Other Ways to Define Occupations

(Paretian Distributions and Income Maximizations)

Benoit Mandelbrot (1962)
Quarterly Journal of Economics, 76(1): pp. 57–85.

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- Assume that each individual must choose *one* of N possible "occupations," $P_n (1 \leqslant n \leqslant N)$ with utility U_n.
- Model of multinomial choice.

- Within the total population, the distributions of the various U_n are random.

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Linear Factor Analysis of the Rental Price of an Undissoluble Bundle of Abilities

- $U_n(1 \leqslant n \leqslant N)$ is the rental price which the occupation P_n is ready to pay for the use of a man's abilities.
- Can be written as a nonhomogeneous *linear* form of F independent *factors* $V_f(1 \leqslant f \leqslant F)$, each of which is randomly distributed in the population, and "measures" one or several "abilities."
- Then one can write:

$$U_n = \sum_{f=1}^{F} a_{nf} V_f + a_{n0} + a_n E_n.$$

The Regions of Acceptance of the Different Offers

- The prospective employee's problem is to determine the n that maximizes the nonlinear function U_n of n.
- Let the random variable W_n designate the incomes *accepted* from the occupation P_n, and
- Let W designate all accepted incomes (observed income distributions).
- The region of acceptance R_n of the offer U_n will be the intersection of the $N - 1$ half-hyperspaces defined by the equations:

$$U_n - U_i \geqslant 0 \qquad (i \neq n).$$

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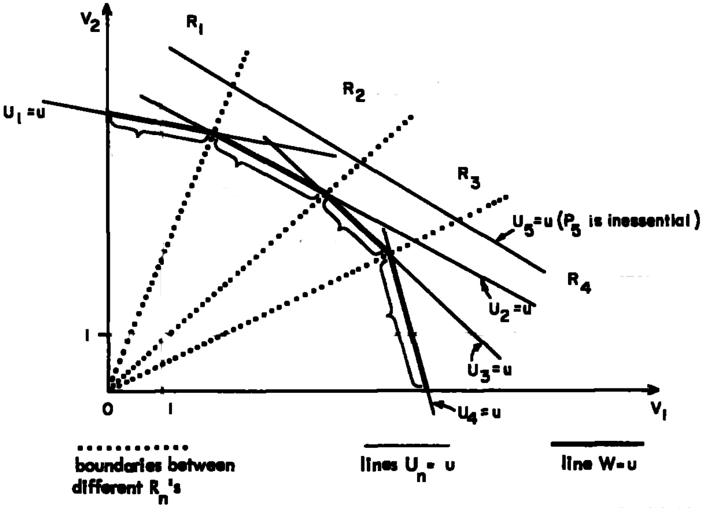
Examples

Homogeneous offers.

- The simplest case, from the viewpoint of R_n and of income iso-surfaces, occurs when the offers are homogeneous forms of the V_f.
- Then, the accepted P_n will obviously depend only upon the relative values of the factors, that is upon what we may call the "factor profile" of the individual.
- For the case $F = 2$, the typical shape of the regions R_n and of the surfaces $W = u$, is given in Figure 1.

Figure 1: Example of Regions of Acceptance in the Case of Two Homogeneous Factors



- Figures 2a to 2e are such that, if one factor is present in very large quantity, it fully determines the chosen occupation, so that changes in the values of other factors are irrelevant.
- But in Figure 2f, we see a case where the occupation is not determined by the value of the high factor V_1 alone, and is greatly influenced by V_2 and V_3, which are present in small quantities; small changes in those other factors can totally modify the value of n that will be chosen.

Figure 2: Six Miscellaneous Examples of the Shape of the Surface $W = u$, in the Case of Three Homogeneous Factors

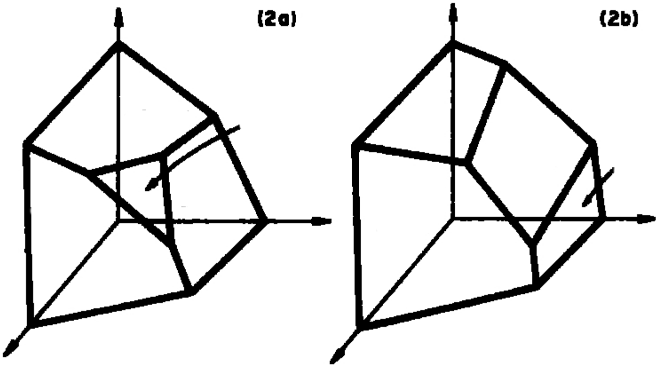
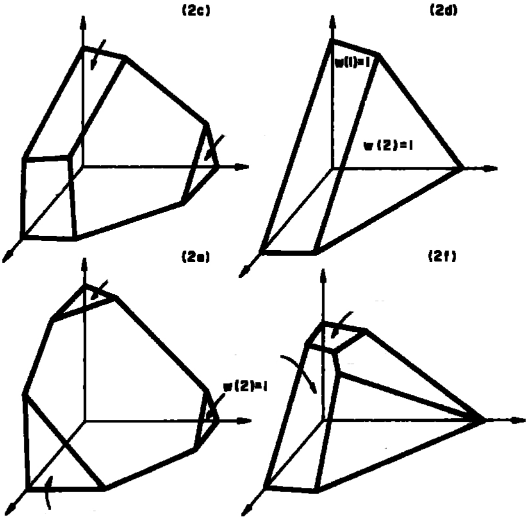


Figure 2: Six Miscellaneous Examples of the Shape of the Surface $W = u$, in the Case of Three Homogeneous Factors



Linear and nonhomogeneous offers.

- In this case, the regions of acceptance R_n need no longer be open cones.
- Some of these regions may be closed, leading to bounded incomes.
- The corresponding occupations will clearly be those which are not very sensitive to the values of the factors.
- On the contrary, fanning-out regions R_n are quite sensitive to the values of the factors.
- An example is given in Figure 3.

Figure 3: Example of Regions of Acceptance in the Case of Two Inhomogeneous Factors

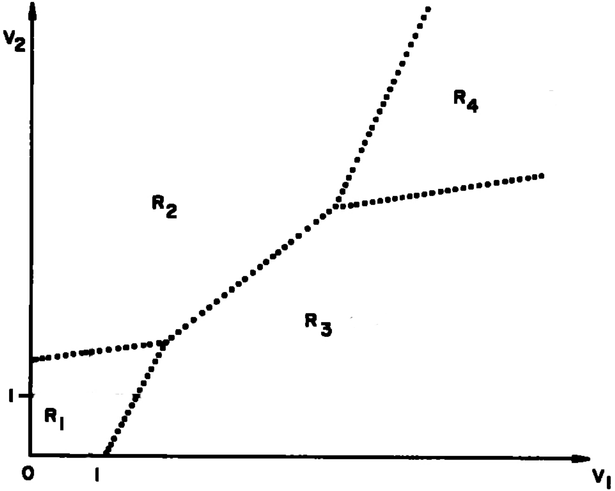
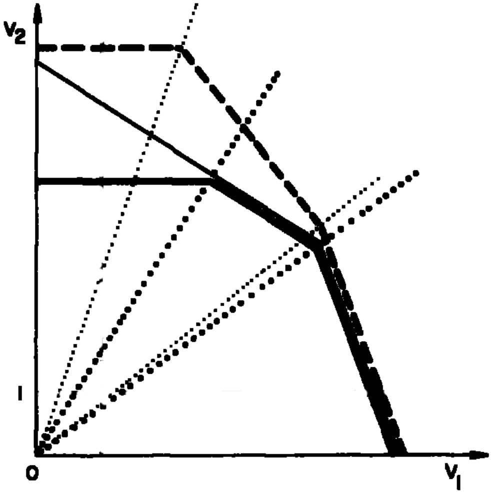


Figure 4: Example of the Change in the Form of the Curve $W = u$, as Factor Loadings Are Modified



solid bold line: situation before the changes in the factor loadings

solid thin line: situation after a change in the factor loadings of U_1 (which thereby becomes inessential)

bold dashed line: situation after the changes in the factor loadings of U_1 and U_2