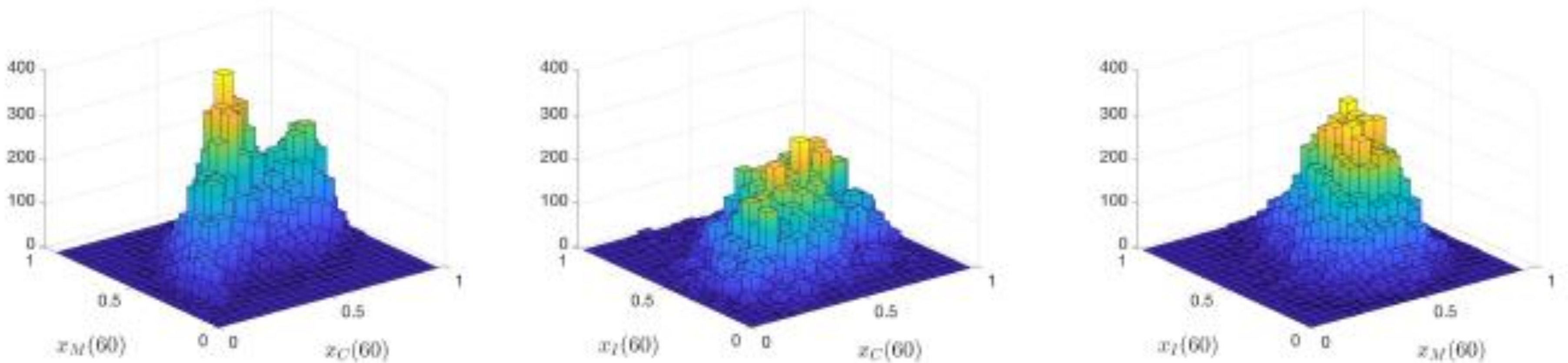
(a) Sampling density, CM (b) Sampling density, CI (c) Sampling density, MI

Figure 3: Distribution of skill requirements and evolution of worker skills with experience



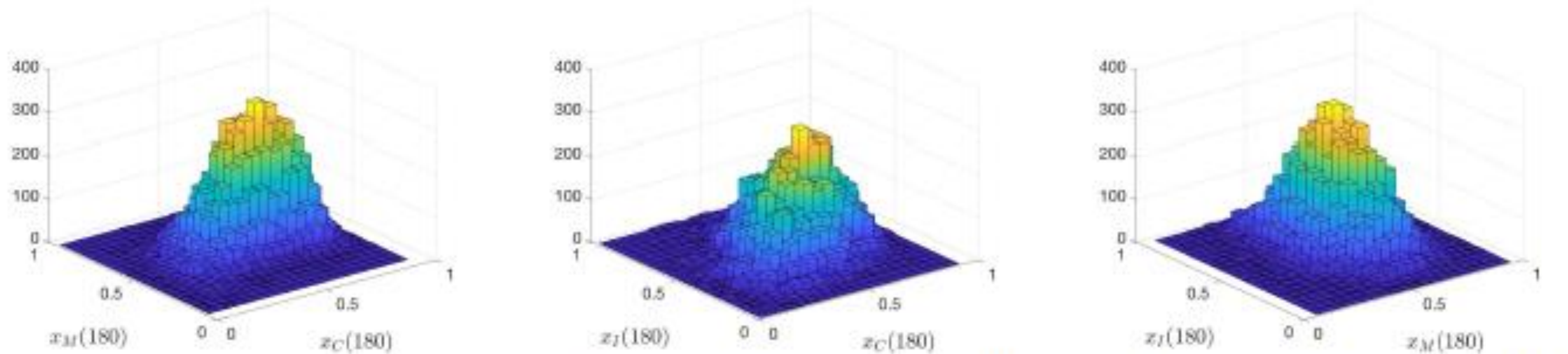
(d) Initial worker skill dist., CM (e) Initial worker skill dist., CI (f) Initial worker skill dist., MI

Figure 3: Distribution of skill requirements and evolution of worker skills with experience



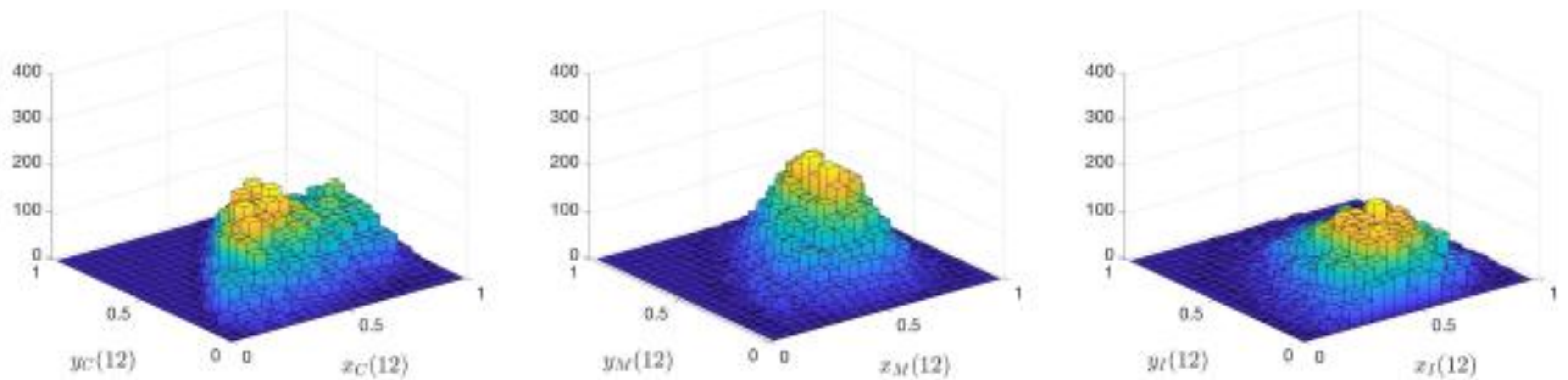
(g) 5 years of experience, CM (h) 5 years of experience, CI (i) 5 years of experience, MI

Figure 3: Distribution of skill requirements and evolution of worker skills with experience



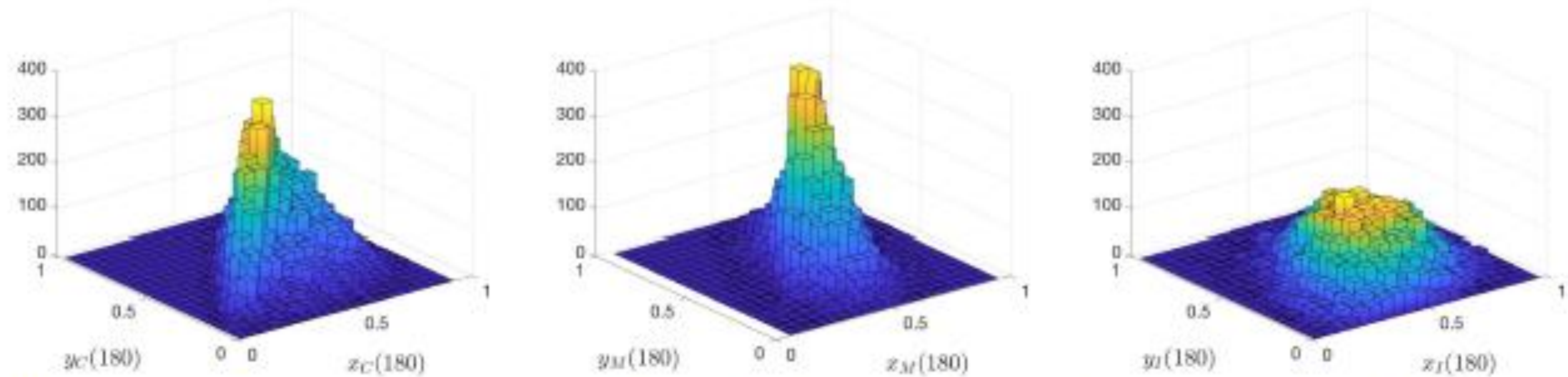
(j) 15 years of experience, CM (k) 15 years of experience, CI (l) 15 years of experience, MI

Figure 4: Sorting



(a) Cognitive, 1 year of exp. (b) Manual, 1 year of exp. (c) Interpersonal, 1 year of exp.

Figure 4: Sorting



(d) Cognitive, 15 years of exp. (e) Manual, 15 years of exp. (f) Interpersonal, 15 years of exp.

7 The Determinants of Social Output

- Analyze the determinants of the social value of output in our model economy.
- Specifically, focus on the expected present discounted sum of future output produced by a worker from experience t onwards (the "experience-t expected career output").
- Consider a worker i with experience t, who is either unemployed (denoted by ℓit = 0) or employed (ℓit = 1) in a job with attributes yit.
- The worker has education (years of schooling) edi, initial skill bundle xi0, unobserved ability ε0i and current skills xit.
- Experience-t expected career output is then defined as:

$$Q_{it} = \mathbf{E}\left[\int_t^{+\infty} \left(\ell_{is}\left[f\left(\mathbf{x}_{is}, \mathbf{y}_{is}\right) - c\left(\mathbf{x}_{is}, \mathbf{y}_{is}\right)\right] + \left(1 - \ell_{is}\right) b\left(\mathbf{x}_{is}\right)\right) e^{-(r+\mu)(s-t)} ds \;\middle|\; \mathbf{x}_{i0}, \mathrm{ed}_i, \varepsilon_{0i}, \mathbf{x}_{it}, \ell_{it}, \mathbf{y}_{it}\right]. \quad (12)$$

Table 5: Decomposition of Var ln Qit

	Share of **Var** $\ln Q_{it}$ due to...			
	initial skills $\mathbf{x}_0$ (term 1)	shocks (term 2)	heterogeneity ε_0 (term 3)	schooling $\mid \mathbf{x}_0$ (term 4)
Whole sample	12.9%	66.6%	20.5%	0.0%
College +	2.65%	76.9%	20.5%	0.0%
Some college	4.37%	74.5%	22.9%	0.0%
Non-college	8.69%	68.4%	22.9%	0.0%

Level of experience: $t = 10$ years.

Table 6: Further decomposition of Var ln Qit

	Share of Var ln Q_{it} due to …			
	$\mathbf{x}_0$	x_{0C}	x_{0M}	x_{0I}
Whole sample	12.9%	11.0%	3.38%	2.50%
College +	2.65%	0.83%	0.38%	0.36%
Some college	4.37%	2.87%	1.45%	0.79%
Non-college	8.69%	7.04%	5.65%	1.30%

Level of experience: $t = 10$ years.

Table 7: Elasticities of Qit

Elasticity of Q_{it} with respect to:	Whole sample	College +	Some college	Non-college
x_{0C}	0.48	0.39	0.49	0.52
x_{0M}	–0.12	–0.12	–0.13	–0.11
x_{0I}	0.01	0.02	0.01	0.00
$\mathrm{E}_{\Upsilon} y_C$	0.15	0.30	0.20	0.03
$\mathrm{E}_{\Upsilon} y_M$	0.20	0.21	0.23	0.19
$\mathrm{E}_{\Upsilon} y_I$	0.18	0.15	0.19	0.20
λ_0	0.13	0.17	0.07	0.13
λ_1	0.17	0.06	0.26	0.20
mismatch	0.88	0.97	0.85	0.83

Level of experience: $t = 10$ years.

8 Extension: Worker Bargaining Power

When workers have bargaining power $\beta \in [0,1]$, the dynamic equations (2) and (3) characterizing, respectively, the value of a match and the value of unemployment, must be amended as follows:

$$(r+\delta+\mu)P(\mathbf{x},\mathbf{y}) = f(\mathbf{x},\mathbf{y}) - c(\mathbf{x},\mathbf{y}) + \delta U(\mathbf{x}) + \mathbf{g}(\mathbf{x},\mathbf{y}) \cdot \nabla_{\mathbf{x}} P(\mathbf{x},\mathbf{y}) + \lambda_1 \beta \mathbf{E} \max\{P(\mathbf{x},\mathbf{y}') - P(\mathbf{x},\mathbf{y}), 0\} \quad (13)$$

and:

$$(r+\mu)U(\mathbf{x}) = b(\mathbf{x}) + \mathbf{g}(\mathbf{x},\mathbf{0}) \cdot \nabla U(\mathbf{x}) + \lambda_0 \beta \mathbf{E} \max\{P(\mathbf{x},\mathbf{y}') - U(\mathbf{x}), 0\}. \quad (14)$$

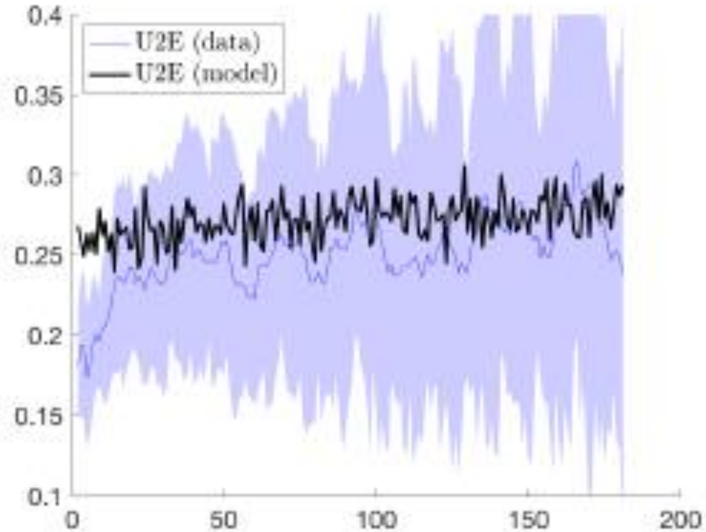
Table 8: Parameter estimates (with worker bargaining power)

	production function										disutility of work			un. inc.
	α_T	α_C	α_M	α_I	α_{CC}	α_{MM}	α_{II}	κ^u_C	κ^u_M	κ^u_I	κ^o_C	κ^o_M	κ^o_I	b
β= 0	137.5	140.3	64.4	92.4	195.6	10.7	15.4	5,165.3	984.5	337.6	54.1	409.6	171.9	137.5
β= 0.5	122.3	116.2	60.7	91.4	280.7	9.7	14.5	3,901.4	641.3	304.5	53.2	376.1	142.5	122.4

	skill accumulation function							general efficiency			
	γ^u_C	γ^o_C	γ^u_M	γ^o_M	γ^u_I	γ^o_I	g	ζ_S	ζ_C	ζ_M	ζ_I
β= 0	7.7e − 3	2.1e − 3	3.4e − 2	7.7e − 3	1.0e − 3	5.8e − 7	2.3e − 3	2.4e − 2	0.18	−0.17	0.20
β= 0.5	7.9e − 3	2.1e − 3	3.9e − 2	8.1e − 3	1.1e − 3	6.1e − 7	2.4e − 3	2.4e − 2	−0.16	−0.22	0.20

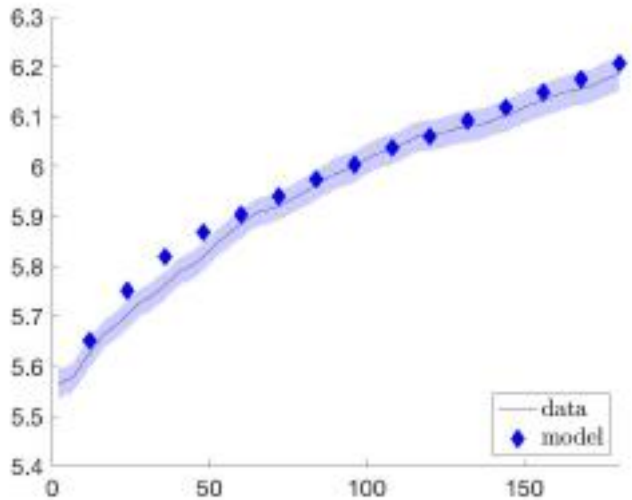
	sampling distribution												trans. rates	
	ξ_C	ξ_M	ξ_I	ρ_{CM}	ρ_{CI}	ρ_{IM}	η^1_C	η^2_C	η^1_M	η^2_M	η^1_I	η^2_I	λ_0	λ_1
β= 0	1.21	0.79	0.88	0.14	0.73	−0.44	1.22	2.86	2.15	2.76	0.93	2.96	0.39	0.16
β= 0.5	1.08	0.76	0.82	0.12	0.72	−0.47	1.23	3.07	2.12	2.90	0.92	3.11	0.41	0.17

Figure 5: Model fit (with worker bargaining power)

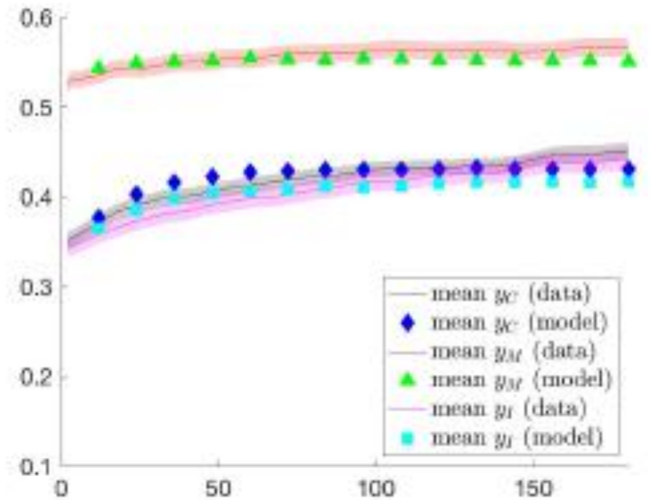


(a) U2E rate (b) E2E rate

Figure 5: Model fit (with worker bargaining power)

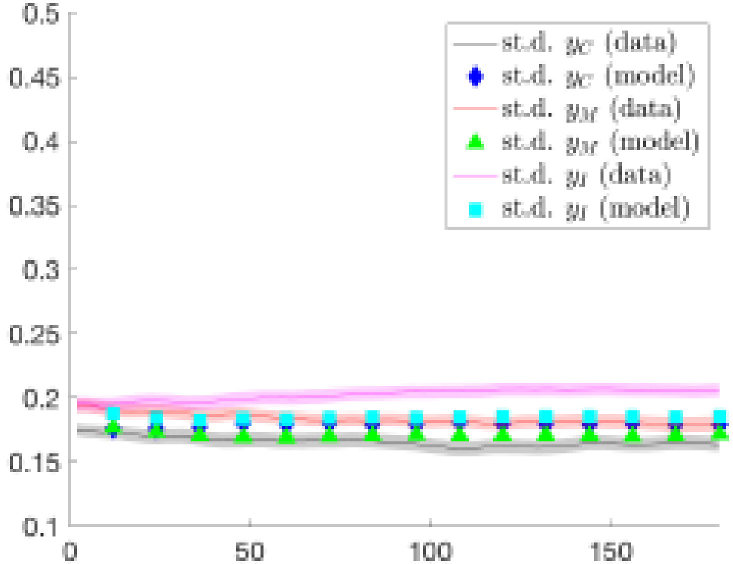


(c) Log wage/experience profile

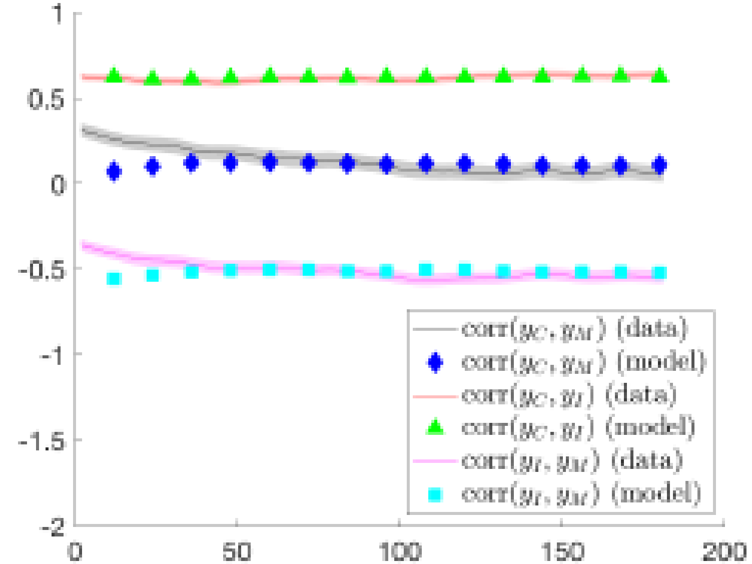


(d) Cross-sectional mean job attributes

Figure 5: Model fit (with worker bargaining power)

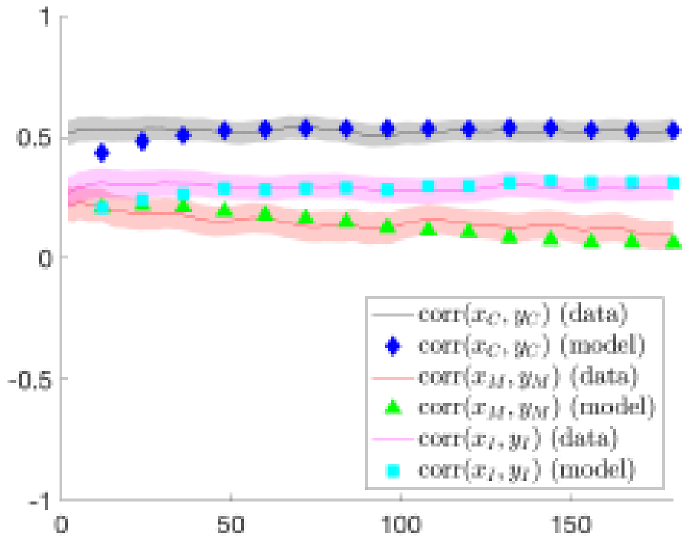


(e) Cross-sectional st.d. of job attributes

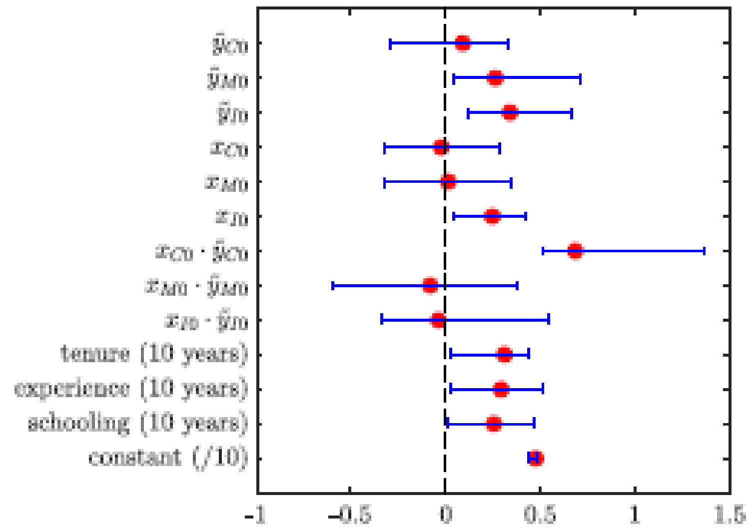


(f) Correlation of job attributes

Figure 5: Model fit (with worker bargaining power)



(g) Corr. of job and worker attributes



(h) Descriptive (log) wage regression

9 Conclusion