## Employer Learning and Statistical Discrimination

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#### I. Introduction



### II. Implications of Statistical Discrimination and Employer Learning for Wages



#### **II.1 A Model of Employer Learning and Wages**



- Our research builds on some previous work, particularly Farber and Gibbons (1996), (hereinafter FG).
- Our model is similar to FG.
- Let  $y_{it}$  be the log of labor market productivity of worker *i* with  $t_i$  years of experience:

$$y_{it} = rs_i + \alpha_1 q_i + \Lambda z_i + \eta_i + H(t_i).$$
(1)  

$$\begin{cases} y_{it} = rs_i + \alpha_1 q_i + \Lambda z_i + \eta_i + H(t_i). \\ y_{it} = \eta_i + \eta_i + \eta_i + H(t_i). \\ y_{it} = \eta_i + \eta_i + \eta_i + H(t_i). \\ y_{it} = \eta_i + \eta_i + \eta_i + H(t_i). \\ y_{it} = \eta_i + \eta_i$$

- In (1) we separate the determinants of productivity into four categories:
- s<sub>i</sub> represents variables that are observed by both the employer and the econometrician;
- *q<sub>i</sub>* includes variables observed by the employer but not seen (or not used) by the econometrician;
- *z<sub>i</sub>* consists of correlates of productivity that are not observed directly by employers but are available to and used by the econometrician;
- and η<sub>i</sub> is an index of other determinants of productivity and is not directly observed by the employers and not observed (or observed but not used) by the econometrician.



- Normalize  $z_i$  so that all the elements of the conformable coefficient vector  $\Lambda$  are positive.
- In addition,  $H(t_i)$  is the experience profile of productivity.
- For now we assume that the experience profile of productivity does not depend on s<sub>i</sub>, z<sub>i</sub>, q<sub>i</sub>, or η<sub>i</sub>.



- In the absence of knowledge of z and η, firms form the conditional expectations E(z|s, q) and E(η|s, q), which we assume are linear in q and s.
- Consequently,

$$z = E(z|s,q) + v = \gamma_1 q + \gamma_2 s + v$$

$$\eta = E(\eta|s,q) + e = \alpha_2 s + e,$$
(2)

- Vector v and the scalar e have mean 0 and are uncorrelated with q and s by definition of an expectation.
- Links from s to z and η may be due in part to a causal effect of s.



- Equations (1) and (2) imply that  $\Lambda \nu + e$  is the error in the employer's belief about the log of productivity of the worker at the time the worker enters the labor market.
- The sum  $\Lambda \nu + e$  is uncorrelated with q and s. (Assume in second )



## The firm observe noisy signal of prosectivity

- ξ<sub>t</sub> = y+ε, where y = y<sub>t</sub> H(t).
  ε<sub>t</sub> reflects transitory variation in the performance of worker *i* and the effects of variation in the firm environment that are hard for the firm to control for in evaluating the worker.
- Employers know q and s.



- Observing ξ<sub>t</sub> is equivalent to observing
   d<sub>t</sub> = ξ<sub>t</sub> − E(y|s, q) = Λν + e + ε<sub>t</sub> which is the sum of the noise
   ε<sub>t</sub> and the error Λν + e in the employer's belief about initial log
   productivity.
- The vector  $D_t = \{d_1, d_2, \dots, d_t\}$  summarizes the worker's performance history.
- Let  $\mu_t$  be the difference between  $\Lambda \nu + e$  and  $E(\Lambda \nu + e|D_t)$ .
- $\mu_t$  is uncorrelated with  $D_t, q$ , and s.
- $\mu_t$  is distributed independently of  $D_t$ , q, and s.
- q, s, and  $D_t$  are known to all employers, as in FG.

Cruciall

• Substituting and taking logs, we arrive at the log wage process:

$$\mathcal{L}(\mathbf{Y}|\mathbf{Y}_{t}S) = (\mathbf{r} + \Lambda\gamma_{2} + \alpha_{2})\mathbf{s} + H^{*}(t) + (\alpha_{1} + \Lambda\gamma_{1})\mathbf{g} + (\mathbf{E}(\Lambda \mathbf{v} + \mathbf{e}|D_{t}) + \zeta_{t},$$
(3)

- $w_t = \log(W_t)$  and  $H^*(t) = H(t) + \log(E(\exp^{\mu t}))$ .
- $E(\Lambda \nu + e|D_t)$  in (3) shows that wages change over time not just because productivity changes with experience, but also because firms learn about errors in their initial assessment of worker productivity.



- Examine the parameters of the conditional expectation of  $w_t$  given s, z, t, and the experience profile  $H^*(t)$ .
- Begin with the case in which z and s are scalars and then turn to the more general cases.
- Consider the conditional expectation function when t = 0, ..., T, with

$$\underline{E(w_t|s,z,t)} = \underbrace{b_{st}s}_{t} + \underbrace{b_{zt}z}_{t} + H^*(t). \tag{4}$$



- To simplify the algebra but without any additional assumptions, we reinterpret s, z, and q as the components of s, z, and q that are orthogonal to H<sup>\*</sup>(t).
- Given that the wage evolves according to (3), the omitted bias formula for least squares regression implies that  $\frac{1}{2} = \frac{1}{2} + \frac{1}{2} +$

$$b_{st} = b_{s0} + \Phi_{st} = [r + \Lambda \gamma_2 + \alpha_2] + \Phi_{qs} + \Phi_{st}$$
  
$$b_{zt} = b_{z0} + \Phi_{zt} = \Phi_{qz} + \Phi_{zt},$$

• where  $\Phi_{qs}$  and  $\Phi_{qz}$  denote the coefficients of the auxiliary regressions of  $(\alpha_1 + \Lambda \gamma_1)q$  on *s* and *z*, respectively, and  $\Phi_{st}$ and  $\Phi_{zt}$  are the coefficients of the regression of  $E(\Lambda v + e|D_t)$ on *s* and *z*.



$$FWL + yRe d an anglement$$

$$but = \frac{Cav(S, W)}{Var(S')} = \frac{Cav(S, w)}{Var(S)}$$

$$\frac{Cav(S, BS - BS + 2 + 4[Av + el0] + F_0)}{Var(S)}$$

$$Be + \frac{Cav(S, BS - BS + 2 + 4[Av + el0] + F_0)}{Var(S)}$$

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$$Be + \frac{Cav(S, BS - BS + 2 + 4[Av + el0] + F_0)}{Var(S)}$$

$$Be + \frac{Cav(S, FS - BS + 2 + 4[Av + el0] + F_0}{Var(S)}$$

$$Cov(S + S) = Cov(S, FS + Ev + el0]$$

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$$Cav(S + Ev + el0]$$

 Using the facts that cov(s, E(Λv + e|D<sub>t</sub>)) = 0 and cov(z, E(Λv + e|D<sub>t</sub>)) = cov(v, E(Λv + e|D<sub>t</sub>)) and the least squares regression formula, one may express Φ<sub>st</sub> and Φ<sub>zt</sub> as

$$\Phi_{st} = \theta_t \Phi_s \quad \text{the true currelulium (6)} \\ \Phi_{zt} = \theta_t \Phi_z, \quad \text{between $5,3$ & $$$$$$$$$$$$$$$$$$$$$$$$$$$$$(N-ne)$$$)}$$

• where  $\Phi_s$  and  $\Phi_z$  are the coefficients of the regression of  $\Lambda v + e$  on s and z and

$$\begin{array}{c}
\theta_{v} = \frac{\operatorname{cov}(E(\Lambda v + e|D_{t}), z)}{\operatorname{cov}(\Lambda v + e, z)} = \frac{\operatorname{cov}(E(\Lambda v + e|D_{t}), v)}{\operatorname{cov}(\Lambda v + e, v)}.
\end{array} (7)$$

Proposition 1. Under the assumptions of the above model,
a the regression coefficient b<sub>zt</sub> is nondecreasing in t, and
b the regression coefficient b<sub>st</sub> is nonincreasing in t.
Proposition 2. Under the assumptions of the above model,

$$\frac{\partial b_{st}}{\partial t} = - \Phi_{zs} \frac{\partial b_{zt}}{\partial t}.$$



However, a matrix version of Proposition 2 still holds

$$\frac{\partial b_{st}}{\partial t} = -\frac{\partial b_{zt}}{\partial t} \Phi_{zs},$$

where Φ<sub>zs</sub> is now the K × J matrix of coefficients of the regression of z on s.



#### **II.2. Statistical Discrimination on the Basis of Race**



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#### **III.** Data and Econometric Specification





#### **IV.** Results for Education



# IV.1. AFQT as a *z* Variable



Figure 1: The Effects of Standardized AFQT and Schooling on Wages

Dependent Variable: Log Wage; OLS estimates (standard errors)

Panel 1 – Experience measure: potential experience						
	ience measu	re: potentia	i experience			
Model:	(1)	(2)	(3)	(4)		
(a) Education	0.0586	0.0829	0.0638	0.0785		
	<b>(</b> 0.0118)	(0.0150)	(0.0120)	(0.0153)		
(b) Black	-0.1565	-0.1553	0.0001	-0.0565		
	(0.0256)	(0.0256)	(0.0621)	(0.0723)		
(c) Standardized AFQT	€ 0.0834	<mark>-0.0060</mark> -	0.0831	0.0221		
	<mark>(0.0144)</mark>	<mark>(0.0360)</mark>	(0.0144)	(0.0421)		
(d) Education *	-0.0032	-0.0234	-0.0068	-0.0193		
experience/10	(0.0094)	(0.0123)	) (0.0095)	(0.0127)		
(e) Standardized AFQT *		0.0752		0.0515		
experience/10	(	(0.0286)		(0.0343)		
(f) Black * experience/I0			-0.1315	-0.0834		
		(	(0.0482)	(0.0581)		
$R^2$	0.2861	0.2870	0.2870	0.2873		
				CHICA		

Figure 2: The Effects of Standardized AFQT and Schooling on Wages

Dependent Variable: Log Wage; OLS estimates (standard errors)

Panel 2 – Experience measure: actual experience						
instrumented by potential experience						
Model:	(1)	(2)	(3)	(4)		
(a) Education	0.0836	0.1218	0.0969	0.1170		
	(0.0208)	(0.0243)	(0.0206)	(0.0248)		
(b) Black	-0.1310	-0.1306	0.0972	0.0178		
	(0.0261)	(0.0260)	(0.0851)	(0.1029)		
(c) Standardized AFQT	0.0925	-0.0361	0.0881	0.0062		
	(0.0143)	(0.0482)	(0.0143)	(0.0572)		
(d) Education *	-0.0539	-0.0952	-0.0665	-0.0889		
experience/10	(0.0235)	(0.0276)	(0.0234)	(0.0283)		
(e) Standardized AFQT *	, , ,	0.1407		0.0913		
experience/10		(0.0514)		(0.0627)		
(f) Black * experience/10		. ,	-0.2670	-0.1739		
			(0.0968)	(0.1184)		
$R^2$	0.3056	0.3063	0.3061	0.3064 <sup>ers</sup>		

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## IV.2. The Sibling Wage and Father's Education as *z* Variables



## Figure 3: The Effects of Father's Education, Sibling Wages, and Schooling on Wages

Dependent Variable: Log Wage; Experience Measure: Potential Experience

Model:	(1)	(2)	(3)	(4)
(a) Education	0.0511	0.0630	0.0568	0.0659
	(0.0160)	(0.0166)	(0.0163)	(0.0167)
(b) Black	-0.2074	-0.2076	-0.0509	-0.0878
	(0.0276)	(0.0276)	(0.0846)	(0.0871)
(c) Log of sibling's wage	0.1802	-0.0260	0.1817	0.0010
	(0.0328)	<mark>(0.0913</mark> )	(0.0329)	(0.0940)
(d) Father's education/10				
		ť		
(e) Education *	0.0107	0.0012	0.0065	-0.0008
experience/10	(0.0131)	(0.0136)	(0.0133)	(0.0136)
(f) Log of sibling's wage *		0.1796	_	0.1571
experience/10		<u>(0.0749)</u>	)	(0.0770)
(g) Father's education * experience/100				
(h) Black * experience/10			-0.1311	-0.1004
			(0.0686)	(0.0704)
$R^2$	0.3183	0.3196	0.3191	0.3200
Observations	10746	10746	10746	10746
Individuals	1441	1441	1441	1441

OLS estimates (standard errors)



## Figure 4: The Effects of Father's Education, Sibling Wages, and Schooling on Wages

Dependent Variable: Log Wage; Experience Measure: Potential Experience

Model:	(5)	(6)	(7)	(8)
(a) Education	0.0666	0.0730	0.0704	0.0734
	(0.0129)	(0.0140)	(0.0130)	(0.0140)
(b) Black	-0.2212	-0.2209	-0.0705	-0.0793
	(0.0250)	(0.0250)	(0.0668)	(0.0692)
(c) Log of sibling's wage	( )	· · · ·	· · · ·	( <i>'</i>
(d) Father's education/10	0.0826	-0.0187	0.0829	0.0314
	(0.0366)	(0.1000)	(0.0364)	(0.1030)
(e) Education *	0.0023	-0.0029	-0.0002	-0.0027
experience/10	(0.0104)	(0.0113)	(0.0105)	(0.0113)
(f) Log of sibling's wage *	(***=**)	()	(******)	()
experience/10				
(g) Father's education *		0.0867		0.0441
experience/100		(0.0813)		(0.0841)
(h) Black * experience/10		(0.0010)	-0.1270	-0.1194
			(0.0541)	(0.0563)
$R^2$	0.2748	0.2750	0.2755	0.2756
Observations	18523	18523	18523	18523
Individuals	2594	2594	2594	2594

#### OLS estimates (standard errors)



## Figure 5: The Effects of Standardized AFQT, Father's Education, Sibling Wage, and Schooling on Wages

Dependent Variable: Log Wage; Experience Measure: Potential Experience

	`		,	
Model:	(1)	(2)	(3)	(4)
(a) Education	0.0505	0.0832	0.0563	0.0780
	(0.0118)	(0.0151)	(0.0120)	(0.0155)
(b) Black	-0.1333	-0.1296	0.0454	-0.0284
	(0.0255)	(0.0257)	(0.0609)	(0.0704)
(c) Standardized AFQT	0.0792	-0.0206	0.0789 <sup>´</sup>	0.0065
	(0.0145)	(0.0361)	(0.0144)	(0.0413)
(d) Log of sibling's wage	0.1602 <sup>´</sup>	0.0560	0.1617	0.0604
	(0.0208)	(0.0352)	(0.0207)	(0.0351)
(e) Father's education/10	0.0362	0.0154	0.0385	0.0295
	(0.0356)	(0.0963)	(0.0354)	(0.0968)
(f) Education *	0.0005	-0.0269	-0.0035	-0.0220
experience/10	(0.0093)	(0.0123)	(0.0094)	(0.0128)
(g) Standardized AFQT		0.0843		0.0614
* experience/10		(0.0285)		(0.0333)
h) Log of sibling wage *		0.1194		0.1151
experience/10		(0.0393)		(0.0393)
(i) Father's education *		0.0176		0.0055
experience/100		(0.0789)		(0.0794)
(j) Black * experience/10			-0.1500	-0.0861
			(0.0474)	(0.0570)
$R^2$	0.2991	0.3014	0.3002	0.3016
				Test.

OLS estimates (standard errors)

## IV.3. The Experience Profile of the Effects of AFQT and Education on Wages



## V. Do Employers Statistically Discriminate on the Basis of Race?



#### **VI. Models with Training**



## Figure 6: The Effects of Standardized AFQT, Father's Education, Sibling Wage, Schooling, and Training on Wages

Model:	(1)	(2)	(3)	(4)
(a) Education	0.0606	0.0802	0.0651	0.0746
	(0.0119)	(0.0151)	(0.0121)	(0.0155)
(b) Black	-0.1159	-0.1135	0.0241	-0.0028
	(0.0265)	(0.0267)	(0.0616)	(0.0722)
(c) Standardized AFQT	0.0334	-0.0199	0.0338	0.0102
	(0.0150)	(0.0363)	(0.0150)	(0.0420)
(d) Log of sibling's wage	0.1594	0.0716	0.1611	0.0759
	(0.0213)	(0.0357)	(0.0213)	(0.0356)
(e) Father's education/10	0.0460	0.0211	0.0482	0.0353
	(0.0356)	(0.0974)	(0.0354)	(0.0977)
(f) Education *	-0.0231	-0.0392	-0.0260	-0.0339
experience/10	(0.0095)	(0.0123)	(0.0096)	(0.0128)
(g) Standardized AFQT *	. ,	0.0460	. ,	0.0207
experience/10		(0.0287)		(0.0339)
(h) Log of sibling's wage *		0.1041		0.1001
experience/10		(0.0402)		(0.0402)
(i) Father's education *		0.0205		0.0084
experience/100		(0.0803)		(0.0805)
(j) Black * experience/10		. ,	-0.1180	-0.0945
			(0.0476)	(0.0583)
(k) Training: <i>R</i> <sub>t</sub>	-0.1143	-0.1095	-0.1115	-0.1091
	(0.0200)	(0.0199)	(0.0199)	(0.0199)
(1) Cumulative training: $\Sigma$	0.1881	0.1830	0.1854	0.1827
$R_{\tau}$	(0.0139)	(0.0139)	(0.0139)	(0.0139)
$R^2$	0.3188	0.3199	0.3195	0.3202

Dependent Variable: Log Wage; Experience Measure: Potential Experience Training Measure: Predicted before 88, Actual After; OLS estimates (standard errors)

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Figure 7: Estimates of the Effects of AFQT, Father's Education, Sibling Wage, and Schooling on Wage Growth with Controls for Training

Dependent Variable:  $\Delta$  log Wage; Experience Measure: Potential Experience

			-	
Model:	(1)	(2)	(3)	(4)
Education *	-0.0060	-0.0694	-0.0106	-0.0729
$\Delta$ experience/10	(0.0833)	(0.0960)	(0.0832)	(0.0959)
AFQT * $\Delta$ experience/10	. ,	0.3025	· · · · ·	0.2975
		(0.1613)		(0.1614)
Log of sibling wage *		0.2153		0.2107
$\Delta$ experience/10		(0.1477)		(0.1477)
Father's education $*$		-0.4306		-0.4215
$\Delta$ experience/10		(0.5034)		(0.5034)
Black * $\Delta$ experience/10	-0.0504	-0.0425	-0.0503	-0.0426
· ,	(0.0484)	(0.0485)	(0.0483)	(0.0484)
Training: $R_t/10$	<b>、</b> ,		0.2468	0.2429
<b>_</b> ,			(0.1024)	(0.1025)
Lag training: $R_{t-1}/10$			-0.0194	-0.0230
<u> </u>			(0.1108)	(0.1108)
S.E.E.	.2965	.2965	.2965´	.2964
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Coefficient estimates (standard errors)

#### VII. Conclusions and a Research Agenda

