

# Efficiency Units, Elementary Hedonic Models (Gorman and Lancaster) With and Without Bundling Restrictions

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## Overview

### Bringing in Selection of Workers to Firms or at Least Some Sectors to Wage Determination

- Pure efficiency units models keep firms in background.
- Let  $\bar{L}$  = aggregate labor,  $\bar{K}$  = aggregate capital.

$$Y = F(\bar{L}, \bar{K})$$

$$W = \frac{\partial F}{\partial \bar{L}} \quad R = \frac{\partial F}{\partial \bar{K}}$$

- No theory of which workers and firms are matched.

- Bring back the identity of firms to develop a theory of matching and heterogeneity.
- Issues: How to match workers to firms?
  - Sorting irrelevant in the case of pure efficiency units models.
  - Becomes important when workers have different efficiency at different firms.
- We start our investigation under the assumptions of perfect certainty on both sides (No private information).
- No transactions costs (mobility costs).

- **Gorman-Lancaster is multi-attributed efficiency units model**
  - An efficiency units model makes the identity of the firm irrelevant (workers equally productive at all firms) – a model of general human capital. Rearrange workers among firms and get no change in output at each firm as long as total efficiency units the same in each firm.
  - A model with comparative advantage emerges if workers have different advantages in different sectors but assignment of a worker to a sector does not preclude any other worker going there. Sectors may be firms or industrial sectors. Now sorting matters – and a nontrivial labor supply function and demand for labor function emerges.

## Assignment Problem (Becker, 1974; Koopmans and Beckmann, 1957; Shapley and Shubik, 1971)

### The Guiding Principle of the Assignment Problem Literature Is Neither Comparative Nor Absolute Advantage

- 1 It is opportunity cost.
- 2 Place worker  $A$  at firm  $\alpha$ .
- 3 Means worker  $B$  can't go to firm  $\alpha$ .
- 4 Not just relative productivity, but who is best relative to the next best allocation determines the assignment. Continuous versions – worker and firms have close substitutes.
- 5 Discrete version (Koopmans–Beckmann) – no close substitutes. (Raises rent division problem). (See handout 6-4.)

- The discrete version requires no notion of comparing the “quality” or “efficiency” of any 2 workers (no need for a scale of labor quality).
- Roy model is a model of comparative advantage but without the 1-1 matching property.

# Models of Wages and the Pricing of Skills

## Standard model of efficiency units

$H$  = human capital measured in efficiency units

$R$  = price per unit efficiency unit

Observed wages are

$$W = RH$$

- a Under competition, all workers receive the same price ( $R$ ) per unit human capital
- b Discrimination, search frictions (including geographical immobility) may create different prices



- c Workers with different productive characteristics  $x$  may have different amounts of human capital

$$H = \phi(x)$$

- d

$$\frac{\partial \ln W}{\partial x} = \frac{1}{\phi(x)} \frac{\partial \phi(x)}{\partial x}$$

a purely technological relationship.

- e Market forces operate only through the intercepts of the log wage equation, not slopes
- f Widely used in empirical labor economics: Heckman and Sedlacek, Keane and Wolpin, etc. Used in multi-attribute matching literature as well.

**Gorman Lancaster Model: Workers have endowments of vectors of traits, each priced like an efficiency unit, at least under certain conditions**

- 1 Workers have a bundle of traits ( $X_i$ ) for worker  $i$ .
- 2 Firms' production functions depend on the aggregate of those traits.

Let  $\hat{X}^j$  be the aggregate of the characteristics of the workforce of firm  $j$ .

- 3  $Y^j = f(\hat{X}^j)$

- 4 Under constant returns to scale, we can represent this as

$$Y^j = N^j f(\bar{X}^j)$$

where  $N^j$  is the number of workers at the firm and  $\bar{X}^j$  is the average quality at the firm. We will assume CRS as does the entire literature on the Edgeworth Box (see Mas-Colell, Whinston, and Green, 1995).

- 5 In the aggregate,

$$Y = G(\hat{X})$$

- 6 Marginal product of an extra unit of  $k$  is

$$\frac{\partial Y}{\partial X_k} = G_k = \pi_k$$

All workers face the same prices;

But now the map between wages and endowments depends on the prices.

- 7 Labor earnings for worker  $i$  are

$$W_i = \sum_{k=1}^K \pi_k X_{i,k}$$

8

$$\ln W_i = \ln \left( \sum_{k=1}^K \pi_k X_{i,k} \right)$$
$$\frac{\partial \ln W_i}{\partial X_k} = \frac{\pi_k}{W_i} \quad k = 1, \dots, K$$

### Mapping not purely technological;

Suppose that there are two sectors with different skill intensities. (Define skill intensity.) (Same ratios of factors in the two sectors have different productivities.)

The Gorman-Lancaster Model: Two production functions for sectors  $A$  and  $B$

$$G^A(\hat{X}^A) \quad \text{and} \quad G^B(\hat{X}^B) \\ \hat{X}^A + \hat{X}^B = \hat{X}$$

Sectoral productivity of factor  $k$  in Sectors  $A$  and  $B$  are, respectively,

$$\frac{\partial G^A(\hat{X}^A)}{\partial X_k} \quad \frac{\partial G^B(\hat{X}^B)}{\partial X_k}$$

As an equilibrium, we know that if workers could unbundle and sell their individual productive characteristics item by item, the law of one price  $\implies$

$$\frac{\partial G^A(\hat{X}^A)}{\partial X_k} = \frac{\partial G^B(\hat{X}^B)}{\partial X_k}$$

But suppose that skills are bundled?

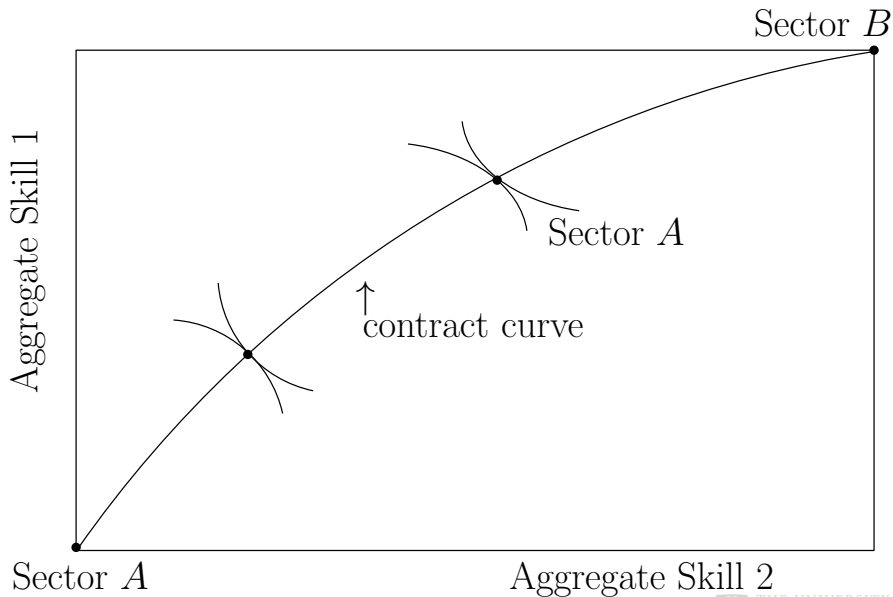
- a Firm buys a *bundle* of skills

$$X_{i,1}, \dots, X_{i,k}, \dots, X_{i,K}$$

when it buys worker  $i$ .

- b All skills used in each sector
- c Consider a case where  $K = 2$ : Full employment of factors.  
Draw up an Edgeworth Box: Assume CRS and that workers can unbundle their skills  
(Box defines the feasible set)



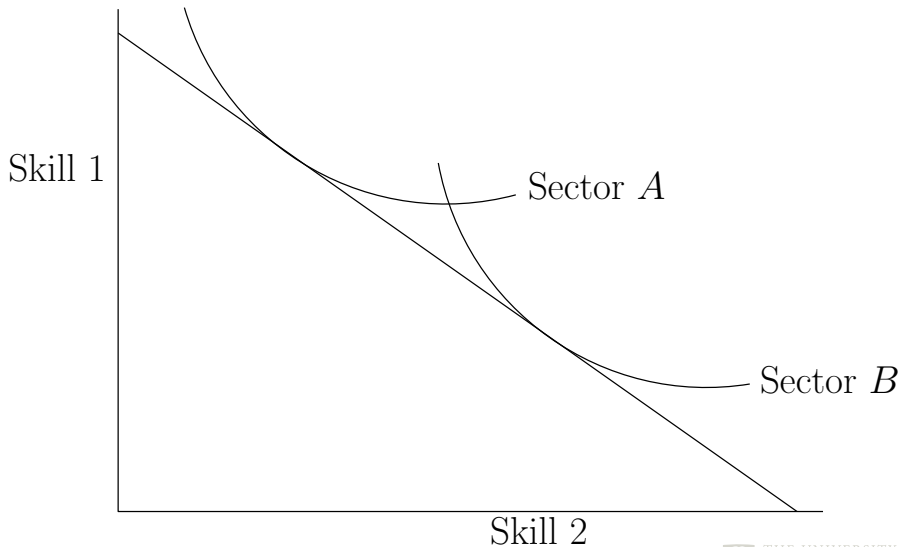


Question: *Why, as you expand Sector A, does the equilibrium price ratio (Skill 1 price to Skill 2 price) increase (i.e., the price of Skill 1 becomes relatively more expensive)?*

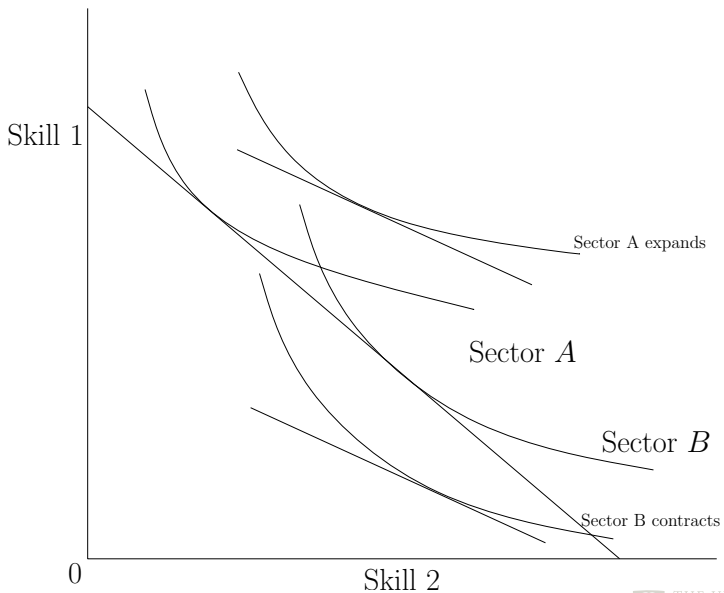
(End of Question.)

- Factor intensities differ across sectors
- As drawn, Sector A has greater Skill 1 intensity, i.e., at the same skills price,  $\pi = (\pi_1, \pi_2)$ , the firm has a bias toward using more of Skill 1.

## An Equilibrium Output: Law of One Price

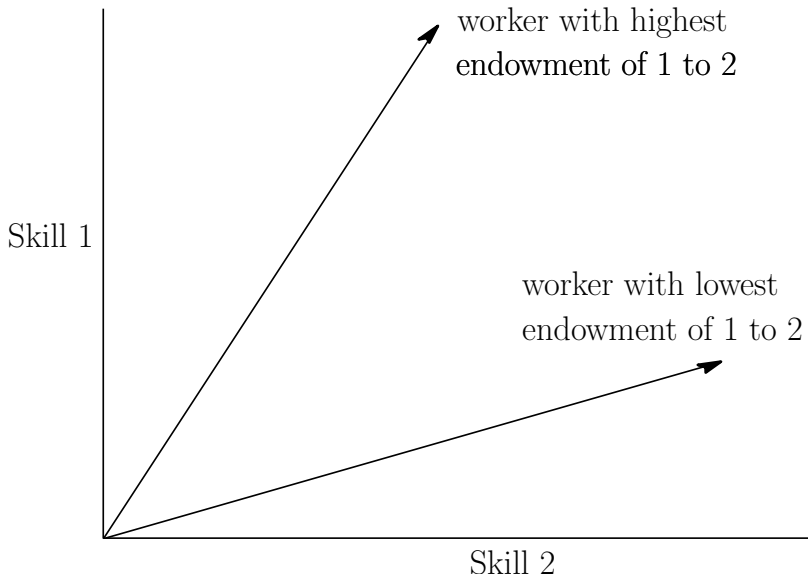


- Notice that as Sector  $A$  expands, the only place it can get workers is from Sector  $B$ .
- $\therefore$  it bids up the skill price of 1 in both sectors.
- Firms substitute toward Skill 2 (cheaper)
- Causes relative price of Skill 1 to expand
- Law of one price still applies.
- Workers are getting one price in both sectors.
- Workers are indifferent as to which sector they go into.



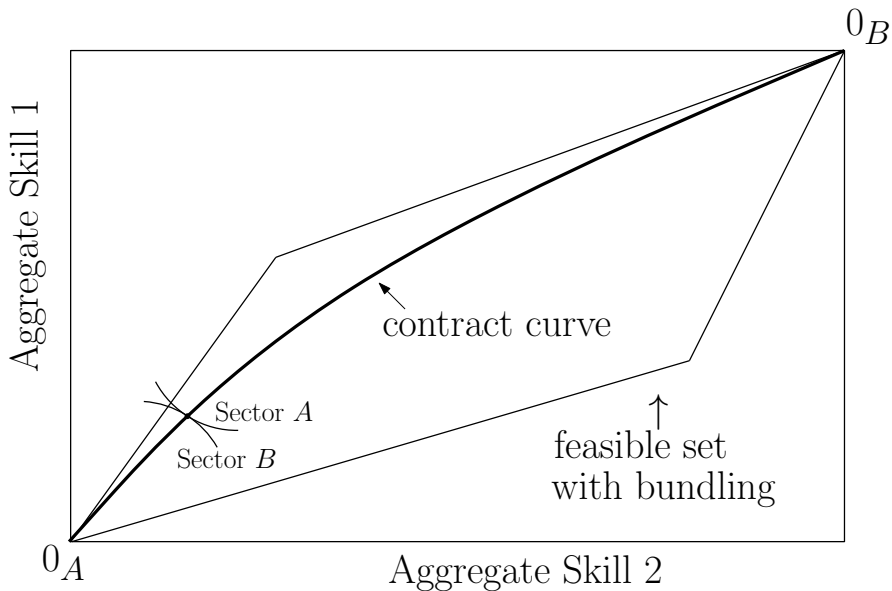
- $A$  is more Skill 1 intensive
- Full employment assumed:
- As the output of Sector  $A$  expands, Sector  $B$  contracts.
- It releases relatively more 2 than 1 because of its skill intensity.
- $\therefore$  Skill price of 2 declines relative to 1.
- (Remember, we assumed constant returns to scale so we do *not* worry about scale effects which may be important.)

- **Suppose now, that workers have bundled skill.**
- Boundaries of Box change: Suppose that range of ratios is as shown



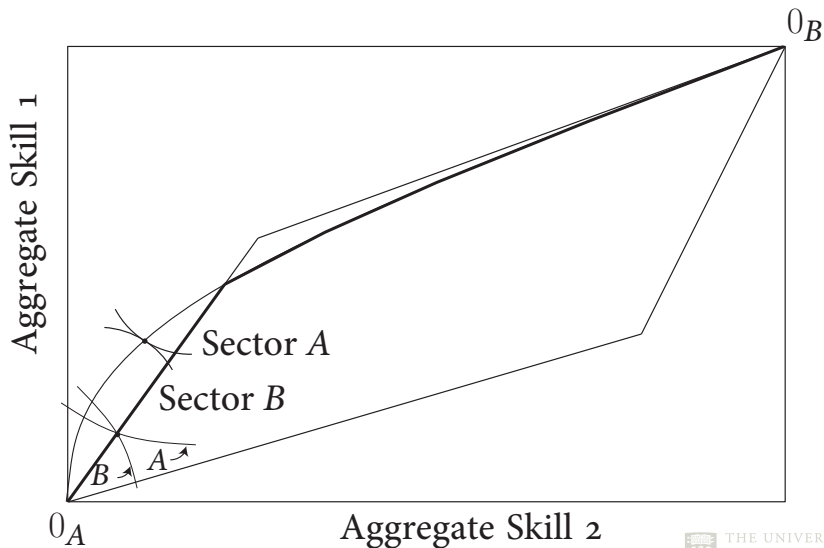


This restricts the range of feasible trades



Suppose that the boundaries are binding and Sector  $A$  is more skill intensive

## Feasible Set



- If you could unbundle workers (so they could sell their personality or their brawn), contract curve would be dotted line above.
- But cannot unbundle.
- Relative price of Skill 1 to Skill 2 is higher in Sector A.
- $\therefore$  unequal prices of skills in the sector

$$\frac{\pi_1^{(A)}}{\pi_2^{(A)}} > \frac{\pi_1^{(B)}}{\pi_2^{(B)}}$$

- Now workers care about which sector they go into.

- Income maximizing worker  $i$  goes into Sector A if

$$\pi^{(A)} X_i > \pi^{(B)} X_i \quad (\text{Discrete choice model})$$

- Worker at the margin is a person with a bundle  $\tilde{X}$  such that

$$\pi^{(A)} \tilde{X} = \pi^{(B)} \tilde{X}$$

- $\therefore$  Now sectoral choice and associated price differences are factors that produce income inequality.
- (Same factor gets a different price in different sectors.)

## Aggregate equilibrium: Workers have

- Demand Equal Supply; Workers sort into sectors
- (May or may not have equal skill prices)

## How to implement this model empirically?

- a Easy if all components of  $X_i$  are observed
- b Difficult if not

See Heckman and Scheinkman (1987) on Reading List for empirical work and derivation under much more general conditions.

- This paves the way to the Roy model of comparative advantage: A basic framework for understanding counterfactuals, wage inequality, and policy variable. Workers have an endowment

$$(X_{iA}, X_{iB})$$

A worker can use only one skill in any sector.  $X_{iA}$  is associated with Sector  $A$ ;  $X_{iB}$  is associated with Sector  $B$ .

- Thus workers have two mutually exclusive endowments.



**The Empirical Importance of Bundling  
A Test of the Hypothesis of Equal Factor Prices  
Across All Sectors**

(From Heckman and Scheinkman,  
*Review of Economic Studies* 54(2), 1987)

- How to estimate the skill prices across sectors when there are unobserved skill prices?
- How to test equality of skill prices across sectors?
- Unobserved traits may be correlated with observed traits

$$Y_{in} = \underbrace{\underline{w}_{no} \underline{x}_{io}}_{\text{observed}} + \{ \underbrace{\underline{w}_{nu} \underline{x}_{iu}}_{\text{unobserved}} + \varepsilon_{in} \}, \quad (1)$$

$$i = 1, \dots, I, \quad n = 1, \dots, N.$$

- Allow for unobserved skills.
- Skills are assumed constant over time for the individual.
- Suppose that persons stay in one sector and we have  $T$  time periods of panel data on those persons.
- Stack these into a vector of length  $T$ .
- Let  $\kappa_u$  be the number of unobserved components.
- Let  $\kappa_o$  be the number of observed components.

In matrix form we may write these equations for person  $i$  as

$$\tilde{Y}_i = \tilde{w}_o \tilde{x}_{io} + \{\tilde{w}_u \tilde{x}_{iu} + \tilde{\varepsilon}_i\}, \quad \text{for each sector } n \quad (2)$$

(Drop the  $n$  subscript for each sector.)

Following Madansky (1964), Chamberlain (1977) and Pudney (1982), assume  $T \geq 2\kappa_u + 1$  and partition (2) into three subsystems:

- We can write a system down for each  $n = 1, \dots, N$ .
- Assume for simplicity  $\underline{x}_{io}$  and  $\underline{x}_{iu}$  are **time invariant**.

$\underline{w}_o (T \times J_0)$        $J_0$  is the number of observed variables

$\underline{w}_u (T \times J_1)$        $J_u$  is the number of unobserved variables

$\underline{x}_{io} (J_0 \times 1)$        $\underline{x}_{iu}$  is  $J_u \times 1$

- The time invariance of  $\underline{x}_{iu}$  is essential (at least for a subset).
- Time invariance of  $\underline{x}_{io}$  is easily relaxed (notationally burdensome).

(i) A basis subsystem of  $\kappa_u$  equations from (2)

$$\underline{Y}_{(1)} = \underline{w}_{o(1)}\underline{x}_{io} + \{\underline{w}_{u(1)}\underline{x}_{iu} + \underline{\varepsilon}_{(1)}\}, \quad n = 1, \dots, N \quad (3a)$$

$\underline{w}_{u(1)}$  is  $\kappa_u \times \kappa_u$

(ii) A second subsystem of equations all of which are distinct from the equations used in (i)

$$\underline{Y}_{(2)} = \underline{w}_{o(2)}\underline{x}_{io} + \{\underline{w}_{u(2)}\underline{x}_{iu} + \underline{\varepsilon}_{(2)}\}, \quad n = 1, \dots, N \quad (3b)$$

(iii) The rest of the equations (at least  $\kappa_u$  in number)

$$\underline{Y}_{(3)} = \underline{w}_{o(3)}\underline{x}_{io} + \{\underline{w}_{u(3)}\underline{x}_{iu} + \underline{\varepsilon}_{(3)}\}. \quad (3c)$$

Assuming that  $\underline{w}_{u(1)}$  is of full rank, the first system of equations may be solved for  $\underline{x}_{iu}$ , i.e.,

$$\underline{x}_{iu} = \underline{w}_{u(1)}^{-1} [\underline{Y}_{(1)} - \underline{w}_{o(1)} \underline{x}_{io} - \underline{\varepsilon}_{(1)}]. \quad (4)$$

Substituting (4) into (3b), we reach

$$\begin{aligned} \tilde{Y}_{(2)} = \tilde{x}_{io} & \left[ \tilde{W}_{o(2)} - \tilde{W}_{u(1)}^{-1} \tilde{W}_{u(2)} \tilde{W}_{o(1)} \right] \\ & + \tilde{W}_{u(1)}^{-1} \tilde{W}_{u(2)} \tilde{Y}_{(1)} + \underbrace{\tilde{\varepsilon}_{(2)} - \tilde{W}_{u(1)}^{-1} \tilde{W}_{u(2)} \tilde{\varepsilon}_{(1)}}_{\text{unobserved error term}}. \end{aligned}$$

- Gets rid of  $\tilde{x}_{iu}$ .
- But OLS fails because, by construction,  $\tilde{\varepsilon}_{(1)}$  is correlated with  $\tilde{Y}_{(1)}$ .



## Internal Instruments

- However, we have an internal instrument
- Use IV to instrument for  $Y_{(1)}$ . The natural instruments are  $Y_{(3)}$ . They are valid as long as  $\underline{w}_{u(3)}$  are nonzero and the rank condition is satisfied.
- Find a lot of evidence against equality of factor prices across sectors.

## Simple Example ( $J_u = 1$ )

- $X_i^0(1)$ : observed variable for  $i$  in the first period
- $X_i^u(1)$ : unobserved in first period (dimension=1)
- $\varepsilon(j)$ : a period  $j$  specific shock uncorrelated with  $X^u(l)$ ,  $X^0(l)1$ , and  $\varepsilon(l)$ ;  $l \neq j$ .

$$Y_i(1) = \beta_1 X_i^0(1) + \lambda_1 X_i^u(1) + \varepsilon_i(1)$$

$$(*) \quad Y_i(2) = \beta_2 X_i^0(2) + \lambda_2 X_i^u(1) + \varepsilon_i(2)$$

$$Y_i(3) = \beta_3 X_i^0(3) + \lambda_3 X_i^u(1) + \varepsilon_i(3)$$

- a  $\beta_j$  is price of observed skills in period  $j$ ;  $X_j$  is price of unobserved skill
- b Remember:  $\varepsilon(j)$  mutually independent, mean zero
- c  $X_i^{(0)}(j) \not\perp X_i^{(u)}(l)$ ; all  $j, l$  (omitted variable bias)
- d Assume  $X_i^u(1) = X_i^u(2) = X_i^u(3)$
- e  $\lambda_j, \beta_j$  and  $X_i^0(j)$  can change with  $j$

- $\varepsilon(l) \perp\!\!\!\perp \varepsilon(k) \quad \forall l \neq k$
- Steps:
  - Step 1: Use equation for  $Y_i(1)$  to solve for  $X_i^u(1)$

$$\frac{Y_i(1) - \beta_1 X_i^0(1) - \varepsilon_i(1)}{\lambda_1} = X_i^u(1)$$

- Assumes  $\lambda_1 \neq 0$  (price of unobserved skill in period 1)
- Step 2: Substitute in the second equation for  $Y_i(2)$

$$Y_i(2) = \beta_2 X_i^0(1) + \frac{\lambda_2}{\lambda_1} (Y_i(1) - \beta_1 X_i^0(1) - \varepsilon_i(1)) + \varepsilon_i(2)$$

- Collect terms

$$* \quad Y_i(2) = (\beta_2 - \frac{\lambda_2}{\lambda_1}\beta_1)X_i^0(1) + \frac{\lambda_2}{\lambda_1}Y_i(1) \\ + \varepsilon_i(2) - \frac{\lambda_2}{\lambda_1}\varepsilon_i(1)$$

- $X_i^u(2) = X_i^u(1)$  eliminated;  $\therefore$  omitted variable eliminated
- From first equation:  $Y_i(1) \not\perp \varepsilon_i$  (out of the frying pan and into the fire)
- Step 3:  $Y_i(3)$  is an instrument for  $Y_i(1)$  in equation (\*)
- Why? (Depends on  $X_i^u(1)$  as does  $Y_i(1)$ )
- $\varepsilon_i(3) \perp (\varepsilon_i(2) - \lambda_2\varepsilon_i(1))$
- Conclusion:  $\therefore$  we get  $(\beta_2 - \frac{\lambda_2}{\lambda_1}\beta_1)$  and  $\frac{\lambda_2}{\lambda_1}$

- Switching the roles of 1, 2, and 3, we can get  $\frac{\lambda_j}{\lambda_k}; j \neq k$
- All assumed to be non-zero
- Notice we need one normalization to separate  $\lambda_j$  from  $X_i^u$  (both unobserved)
- Set  $\lambda_1 = 1$ ,  $\therefore$  we know  $\lambda_2, \lambda_3$
- This normalization is essential: we do not directly observe  $X_i^u(1), X_i^u(2)$  or  $X_i^u(3)$  or the  $\lambda$ .
- They enter the wage equation as  $[\lambda_1 X_i^u(1)], [\lambda_2 X_i^u(2)], [\lambda_3 X_i^u(3)]$ .

$$\left. \begin{aligned} \beta_3 - \lambda_3\beta_1 &= \phi_{31} \\ \beta_3 - \lambda_3\beta_1 &= \phi_{32} \\ \beta_1 - \lambda_1\beta_2 &= \phi_{12} \\ \beta_1 - \lambda_1\beta_3 &= \phi_{13} \\ \beta_2 - \lambda_2\beta_1 &= \phi_{21} \\ \beta_2 - \lambda_2\beta_3 &= \phi_{23} \end{aligned} \right\} \phi_{l,k} \text{ all known}^1$$

- <sup>1</sup>But not necessarily the individual parameters on the left hand side (except  $\lambda_j$ )
- From previous analysis, the  $\phi_{ij}$  all known as are  $\lambda_j$
- 3 equations; 3 unknowns
- $\therefore \beta_1, \beta_2, \beta_3$  known (rank condition requires “sufficient” variation in prices of skills)
- Everything identified (prices of observed and unobserved skills) up to normalization.

TABLE I

*(Basis described in the appendix)*

(1) Sector	(2) System MSE	(3) Test	(4) $F(\text{DFN}, \text{DFD}) =$	(5) Prob > $F$	(6) Number of observations in each year
Durable vs. Nondurable	3.208210	1	(117, 1143) = 1.1448	0.1491	153
		2	(90, 1143) = 0.9213	0.6840	
		3	(27, 1143) = 1.7777	0.0087	
Manufacturing vs. Service	3.447400	1	(117, 3411) = 1.6754	0.0001	405
		2	(90, 3411) = 0.7336	0.9717	
		3	(27, 3411) = 3.0062	0.0001	
Blue vs. White Collar	2.600956	1	(156, 6648) = 2.4197	0.0006	580
		2	(120, 6648) = 1.2943	0.0176	
		3	(36, 6648) = 3.0714	0.0001	
North vs. South	2.299067	1	(156, 7056) = 1.9586	0.0001	614
		2	(120, 7056) = 1.4981	0.0007	
		3	(36, 7056) = 3.0844	0.0008	
Manufacturing vs. Non-mfg	4.746601	1	(117, 5787) = 1.4411	0.0015	669
		2	(90, 5787) = 1.1062	0.2323	
		3	(27, 5787) = 3.0978	0.0001	

*Notes.*

1. Test 1 tests equality of the coefficients of (12) in both sectors.

Test 2 tests equality of the coefficients associated with observed characteristics in (12).

Test 3 tests equality of the coefficients associated with the unobserved characteristics in (12) ( $w_{u(1)}^{-1}, w_{u(2)}$ ).

*Notes.*

1. Test 1 tests equality of the coefficients of (12) in both sectors.  
Test 2 tests equality of the coefficients associated with observed characteristics in (12).  
Test 3 tests equality of the coefficients associated with the unobserved characteristics in (12) ( $w_{u(1)}^{-1}$ ,  $w_{u(2)}$ ).
2. Durable: Metal Industries, Machinery including Electrical, Motor Vehicles and other Transportation Equipment, other durables.  
Non Durable: Food, Tobacco, Textile, Paper, Chemical and other Non Durables.  
Manufacturing: All Durable and Non Durable plus "manufacturing unknown".  
Services: Retail Trade, Wholesale Trade, Finance, Insurance, Real Estate, Repair Service, Business Service, Personal Service, Amusement, Recreation and Related Services, Printing, Publishing and Allied Services, Medical and Dental Services, Educational Services, Professional and Related Services.  
North: Conn., Del., Ill., Ind., Maine, Mass., Mich., Minn., N.H., N.J., N.Y., Ohio, Penn., R.I., W. Va., Wis., Vermont.  
South: Alab., Ark., Fla., Geo., Ky., La., Miss., N.C., S.C., Tenn., Tex., Va., Ok.  
White Collar: Professional, Technical and Kindred; Managers, Officials and Proprietors; Self Employed Businessmen; Clerical and Sales Work.  
Blue Collar: Craftsmen, Foremen and Kindred Workers; Operatives and Kindred Workers; Labourers and Service Workers, Farm Labourers.



TABLE II  
4 Factor models

(1) Sector	(2) System MSE	(3) Test	(4) $F(\text{DFN}, \text{DFD}) =$	(5) Prob > F	(6) Number of observations in each year
Durable vs. Nondurable	1.480446	1	(144, 1089) = 1.2902	0.0166	153
		2	(108, 1089) = 1.1722	1.1197	
		3	(36, 1089) = 1.3644	0.0756	
Manufacturing vs. Service	1.271277	1	(144, 3357) = 2.6513	0.0001	405
		2	(108, 3357) = 1.2957	0.0231	
		3	(36, 3357) = 6.6334	0.0001	
Blue vs. White Collar	3.830300	1	(192, 6576) = 1.7228	0.0001	580
		2	(144, 6576) = 1.3400	0.0045	
		3	(48, 6576) = 1.8698	0.0003	
North vs. South	2.456318	1	(192, 6984) = 1.9893	0.0001	614
		2	(144, 6984) = 0.8240	0.9381	
		3	(48, 1836) = 2.3018	0.0001	
Manufacturing vs. Non-mfg.	1.617166	1	(180, 1836) = 1.7121	0.0001	669
		2	(132, 1836) = 1.4107	0.0020	
		3	(48, 1836) = 2.0701	0.0001	

TABLE III  
5 Factor models

(1) Sector	(2) System MSE	(3) Test	(4) $F(DFN, DFD) =$	(5) Prob > $F$	(6) Number of observations in each year
Blue vs. White Collar	1.573852	1	(228, 6912) = 2.0534	0.0001	580
		2	(168, 6912) = 1.6639	0.0001	
		3	(60, 6912) = 3.8733	0.0001	
North vs. South	1.418750	1	(228, 6504) = 3.8840	0.0001	614
		2	(168, 6504) = 2.2027	0.0001	
		3	(60, 6504) = 10.0017	0.0001	

## APPENDIX

For the 3 factor models we adopt the following basis:

Years for wages ( $Y_{(2)}$ )	Basis years
1968, 1969, 1970	1971, 1972, 1973
1971, 1972, 1973	1968, 1969, 1970
1974, 1975, 1976	1971, 1972, 1973
1977, 1978, 1979	1974, 1975, 1976

For the 4 factor models we adopt the following choice of basis:

Years for wages ( $Y_{(2)}$ )	Basis years
1968, 1969, 1970, 1971	1972, 1973, 1974, 1975
1972, 1973, 1974, 1975	1968, 1969, 1970, 1971
1976, 1977, 1978, 1979	1972, 1973, 1974, 1975

For the 5 factor models we adopt the following choice of basis:

Years for wages ( $Y_{(2)}$ )	Basis years
1968, 1969, 1970, 1971, 1972	1973, 1974, 1975, 1976, 1977
1973, 1974, 1975, 1976, 1977	1968, 1969, 1970, 1971, 1972
1978, 1979	1968, 1969, 1970, 1971, 1972