Post Schooling Wage Growth: Models of Wage Growth Part II

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Models of Wage Growth

- A basic tenet of modern labor economics is that the observed life cycle wage patterns are, to a large extent, a matter of choice.
- Thus, each worker can influence his future wage by going to school, by choosing an occupation and by searching for a better job.
- Of course, wage levels and wage growth are also influenced by factors beyond the worker's control, such as aggregate demand and supply, technology, degree of competition and the institutional framework.
- Nevertheless, individual choice in a given market situation is an important part of the equilibrium analysis of wage outcomes.

Investment

- Workers have a finite life, T, and time is discrete.
- Let Y_t denote the earning capacity of the worker with the *current* employer, $t, t = 1, 2., ... T$.
- We assume that

$$Y_t = R_t K_t, \tag{1}$$

  where K_t is the worker's human capital and R_t is the rental rate.
- In a competitive world, without friction, all firms pay the *same* rental rate.

- Workers can accumulate human capital by investment on the job.
- Let I_t, be the proportion of earnings capacity that is forgone when the worker learns on the job.
- Hence, current earnings are

$$y_t = R_t K_t (1 - I_t). \quad (2)$$

- Following the Ben-Porath (1967) model, suppose that human capital evolves according to

$$K_{t+1} = K_t + g(I_t K_t), \quad (3)$$

where $g(.)$ is increasing and concave with $g(0) = 0$.

- Thus, a worker who directs a larger share of his existing capital to investment has lower current earnings but a higher future earning capacity.

- Here we consider only the behavior of workers for a given "production function" $g(.)$.
- In a more general analysis, this function would be influenced by market forces (see Rosen, 1972, and Heckman et al., 1998), but we do not attempt to close the model by deriving the equilibrium trade-off between current and future earnings.

- To determine a worker's investment, we form the Bellman equation

$$V_t(K_t) = \underset{I_t}{Max}[R_t K_t(1 - I_t) + \beta V_{t+1}(K_t + g(I_t K_t))], \quad (4)$$

  where β represents the discount factor and $\beta < 1$.
- This equation states that the value of being employed in period t consists of the current earnings with this employer and the *option* to augment human capital through learning on the job.
- Each of these terms depends on the level of investment of the worker, and one considers only the optimal choices of the worker in calculating the value of the optimal program.

- The first-order condition for I_t in an interior solution is

$$\frac{R_t}{g'(I_t K_t)} = \beta V'_{t+1}(K_{t+1}). \quad (5)$$

- The left-hand side of (5) describes the marginal costs of investment in terms of forgone current earnings, while the right-hand side is the marginal value of additional future earnings.
- In the last period, T, investment is zero because there are no future periods left in which to reap the benefits.

- Differentiating both sides of (4) w.r.t K_t and using (5) we obtain the rule of motion for the marginal value of human capital

$$V_t'(K_t) = R_t + \beta V_{t+1}'(K_{t+1}). \tag{6}$$

- Using the end condition that $V_{T+1}(K_{T+1}) = 0$ for *all* K_{T+1}, meaning that human capital has no value beyond the end of the working period, we obtain

$$V_T'(K_T) = R_T. \tag{7}$$

- The standard investment model assumes stationary conditions; hence, R_t is a constant that can be normalized to 1.
- Then, using (7) and solving (6) recursively, the value of an additional unit of human capital at time t is

$$V_t'(K_t) = \frac{1 - \beta^{T+1-t}}{1 - \beta}, \tag{8}$$

which is *independent* of K_t.

- It follows that the value of being employed at a given current wage *declines* with time, that is, $V'_t(K_t) \geq V'_{t+1}(K_{t+1})$ for all periods $t = 1, 2, ., ., T$.
- The shorter the remaining work horizon, the less valuable is the current stock of human capital and the lower the incentive to augment that stock.
- The lack of dependence on history, implicit in the Ben-Porath (1967) specification, is sufficient but not necessary for the result of declining investment, which holds under more general conditions (see Weiss, 1986).

- The model can be easily generalized to the case in which R_t is variable over time.
- In this case, equation (8) becomes

$$V_t'(K_t) = \sum_{\tau=t}^{T} \beta^{\tau-t} R_\tau. \tag{8'}$$

- Comparing these expressions, it is seen that if R_t rises with time, then the investment in human capital is higher at each period.
- The reason is that investment occurs when a worker receives a relatively lower price for his human capital, so that the forgone earnings are relatively low.
- If the rental rate rises with time at a decreasing rate, this relative price effect weakens with time and investment declines.

The observable implications of this model are clear:

- For a constant R, investment declines as the worker ages and approaches the end of his working life.
- Earnings rise along an optimal investment path. This is caused by two effects that reinforce each other; positive investment increases earning capacity and declining investment induces a rise in its utilization rate.
- If R varies with time, workers that expect exogenous growth in their earning capacity invest at a higher rate and their wage rises at a higher pace. Investment declines if the rate of growth in the rental rate decreases.

Investment in school and on the job

- Investment in school and on the job can be viewed as two alternative modes of accumulation of human capital that complement and substitute each other.
- Complementarity arises because human capital is self-productive, so that human capital accumulated in school is useful for learning on the job.
- Substitution arises because life is finite and if more time is spent in school, there is less time left for investment on the job.
- Although the focus of this survey is on post-schooling investments, the fact that these two modes are to some extent jointly determined leads us to expect interactions, whereby individuals completing different levels of schooling will invest differentially on the job and therefore display different patterns of wage growth.

- Investment on the job is usually done *jointly* with work, while schooling is done separately.
- As a consequence, one foregoes less earning when training on the job than in school.
- However, in school, one typically specializes in the acquisition of knowledge and human capital is consequently accumulated at a faster rate.
- One can capture these differences by assuming different production (and cost) functions for the two alternative investment channels.

- Let p_t be a labor force participation indicator such that $p_t = 1$ if the individual works in period t and $p_t = 0$, otherwise.
- Suppose that when the individual does not work he goes to school and then accumulates human capital according to $K_{t+1} = K_t(1+\gamma)$ where γ is a fixed parameter such that $\gamma K_t > g(l_t K_t)$.
- We also assume that $(1+\gamma) > \frac{1}{\beta}$, which means that the rate of return from investment in human capital γ exceeds the interest rate.
- Otherwise, such investment would never be optimal.
- Assume stationary conditions and let $R_t = 1$.
- We can now rewrite the Bellman equation in the form

$$V_t(K_t) = \underset{p_t, l_t}{Max}[p_t K_t(1-l_t) + \beta V_{t+1}(K_t + p_t g(l_t K_t) + (1-p_t)\gamma K_t)]. \tag{9}$$

- School is the preferred choice in period t if

$$\beta V_{t+1}(K_t(1+\gamma)) > K_t(1-l_t^*) + \beta V_{t+1}(K_t + g(l_t^* K_t)), \quad (10)$$

where the optimal level of training on the job, l_t^*, is determined from (5). Finally, the law of motion for the marginal value of human capital is modified to

$$V_t'(K_t) = p_t + \beta V_{t+1}'(K_{t+1})(1+(1-p_t)\gamma). \quad (11)$$

This extension has several implications:

- Specialization in schooling occurs, if at all, in the first phase of life. It is followed by a period of investment on the job. In the last phase of the life cycle, there is no investment at all.
- During the schooling period, there are no earnings, yet human capital is accumulated at the maximal rate $(1+\gamma)$. During the period of investment on the job, earnings are positive and growing. In the last phase (if it exists), earnings are constant.
- A worker leaves school at the first period in which (10) is reversed. At this point it must be the case that $I_t^* < 1$, which means that at the time of leaving school, earnings must *jump* to a positive level. This realistic feature is present only because we assume different production (and cost) functions in school and on the job, whereby accumulation in school is faster but requires a larger sacrifice of current earnings.

- A person with a larger initial stock of human capital, K_0, will stay in school for a shorter period and spend more time investing on the job. He will have higher earnings and the same earnings growth throughout life.
- A person with a larger scholastic learning ability, γ, will stay in school for a longer period and spend less time investing on the job. He will also have higher earnings and the same earning growth throughout life.

- Although these results depend heavily on the particular form of the production function (3), they illustrate that unobserved characteristics of economic agents can create a negative correlation between the amounts of time spent investing in school and on the job, while there need be no correlation between completed schooling and post schooling wage growth.
- It should be noted, however, that wage growth is often higher for the more educated, which casts some doubt on the neutrality implied by (3).
- Uncertainty and unexpected shocks can also affect the correlation between schooling and investment.
- For instance, the introduction of computers may raise the incentive to invest on the job among educated workers to a larger extent than among uneducated workers because the investment's payoff may be lower for the second group.

Search

- In a world with limited information and frictions, firms may pay a different R because workers cannot immediately find the highest paying firm and must spend time and money to locate employers.
- If a worker meets a new employer, he obtains a random draw $\tilde{R}$ from the given distribution of potential wage offers $F(R)$. The worker decides whether to accept or reject this offer.
- To simplify, we assume here that workers are relatively passive in their search for jobs.
- They receive offers at some fixed exogenous rate λ, but do not initiate offers through active job search.

- We discuss here the case with homogenous workers and firms, assuming that workers are equally productive in all firms and their productivity is constant over time.
- However, firms may pay different wages for identical workers.
- Specifically, if K is the worker's human capital, then the profits of a firm that pays the worker R are $K - RK$.
- Firms that post a high R draw more workers and can coexist with a firm that posts a low R and draws few workers.
- In equilibrium, all firms must have the same profits (see Mortensen and Pissarides, 1999).
- Here we consider only the behavior of workers for a given wage distribution, $F(R)$, and do not attempt to close the model by deriving either the equilibrium wage offer distribution or the equilibrium trade-off between current and future earnings.
- In a more general analysis, the wage distribution is determined by market forces (see Wolpin, 2003).

- Lets us momentarily ignore investment and look solely at the implications of search.
- Consider a worker who receives a rental rate R_t for his human capital from his current employer in period t, so that $Y_t = KR_t$.
- Now imagine that during period t, the worker is matched with a new employer offering another rental rate, R.
- Because the worker can follow the same search strategy wherever he is employed, it is clear that the offer will be accepted if $R > R_t$ and rejected if $R < R_t$.
- If the worker rejects the offer and stays with the current employer, his earning capacity remains the same and $Y_{t+1} = Y_t$.
- If the worker accepts the outside offer and moves to the new employer, his new wage, $Y_{t+1} = RK$, must exceed Y_t.
- The probability that the worker will switch jobs is $\lambda(1 - F(R_t))$ and is *decreasing* in R_t.

The observable implications of this model are:

- A job has an *option value* to the worker. In particular, he can maintain his current wage and move away when he gets a better offer. Consequently, earnings rise whenever the worker switches jobs and remain constant otherwise.
- The higher the worker's current wage, the more valuable is the current job; hence, the offers that the workers accepts must exceed a higher reservation value. Therefore, the quit rate and the expected wage growth decline as the worker accumulates work experience and climbs up the occupational ladder.
- A straight-forward extension is to add involuntary separations. Such separations are usually associated with wage reduction and are more likely to occur at the end of the worker's career, which may explain the reduction in average wages towards the end of the life cycle.

- This model can be generalized by allowing the worker to control the arrival of new job offers by spending time on the job in active search (see Mortensen, 1986).
- Search effort declines as the worker obtains better jobs, so that the arrival rate of job offers and wage growth decline, too.
- Towards the end of the career, a worker may reduce his search effort to a level that generates no job offers.
- Consequently, voluntary quits and wage growth cease.

- The same search model can be motivated slightly differently by assuming that workers and firms are heterogenous.
- Let workers be ranked by their skill, K.
- Let firms be ranked by their *minimal* skill requirement R (see Weiss et al., 2003).
- Assume that worker K employed by firm R produces R if $K \geq R$ and 0 otherwise.
- Because workers with $K \geq R$ on job R produce the same amount, irrespective of their K, we can set their wages to R (assuming zero profits).
- A worker K who is now employed at firm R_t and meets (with probability λ) a random draw from the population of employers, R, is willing to switch if and only if $R > R_t$.
- However the employer is willing to accept him only if $K \geq R$.
- Transition into a better job thus occurs with probability $\lambda(F(K) - F(R_t))$.

Comparison of investment and search

- The investment and search models have similar empirical implications for *average* growth in earnings, i.e., positive and declining wage growth.
- In the investment model, the reason for wage growth is that the worker chooses to spend some of his time learning.
- However, investment declines as a result of the shortened remaining work period, which causes wage growth to taper off.
- In the search model, wage growth is an outcome of the option that workers have to accept or reject job offers.
- Acceptance depends on the level of earnings that the worker attained by time t, so that history matters.

- Two workers of the same age may behave differently because of their different success records in meeting employers.
- But the general trend is for wage growth to decline because workers who attained a higher wage have a lower incentive to search and are less likely to switch jobs.

- Although investment and search have similar implications for wage growth, they can be distinguished by their different patterns in the *variance* of wages and the correlation between wages at different points of the life cycle.
- As shown by Mincer (1974), the variance in wages first declines and then rises, as we move across age groups in a cross section or follow a cohort.
- The reason is that a current low wage is compensated for by a future high wage, so that workers who invest more intensely will *overtake* those with a lower investment rate.
- The minimal variance occurs in the middle range of experience, where individual earning profiles cross.
- Under search, the cause for variability is not differential investment but different success record in locating suitable job matches and the variability in accepted wage offers.

- Homogenous workers become increasingly heterogeneous due to their longer exposure to random job offers.
- However, selection modifies the impact of such shocks on wages, because wages do not go down when the worker keeps the job and those who have high wages are less likely to get a better offer.
- Thus, the variance first increases and then declines as workers are gradually climbing up the income distribution.
- If workers are initially heterogeneous, the variance may also first increase and then decline as workers are gradually sorted into their "right" place.
- The investment model suggests a negative correlation between wage level and wage growth at the beginning of the worker's career and a positive correlation between wage growth and wage level late in the worker's career, whereas the search model implies a negative correlation between current wage and wage growth at *any* point of the life cycle.

- Search and investment also have similar implications for quits, especially if investment has a firm-specific component.
- To the extent that specific investment can be described by a stochastic learning process on the job, as in Jovanovic (1984) and Mortensen (1988), then both wage growth and mobility can be outcomes of either internal shocks in the form of changes in the quality of a match, or external shocks in the form of outside offers.
- The average patterns of wage growth and separations will be the same under specific investment or search.
- However, higher moments, such as the wage variances among stayers and movers, can indicate the importance of specific capital and search, respectively.

Putting the two together

- We now consider the possible interaction between search and investment behavior.
- To simplify, we continue to assume that workers can reject or accept offers as they arrive at an exogenous rate λ, but cannot initiate offers by investing in search.
- However, the option of passive search changes the incentives to invest in human capital.

- The Bellman equation becomes

$$V_t(R_t, K_t) = \underset{I_t}{Max}\{R_t K_t(1 - I_t) + \beta[\lambda E\{\max[V_{t+1}(R_t, K_{t+1}), V_{t+1}(R, K_{t+1})] + (1 - \lambda)V_{t+1}(R_t, K_{t+1})]\}. \tag{12}$$

- Because a worker with a given K can follow the same search and investment strategy on any job, it is clear that he will switch jobs if $R > R_t$.

- Given this reservation value strategy, we can write

$$E\{\max\left[V_{t+1}(R_t, K_{t+1}), V_{t+1}(\tilde{R}_{t+1}, K_{t+1})\right]\} =$$

$$F(R_t)V_{t+1}(R_t, K_{t+1}) + \int_{R_t}^{\infty} V_{t+1}(R, K_{t+1})f(R)dR, \qquad (13)$$

where $f(R)$ is the density of wage offers.

- The first-order condition for l_t is now

$$\frac{R_t}{g'(l_t K_t)} = \beta V_{k,t+1}\left(R_t, K_{t+1}\right)$$

$$+ \lambda\beta \int_{R_t}^{\infty} \left(V_{k,t+1}\left(R, K_{t+1}\right) - V_{k,t+1}(R_t, K_{t+1})\right)f(R)dR, \qquad (14)$$

where $V_{k,t}$ denotes the partial derivative of $V_t(.,.)$ with respect to K_t.

- The interaction between investment and search decisions is captured by the second term in equation (14) which shows that the incentives to invest now include the *capital gains* that the worker obtains if he changes employers.
- The higher K_t, the more one gains from a favorable draw of R; therefore, the incentive to accumulate human capital is stronger.

This extended model has the following features:

- As long as the worker stays with the same firm, investment in human capital declines because of the shortened work period.
- On any such interval, the worker invests more than he would without search and a fixed R. This result reflects the upward drift in the R which is inherent in the search model and qualitatively similar to the result in the regular investment model when R rises exogenously.
- Investment drops when the worker switches to a new job with a higher R, because the option of switching to a new job becomes less valuable.

Human capital and skills

- Human capital K is an aggregate that summarizes individual skills in terms of production capacity.
- Different skills are rewarded differentially in different occupations.
- We assume that this aggregate may be represented as

$$\ln K_j = \sum_s \theta_{sj} S_s, \tag{15}$$

  where S_s is the quantity of skill s possessed by the individual and θ_{sj} is a non-negative parameter that represent the contribution of skill s to occupation j.
- Firms reward individual skills indirectly by renting human capital at the market-determined rental rate, R.

- Thus, the parameter θ_{sj} is the proportional increase in earning capacity associated with a unit increase in skill x_s if the individual works in occupation j.
- Having assumed that θ_{sj} is independent of the quantity of skill s possessed by the individual, these coefficients may be viewed as the implicit "prices" (or "rates of return") of skill s in occupation j.

- Because we are interested here in the timing of occupational changes, it will be convenient to set the problem in continuous time.
- We denote by T the duration of the worker's lifetime and by t a point in time in the interval $[0, T]$.
- We define $h_j(t)$ as the portion of available time spent working in occupation j at time t, so that $0 \leq h_j(t) \leq 1$ and $\sum_j h_j(t) = 1$.
- The worker will typically work at one particular occupation in each point in time but is free to switch occupations at any time.
- The worker's earning capacity is

$$Y(t) = R \sum_j h_j(t) K_j(t). \tag{16}$$

- Skills are initially endowed and can then be augmented by acquiring experience.
- We consider here a "learning by doing" technology whereby work at a rate $h_j(t)$ in a particular occupation j augments skill s by $\gamma_{sj}h_j(t)$.
- Thus, the change in skill s at time t is

$$\dot{S}_s = \sum_j \gamma_{sj}h_j(t). \tag{17}$$

- Note the joint production feature of this technology.
- Working in any one occupation j can influence many skills that are useful in other occupations.
- Yet, such experience may be more relevant to some particular skills.
- In this way, we obtain that work experience is transferable but not necessarily general.

- In the static version of this model (the Roy model), individual skills are constant ($\gamma_{sj} = 0$ for all s and j) and the main issue is the mapping between skills and earnings that results from the different occupational choices of workers with different skills.
- The basic principle that applies there is that each individual will spend *all* his work time in the occupation in which his bundle of skills commands the highest reward [see Willis (1986) and Heckman and Honore, 1990].
- Unexpected changes in the prices of skills, θ_{sj}, can cause the worker to switch occupations; however,under static conditions there is no occupational mobility.
- In the dynamic set up that we outline here, skills vary with time, and this variation is influenced by the worker's career choices.
- In such a context, *planned* occupational switches can arise, even in the absence of shocks, if experience is sufficiently transferable across occupations.

- To simplify the exposition, we consider the case of two occupations and two skills and examine the conditions for a single switch.
- Given our simplifying assumptions, the earnings capacity of a worker in different occupations, K_j grows at constant rates that depend on the occupation in which the worker specializes.
- Suppose that the worker switches from occupation 1 to occupation 2 at time x and then stays there for the rest of his life.
- Then, in the early phase, prior to time x, $h_1(t) = 1$ and

$$\frac{\dot{K}_1}{K_1} = \theta_{11}\gamma_{11} + \theta_{21}\gamma_{21} \equiv g_{1,1}, \tag{18}$$

$$\frac{\dot{K}_2}{K_2} = \theta_{12}\gamma_{11} + \theta_{22}\gamma_{21} \equiv g_{2,1}.$$

- In the later phase, after x, $h_2(t) = 1$ and

$$\frac{\dot{K}_1}{K_1} = \theta_{11}\gamma_{12} + \theta_{21}\gamma_{22} \equiv g_{1,2}, \tag{19}$$

$$\frac{\dot{K}_2}{K_2} = \theta_{12}\gamma_{12} + \theta_{22}\gamma_{22} \equiv g_{2,2}.$$

- The expected lifetime earnings of the worker is

$$V(x) = R\{K_1(0) \int_0^x e^{-rt+g_{1,1}t} dt + K_2(0) \int_x^T e^{-rt+g_{2,1}x+g_{2,2}(t-x)} dt\}. \tag{20}$$

- For a switch at time x to be optimal, it is necessary that $V'(x) = 0$ and for $V''(x) < 0$.

- It can be shown that if work experience in each occupation raises the worker's earnings in that same occupation by more than in the alternative occupation (that is, $g_{1,1} > g_{2,1}$ and $g_{2,2} > g_{2,1}$) then $V'(x) = 0$ implies that $V''(x) > 0$, so that the worker will *never* switch occupations.
- Instead, the worker will specialize in one occupation throughout his working life and concentrate all his investments in that occupation (see Weiss, 1971).
- However, some occupations require a preparation period in other occupations, that serve as stepping stones (see Jovanovic and Nyarko, 1997).
- For instance, it is not uncommon that successful managers start as engineers or physicians rather than junior managers.

Specifically, suppose that

$$\gamma_{11} > \gamma_{12}, \quad \gamma_{21} > \gamma_{22}, \quad \theta_{11} < \theta_{12}, \quad \theta_{21} < \theta_{22}. \tag{21}$$

- Then it is easy to verify that, depending on initial conditions, the worker may start in occupation 1 and then switch to occupation 2 because skill 1 is more important in occupation 2, i.e., $\theta_{12} > \theta_{11}$, but occupation 1 is the better place to acquire skill 1, i.e., $\gamma_{11} > \gamma_{12}$.
- It does not pay to specialize in occupation 1 because the worker will not exploit his acquired skills that are more useful in occupation 2.
- Nor is it usually optimal to specialize in occupation 2, because then the worker will not acquire sufficient skills.
- However, a worker with a large endowment of skill 1 or skill 2 may specialize in occupation 2 immediately.

- This model illustrates quite clearly the main features of occupations that serve as stepping stones.
- Basically, these occupations enable the worker to acquire skills that can be used later in other occupations in a cheaper or more effective way.
- Although these jobs pay less for *all* workers with *given* skills, some workers may still enter them as an investment in training.

- The pattern of earnings growth that is implied by this sequence of occupational choices is easy to summarize.
- At the point of switch, x, earnings *rise* instantaneously, where the proportional jump is $S_1(0)(\theta_{11} - \theta_{12}) + S_2(0)(\theta_{21} - \theta_{22}) + (g_{1,1} - g_{2,1)}x$.
- The growth rate of earnings may either rise or decline following this change, because the restrictions in (21) are consistent with either $g_{1,1} > g_{2,2}$ or $g_{1,1} < g_{2,2}$.
- If we assume, however, that the differences between the two occupations in the learning coefficients (the $\gamma' s$) are more pronounced than the differences in the prices of skills (the $\theta' s$) then $g_{1,1} > g_{2,2}$ and the growth rate in earnings will decline, which is the more realistic case.

Wages, productivity and contracts

- The presumption, so far, was that a worker's wage is closely tied to his productivity.
- However, the relation between these two variables may be quite complex, especially when workers and firms develop durable relationships.
- In such a case, wages and productivity are still tied in terms of long-term averages but, in the short run, systematic differences between wages and productivity may appear that represent credit and risk sharing arrangements, or incentives to exert effort.
- We shall not attempt to describe the complex issues associated with incentives for effort, about which several excellent recent surveys exist.
- However, the issues associated with credit and risk sharing are easy to explain.

- Trade between workers and employers that extends over time is motivated by some basic asymmetry between the parties.
- Specifically, firms may have better access to the capital market and may be able to pool some risks.
- If a worker's output varies over time, and if he has no access to the capital market, the firm may smooth his consumption by offering a flat wage profile which effectively means that the worker borrows from the firm.
- Similarly, if a worker's output is subject to shocks, the firm may accept these risks and provide the worker with insurance that stabilizes his income.
- As we shall now show, the ability of firms to provide such credit or insurance arrangements is limited by the commitments that workers (and firms) can make.

- Consider a worker with a fixed bundle of skills and suppose that because of random variations in the prices of skills, his/her human capital is subject to capital gains or losses.
- Specifically,

$$K_{t+1} = \begin{cases} K_t(1+g) & \text{with probability} \quad p \\ K_t(1-\delta) & \text{with probability} \quad 1-p \end{cases}, \tag{22}$$

where g and δ are fixed parameters that govern the size of capital gains and losses, respectively.

- We denote by $Q_t(K_{t-1})$ the expected present value of the worker's output over the remainder of his work life, $T-t$.
- Let h_t be a sequence of zeros and ones, where 1 for the τ element, $\tau = 1, 2, ...t$ indicates the occurrence of a positive shock and a 0 indicates the occurrence of a negative shock in period τ.

- We refer to such a sequence as the history or sample path.
- Let $y_t(h_{t-1})$ be the wage that a firm promises to pay a worker with history h_{t-1} in period t and let $Y_t(h_{t-1})$ be the present value of the expected payments over the remainder of the working life, from t to T.
- We can think of $Y_t(h_{t-1})$ as the worker's *contractual* assets.

- A risk-neutral firm is indifferent between all contingent contracts that yield the same expected value.
- However, a risk-averse worker with no access to the capital or insurance markets would prefer that the payment stream will be as stable as possible.
- If the worker can commit to stay with the firm, the competition among firms will force them to offer wage contracts that smooth the wage payments over time and across states of nature.
- In practice, workers cannot legally bind themselves to a firm; their option to leave the firm limits the insurance and consumption smoothing that firms can provide (see Harris and Holmstrom, 1982; Weiss, 1984).

- A competitive payment scheme must maximize the expected utility of the worker given the firm's expected profits and the worker's outside options.
- Therefore, the contract that survives must solve the following program

$$V_t(K_{t-1}, Y_{t-1}) = \tag{23}$$
$$\underset{y,x_1,x_0}{Max}\{(u(y) + pV_{t+1}(K_{t-1}(1+g), Y_{t-1} + x_1)$$
$$+ (1-p)V_{t+1}(K_{t-1}(1-\delta), Y_{t-1} + x_0)\},$$

subject to

$$y + px_1 + (1-p)x_0 = 0, \tag{24a}$$
$$Y_{t-1} + x_1 \geq Q_{t-1}(K_{t-1})(1+g)) - a, \tag{24b}$$
$$Y_{t-1} + x_0 \geq Q_{t-1}(K_{t-1})(1-\delta)) - a, \tag{24c}$$

where a is a parameter that represents the costs of mobility across firms, such as loss of firm-specific capital.

- The state variables at period t are the worker's human capital and the expected payments from the firm under the existing contract (including current obligations $y_t(h_{t-1})$).
- The control variables, y, x_1, x_0 represent possible revisions of that contract that can make the worker better off, keeping the firm's expected profits constant and keeping the worker with the firm.
- Constraint (24a) requires that the revisions maintain the cost of the contract to the firm (because Q_{t-1} is fully determined by K_{t-1}, this implies that expected profits are unchanged).
- Constraints (24b) and (24c) imply that other firms cannot bid workers away.
- If the firm changes the contract in such a manner that its obligation falls short of the worker's expected output, it cannot retain the worker because another firm can offer a superior contract and still make non-negative profits.

- The first order conditions are

$$u'(y) - \lambda = 0, \tag{25a}$$

$$\frac{\partial V_{t+1}(K_{t-1}(1+g), Y_{t-1} + x_1)}{\partial Y_t} - \lambda + \frac{\mu_1}{p} = 0, \tag{25b}$$

$$\frac{\partial V_{t+1}(K_{t-1}(1-\delta), Y_{t-1} + x_0)}{\partial Y_t} - \lambda + \frac{\mu_2}{1-p} = 0, \tag{25c}$$

where λ, μ_1, μ_2 are the time-variable non-negative Lagrange multipliers that are associated with the constraints (24a), (24b) and (24c), respectively.

- Differentiating (23) with respect to Y_{t-1} and using conditions (25a)-(25c), we have

$$\frac{\partial V_t(K_{t-1}, Y_{t-1})}{\partial Y_{t-1}} = \lambda, \tag{26}$$

  which implies that in each period and at any possible state, the marginal utility of consumption, $u'(y)$, is equated to the marginal value of the worker's contractual assets, $\frac{\partial V_t(K_{t-1}, Y_{t-1})}{\partial Y_{t-1}}$.
- Because the Lagrange multipliers μ_1 and μ_2 are non-negative, it follows from conditions (25b) and (25c) that the payment stream is arranged in such a way that the marginal value of contractual assets never rises.
- This also means that wage payments never decline as successive realizations of human capital unfold.

- These results have a simple economic interpretation.
- Workers who may suffer either capital gains or capital losses, when skill prices change, would like the firm to transfer wages from "good" states when income is high and marginal utility of income is low to "bad" states when income is low and marginal utility of income is high.
- The firm is willing to do so only if the expected present value of wage payments does not rise in consequence.
- Thus, paying a higher current wage in a bad state implies a wage reduction in some future good state.
- However, the firm can commit to such a transfer policy only if it is able to retain the worker and collect the payment for the insurance that it provides the worker now.

- If the cost of mobility across firms, *a*, is sufficiently high to prevent mobility, then constraints (24b)- (24c) are not binding and $\mu_1 = \mu_2 = 0$.
- Then, the optimal contract implies that *y* is a constant, which means that the firm provides *perfect* insurance and consumption smoothing.
- However, if the cost of mobility across firms, *a*, is sufficiently low, the constraint (24c) which corresponds to a positive shock is binding, because such a shock makes the worker more attractive to other firms.
- The wage profile that emerges in this case is one in which the wage rises when workers receive a positive shock but remains unchanged when they receive a negative shock.
- In this way, the workers receive *partial* insurance from the firm.

- When a positive shock occurs, wages are raised to the minimal level required to retain the worker.
- When a negative shock occurs, wages are set above the worker's productivity.
- This policy requires that workers pay for the insurance by accepting initial wages that fall short of their productivity upon joining the firm.

- If the costs of mobility across firms are low, and workers must be induced to stay with the firm, then their *average* wages rise *faster* than their average productivity.
- This result is reversed if there are substantial costs of mobility across firms and the workers are locked to the firm, a condition that allows the firm to provide perfect insurance.
- In this case, of course, average wages rise at a *lower* rate than does productivity.

- In equilibrium, there is no mobility across firms.
- However the workers' *option* to leave the firm affects wage growth.
- Paradoxically, workers are better off when the costs of mobility are high.
- This holds for two related reasons.
- First, with high mobility costs, workers are effectively locked in with the firm so that the firm can provide perfect rather than partial insurance.
- Second, because information is public and workers are equally productive in all firms, mobility serves no productive role.
- Thus the most efficient arrangement is for workers to stay with their employers.

- A more complex situation arises if workers can influence skill acquisition and use via occupational switches.
- Then, workers will receive less insurance from the firm but obtain higher wage growth resulting from investment in skills acquisition.
- In addition, workers may try to create a more balanced portfolio of skills, a factor supporting mobility and, possibly, multiple job holding.

- An important feature of the optimal wage contract is that wages in period t generally depend on the entire history of shocks and not simply on the accumulated human capital at time t.
- Specifically, $y_t(h_{t-2}, 1, 0)$ may exceed $y_t(h_{t-2}, 0, 1)$.
- While workers have the same productive capacity in period t in both cases, there are wage gains from having *early* success.
- This is because early success provides opportunities for sharing risk with potentially more productive realizations in the future, an option not available to workers who experienced early failure.
- More generally, conditions at the time at which the commitments are taken e.g., when workers entered the firm, can cause wage differences between workers who are equally productive.

Unobserved productivity and learning

- A particular worker's productivity may be unknown to the worker and potential employers.
- Over time, the worker's performance is observed; one may use this information to make inferences about the worker's "true" skills.
- This learning process can create negative and positive shocks to the worker's perceived productivity, similar to those discussed above.
- However, the learning model has further implications concerning mobility.
- That is, workers can experiment in an occupation where learning about ability is possible and then, as their abilities are gradually revealed, sort themselves into different occupations, based on their realized performance.

- Let there be two occupations, one low skill, one high skill, and let there be two types of workers, those of high ability and those of low ability.
- All workers perform equally well in the low-skill occupation and produce one unit of output per period, irrespective of ability.
- Workers differ in their ability to perform the required jobs in the high-skill occupation; we denote the expected output, per period of time, as q_l and q_h for the low and high ability workers, respectively.
- However, neither the workers nor their employers know whether a particular worker is of high or low ability.
- The common prior probability that a specific worker is of low ability is denoted by π_0.

- With time, as a worker's performance is observed by all agents (including the worker himself), all agents modify this common prior.
- Although a worker's productivity remains constant over time, the new information can affect his wages and employment.

- We may model the realized output as a simple Bernoulli trials so that q_i is the fixed probability that type i, $i = l, h$, will produce one unit of output in period t and $1 - q_i$ is the probability that type i will produce nothing in period t.
- Let $n(t)$ be the (random) number of successes that a worker has accumulated up to period t.
- Based on this information, one can update the probability that he is of the low ability type.

- Specifically, the posterior probability is

$$\pi(t,r) \equiv \Pr\{q = q_l / n(t) = r\} = \frac{\pi_0 q_l^r (1-q_l)^{t-r}}{\pi_0 q_l^r (1-q_l)^{t-r} + (1-\pi_0) q_h^r (1-q_h)^{t-r}}, \tag{27}$$

and the updated expected output per period is

$$q(t,r) = q_l \pi(t,r) + q_h[1-\pi(t,r)]. \tag{28}$$

- From (27) it follows that $\pi(t, r)$ rises in t for a given r and declines with r for a given t.
- That is, if a worker did not perform well, a low $n(t)$ up to a given time t, the posterior probability that he is of low ability increases.
- In contrast, if the worker has a favorable record, the posterior probability that he is of high ability increases.
- The perceived (expected) output of the worker is correspondingly modified downwards or upwards.
- (In this respect, the model is similar to the one discussed in the previous section, except that the informational value of the shocks (success or failure) decays over time.)
- With sufficient time, the process reveals the true identity of the worker.

- Consider first the case in which workers are risk-neutral and assume that workers are paid their current perceived output at each point of time.
- Because all workers are ex ante identical, they will all start at the risky high skill occupation, while attempting to learn their true ability.
- As the public information about each worker accumulates, workers are separated in terms of wages and employment.
- Those with inferior performance will receive lower wages and some of them will choose to leave.
- Those with superior records will receive higher wages and will choose to stay.

- Because of the finite time horizon and costs of mobility, workers will not move at the end of their career even though their perceived output and wages continue to fluctuate.
- This mobility pattern continues to hold if workers are risk-averse and if firms provide partial insurance so that wages are rigid downwards.
- However, an important difference is that such insurance can induce the workers to stay in the skilled sector even if their output in that occupation is low.
- With efficient contracts, such workers must be forced out, i.e., denied tenure (see Harris and Weiss, 1984).

- The "pure" learning model has some strong implications for wage growth that hold for any distribution of shocks provided that we continue to assume that the shocks are independent across time.
- Suppose that worker *i*'s performance in period t is given by

$$y_{it} = \eta_i + \varepsilon_{it}, \tag{29}$$

where η_i is a fixed parameter that is unknown to the firm, and ε_{it} is a random *iid* shock with zero mean.

- Now if firms pay wages based on workers perceived output at time t, $w_{it} = E(y_{it}/I_t) = E(\eta_i/I_t)$, where I_t is any information available at t.

- Then, because expectations are linear operators, it follows that $E(\eta_i/I_t) = E(E(\eta_i/I_{t+1})/I_t)$ and

$$w_{it} = E(w_{i,t+1}/w_{it}). \tag{30}$$

- This martingale property implies that innovations in the wage process $w_{i,t+1} - E(w_{i,t+1}/I_t) = w_{i,t+1} - w_{it}$ are serially uncorrelated.
- Intuitively, any particular piece of the agents' information that the researcher observes has already been used by the agents and cannot change the predicted outcome (see Farber and Gibbons, 1996).
- However, if one adds contracting and downward rigidity due to risk aversion, then, conditioned on the current wage, history matters.
- In particular, early success implies higher wages throughout the worker's career.

- Nevertheless, if a person with an early success is compared to a person with a late success, but both receive the *same current wage* then the late beginner will have the higher future expected wage (see Chiappori et al, 1999).
- That is, the fact that the early beginner has the same wage as a late beginner speaks against him.
- In this respect, “what have you done for us lately” matters more.

- Farber and Gibbons (1996) and Altonji and Pierret (2001) discuss further empirical implications of such models of public learning.
- Importantly, they distinguish between information available to an outside observer (econometrician) and the information available to the economic agents.
- If the econometrician can observe a variable that is correlated with ability, even if not observed by the agents, then this variable will have an affect on wages which rises with time, reflecting the accumulation of information by the agents.
- In contrast, the effects of outcomes that employers observe, other than the worker's output, and that are correlated with ability (such as schooling) will decline over time as their marginal informational content diminishes.