Life Cycle Earnings Dynamics

Alexandre Simoes, Samuel Higbee, Sidharth Sah

March 10, 2021

Econ 35003: Human Capital, Markets, and the Family

Table of contents

Permanent Income Hypothesis

Overview

Starting point is Friedman (1957). More recent work adds heterogeneity, borrowing constraints, and examines inequality. Several models of income processes have been proposed.

Road map:

- Gain intuition from quadratic utility example
- Posit more realistic preferences and income processes
- Add heterogeneity and borrowing constraints (Blundell et al 2008, Kaplan and Violante 2009)
- Examine permanent income inequality (Abbot and Gallipoli 2019)
- Consider a more nonparametric approach (Arellano, Blundell, and Bonhomme 2017)

PIH with Quadratic Preferences

Following example illustrates the main features

- quadratic utility
- discount rate $\frac{1-\beta}{\beta}$
- single risk-free asset with return r
- Finite horizon, no bequest, end-of-life solvency constraint

$$\Delta C_{i,a,t} = \pi_a \sum_{j=0}^{A} \frac{\mathbb{E}[Y_{i,a+j,t+j} \mid \mathcal{F}_{i,a,t}] - \mathbb{E}[Y_{i,a+j,t+j} \mid \mathcal{F}_{i,a-1,t-1}]}{(1+r)^j} \tag{1}$$

$$\pi_a = \frac{r}{1+r}\left[1 - \frac{1}{(1+r)^{A-a+1}}\right]^{-1} \tag{2}$$

Only innovations in the information set matter

Permanent/Transitory Income

Model income process as the sum of a permanent and transitory component

$$Y_{i,a,t} = p_{i,a,t} + \epsilon_{i,a,t} \tag{3}$$
$$p_{i,a,t} = p_{i,a-1,t-1} + \eta_{i,a,t} \tag{4}$$

Then the income response is

$$\Delta C_{i,a,t} = \pi_a \epsilon_{i,a,t} + \eta_{i,a,t} \tag{5}$$

- Consumption responds one-to-one with permanent shocks
- Transitory shocks are smoothed through saving/borrowing
- At the end of the horizon, transitory shocks treated like permanent shocks

Empirical evidence

Blundell et al (2008)

- CRRA preferences
- Estimate with PSID and CEX data
- model nests full insurance and autarky as special cases

Empirical evidence

Model Overview

$$Y_{i,t} = p_{i,t} + \nu_{i,t} \tag{6}$$

$$p_{i,t} = p_{i,t-1} + \eta_{i,t} \tag{7}$$

$$\nu_{i,t} = \sum_{j=0}^{q} \theta_j \epsilon_{i,t-j} \tag{8}$$

$$\implies \Delta C_{i,t} = \phi_{i,t}\eta_{i,t} + \psi_{i,t}\epsilon_{i,t} + \xi_{i,t} \tag{9}$$

Y is log-income, covariates partialled out. C is log-consumption, covariates partialled out. ξ_{it} is preference shock or measurement error.

Empirical evidence

Identification

$$Var(\Delta Y_t) = Var(\eta_t) + Var(\Delta \nu_t) \tag{10}$$

$$Cov(\Delta Y_t, \Delta Y_{t+s}) = Cov(\Delta \nu_t, \Delta \nu_{t+s}) \qquad s > 0 \tag{11}$$

$$Var(\Delta C_t) = \phi_t^2 Var(\eta_t) + \psi_t^2 Var(\epsilon_t) + Var(\xi_t) \tag{12}$$

$$Cov(\Delta C_t, \Delta C_{t+s}) = 0 \qquad s > 0 \tag{13}$$

$$Cov(\Delta C_t, \Delta Y_t) = \phi_t Var(\eta_t) + \psi_t Var(\epsilon_t) \tag{14}$$

$$Cov(\Delta C_t, \Delta Y_{t+s}) = \psi_t Cov(\epsilon_t, \Delta \nu_{t+s}) \qquad s > 0 \tag{15}$$

Empirical evidence

Blundell et al (2008)

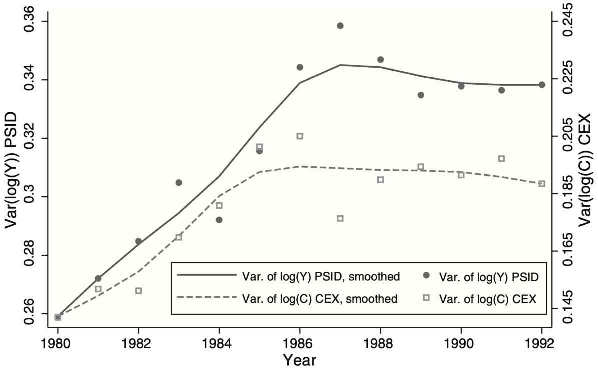


FIGURE 1. OVERALL PATTERN OF INEQUALITY

Empirical evidence

Blundell et al (2008)

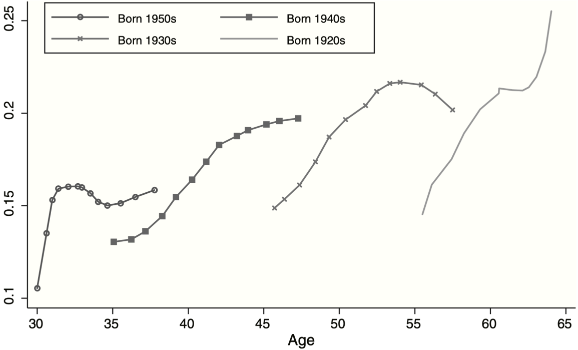


FIGURE 2. VARIANCE OF LOG CONSUMPTION OVER THE LIFE CYCLE

Empirical evidence

Blundell et al (2008)

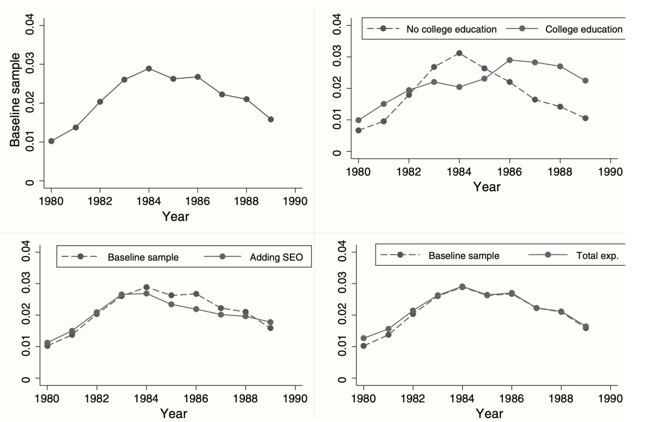


Figure 7. Variance of Permanent Shocks in Various Specifications and Samples

Empirical evidence

Blundell et al (2008)

TABLE 6—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

	Whole sample	No college	College	Born 1940s	Born 1930s
ϕ	0.6423	0.9439	0.4194	0.7928	0.6889
(Partial insurance perm. shock)	(0.0945)	(0.1783)	(0.0924)	(0.1848)	(0.2393)
ψ	0.0533	0.0768	0.0273	0.0675	−0.0381
(Partial insurance trans. shock)	(0.0435)	(0.0602)	(0.0550)	(0.0705)	(0.0737)

Note: value shown is proportion uninsurable

Adding Borrowing Constraint

Kaplan and Violante (2010)

- Starting point: Blundel et al (2008)
- Augment with heterogeneous agents/ borrowing constraint / social security
- Calibrate
- Use Blundel et al methodology to estimate
- Does Blundel methodology get close?

Adding Borrowing Constraint

Model Overview

- Agents work until retirement T^{ret},
- Survive to date t with probability α_t. $\alpha_s = 1$ for $s < T^{ret}$, $\alpha_s = 0$ for $s > T$.
- Utility $\mathbb{E}_0 \sum_{t=1}^{T} \beta^{t-1} \alpha_t u(C_{i,t})$
- $Y_{i,t} = p_{i,t} + \epsilon_{i,t}$ and $p_{i,t} = p_{i,t-1} + \eta_{i,t}$
- Budget constraint:

$$C_{i,t} + A_{i,t+1} = (1+r)A_{i,t} + Y_{i,t} \qquad t < T^{ret} \qquad (16)$$

$$C_{i,t} + \frac{\alpha_t}{\alpha_{t+s}} A_{i,t+1} = (1+r)A_{i,t} + P(\tilde{Y}_i) \qquad t \geq T^{ret} \qquad (17)$$

Adding Borrowing Constraint

Kaplan and Violante (2010)

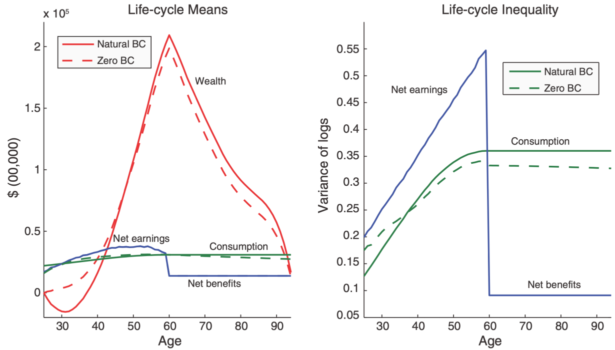


FIGURE 1. LIFE-CYCLE PROFILES FOR MEANS AND VARIANCES IN THE NBC AND ZBC ECONOMIES

$$\text{Natural BC} \implies \text{solvency at } T.$$

$$\text{Zero BC} \implies A > 0.$$

Adding Borrowing Constraint

Kaplan and Violante (2010)

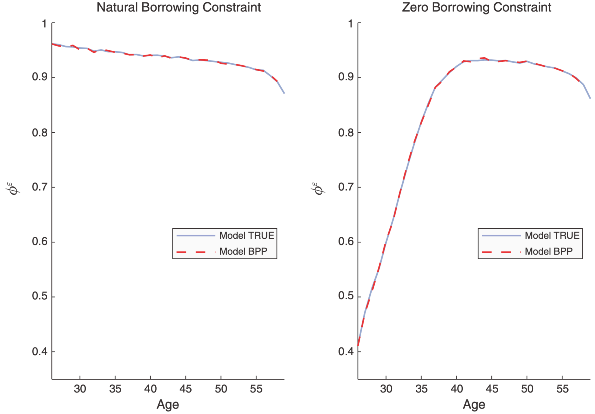


FIGURE 2. AGE PROFILES OF INSURANCE COEFFICIENTS FOR TRANSITORY SHOCKS

Adding Borrowing Constraint

Kaplan and Violante (2010)

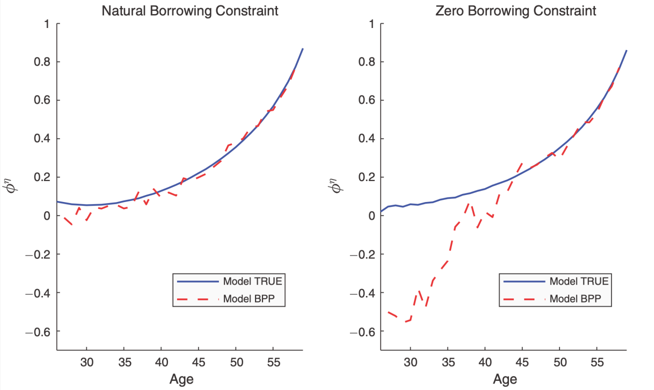


FIGURE 3. AGE PROFILES OF INSURANCE COEFFICIENTS FOR PERMANENT SHOCKS

Adding Borrowing Constraint

Kaplan and Violante (2010)

TABLE 1—RESULTS FROM THE BENCHMARK MODELS WITH NBC AND ZBC

	Permanent shock			Transitory shock		
	Data BPP	Model BPP	Model TRUE	Data BPP	Model BPP	Model TRUE
Natural BC	0.36 (0.09)	0.22	0.23	0.95 (0.04)	0.94	0.94
Zero BC	0.36 (0.09)	0.07	0.23	0.95 (0.04)	0.82	0.82

Note: value shown is proportion insurable.

Intuition: Orthogonality condition for consumption growth and transitory shock fails near BC.

Permanent Income Inequality

Abbot and Gallipoli (2019)

- Nonparametric estimation of permanent income, including human wealth and assets
- PIH sheds light on inequality of several forms
- PSID 1967-2016

Standard asset pricing approach used to quantify human wealth:

$$\theta_{i,t} = \mathbb{E}_{it}\left[\beta \frac{u_c(c_{i,t+1}, v_{i,t+1})}{u_c(c_{i,t}, v_{i,t})}(y_{i,t+1} + \theta_{i,t+1})\right] \tag{18}$$

Identified using Euler equation $u_c(c, v) = \beta\mathbb{E}[u_c(c', v')R' \mid c, v]$ for some asset return R'.

Abbot and Gallipoli (2019)

Figure 1: Marginal utility as a function of consumption expenditures.

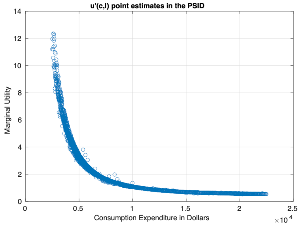
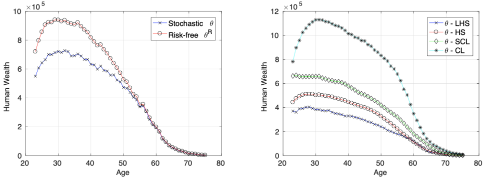
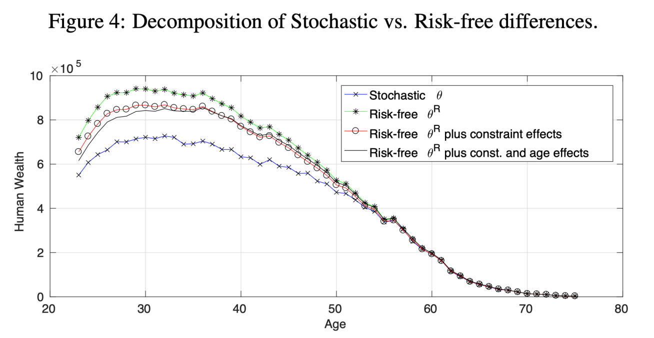


Figure 2: Average human wealth over the life cycle. Values in 2016 dollars. LHS denotes less than high school education; HS is high school degree only; SCL is some college; and CL is college degree or higher.



Permanent Income Inequality

Abbot and Gallipoli (2019)



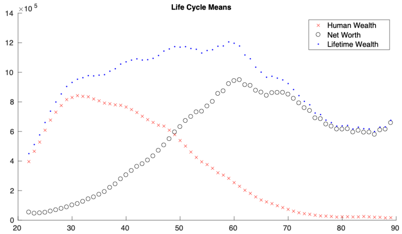
Difference between stochastic and risk-free valuation is welfare cost of market incompleteness.

- Mean $104k$
- Median $54k$

Permanent Income Inequality

Abbot and Gallipoli (2019)

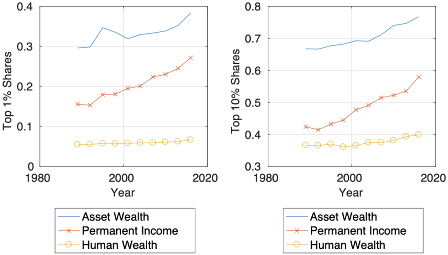
Figure 5: Average human wealth, net worth and lifetime wealth (the sum of human wealth and net worth) over the lifecycle. Values in 2016 dollars.



Permanent Income Inequality

Abbot and Gallipoli (2019)

Figure 10: Concentration of net worth, human wealth and permanent-income by year (1989 to 2016. Left panel top 1%; right panel top 10%). Each plot reports the share in the hands of households at the top of the respective distribution, e.g. share of human wealth held by the top 10% of the human wealth distribution.



Abbot and Gallipoli (2019)

Year	Net Worth	Human Wealth	Lifetime Wealth	Earnings	Permanent-income
	(1)	(2)	(3)	(4)	(5)
1989	0.296	0.055	0.131	0.101	0.156
1998	0.336	0.057	0.162	0.102	0.181
2007	0.333	0.059	0.197	0.144	0.224
2016	0.384	0.067	0.244	0.194	0.272

Table 6: This table reports the share of variable "X" in the hands of the households in the top 1% of the distribution of that same variable "X". For example, the share of permanent-income held by the households in the top 1% of the distribution of permanent-income. In Appendix C we report results for all sample years.

Year	Net Worth	Human Wealth	Lifetime Wealth	Earnings	Permanent-income
	(1)	(2)	(3)	(4)	(5)
1989	0.296	0.011	0.130	0.039	0.153
1998	0.336	0.016	0.161	0.053	0.176
2007	0.333	0.016	0.196	0.066	0.212
2016	0.384	0.019	0.243	0.107	0.265

Table 7: This table reports the share of variable "X" in the hands of the households in the top 1% of the distribution of net worth. For example, the share of permanent-income held by the households in the top 1% of the distribution of net worth. In Appendix C we report results for all sample years.

Permanent Income Inequality

Abbot and Gallipoli (2019)

Year	Net Worth (1)	Human Wealth (2)	Lifetime Wealth (3)	Earnings (4)	Permanent-income (5)
1989	0.668	0.366	0.405	0.372	0.424
1998	0.683	0.361	0.430	0.363	0.445
2007	0.712	0.375	0.488	0.416	0.515
2016	0.768	0.399	0.543	0.472	0.579

Table 1: This table reports the share of variable "X" in the hands of the households in the top 10% of the distribution of that same variable "X". For example, the share of permanent-income held by the households in the top 10% of the distribution of permanent-income. In Appendix C we report results for all years.

Year	Net Worth (1)	Human Wealth (2)	Lifetime Wealth (3)	Earnings (4)	Permanent-income (5)
1989	0.668	0.125	0.351	0.203	0.385
1998	0.683	0.129	0.381	0.217	0.409
2007	0.712	0.122	0.457	0.267	0.490
2016	0.768	0.125	0.521	0.311	0.564

Table 2: This table reports the share of variable "X" in the hands of the households in the top 10% of the distribution of net worth. For example, the share of permanent-income held by the households in the top 10% of the distribution of net worth. In Appendix C we report results for all sample years.

Abbot and Gallipoli (2019)

Year	Net Worth	Human Wealth	Lifetime Wealth	Earnings	Permanent-income
	(1)	(2)	(3)	(4)	(5)
1989	0.300	0.602	0.499	0.536	0.473
1998	0.285	0.601	0.468	0.524	0.445
2007	0.262	0.591	0.417	0.495	0.388
2016	0.219	0.580	0.385	0.464	0.349

Table 8: This table reports the share of variable "X" in the hands of the households ranked between the 50th and 90th percentiles of the distribution of that same variable "X". For example, the share of permanent-income held by households in the 50-90 percentiles of the distribution of permanent-income.

More Flexible Models and Estimates

Arellano, Blundell, Bonhomme (2018)

Does persistence of shocks differ across the income distribution?

$$Y_{i,t} = p_{i,t} + \epsilon_{i,t} \tag{19}$$

We allow p to follow a general first-order Markov process. Let $p_{i,t} = Q_t(p_{i,t-1}, u_{i,t})$ where $u_{i,t}$ uniform. Measure persistence as

$$\frac{\partial Q_t(p_{i,t-1}, \tau)}{\partial p}$$

Intuitively: how does a rank τ shock affect permanent component of income at different levels of initial income?

More Flexible Models and Estimates

Arellano, Blundell, Bonhomme (2017)

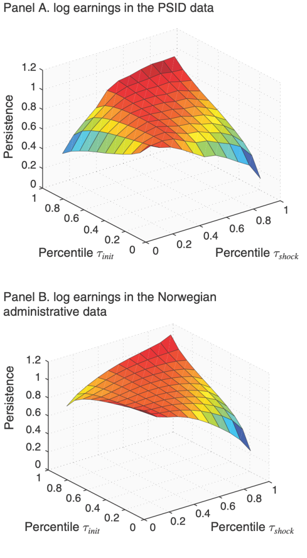


FIGURE 1. QUANTILE AUTOREGRESSIONS

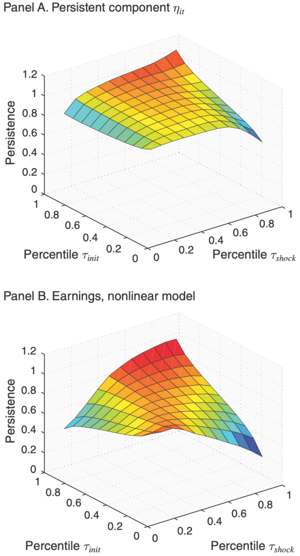


FIGURE 2. NONLINEAR PERSISTENCE

Heterogeneity and Uncertainty

Heterogeneity and Uncertainty

The idiosyncrasies of the income process can be broadly separated into two components:

- Heterogeneity: differences in characteristics (e.g. schooling level) lead to differences in initial states and income profiles
- Uncertainty (risk): idiosyncratic shocks that can be permanent/transitory (persistence and variability can also be heterogeneous)

Heterogeneity and Uncertainty

The literature investigates:

- What is the true income process?
- How to incorporate heterogeneity?
- How to separate uncertainty from predictable heterogeneity?

Different choices of income processes can lead to very different welfare and policy implications.

Heterogeneous vs. Restricted Income Process

HIP (moderately persistent shock, heterogeneous trend):

$$Y_{i,h,t} = \alpha_i + \beta_i h + p_{i,h,t} + \epsilon_{i,h,t} \tag{20}$$

$$p_{i,h,t} = \rho p_{i,h-1,t-1} + \eta_{i,h,t} \tag{21}$$

Where h is potential experience and $\rho < 1$

RIP (highly persistent shock - random walk, homogeneous trend):

$$Y_{i,h,t} = \alpha_i + \beta h + p_{i,h,t} + \epsilon_{i,h,t} \tag{22}$$

$$p_{i,h,t} = p_{i,h-1,t-1} + \eta_{i,h,t} \tag{23}$$

Why HIP or RIP?

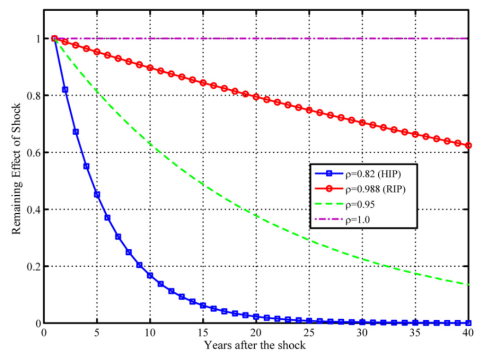
Early papers (e.g. Lillard and Weiss, 1979) fit the encompassing model shown in HIP and consistently found:

- $0.5 < \rho < 0.7$
- $\sigma_\beta >> 0$

Later papers, most notably MaCurdy (1982), tested the hypothesis that $\sigma_\beta = 0$ and were not able to reject the null.

After replacing β_i with β, these papers estimated $\rho \approx 1$

Welfare Implications



Remember:

$$\Delta C_{i,a,t} = \pi_a \sum_{j=0}^{A} \frac{\mathbb{E}[Y_{i,a+j,t+j} \mid \mathcal{F}_{i,a,t}] - \mathbb{E}[Y_{i,a+j,t+j} \mid \mathcal{F}_{i,a-1,t-1}]}{(1+r)^j}$$

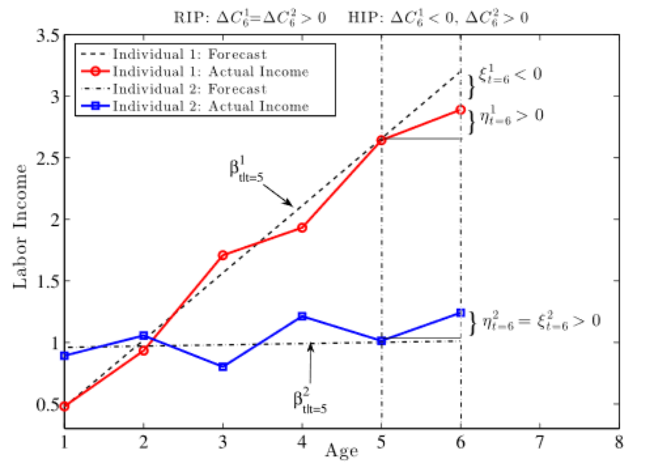
Welfare Implications

$$Y_{i,a,t} = \rho Y_{i,a-1,t-1} + \epsilon_{i,a,t} + \theta \epsilon_{i,a-1,t-1}$$
$$\Delta C_{i,a,t} = \kappa(r, \rho, \theta, A - a)\epsilon_{i,a,t}$$

Fixing $r = 0.02$

ρ	θ	$A - a$	κ
1	−0.2	40	0.81
1	0	10	1
0.99	−0.2	40	0.68
0.95	−0.2	40	0.39
0.8	−0.2	40	0.13
0.95	−0.2	30	0.45
0.95	−0.2	20	0.53
0.95	−0.2	10	0.65
0.95	−0.1	40	0.44
0.95	−0.01	40	0.48
1	0	∞	1
0	−0.2	40	0.03

Welfare Implications



Policy Implications

Consider that a policymaker is trying to reduce consumption inequality.

- HIP: stochastic shock have moderate persistence and income growth is highly heterogeneous.
  - Investment in human capital (initial conditions) are expected to bring good returns and less uncertainty (if β is known).
- RIP: all individuals experience the same long-term trend and are subject to highly persistent shocks.
  - Informing agents about the insurance market or providing social insurance would be valuable as well.

Evidence for HIP

Guvenen (2008) formally introduces the debate and provides evidence supporting HIP.

Misspecification as RIP bias the persistence parameter towards unity (analytical proof and simulation evidence):

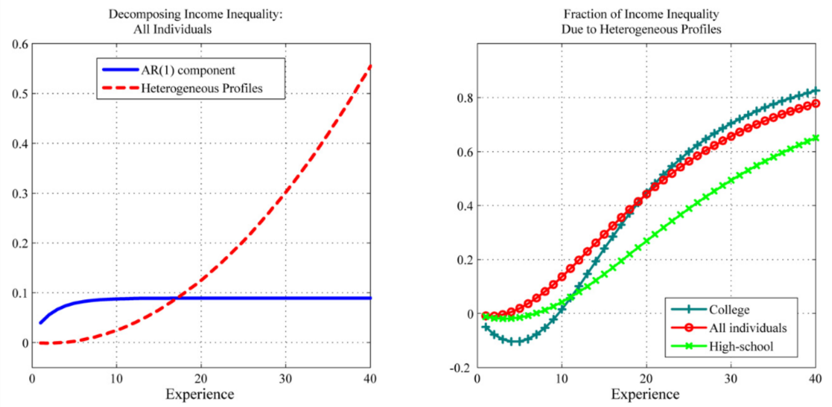
- Generates data using the HIP model with $\rho = 0$ (other parameters are close to estimates from data)
- Testing different age distributions, estimates an RIP model with $\rho \approx 0.9$

Guvenen (2008) - Empirical Results

Data: PSID 1968-1993; male head of household with at least 20 years of data; positive labor earnings; worked between 520 and 5110 hours; average hourly earnings within a preset range; no SEO (poor individuals). Total of 1270 heads.

	Sample	ρ	σ^2_α	σ^2_β	$corr_{\alpha\beta}$
			Panel A: σ^2_β *restricted* to be zero (RIP process)		
(1)	All	.988	.058	–	–
		(.024)	(.011)		
(2)	College	.979	.031	–	–
		(.055)	(.021)		
(3)	High-school	.972	.053	–	–
		(.023)	(.015)		
			Panel B: σ^2_β *unrestricted* (HIP process)		
(4)	All	.821	.022	.00038	−.23
		(.030)	(.074)	(.00008)	(.43)
(5)	College	.805	.023	.00049	−.70
		(.061)	(.112)	(.00014)	(1.22)
(6)	High-school	.829	.038	.00020	−.25
		(.029)	(.081)	(.00009)	(.59)
(7)	All (large sample)	.842	.072	.00043	−.33
		(.024)	(.055)	(.00007)	(.40)
(8)	All (first 10 cov.)	.899	.055	.00055	−.73
		(.042)	(.060)	(.00013)	(.38)

Guvenen (2008) - Empirical Results



Evidence for RIP

Hryshko (2012) performs several additional simulations and repeat the analysis by Guvenen (2008) finding support for RIP.

Misspecification as HIP bias towards heterogeneity of the growth parameter (analytical proof and simulation evidence)

- Part of the persistent shocks will be interpreted as idiosyncratic income growth

Parameters/Trans. Comp.	(1) ARMA(1, 1) $\sigma_\beta^2 = 0, \sigma_\xi^2 = 0.02$	(2) AR(1) $\sigma_\beta^2 = 0, \sigma_\xi^2 = 0.02$	(3) MA(1) $\sigma_\beta^2 = 0, \sigma_\xi^2 = 0.02$
Heterog. growth, $\hat{\sigma}_\beta^2$	0.0005	0.00052	0.0007
	(0.00009)	(0.00008)	(0.00009)
Var. perm. shock, $\hat{\sigma}_\xi^2$	0.00	0.00	0.00
	—	—	—
AR, $\hat{\phi}$	0.762	0.675	—
	(0.044)	(0.032)	—

Simulations used $\rho = 1$.

Hryshko (2012) - Empirical Results

Data: PSID 1968-1997; male head of household pf ages 25-64; at least 9 consecutive observations; drops cases with extreme variations in consecutive years; valid and above zero labor earnings; no SEO. Total of 1916 heads.

	(1) HIP	(2) Add RW	(3) Est. Pers.
$\hat{\sigma}^2_\beta$	0.0004	0.00	0.00
	(0.00004)	(0.00006)	(0.001)
$\hat{\sigma}^2_\xi$	0.00	0.015	0.016
	—	(0.002)	(0.002)
$\hat{\phi}$	0.712	0.367	0.343
	(0.029)	(0.115)	(0.194)
$\hat{\theta}$	−0.187	−0.091	−0.081
	(0.024)	(0.08)	(0.113)
$\hat{\sigma}^2_\varepsilon$	0.046	0.028	0.027
	(0.001)	(0.002)	(0.005)
$\hat{\phi}_{rw}$	0.0	1.0	0.992
	—	—	(0.158)
χ^2	793.32	697.05	694.38
(d.f.)	(431)	(430)	(429)

A Note on Estimation Differences

The estimation methods differ in a few aspects.

For example:

- Hryshko (2012) - first differences (variation in income across consecutive years)
- Guvenen (2008) - log labor earnings levels (can be affected by specifications of initial conditions)

Through several simulations Hryshko (2012) shows that his method correctly estimates all parameters in an encompassing model for different true DGPs.

Pervasive Heterogeneity and Codependence

Motivated by this debate, Browning and Ejrnæs (2013) propose a generic model ARMA(1,2) with two interesting features:

- Pervasive heterogeneity - all parameters are allowed to be individual specific
- Codependence - parameters come from joint distributions allowing them to be correlated

Browning and Ejrnæs (2013)

A simplified version of the model:

$$Y_{i,t} = \alpha_i + \beta_i t + \tau_i t^2 + p_{i,t} + \epsilon_{i,t} \tag{24}$$

$$p_{i,t} = \rho_i p_{i,t-1} + \eta_{i,t} \tag{25}$$

$$\epsilon_{i,t} = \xi_{i,t} + \theta_{i1}\xi_{i,t-1} + \theta_{i2}\xi_{i,t-2} \tag{26}$$

The last parameter is the standard deviation of the distribution (translated hyperbolic sine) of $\eta_{i,t}$, defined as ν_i.

Empirical Results

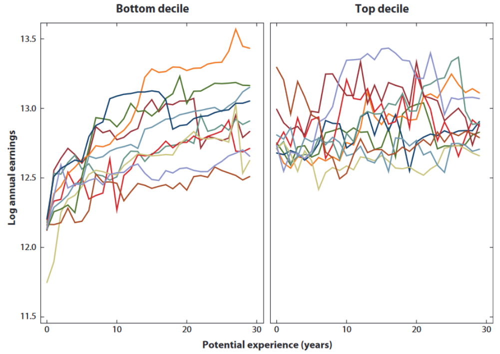
Data: Danish administrative dataset (tax records)

Relevant correlations:

- Initial earnings and growth in the beginning of career (defined as functions of the parameters): -0.7
- Persistence (ρ) and variance of shocks (ν): 0.25

Empirical Results

The first result is consistent with patterns observed in the data and supports the HIP intuition of income growth heterogeneity.



The second suggests that self-insurance is more costly for individuals that face high variance since they also face higher persistence.

Forecastable and Unforecastable Variability

- Previous models: variability in income is incorporated into consumption *ex-post*.
- Temporary/permanent decomposition does not separate sources of variability predictable *ex-ante*.
- Cunha et al. (2005): future variability is partially incorporated ex-ante and partially unknown (incorporated ex-post). Heterogeneity versus uncertainty.
- Identification of the agent's information set given income data and economic choices (consumption, schooling, etc.).

Identification of the Information Set

Consider the simplified consumption function:

$$C_{i,t} = Y_{i,t} + E[Y_{i,t+1}|\Omega_{i,t}] \tag{27}$$

where $\Omega_{i,t}$ is the agent's information set.

We assume quadratic preferences, ignore any self-insurance and future discounting, and consider that the agent only receives income in periods t and $t+1$.

Identification of the Information Set

We write the income $Y_{i,t+1}$ as:

$$Y_{i,t+1} = X_{i,t+1}\beta + \zeta^{A}_{i,t} + \zeta^{U}_{i,t} \tag{28}$$

where $X_{i,t+1}\beta$ is observed by both the econometrician and the agent, $\zeta^{A}_{i,t}$ is observed by the agent, and $\zeta^{U}_{i,t}$ is unobserved.

Identification of the Information Set

We can define deviation functions:

$$z^C_{i,t} = C_{i,t} - Y_{i,t} - X_{i,t+1}\beta \tag{29}$$

$$z^Y_{i,t} = Y_{i,t+1} - X_{i,t+1}\beta \tag{30}$$

If $\zeta^A_{i,t} \neq 0$, then $cov(z^C_{i,t}, z^Y_{i,t}) \neq 0$.

Agent has superior information.

Uncertainty and Inequality

Cunha and Heckman (2016) use an analogous identification strategy in the context of schooling choices to decompose trends in inequality and earnings variance

Data: National Longitudinal Survey of Youth (NLSY)

Evolution of Uncertainty

	College	High School	Returns
NLS/1966:			
Total variance	195.882	136.965	611.245
Variance of unforecastable components	76.332	31.615	167.187
Variance of forecastable components	119.550	105.350	444.058
NLSY/1979:			
Total variance	292.368	165.350	823.200
Variance of unforecastable components	84.464	48.137	221.976
Variance of forecastable components	207.904	117.214	601.223
Evolution:			
Percentage increase in total variance	49.26	20.72	34.68
Percentage increase in variance of unforecastable components	10.65	52.26	32.77
Percentage increase in variance of forecastable components	73.90	11.26	35.39
Percentage increase in total variance by source:			
Percentage increase in total variance due to unforecastable components	8.43	58.20	25.85
Percentage increase in total variance due to forecastable components	91.57	41.80	74.15

Human Capital

Johnson (1978) - Theory of Job Shopping

- Individuals entering job market have uncertainty about their "general" ability level and job-specific abilities
- Workers receive earnings from job i:

$$E_i = \alpha + b_i\theta + \mu_i$$

- α is mean earnings, b_i return rate to general ability, θ general ability of individual, μ_i job-specific ability
- If worker starts with job 1, will switch to job 2 if
    - "Learning" mobility: $E_1 > \alpha$ and $\frac{b_1\sigma_\theta^2}{b_1^2\sigma_\theta^2+\sigma_{\mu_1}^2}b_2 > 1$
    - "Search" mobility: $E_1 < \alpha$ and $\frac{b_1\sigma_\theta^2}{b_1^2\sigma_\theta^2+\sigma_{\mu_1}^2}b_2 < 1$
- Absent a sufficiently high switching cost, worker starts with more variable job ($\sigma_{\mu_i}^2 > \sigma_{\mu_j}^2$)

Johnson (1978) - Theory of Job Shopping

- Model consistent with a few notable empirical facts of life-cycle earnings dynamics:
    - Higher rates of job mobility among less experienced workers
    - Earnings dispersion falls during initial years of work experience - it is optimal to start with riskier job (absent large enough mobility cost)

Weitzman (1979) - Optimal Search for the Best Alternative

- Model of search among many "boxes" with varying cost (c_i) and time to open and distribution of reward ($F_i(x_i)$)
- Solution: assign each box a reservation price (z_i) exactly equal to value of opening that box:

$$c_i = \beta \int_{z_i}^{\infty} (x_i - z_i) dF(x_i) - (1 - \beta) z_i$$

- Pandora Rule:
  1. Open box with highest reservation price
  2. Terminate search when reward exceeds reservation price of remaining boxes
- More variable boxes opened first (a mean-preserving spread will increase z_i)

Yamaguchi (2012) - Tasks and Heterogeneous Human Capital

- Workers select occupations from a K-dimensional space of task complexity (x_t)
- Worker skills (s_t) contribute more to production when corresponding task is complex:

$$\log w_t = p_0 + p_1' x_t + [p_2 + P_3' x_t]' s_t + \eta_t$$

- Worker skills evolve based on depreciation (D), complexity of tasks, and worker characterstics (d):

$$s_{t+1} = D s_t + a_0 + A_1 x_t + A_2 d + \varepsilon_{t+1}$$

- A_1 allowed to be fully flexible, estimated to be positive along diagonal

Yamaguchi (2012)

- Workers have preferences over task complexity and change from prior average task compexity ($\bar{x}_t$):

$$v_t = (g_0 + G_1 d + G_2 s_t + \tilde{v}_t)' x_t + x_t' G_3 x_t + (x_t - \bar{x}_t)' G_4 (x_t - \bar{x}_t)$$

- Preference for task complexity can vary by worker skill, other characteristics
- Workers optimize considering preferences for task complexity, wage, and effect of task complexity on skill evolution:

$$\begin{aligned} V_t(s_t, \bar{x}_t, \tilde{v}_t, \eta_t | d) = & \max_{x_t} \log w(x_t, s_t, \eta_t) + v_t(x_t, \bar{x}_t, s_t, \tilde{v}_t | d) \\ & + \beta E V(s_{t+1}, \bar{x}_{t+1}, \tilde{v}_{t+1}, \eta_{t+1} | d) \end{aligned}$$

Yamaguchi (2012)

- Task complexity of occupations measured from Dictionary of Occupational Titles
- Skills (and tasks) reduced to cognitive and motor
- Specified skill approach is less computationally expensive than allowing each occupation to have arbitrary returns to skills
- Kalman filtering used - wages are a noisy signal of skill
- Finds more complex skills lead to greater skill growth and greater skill associated with preference for complexity

Yamaguchi (2012)

Table 6
Log Wage Variance When Initial Conditions Are Homogeneous

		Homogeneous			
Year	Benchmark	Preference	Initial Skills	Learning Ability	All
1	.206	.204	.061	.206	.061
10	.292	.260	.234	.241	.190
20	.359	.297	.335	.257	.232

NOTE.—Author's estimates from the National Longitudinal Survey of Youth 1979–2000. Sample consists of 2,417 men.

- At beginning of life-cycle initial skill explains much of wage variation
- By late career initial skill explains little - task preference and learning ability predominate

Yamaguchi (2012)

Table 8
Mean Skill Profiles by Education

Year	All Men	High School Dropouts	High School	College
Cognitive skills:				
1	.000	−.813	−.269	.498
10	.631	−.650	.206	1.405
20	.996	−.539	.489	1.923
Motor skills:				
1	.000	.731	.240	−.448
10	−.066	.871	.240	−.637
20	−.108	.950	.238	−.750

NOTE.—Author's estimates from the National Longitudinal Survey of Youth 1979–2000. Sample consists of 325 high school dropouts, 1,009 high school graduates, and 1,083 college workers.

- Cognitive skill grows for all groups - most rapidly for college educated
- Motor skill evolution differs in direction across groups

Lochner, Park, and Shin (2018) - Wage Dynamics and Returns to Unobserved Skill

- Wage variance is increasing even when conditioning on observables - is variance in unobserved skill or returns to unobserved skill increasing (or both)?
- Wages are a function of observable characteristics (d_t) and unobserved skill ($s_{u,t}$):

$$\log w_t = f_t(d_t) + \mu_t s_{u,t} + \eta_t$$

- Random shock assumed iid and independent of skill, so covariances of wage residuals depends on returns to unobserved skill and covariances of unobserved skill:

$$Cov(\log w_t - f_t(d_t), \log w_{t'} - f_{t'}(d_{t'})) = \mu_t \mu_{t'} Cov(s_{u,t}, s_{u,t'})$$

Lochner, Park, and Shin (2018)

- Skill evolution based on individual growth factor (δ), and time and cohort (c) varying factor:

$$s_{u,t} - s_{u,t-1} = \tau_t(c)\delta + \varepsilon_t$$

- Random shock again independent of everything:

$$Cov(s_{u,t}, s_{u,t'}|c) = Var(s_{u,t'}|c) + \sum_{j=t'+1}^{t} \tau_j(c) Cov(s_{u,t'}, \delta|c)$$

- Identifying assumption: no unobserved heterogeneity in skill growth among most experienced workers, $\tau_t(c) = 0$ if $t - c \geq \overline{e}$

Lochner, Park, and Shin (2018)

- Can then identify returns to skill from covariances of wage residuals:

$$\frac{Cov(\log w_t - f_t(d_t), \log w_{t'} - f_{t'}(d_{t'}))}{Cov(\log w_{t-1} - f_{t-1}(d_{t-1}), \log w_{t'} - f_{t'}(d_{t'}))} = \frac{\mu_t}{\mu_{t-1}}$$

- Model estimated using PSID data

Lochner, Park, and Shin (2018)

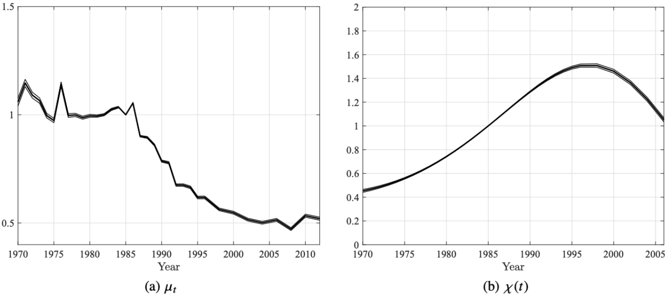


Figure 18: Estimated μ_t and $\chi(t)$ Accounting for Time-Varying Variance of Heterogeneous Skill Growth

- See estimated returns to unobserved skill falling sharply over period
- χ_t is component of $\tau_t(c)$ attributable to year, not experience (assuming separability)