# Heterogeneity in the Dynamics of Labor Earnings

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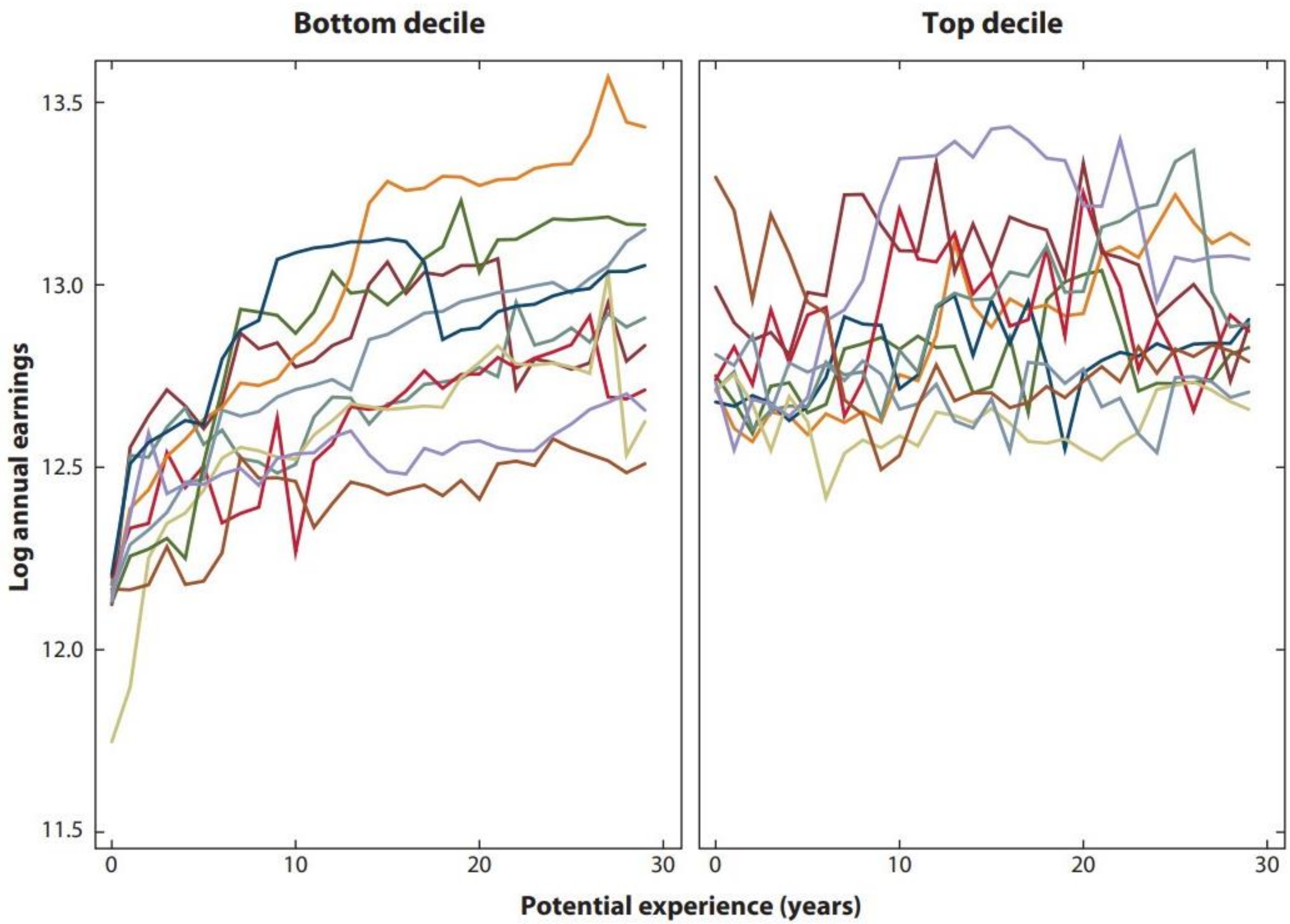
# 1. Introduction

- During the past 35 years, a large empirical literature on individual earnings dynamics has developed.
- **Figure 1** shows earnings paths for 20 Danish men with vocational training who were born in 1958; they are followed from close to the beginning of their labor market careers in 1980 until 30 years.
- The second motivation for allowing for pervasive heterogeneity derives from our view that models of the earnings process, even though atheoretic, should not rule out widely posited theory models.
- Third, most interesting questions cannot be properly addressed unless heterogeneity is explicitly taken into account.
- The final reason why it is important to consider heterogeneity is that it has a major impact on the validity of econometric modeling and estimation procedures.

# Figure 1: Individual earnings path for 20 Danish men with vocational training born in 1958.




Data taken from the Danish register (1980–2009).

# 2. PARAMETRIC EARNINGS PROCESSES

## *2.1. An ARMA(1, 2) for Each Person*

- We assume that the deterministic component of the earnings process has a steady-state mean given by

$$E(y_{it}) = \mu_i + a_i(t-1) + \tau_i(t-1)^2 \qquad (1)$$

$$\begin{aligned} \varepsilon_{it} &= p_{it} + u_{it} \qquad (2) \\ p_{it} &= \rho_i p_{i,t-1} + \eta_{it} \end{aligned}$$

- Combining Equations 1 and 2, we have

$$\begin{aligned} y_{it} = {} & [\mu_i(1-\rho_i) + \rho_i(a_i - \tau_i)] + \rho_i y_i + [a_i(1-\rho_i) + 2\rho_i\tau_i](t-1) \\ & + \tau_i\,(1-\rho_i)(t-1)^2 + u_{it} - \rho_i u_{i,t-1} + \eta_{i,t} \qquad (3) \end{aligned}$$

$$\begin{aligned} y_{it} = {} & [\mu_i(1-\rho_i) + \rho_i(a_i - \tau_i)] + \rho_i y_i + [a_i(1-\rho_i) + 2\rho_i\tau_i](t-1) \\ & + \tau_i\,(1-\rho_i)(t-1)^2 + \xi_{it} + \theta_{i1}\xi_{i,t-1} + \theta_{i2}\xi_{i,t-2} \qquad (4) \end{aligned}$$

$$\Delta y_{it} = (a_i - \tau_i) + 2\,\tau_i\,(t-1) + \xi_{it} + \theta_{i1}\xi_{i,t-1} + \theta_{i2}\xi_{i,t-2} \qquad (5)$$

## 2.2. The Properties of the Shock

- The most common assumption for parametric models is

$$\xi_{it} \sim \mathrm{N}\left(0, v_i^2\right) \tag{6}$$

- A convenient distribution is the translated hyperbolic sine (THS) transformation:

$$\begin{aligned} &\xi_{it} = a_i + \nu_i \ \sinh\{c + dN(0,1)\}, \\ &\text{with } \ a_i = \frac{\nu_i}{2}(e^{-c} - e^{c})e^{(d^2/2)} \ \text{ and } \ \nu_i,\, d > 0, \end{aligned} \tag{7}$$

$$\mathrm{std}(\xi_{it}) = \frac{\nu_i}{2}\sqrt{2 + \exp(d^2 - 2c) + \exp(d^2 + 2c)}\sqrt{\exp(d^2) - 1}. \tag{8}$$

## *2.3. Initial Conditions*

- For $t > 1$, the process in Equation 4 can be written:

$$y_{it} = \mu_i + \alpha_i(t-1) + \tau_i(t-1)^2 + \rho_i^{t-1}(y_{i1} - \mu_i) + \sum_{s=0}^{t-2} \rho_i^s \left(\xi_{i,t-s} + \theta_{1i}\xi_{i,t-s-1} + \theta_{2i}\xi_{i,t-s-2}\right), \tag{9}$$

- For mean and covariance stationarity, we require:

$$\begin{aligned} m_1 &= 0, \\ c_0 &= 1, \\ c_{i1} &= \rho_i + \theta_{i1}, \\ c_{i2} &= \frac{\rho_i^2 + \rho_i\theta_{i1} + \theta_{i2}}{\sqrt{1-\rho_i^2}}. \end{aligned} \tag{10}$$

# *2.4. Heterogeneity in the Parameters*

# *2.4.1. Unit roots and heterogeneous trends*

- We can use the deterministic component of the conditional model in Equation 4, where, for convenience, we drop the quadratic trend ($\tau_i$) and assume that the AR parameter is homogeneous to line up with the discussion in the previous literature:

$$y_{it} = [\mu_i(1-\rho) + \rho\alpha_i] + \rho y_{i,t-1} + \alpha_i(1-\rho)(t-1). \tag{11}$$

- **Table 1** presents the four possible cases

- **Table 2** summarizes the implications of different assumptions for the cross-sectional variance.

# Table 1: Hypotheses on stability and heterogeneity

| | Trend | |
|---|---|---|
| | Homogeneous | Heterogeneous |
| Stable | $\rho < 1, \alpha_i = \alpha$ | $\rho < 1$ (HIP) |
| Unit root | $\rho = 1, \alpha_i = \alpha$ (RIP) | $\rho = 1$ |

Abbreviations: HIP, heterogeneous income profiles;
RIP, restricted income profiles.

## Table 2: Different assumptions for cross-sectional variance in heterogeneous income profiles and restricted income profiles models

| | $\rho = 1$ | $\rho < 1$ | $\rho < 1$ |
|---|---|---|---|
| Heterogeneity | Unit root | Stable | Stationary |
| No heterogeneity $\alpha_i = \alpha$ | Increasing linear | Monotone | Constant |
| $\alpha_i$ (not correlated with intercept $\mu_i$) | Increasing | Nonmonotone | Increasing |
| $\alpha_i$ (correlated with intercept $\mu_i$) | U-shaped | Nonmonotone | U-shaped |

# *2.4.2. Incorporating codependent latent heterogeneity*

- To model the joint distribution of these model parameters, we follow Browning et al. (2010) and employ a triangular factor model with standard normal factors denoted by $\eta_{ik}, k = 1, \dots 7$. The system is given by:

$$
\begin{aligned}
\nu_i &= \exp[\phi_1 + \exp(\psi_{11})\eta_{i1}], \\
\mu_i &= \phi_2 + \psi_{21}\eta_{i1} + \exp(\psi_{22})\eta_{i2}, \\
\alpha_i &= \phi_3 + \sum_{k=1}^{2}\psi_{3k}\eta_{ik} + \exp(\psi_{33})\eta_{i3}, \\
\tau_i &= \phi_4 + \sum_{k=1}^{3}\psi_{4k}\eta_{ik} + \exp(\psi_{44})\eta_{i4}, \\
\rho_i &= \ell\left[\phi_5 + \sum_{k=1}^{4}\psi_{5k}\eta_{ik} + \exp(\psi_{55})\eta_{i5}\right], \\
\theta_{1i} &= 2l\left[\phi_6 + \sum_{i=1}^{5}\psi_{6k}\eta_{ik} + \exp(\psi_{66})\eta_{i6}\right] - 1, \\
\theta_{2i} &= 2l\left[\phi_7 + \sum_{i=1}^{6}\psi_{7k}\eta_{ik} + \exp(\psi_{77})\eta_{i7}\right] - 1,
\end{aligned}
\tag{12}
$$

- The initial value is specified as discussed in Section 2.3 but with an extension to allow for dependence on the other model parameters:

$$y_{i1} = \mu_i + m_1 + \tilde{c}_0 \xi_{i1} + \tilde{c}_1 (\rho_i + \theta_{i1}) \xi_{i0} + \tilde{c}_2 \left( \frac{\rho_i^2 + \rho_i \theta_{i1} + \theta_{i2}}{\sqrt{1 - \rho_i^2}} \right) \xi_{i,-1} \tag{13}$$

$$+ \psi_{81} \eta_{i1} + \psi_{83} \eta_{i3} + \psi_{84} \eta_{i4}.$$

- Following Equation 10, stationarity implies

$$\begin{aligned} m_1 = 0,\ \tilde{c}_0 = \tilde{c}_1 = \tilde{c}_2 = 1, \\ \psi_{81} = \psi_{83} = \psi_{84} = 0. \end{aligned} \tag{14}$$

- From Equation 9 (with, once again, the quadratic trend term set to 0), expected earnings growth between t 1 and t is given by

$$E(\Delta y_{it} | y_{i1}) = \alpha_i + \rho_i^{t-2} (\rho_i - 1)(y_{i1} - \mu_i)\ t > 3. \tag{15}$$

## *2.4.3. Incorporating observed heterogeneity*

# *2.4.4. First-round regression with time dummies*

- In a model with heterogeneous model parameters, an FRR would tend to induce bias in estimates of the distribution of the model parameters. To illustrate this point, consider a simple model with $\mu_i = a_i = \tau_i = \theta_{i1} = \theta_{i2} = 0$ and common time effects:

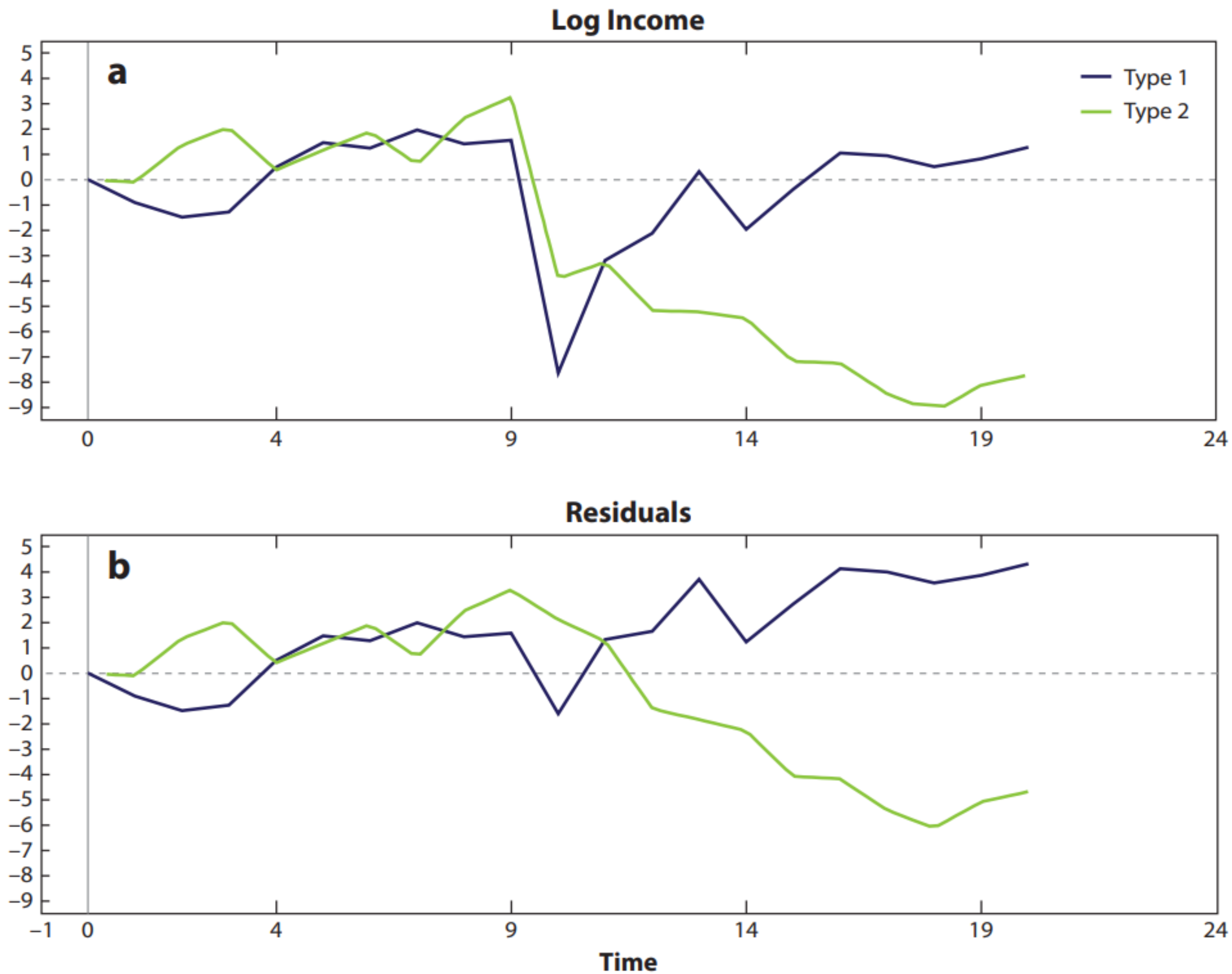
$$y_{it} = \rho_i y_{it-1} + \omega_t + \xi_{it}, \tag{16}$$

- To simplify the calculations, we assume that there are two types of individuals:

$$\begin{aligned} \text{Type 1}: y_{it} &= 0.5 y_{it-1} + \omega_t + \xi_{it}, \\ \text{Type 2}: y_{it} &= y_{it-1} + \omega_t + \xi_{it}. \end{aligned}$$

- **Figure 2** illustrates these points by simulating the model above

# Figure 2: The impact of a first-round regression



# 3. THE DATA AND ESTIMATION

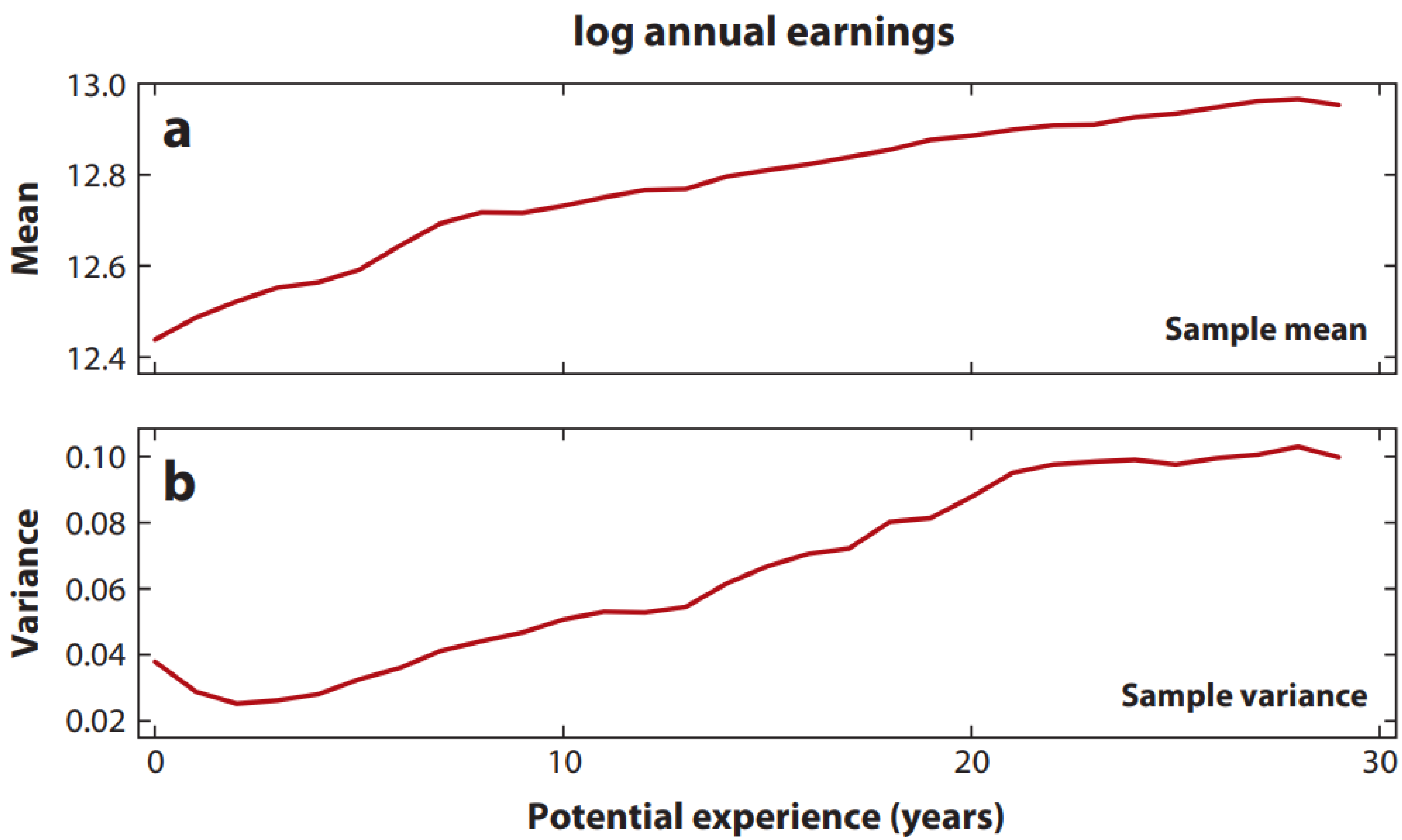
## *3.1. The Danish data*

- We use Danish administrative data to illustrate the estimation of the earnings model outlined in Section 2.4. Our estimation sample is a narrowly defined sample, which is selected to make it as homogeneous as possible.

- **Figure 3** shows the sample mean and variance of earnings as a function of potential experience. The mean is increasing in potential experience.

- **Table 3** displays the autocorrelations for earnings growth (not levels). The table shows that our sample exhibits the general pattern found in the literature.

# Figure 3: The sample mean and variance of log annual earnings




Data taken from the Danish register (1980–2009)

# Table 3: The autocorrelations for earnings growth in the Danish data sample

| Order | Coefficient | $p$ value | Order | Coefficient | $p$ value |
|---|---|---|---|---|---|
| 1 | −0.257 | 0.00 | 9 | −0.005 | 0.35 |
| 2 | −0.024 | 0.00 | 10 | 0.005 | 0.36 |
| 3 | −0.022 | 0.00 | 11 | −0.001 | 0.30 |
| 4 | −0.020 | 0.00 | 12 | 0.017 | 0.00 |
| 5 | −0.018 | 0.00 | 13 | 0.002 | 0.76 |
| 6 | −0.000 | 0.99 | 14 | −0.006 | 0.33 |
| 7 | 0.000 | 0.98 | 15 | −0.012 | 0.03 |
| 8 | −0.001 | 0.20 | | | |

# *3.2. Estimation*

## *3.2.1. Choosing auxiliary parameters*

## *3.2.2. Cross-sectional variance*

## 3.2.3. Autocorrelations

# *3.2.4. Transitions*

## *3.2.5. First-period observation*

## *3.2.6. Individual regression–based auxiliary parameters*

- The first step, which is analogous to Equation 1, employs a regression of log earnings on a quadratic of experience for each agent:

$$y_{it} = b_{1i} + b_{2i}t + b_{3i}t^2 + r_{it}. \tag{17}$$

- In the second step, we take the estimated residuals from this regression,

$$\widehat{r}_{it} = y_{it} - \left(\widehat{b}_{1i} + \widehat{b}_{2i}t + \widehat{b}_{3i}t^2\right), \quad t = 1, \ldots, T, \tag{18}$$

- Finally, we take the estimated residuals from the second regression,

$$\widehat{u}_{it} = \widehat{r}_{it} - \widehat{b}_{4i}\widehat{r}_{it-1}, \quad t = 2, \ldots, T, \tag{19}$$

# 4. ESTIMATION RESULTS

## *4.1. The Fit of Different Models*

- For the estimation, we use SMD, and the APs described in Section 3. We have in total 86 APs. We use the 46 IRB APs and the 9 APs for the initial observation for estimation (that is, these APs are matched to the data using a conventional weighting matrix). The remaining 31 APs from the first three sets are kept back for GF tests.

- **Table 4** presents results for four stable models and a unit root model.

# Table 4: Specification tests

| | Stable models | | | | Unit root |
|---|---|---|---|---|---|
| | Preferred model | Homogeneous AR | No heterogeneity in quadratic trend | No codependence | |
| No parameters | 41 | 34 | 28 | 20 | 29 |
| Degrees of freedom | 14 | 21 | 27 | 35 | 26 |
| Overidentifying (OI) test statistic | 31.0 | 145.8 | 226.9 | 456.4 | 287.5 |
| Goodness of fit [$\chi^2(31)$] | 90.7 | 205.3 | 164.5 | 226.4 | 301.7 |

# *4.2. The Implications of the Preferred Model*

- Although some of these results are very similar to those found for the PSID in Browning et al. (2010), there are also results in which the sign and size of the parameters are particular to this sample.

- The magnitude of the heterogeneity found in the preferred model is displayed by the first, fifth, and ninth deciles of the model parameters presented in the top panel of **Table 5**.

# Table 5: The parameter distribution

| | $y_1$ | $\nu$ | $\mu$ | $\alpha \times 10$ | $tp$[a] | $\rho$ | $\theta_1$ | $\theta_2$ |
|---|---|---|---|---|---|---|---|---|
| **Marginal distributions** | | | | | | | | |
| 1st decile | 12.21 | 0.042 | 12.38 | 0.04 | 29.0 | 0.11 | −0.309 | 0.011 |
| Median | 12.43 | 0.068 | 12.45 | 0.32 | 41.2 | 0.63 | 0.021 | 0.100 |
| 9th decile | 12.67 | 0.109 | 12.52 | 0.58 | 50.3 | 0.96 | 0.347 | 0.188 |
| **Correlations** | | | | | | | | |
| $corr(y_1)$ | 1 | 0.26 | 0.15 | −0.04 | −0.16 | 0.27 | −0.31 | −0.70 |
| $corr(\nu)$ | | 1 | 0.13 | 0.53 | −0.74 | 0.25 | −0.32 | −0.38 |
| $corr(\mu)$ | | – | 1 | −0.70 | 0.25 | 0.17 | −0.21 | −0.47 |
| $corr(\alpha)$ | | – | – | 1 | −0.84 | 0.22 | −0.09 | −0.03 |
| $corr(tp)$ | | – | – | – | 1 | −0.52 | 0.27 | 0.44 |
| $corr(\rho)$ | | – | – | – | – | 1 | −0.43 | −0.23 |
| $corr(\theta_1)$ | | – | – | – | – | – | 1 | 0.54 |

$tp$ is defined as the turning point of the quadratic trend function in years

# 5. CONCLUDING REMARKS

- **First**, we derive some broad lessons on earnings dynamics based on the Danish sample.
- **Our principal conclusion** is that there is strong evidence for pervasive codependent heterogeneity.
- **Our second contribution** is to emphasize the usefulness of IRB statistics, which more effectively exploit the individual time-series information.
- **Third**, we reiterate the general point that if a parametric model is to have any credibility, it must be checked against a wide range of discerning GF measures.
- **Finally**, we show that pervasive heterogeneity can have a major impact on econometric modeling and estimation procedures.