

American Inequality and Social Mobility Viewed Through a Danish Prism

James J. Heckman
CEHD: The University of Chicago

Rasmus Landersø
The Rockwool Foundation

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**CENTER FOR THE ECONOMICS
OF HUMAN DEVELOPMENT**
The University of Chicago

Research Partners:

Sadegh Eshaghnia, Miriam Gensowski, Kristian Karlson, and Rafeh Qureshi

Inequality and Social Mobility

Figure 1: Intergenerational Mobility and Inequality: The “Gatsby Curve”

$$\underbrace{\ln Y_1}_{\text{income of child}} = \alpha + \underbrace{\beta}_{\text{IGE}} \underbrace{\ln Y_0}_{\text{income of parent or parents}} + \varepsilon$$

$\beta \uparrow$, Mobility \downarrow



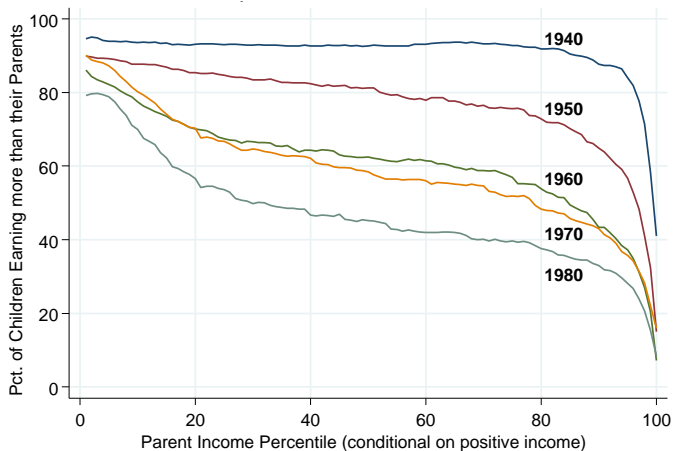
Source: Corak (2016), "Inequality from Generation to Generation: The United States in Comparison"

“The American Dream is now spoken with a Scandinavian accent.”

– Washington Post (2014)

**Unlike in the U.S.,
Recent Danish Cohorts Are Doing More or Less The Same
as Their Parents**

Figure 2: Absolute Mobility: Probability Children Do Better Than Parents by Cohort, U.S. – Pre-Tax and Transfer



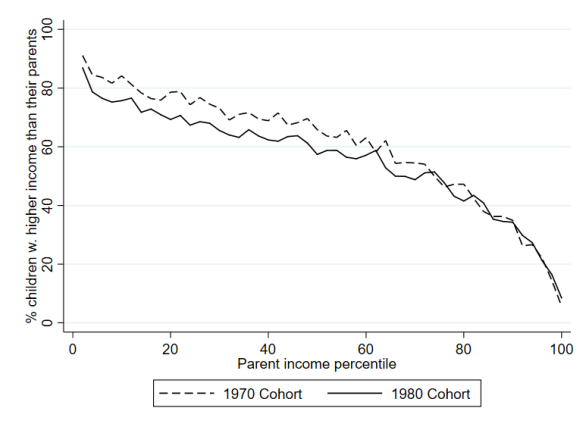
Source: Chetty et al. (2017)

Figure 3: Absolute Mobility: Probability Children Do Better Than Parents) by Cohort: U.S – Pre-Tax and Transfer



Source: Chetty et al. (2017)

Figure 4: Absolute Mobility: Probability Children Do Better Than Parents) by Cohort: Denmark – Pre-Tax and Transfer



Source: Own calculations based on data from Statistics Denmark

What Can Be Learned from Denmark About Reducing Inequality and Promoting Social Mobility?

Specifically, Should America Emulate the Scandinavian Model?

Denmark is a Laboratory for Understanding the Sources of Inequality and Social Mobility

- Reducing inequality and promoting social mobility is a central focus of the modern Danish welfare state.
- Many traditional explanations of inequality and social immobility do not hold in Denmark.
- Suggest a fresh look at the origins of inequality and social immobility.
- Equality in services offered is mandated.
- Health care; teachers paid the same everywhere; free daycare; free college.
- Greater social cohesion (witness U.S. versus Danish response to COVID-19).
- **Post tax and transfers**, income inequality is low and income mobility is high.

- Not due to superior production of human capital.
- Educational mobility is remarkably similar in the U.S. and Denmark.
- There are substantial skill and education gaps across families by background.

- Advantages from Denmark's universal access to services are reaped relatively more by the affluent rather than by the disadvantaged (Matthew Effects) who don't often know or find it more difficult to use these services.
- We find strong evidence of sorting of families by parental income and education.
- "Power of place" is a consequence of family sorting and family influence.
- Families purposefully **choose** neighborhoods and timing of moves.
- Not random (in contrast to influential claims otherwise).
- Sorting in Denmark is comparable to that in the U.S.
- This sorting affects estimated IGEs.

- IGE \downarrow average family income \uparrow ; more affluent places have lower IGEs but higher initial conditions.
- Sorting \Rightarrow strong family income gradients on child outcomes.
- Sorting by teachers into more advantaged districts.
- Despite equal wages across neighborhoods, payment to teachers is in quality of students taught.

Our Argument

- 1 A central premise of the welfare state (since the writings of Max Weber) is the equality of access -- in Denmark this is mandated by law, yet equal access by the state does not imply equal use of public services.
 - 1 Equality in the law does not imply equality in use of services.
 - 2 “Matthew effects” (to those who have, more is given) play a powerful role.
 - i Parents reinforce (or substitute) public services delivered.
 - ii Pick neighborhoods that offer better public services.
 - iii Enforce delivery of services.
 - 3 Karlson and Landerso show how effective targeting can be: the move to universal access (rather than focus on disadvantage) **increased** the education IGE.
- 2 Denmark has a free housing market, as in U.S.—sorting is large and increasing.

Summary (Cont'd)

- 3 Sorting by parental income and resources plays a powerful role in the U.S. and Denmark.
- 4 The **choice** of the neighborhood of residence to raise children has a powerful role in explaining intergenerational inequality, which has been ignored in much recent work.
- 5 The IGE regression $\ln Y_1 = \alpha + \beta \ln Y_0 + \varepsilon$ has focused on β across countries.
- 6 We look at neighborhoods within countries $n \in \mathcal{N}$:

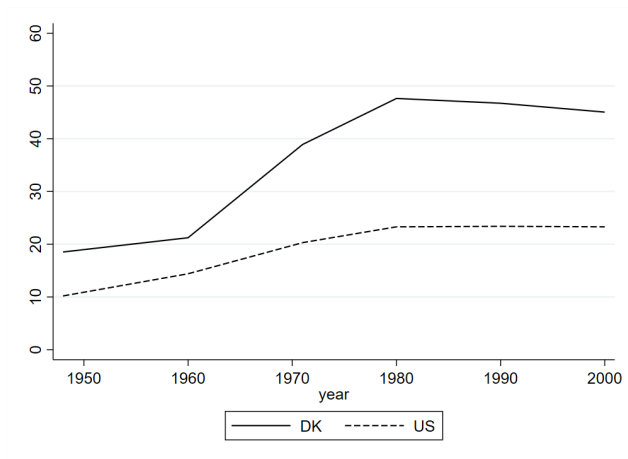
$$Y_{1n} = \alpha_n + \beta_n \ln Y_{0n} + \varepsilon_n$$

- 7 Child outcomes depend on (α_n, β_n) **pair**, not just β_n .
- 8 α_n is initial condition associated with n .
- 9 Parents **purposefully** select neighborhoods when children are very young often well before schooling begins (not random with regard to age of child).

Summary (Cont'd)

- ⑩ Two notions of family influence:
 - ① Transmission of lifetime value functions across generations
 - ② Impact of timing and column of volume of family resources on child outcomes over the lifecycle
- ⑪ Access to register data enables us to investigate appropriate measures of lifetime well-being for measuring family influence.
 - ① Which income concept? Does it matter?
 - ② Consumption?
 - ③ Average income around age 35 (as traditionally used?)
 - ④ Lifetime resources (value function)? (valuing uncertainty; leisure; accounting for credit constraints)
 - ⑤ PDV (discounted family income)?
- ⑫ PDV is best predictor of child outcomes.
- ⑬ IGE based on lifetime measures (after including tax and transfers) is very high (much higher than traditional measures).

Figure 5: Public spending and transfer payments, as percentage of GDP, Denmark and U.S.



Source: U.S. is based on Federal Budget of the United States, from 1962-2000, and historical statistics of the Federal reserve FRASER data series prior). Denmark is based on public data.

Note: The figure shows public expenditures and transfer payments (excl. defense and interest payments) as percentage of GDP for Denmark and the U.S.

Denmark Spends Generously on Public Education

Table 1: Expenditure on educational institutions as a percentage of GDP, by source of funding and level of education

	<i>Pre-primary</i>			<i>Prim., secon., and post-secon. non-tertiary</i>		
	Public ¹	Priv. ²	Total	Public ¹	Priv. ²	Total
Denmark	1.30	0.11	1.41	4.3	0.1	4.4
U.S.	0.33	0.14	0.47	3.4	0.3	3.7

Note: Table shows public, private, and total expenditures on education as percentages of GDP in 2013 for Denmark and the U.S.

¹: Including public subsidies to households attributable for educational institutions, tuition and fees (U.S.), and direct expenditure on educational institutions.

²: Net of public subsidies attributable for educational institutions.

Source: OECD: Education at a glance 2014.

Denmark Spends Generously on Public Education, Cont'd

Table 1: Expenditure on educational institutions as a percentage of GDP, by source of funding and level of education

	<i>Tertiary</i>			<i>All levels</i>		
	Public ¹	Priv. ²	Total	Public ¹	Priv. ²	Total
Denmark	1.8	0.1	1.9	7.5	0.4	7.9
U.S.	0.9	1.8	2.7	4.7	2.2	6.9

Note: Table shows public, private, and total expenditures on education as percentages of GDP in 2013 for Denmark and the U.S.

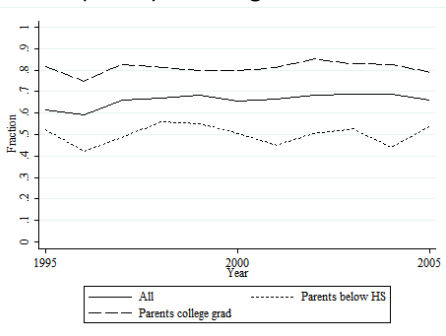
¹: Including public subsidies to households attributable for educational institutions, tuition and fees (U.S.), and direct expenditure on educational institutions.

²: Net of public subsidies attributable for educational institutions.

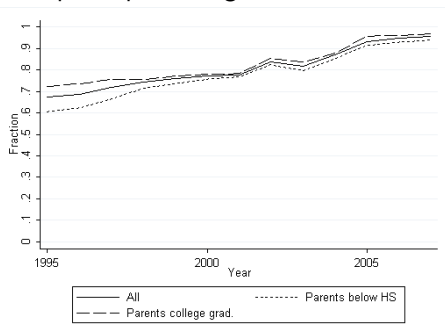
Source: OECD: Education at a glance 2014.

Figure 6: Daycare and Preschool Use

(a) Trend in pre-school participation, age 4, U.S.



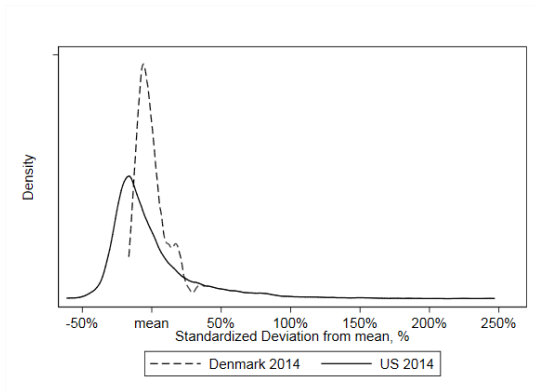
(b) Trend in pre-school participation, age 4, Denmark



Source: Figure 4 Lundersø and Heckman (2018).

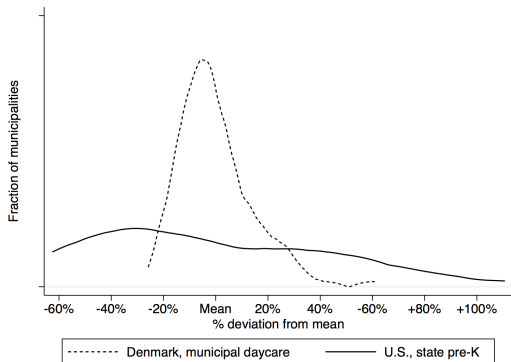
School expenditures much more equal geographically in Denmark than in the U.S.:

Figure 7: Gross expenditures per student, public schools, deviation from mean



Pre-K expenditures much more equal geographically in Denmark than in the U.S.:

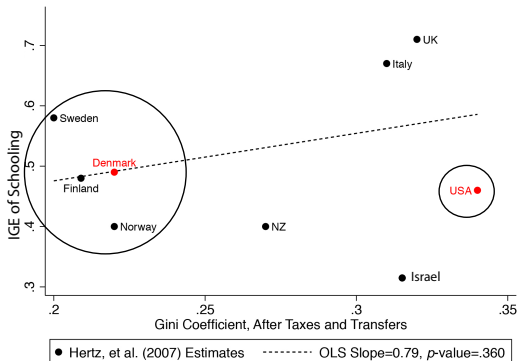
Figure 8: Expenditures per child, state pre-K / municipal daycares, deviation from mean



Educational Mobility

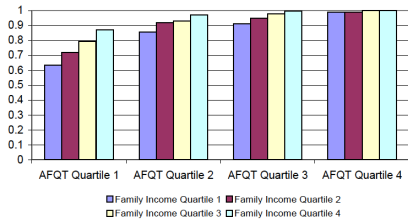
In Terms of IGE in Education, Denmark Not Better Than U.S. Despite its Generous Welfare State

Figure 9: Intergenerational Educational Mobility and Inequality



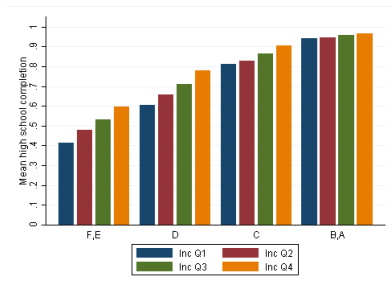
Source: Setzler (2015).

High School Completion U.S.



US: Belley and Lochner (2007)

High School Completion Denmark

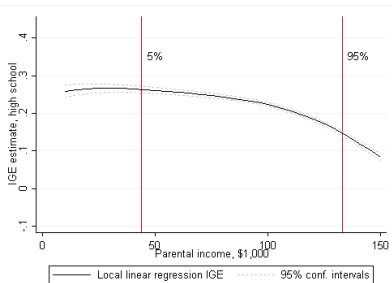


Denmark: Landersø and Heckman (2017)

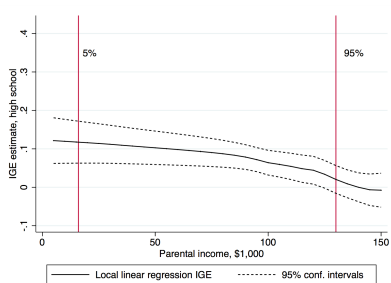
Relationships Between Child Education and Parental Income Stronger in Denmark Than U.S.

Figure 10: Local Intergenerational Elasticities between children's education and parental log gross income including transfers, absolute income weights, Denmark and the U.S.

(a) High school completion, Denmark



(b) High school completion, U.S.

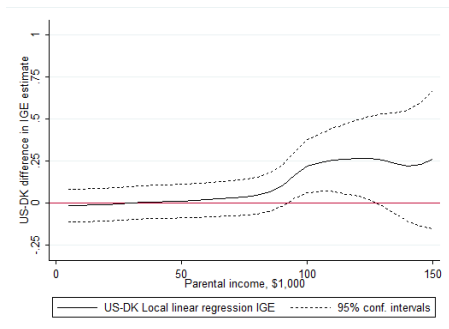


Source: Figure 7 Landersø and Heckman (2018).

Figure 11: U.S.-Denmark differences in income mobility across parents' income:

$$\min_{\alpha[\ln(Y_0^P)], \beta[\ln(Y_0^P)]} \sum_{i=1}^N K_{h_\lambda}(Y_0^P, Y_i^P) \cdot \{ \ln(Y_i^C) - \alpha[\ln(Y_0^P)] - \beta^{IGE}[\ln(Y_0^P)] \ln(Y_i^P) \}^2$$

By **market income**, IGE is only higher in U.S. for affluent families

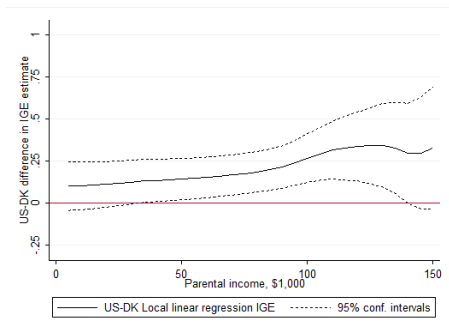


Note: Child income on parents' income for cohorts of 1973-75. Source: Landersø and Heckman (2017).

Figure 12: U.S.-Denmark differences in income mobility across parents' income:

$$\min_{\alpha[\ln(Y_0^P)], \beta[\ln(Y_0^P)]} \sum_{i=1}^N K_{h_\lambda}(Y_0^P, Y_i^P) \cdot \{ \ln(Y_i^C) - \alpha[\ln(Y_0^P)] - \beta^{IGE}[\ln(Y_0^P)] \ln(Y_i^P) \}^2$$

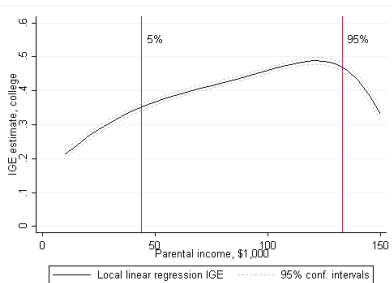
By **post transfer income**, IGE differences emerge at low levels too



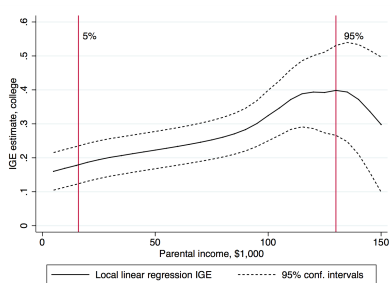
Note: Child income on parents' income for cohorts of 1973-75. Source: Landersø and Heckman (2017).

Figure 13: Local Intergenerational Elasticities between children's education and parental log gross income including transfers, absolute income weights, Denmark and the U.S.

(d) College completion, Denmark



(e) College completion, U.S.



Source: Figure 7 Landersø and Heckman (2018).

Figure 14: College completion and cognitive test scores, by parental background, year of birth, and country

a) College completion by parents' education

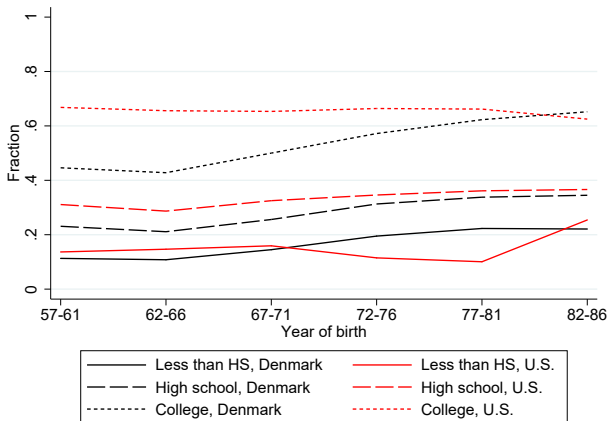
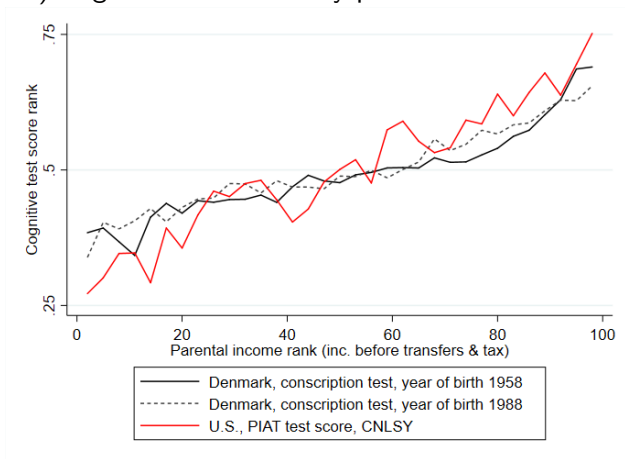


Figure 14: College completion and cognitive test scores, by parental background, year of birth, and country, Cont'd

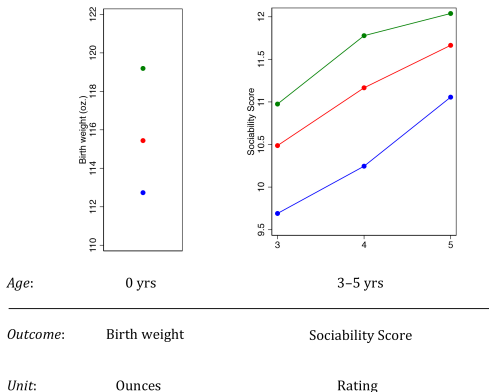
b) Cognitive test scores by parents' income rank



Substantial Gaps in Life Outcomes Across Children With Mothers With Different Education Levels

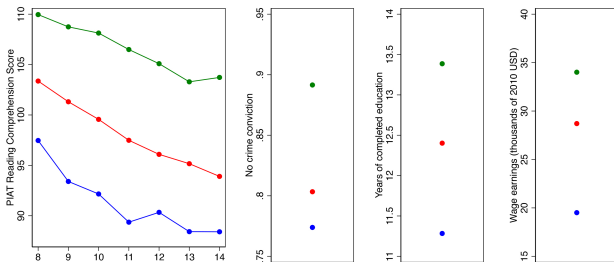
Gaps by Mother's Education

Figure 15: U.S. CNLSY



- Green: College educated
- Red: High school
- Blue: Less than high school

Figure 16: U.S. CNLSY, Cont.



Age:	8-14 yrs	25 yrs	30 yrs	40 yrs
Outcome:	PIAT Reading Comprehension Score	No criminal conviction	Years of schooling	Wage Earnings
Unit:	Test score	Fraction	Years	Thousands of 2010 U.S. Dollars

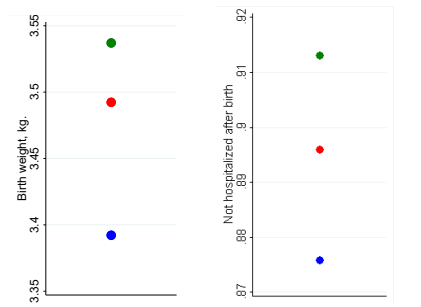
- Green: College educated
- Red: High school
- Blue: Less than high school

Danish Counterparts

Figure 17: Danish Gaps

0 ys
Birth weight

0 ys
Not admitted
to neonatal ward



- Green: College educated
- Red: High school
- Blue: Less than high school

Figure 17: Danish Gaps, Cont'd

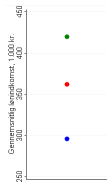
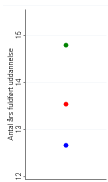
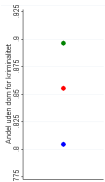
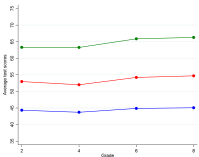
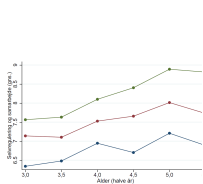
3-5 yo.
Assessed
skills

8-14 yo.
Test scores,
reading

25 yo.
No crime
conviction

30 yo.
Years of
Schooling

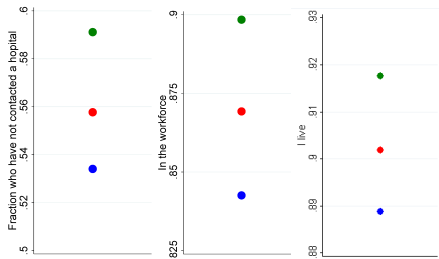
40 yo.
Income



- Green: College educated
- Red: High school
- Blue: Less than high school

Note: Figure shows average outcomes by mother's highest completed education. In the figures with three levels, mother's education is defined as: BLUE, only compulsory schooling; RED, high school; GREEN, college.

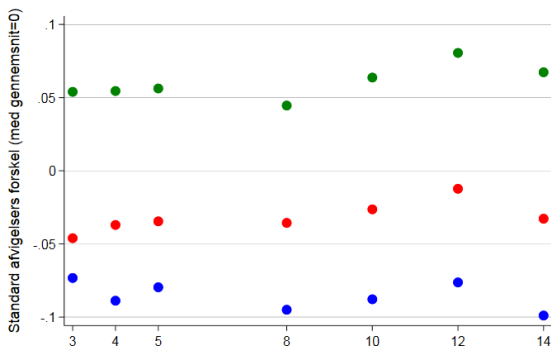
40-50 yo. Not contacted hospital
 54 yo. In the labor force
 60 yo. Alive



- Green: College educated
- Red: High school
- Blue: Less than high school

Note: Figure shows average outcomes by mother's highest completed education. In the figures with three levels, mother's education is defined as: BLUE, only compulsory schooling; RED, high school; GREEN, college.

Figure 18: Equally aged peers' language test scores, by mother's education

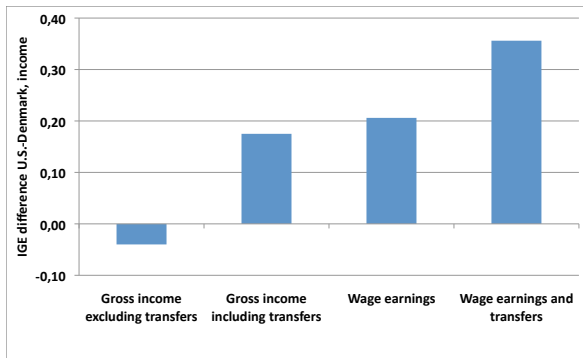


Source: Own calculations based on data from Tryk Fonden and Statistics Denmark.

Why Do These Gaps Arise?

Greater Income Mobility is Largely Due to the Highly Progressive Danish Tax-Transfer System Not Because of a Better Production of Child Skills

Figure 19: Differences in income mobility (IGEs) between the U.S. and Denmark: $\ln Y_i^C = \alpha + \beta^{IGE} \ln Y_i^P + \varepsilon_i$ for different income definitions



Note: Child income on parents' income for cohorts of 1973-75.

Source: Landersø and Heckman (2017).

Lower Income Inequality and Social Mobility Not Skills-Based

Tax and Transfer System Based Equality

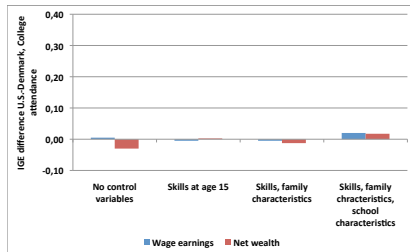
Summarizing

Figure 20: U.S.-Denmark differences in Income and Education IGEs

Income IGE



College attendance IGE



Source: Tables 1 and 4 Landersø and Heckman.

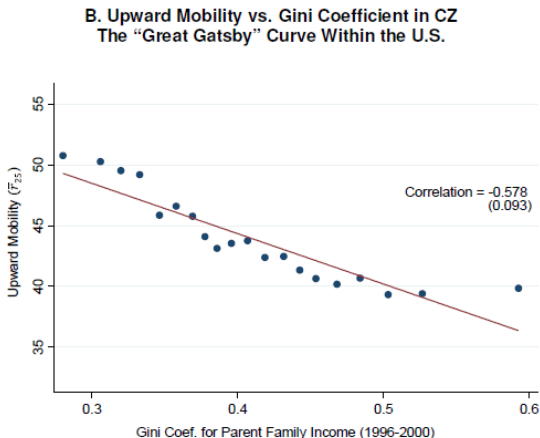
Sorting by Income and Education: Endogenous Neighborhoods

- Outcomes by family background in Denmark suggest that something else besides public expenditure is at work and strongly so, despite near equality of public expenditure.
- Equalizing expenditure is not enough to reduce gaps.

- Evidence on the early emergence of gaps leaves open the question of what aspects of families are responsible for producing these gaps.
- The evidence from a large body of research demonstrates an important role for investments and family and community environments in determining adult capacities above and beyond the role of genes.
- Home investments matter.
- But the choice of **neighborhoods**: peers and public goods also matters.
- **The element of family choice of neighborhoods has been neglected in recent literature that ignores such choices and treats neighborhoods and timing of moves to neighborhoods as randomly assigned.**

American Neighborhood Effects Have Been Heavily Featured in the Press (e.g., Atlas of Opportunity)

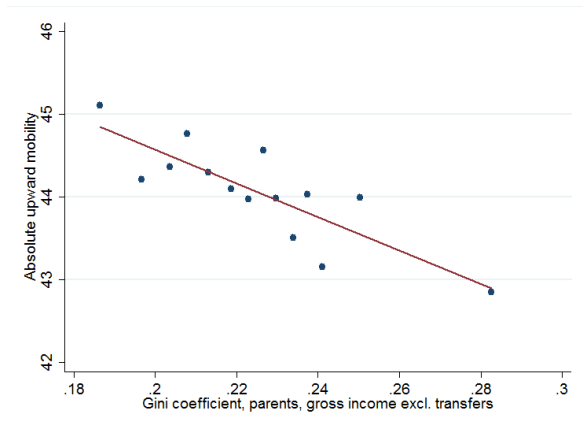
Figure 21: The Great Gatsby Curve, within the U.S.



Source: Chetty et al. (2014).

Note: \bar{F}_{25} is the relative mobility in rank at the 25th percentile.

Figure 22: The Great Gatsby Curve in Denmark Across Municipalities



Note: Birth cohorts 1971–1976 (parental income measured as 9-year averages during child generations; childhood; children's income measured at ages 35–37, ..., 40–42 depending on cohort). Figure shows a scatter plot of "absolute upward mobility" (defined as the expected child rank at parents' 25th percentile, where ranks are defined in terms of gross income excluding transfers in full population) across municipality-specific Gini coefficients. 15 bins of 6.67% of municipalities.

Sorting and Segregation

- Segregation: How similar are families who live in the same neighborhood?
 - Can be measured in many different ways
 - Different dimensions of segregation: native / immigrant (binary), education (discrete), income (continuous).
 - Different definitions of areas
- Measure of segregation in income in neighborhoods: Theil (1972), Reardon and Bishoff (2011), can be used to form a scale from 0-1:
 - 0 is no income segregation, 1 is full income segregation
 - 0: All income percentiles equally represented in all neighborhoods.
 - 1: Each neighborhood consists of families from same part of income distribution.

In the U.S., Sorting is High at Both Ends of the Income Distribution and Sorting Increasing

Figure 23: Income Segregation Patterns in the U.S.

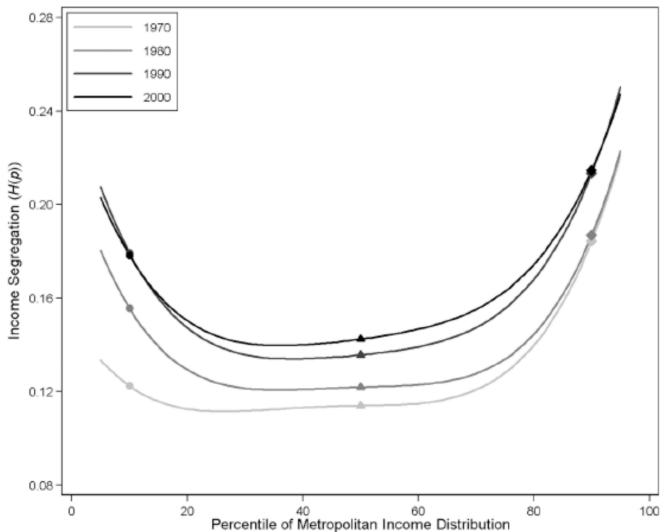


Figure 24: Income segregation by gross income excl. transfers across primary school Catchment Areas by year, Denmark

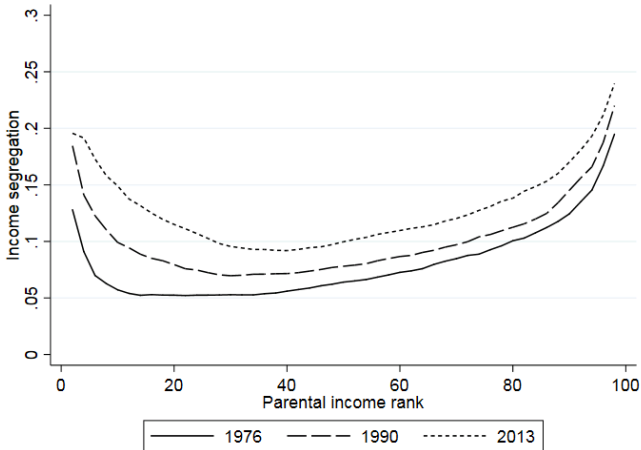
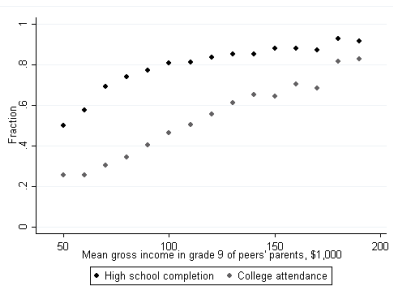
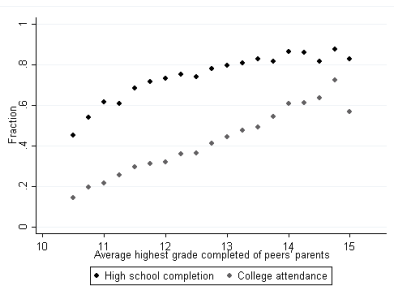


Figure 25: Strong Socioeconomic Gradients in Neighborhood Quality in Educational Attainment in Denmark

(A) High school completion and college attendance rates across average gross income of school peers' parents

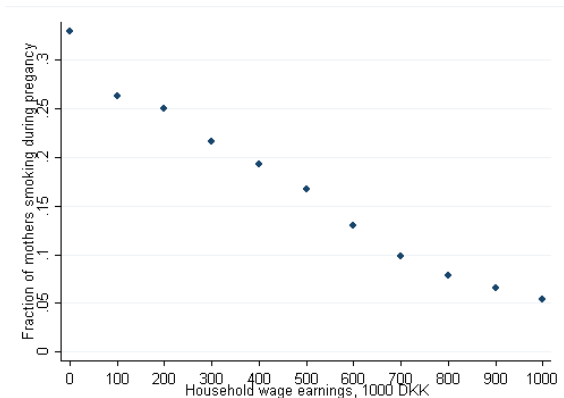


(B) High school completion and college attendance rates across average highest grade completed of school peers' parents



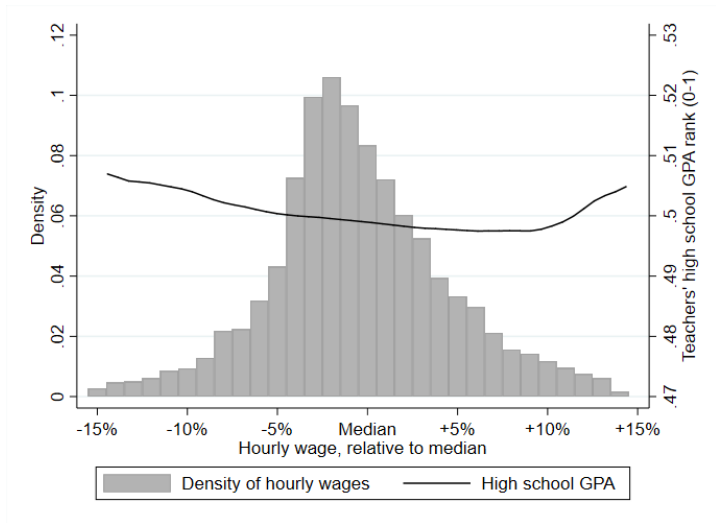
Danish Family Environments Fundamentally Unequal Across the Income Distribution

Figure 26: Fraction of mothers smoke during pregnancy by household wage earnings year prior to childbirth



Sorting of Teacher Quality Across Neighborhoods

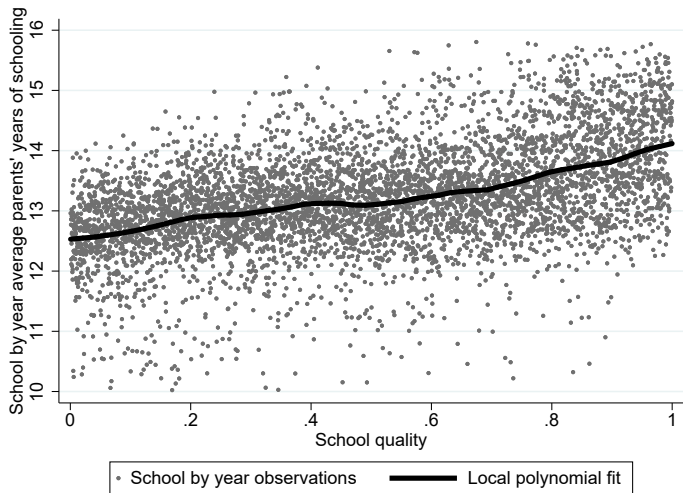
Figure 27: Teachers' hourly wage distribution and high school GPA



Source: Gensowski et al. (2020).

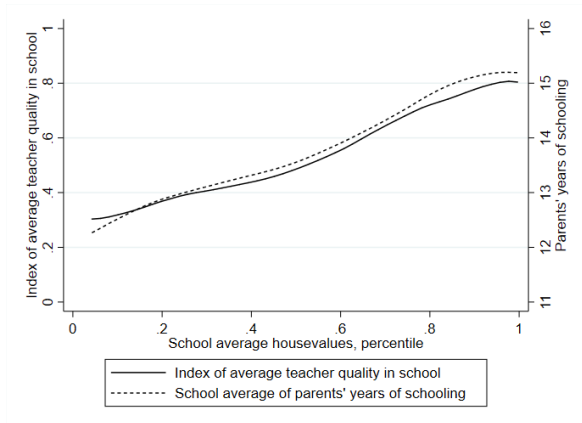
- Consider how teachers sort. All teachers more or less paid the same.
- Non-price allocation mechanism at work \implies sorting by student quality
- Sorting is a non-price mechanism. Best teachers sorted to best neighborhoods.
- Parents (through school boards) also have say on hires.

Figure 28: Parents' years of schoolings by average teacher quality in schools



Source: Gensowski et al. (2020).

Figure 29: Average teacher quality in schools and parent's education, by housing values



Source: Gensowski et al. (2020).

Another Role for Parental Influence: Parents May Reinforce School Quality or Substitute Away From It

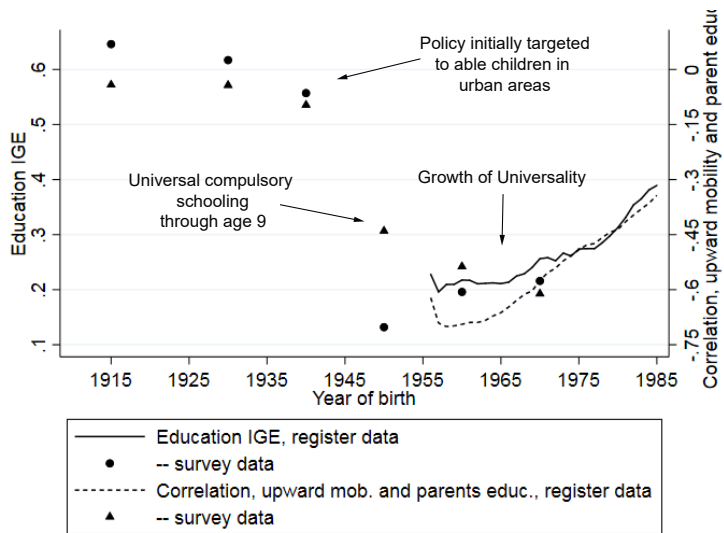
1. “Equal” access downstream but not equal skill formation upstream:
 - More advantaged children enter schools with greater skill
 - Two (stylized) potential barriers for education: parental costs and parental skills.
 - Denmark eliminates costs but retains importance of parental skills
 - Implicitly favors investments of children of high skilled parents.

2. How do parents respond to public investments?
 - Do they adjust their own investments?
 - Reinforcement or substitution?
3. Parents maximizing child skills by own investments at home and sorting into better institutions:
 - The role of i) family inputs, ii) institutions, and iii) sorting possibilities are closely linked.
 - **Gensowski and Landerso (2020)** show that in Denmark parents on net **substitute** their inputs in the presence of better quality schools.

Public Policy Shift (Karlson and Landerso, 2020)

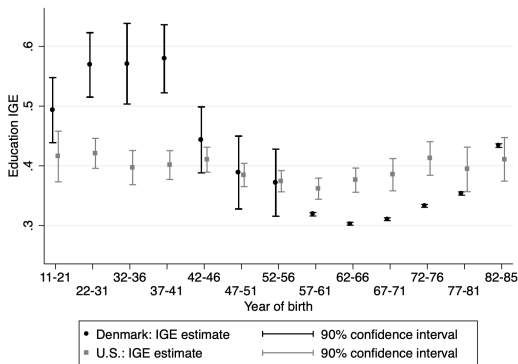
- A strong disconnect between the expansion of the timing of the welfare state from the 1960s onwards, and the timing when educational mobility peaked in Denmark.
 - Denmark saw massive expansion of education in the bottom from 1940s - mid 1960s cohorts \implies high mobility. Lower tail expansion driven by those from low-resource families.
 - Expansion in college and university from cohorts born during the 1970s onward \implies lower mobility. Upper tail expansion driven by those from affluent families.

Figure 30: Correlation between upward educational mobility and parents' education, back to 1910



$$\underbrace{y^C}_{\text{years of schooling child}} = \alpha + \beta \underbrace{y^P}_{\text{years of schooling parents}} + u$$

$\beta \uparrow$, Mobility \downarrow



Source: Karlson and Landersø (2020) U.S. estimates from GSS data, similar to Hilger (2017), Hout & Janus (2011).

Compulsory School Reforms Increased Educational Mobility

**Universality: Helps Children From Better Family
Backgrounds Relatively More**

Targeting Disadvantaged Children Yields Highest Benefits

- Elango et al. (2016): Early Childhood Interventions most effective for disadvantaged.
- Kline and Walters (2016) show effects of Head Start for disadvantaged.
- Havnes and Mogstad (2011): Introduction of universal daycare in Norway: program effects are biggest for most disadvantaged.
- Dustmann et al. (2017) study an expansion of childcare in Germany and find similar results.
- Walters (2018) shows similar evidence for choice of charter schools.

New Evidence on Neighborhood Effects

Estimating Neighborhood-level Mobility

- Estimate neighborhood-specific intercepts and slopes (with no controls)

$$y_{in}^c = \alpha_n + \beta_n^{IGE} y_{in}^p + u_{in}$$

using market income (labor earnings and capital income), before transfers and taxes, of children and parents.

- Let $n \in \mathcal{N}$ index neighborhoods: Danish parishes
 - Parishes are administrative units from the Church of Denmark
 - On average, home to 2,500 residents (comparable to a small U.S. Census tract or small zip code area)
- For much of the presentation, assign children to the parish they spent the longest time during childhood (ages 0-17)
 - However social, neighborhood mobility estimates are robust when accounting for exposures to different neighborhoods during childhood
- Let y_{in}^c : log of long-run average income between ages 30-45; y_{in}^p is log of child's family when they are 0-17
- Main measure of income: market income (labor earnings and capital income). We use alternative measures from register data: before or after transfers and taxes
- We also use PDV of disposable income

Sample construction for empirical analysis:

Analysis	Neighborhood	Permanent Inc.
Source	Danish registers	registers/survey
Sample	birth cohorts 1973/83	birth cohorts 1981/82
Years	1980-2018	1980-2018
Unit	family	father/family
Age: -Child	30 onward (up to 45)	30-35
-Parent	0-17 of child	0-17 of child

Figure 31: Empirical distribution of $\hat{\beta}_n^{IGE}$

(a) Distribution of $\hat{\beta}_n^{IGE}$ with and without family controls

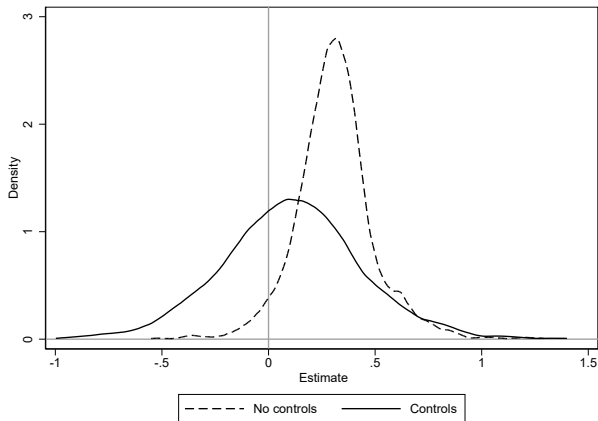
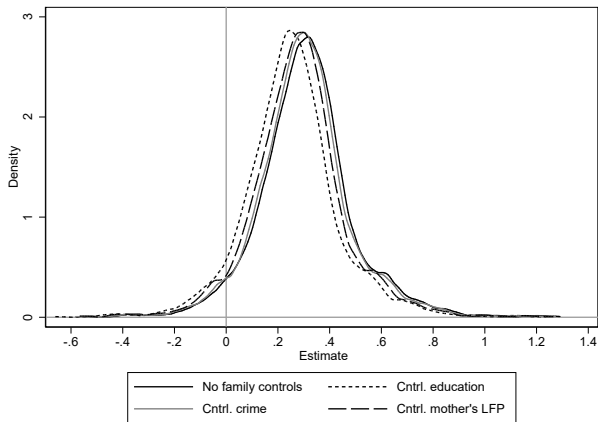


Figure 31: Empirical distribution of $\hat{\beta}_n^{IGE}$, Cont'd

(b) Illustration of how the distribution of $\hat{\beta}_n$ changes using selected family controls

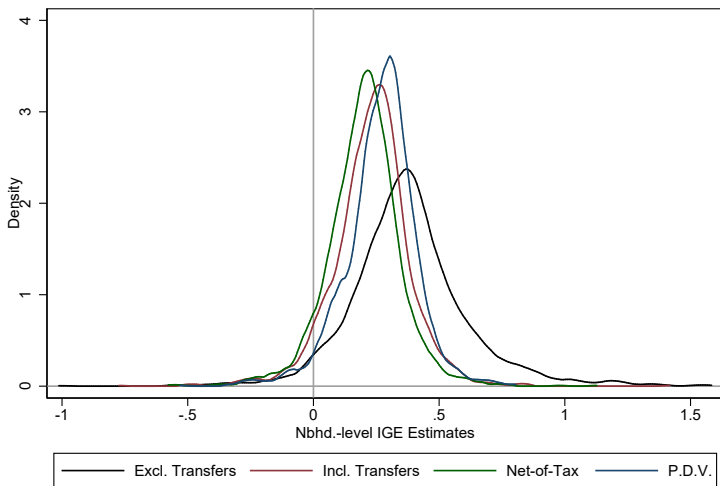


Source: Cholli (2021).

Note: The figure shows distributions of IGE estimates for parishes in Denmark (around 2,000 in Denmark; roughly the size of small U.S. Census tracts). Figure a) shows the densities for Income W/o transfers with and without controlling for family characteristics in each neighborhood: parents' age, child gender, household assets, mother's labor supply, parents' education, household size, marital status, parents' hospitalizations, parents' crime. Figure b) illustrates the impact of individual family variables.

Redistribution (i.e., After-Tax and Transfer Income) Greatly Reduces $\hat{\beta}_n^{IGE}$ and its Variability Across Neighborhoods

Figure 32: Empirical Distribution of $\hat{\beta}_n^{IGE}$ (parishes), by Income Measure Used



- **Variation in $\hat{\beta}_n^{IGE}$ appears to increase for more granular neighborhoods.**
- **Fewer families at smaller neighborhood units.**
- **Greater sampling error?**

Focus on:

- Parishes are neighborhood unit; Like zip codes.
- Market income before transfers and taxes as measure or child and parent income

Note: results that follow are qualitatively similar across alternative income measures

Lots of Sampling Error:
Variance of Estimated β_n

$$\text{Var}(\hat{\beta}_n^{IGE}) = \underbrace{\mathbb{E} \left[\hat{\sigma}^2(\hat{\beta}_n^{IGE}) \right]}_{\text{sampling error}} + \underbrace{\text{Var}(\beta_n^{IGE})}_{\substack{\text{population} \\ \text{heterogeneity} \\ \text{"signal"}}}$$

- 65% of the variation in α_n, β_n^{IGE} is due to sampling error
- Even for statistically significant estimates, 60% of variance among n is due to sampling error
- Consistent with Mogstad et al. (2020)

Contribution of the Neighborhoods to Inequality in Child Income as Adult

Variance decomposition between vs. within neighborhoods
(accounting for sampling error)

$$\text{Var}(y_{in}^c) = \underbrace{\text{Var}(\bar{y}_n^c)}_{\text{between nbhd. 10\%}} + \underbrace{\mathbb{E}[\text{Var}(y_{in}^c | i \text{ lives in } n)]}_{\text{within nbhd. 90\%}}$$

**More Advantaged Families associated with
Lower Neighborhood IGEs**

Purposive Choice of Neighborhood to Raise Children by Education of Mother

Quality of Neighborhood for Child Rearing Improves With Education of the Mother

Figure 33: Average income in area of residence and moving pattern, by time to/from birth of first child

(a) Average income in area of residence relative to country average

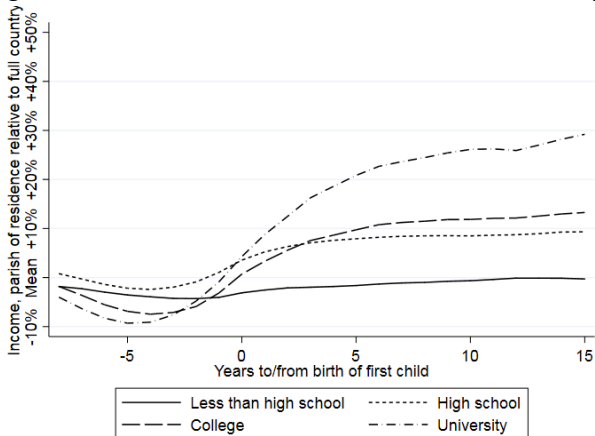


Figure 33: Average income in area of residence and moving pattern, by time to/from birth of first child, Cont'd

(b) Fraction that move each year

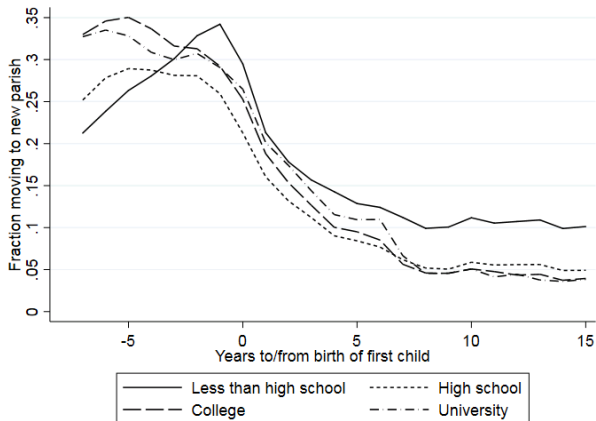
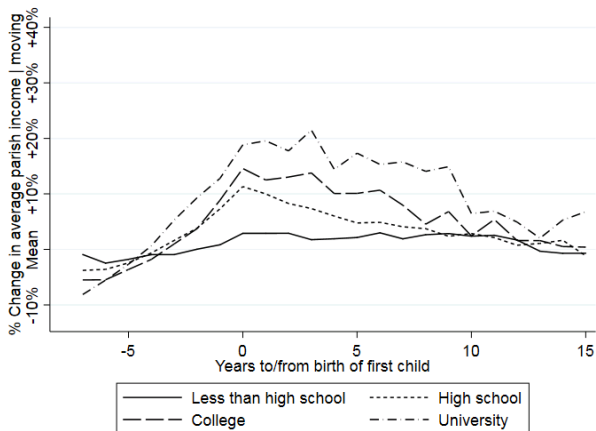


Figure 33: Average income in area of residence and moving pattern, by time to/from birth of first child, Cont'd

(c) Average income, destination-origin



Obviously Family Moves Not Random

The Assumption of Random Moves by Parents is a Key Identifying Assumption in Recent Work on Neighborhood Effects

Quality of Neighborhood for Child Rearing Improves With Education of the Mother

Summary of Location Choice

- 1 Most moves made by young parents prior to the start of school.
- 2 First-born children experience moves more often than second- and third-borns.
- 3 Highly educated mothers make fewer moves after arrival of children with large positive changes in neighborhood quality.
- 4 Gaps in neighborhood quality remain large and persist during adolescence.

**Sorting Can Create Artificial Neighborhood Effects if
Parental Income Not Well Measured (As it is Not in the
U.S. Census/IRS Data)**

Through Sorting Neighborhood Proxies Parent Income and Family Characteristics

Figure 34: Slopes of Non-linear $\ln Y^C$ on $\ln Y^P$ Curve as a Function of Y^P , Danish Cohorts 1972–1984

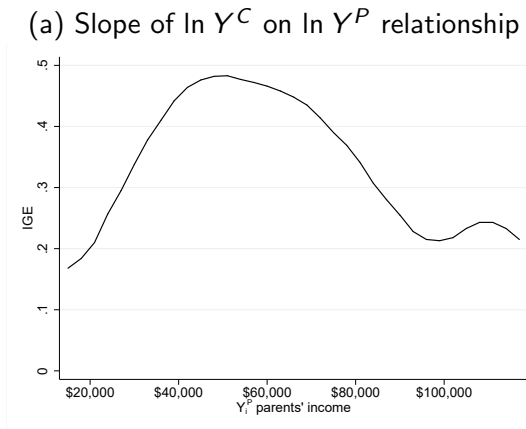
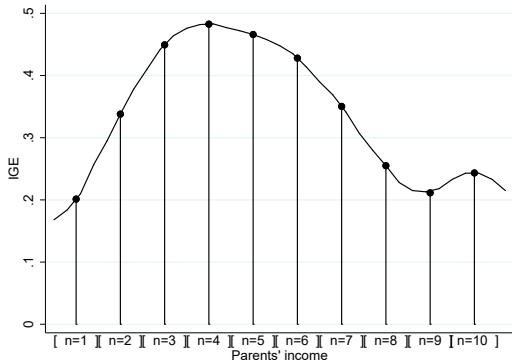


Figure 34: Slopes of Non-linear $\ln Y^C$ on $\ln Y^P$ Curve as a Function of Y^P , Danish Cohorts 1972–1984, Cont'd

(b) IGEs for the case of perfect sorting



Intergenerational Transmission of Utility

Toward Lifetime Measures of IGE

IGE of Value Functions

- The traditional literature on the IGE focuses on matching the income of father to that of son over common windows of age.
- It sometimes matches income of child's family income with family income of parent.
- We construct **lifetime measures** (both for individuals and on families).
- Compute IGEs of value functions and other approximations of value functions.
- Investment in child skills is the outcome of a lifetime investment strategy by parents which we model.
- Account for:
 - ❶ Age of marriage and cohabitation
 - ❷ Onset of fertility
 - ❸ Timing and spacing of births
 - ❹ Divorce

Conventional measures

- Wage income
- Disposable income
- Income (with and without) tax/transfers
- Consumption (with and without equivalence scale)

Lifetime measures

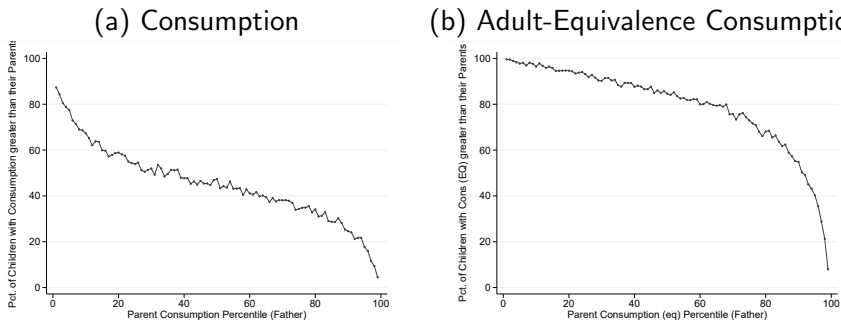
- Human wealth (the value of human capital)
- Expected PDV of disposable income
- Value functions at common ages

Consumption

- Impute total household expenditures from the relationship between Danish Expenditure Survey and Danish register data
- Compute equivalence scale consumption by adjusting imputed consumption for household composition
- Alternatively, consumption can be imputed using accounting identity by only using registers (Browning et. al. (2003)).

Importance of Adjusting for Family Composition

Figure 35: Pct. of Children with Greater Consumption than their Parents



Note: This figure shows the percentage of children (of 1981–1982 birth cohorts) whose consumption expenditures are greater than that of their parents (father's family), by percentile of parents' consumption distribution. Consumption is averaged over ages 30–35 for both parents and children. We use survey imputed consumption measure. Panel A uses raw total household consumption expenditures. Panel b uses an equivalence scale to adjust for family size.

Human Wealth Measure

The Value of an Individual's Human Capital (Huggett and Kaplan, 2016):

- Human wealth (HW) θ_j measures the asset value of human capital at age j :

$$\theta_j \equiv E \left[\sum_{k=j+1}^J m_{j,k} d_k \right] \quad (1)$$

where $m_{j,k}$ is the stochastic discount factor and d_k “dividends” at age k

- “Dividends” d_k includes earnings and the value of leisure

- Derived from a thought experiment
- Maximize lifetime utility accounting for consumption and labor supply (value of leisure)
- Prices and asset rate of returns are external parameters facing agents
- θ_j is the monetary equivalent value of lifetime program:
how much a person would be willing to pay (or sell) for his/her lifetime program.

Link to Human Wealth in Dynastic Framework

Human Wealth and Economic Well-being:

- Human wealth captures impacts of:
 - **Credit constraints:** For borrowing constrained agents, the value tends to be lower.
 - Future earnings are valued less by individuals who cannot access them in advance.
 - **Income uncertainty:** Risk aversion and uncertainty reduce lifetime utility.
- Closer link to economic decisions than per period realized income (or average over a few periods)

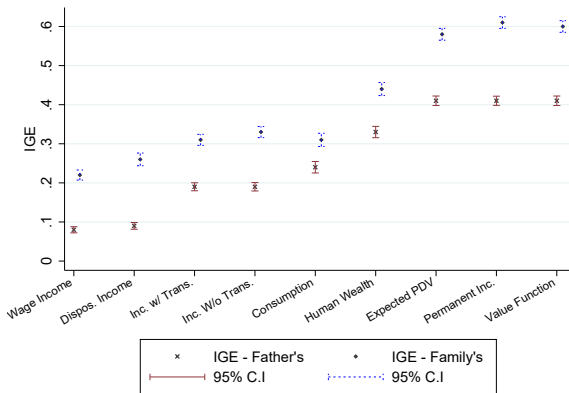
Permanent Income

Annuitized Human Wealth and Financial Wealth Discounted Value of Income

- Present Discounted Value (PDV) of disposable income is similar to HW.
- Instead of stochastic discount factor use a risk-free rate to discount future inc.

Figure 36: Log-Log IGE Estimates

(At Ages 30-35 for Both Generations)



Note: This figure depicts the IGE estimates. The sample includes fathers and children of 1981-1982 birth cohorts. Outcomes are measured at ages 30-35 for both generations.

Table 1: IGE Estimates- Ages 30-35 for both Parents and Children

	Father	Family
Traditional Measures		
Wage Income	0.08*** (0.005)	0.24*** (0.009)
Dispos. Income	0.09*** (0.005)	0.26*** (0.010)
Inc. w/ Trans.	0.19*** (0.006)	0.31*** (0.009)
Inc. w/o Tax/Trans.	0.17*** (0.006)	0.34*** (0.009)
HH. Cons. (EQ)	0.13*** (0.007)	0.22*** (0.008)
HH. Cons.	0.25*** (0.008)	0.32*** (0.010)

Note: The IGE is the slope coefficient from the log-log regression of child measure on father (family) measure: $\log(\bar{y}_i^c) = \alpha + \beta * \log(\bar{y}_i^f)$ where \bar{y}_i^c denotes the average (over 30-35) of child measure, and \bar{y}_i^f denotes the average of father (family) measure when the child was 30-35 years old.

Table 1: IGE Estimates- Ages 30-35 for both Parents and Children,
Cont'd

	Father	Family
Lifetime Measures		
Human Wealth	0.33*** (0.008)	0.44*** (0.010)
Expected PDV	0.41*** (0.007)	0.58*** (0.009)
Permanent Inc. (annuit. HW+assets)	0.31*** (0.007)	0.49*** (0.009)
Permanent Inc. (annuit. PDV+assets)	0.40*** (0.007)	0.60*** (0.009)
Value Function		
Value Function (CRRA utility w/ $\rho = 1.3$)	0.30*** (0.006)	0.51*** (0.009)
Value Function (CRRA utility w/ $\rho = 0.67$)	0.40*** (0.007)	0.59*** (0.009)
Value Function (linear utility)	0.42*** (0.007)	0.59*** (0.008)

Note: The IGE is the slope coefficient from the log-log regression of child measure on father (family) measure: $\log(\bar{y}_i^c) = \alpha + \beta * \log(\bar{y}_i^f)$ where \bar{y}_i^c denotes the average (over 30-35) of child measure, and \bar{y}_i^f denotes the average of father (family) measure when the child was 30-35 years old.

$\rho \uparrow$ **Risk Aversion** \uparrow

Table 2: Educational Outcomes of Child by Measures of Parental Resources

Unit of Parental Measures	Father		Family	
	Coefficient	R Square	Coefficient	R Square
Wage Income	0.30***	0.012	0.78***	0.038
Dispos. Income	0.61***	0.011	1.17***	0.016
Inc. w/ Trans.	1.19***	0.036	2.06***	0.052
Inc. w/o Trans.	0.36***	0.020	1.40***	0.074
HH. Cons.	2.24***	0.032	2.70***	0.037
HH. Cons. (EQ)	0.93***	0.005	1.83***	0.015
Human Wealth	2.20***	0.038	3.06***	0.053
Expected PDV	3.75***	0.101	5.39***	0.132
Permanent Inc. (annuit. HW+assets)	1.43***	0.030	2.94***	0.061
Permanent Inc. (annuit. PDV+assets)	2.92***	0.087	4.86***	0.133
Value Function ($\rho = 1.3$)	26.12***	0.088	45.41***	0.121
Value Function ($\rho = 0.67$)	14.77***	0.096	22.39***	0.136
Value Function (linear)	6.99***	0.093	10.15***	0.136

Note: Sample is restricted to native Danes of 1981-82 birth cohorts and their parents. The slope coefficient reported in this table is estimated as follows: $edu_i^f = \alpha + \beta * \log(\bar{y}_i^f)$ where edu_i^f denotes the child's years of schooling, and \bar{y}_i^f denotes the average of father (family) measure over ages 30-35. Family outcomes are the sum of mother's and father's outcome. Standard errors are reported in parentheses.

* $p < .1$, ** $p < .05$, *** $p < .01$

Table 3: Expected PDV of Child by Measures of Parental Resources

Unit of Parental Measures	Father		Family	
	Coefficient	R Square	Coefficient	R Square
Wage Income	0.03***	0.011	0.08***	0.031
Dispos. Income	0.07***	0.011	0.13***	0.016
Inc. w/ Trans.	0.14***	0.038	0.24***	0.053
Inc. w/o Trans.	0.04***	0.020	0.16***	0.067
HH. Cons.	0.27***	0.034	0.33***	0.040
HH. Cons. (EQ)	0.11***	0.005	0.21***	0.015
Human Wealth	0.24***	0.034	0.33***	0.047
Expected PDV	0.39***	0.081	0.56***	0.106
Permanent Inc. (annuit. HW+assets)	0.16***	0.027	0.32***	0.054
Permanent Inc. (annuit. PDV+assets)	0.31***	0.072	0.51***	0.108
Value Function ($\rho = 1.3$)	2.77***	0.073	4.75***	0.098
Value Function ($\rho = 0.67$)	1.53***	0.076	2.30***	0.107
Value Function (linear)	0.72***	0.072	1.04***	0.106

Note: Sample is restricted to native Danes of 1981-82 birth cohorts and their parents. The slope coefficient reported in this table is estimated as follows: $\log(P\bar{D}V_i^c) = \alpha + \beta * \log(\bar{y}_i^f)$ where $P\bar{D}V_i^c$ denotes the child's expected PDV of disp. income, and \bar{y}_i^f denotes the average of father (family) measure over ages 30-35. Family outcomes are the sum of mother's and father's outcome.

Standard errors are reported in parentheses.

* $p < .1$, ** $p < .05$, *** $p < .01$

Summary of Value Function IGEs

- Measures of lifetime resources show a stronger link across generations than traditional measures:
- Compared to measures at father's income, family resources better predict child outcomes.
- Lifetime resources have a closer connection to economic decisions (e.g. investments in children) than resources averaged over a short panel.
- Lifetime measures of IGE paint a different picture of income mobility in Denmark: mobility is significantly lower than previously thought.

Summary

- Denmark is widely perceived to be a Garden of Eden by many politicians, public figures, and “informed” citizens around the world.
- Danish policies have been widely advocated.

- Our previous work (SJE, 2017) examined Danish policy and evidence.
 - a For Denmark: less inequality and greater social mobility, in terms of income.
 - b Equality in earnings and IGE in earnings is a consequence of tax and transfer policy.
 - c This equalizes income and at the same time reduces the incentives of children to acquire skills.
 - d This equality is not a result of education and skills policies.
 - e They are generous and offered equally to all:
 - i Universal pre-K
 - ii Equal pay and financial resources for all schools everywhere
 - iii Extensive job training and retraining associated with its carrot-and-stick policy for unemployment insurance
 - iv Universal health care
 - v Free college
 - vi Generous work leaves for parents with newly-born children.

- (f) However, welfare state policies distort incentives to acquire skills (see SJE, 2018).
- (g) Gaps in skills and lifetime outcomes (e.g., earnings, health, and crime) of children of the less educated and the more educated mothers about the same for the U.S. and Denmark, both quantitatively and qualitatively.

- We find strong evidence of sorting of families on income and education
- Advantages from universal access to services are reaped relatively more by the affluent rather than by the disadvantaged (Matthew Effects)
- “Power of place” is due to family sorting
 - Family **choice** of neighborhoods
 - Timing of choices not random (in contrast to influential claims otherwise)
 - Sorting patterns comparable to U.S.
 - IGE \downarrow Family income \uparrow ; More affluent places have lower IGEs
 - The sorting shows up most strongly in initial conditions
 - Sorting \implies Strong family income gradients on child outcomes

- Sorting by teachers into more advantaged districts
- Despite equal wages for teachers; payment is in quality of students taught.
- Neighborhood effects large through parental choices, not some intrinsic property of an address.

- A life cycle – human wealth approach to measuring family influence.
- Long-term measures of family income (value functions) much more predictive of child outcomes than currently used measures.
- IGEs higher for life cycle measures of family resources than traditional sources.

Thank You for Your Attention

Human Wealth in Dynastic Framework (Appendix)

- Agents have preferences over consumption c and leisure n , and that they also care about their child's utility in adulthood, which depends on their initial assets (bequests) and human capital.
- They maximize expected utility subject to a budget constraint, an earnings process and a technology for their child's human capital.

$$\max_{c,n,a,y,l,\tau_I} U(c, n) + \delta V(b_c, h_c) \quad (2)$$

$$s.t \quad c_j + B_j + \sum_i a_{j+1}^i = \sum_i a_j^i R_j^i + e_j \quad (3)$$

$$B_j = \begin{cases} l_j & \text{if } j \in \{J_f, \dots, J_c - 1\} \\ b_c^0 & \text{if } j = J_c \\ 0 & \text{if } j < J_f \text{ or } j > J_c \end{cases} \quad (4)$$

$$e_j = G_j(y^j, n^j, z^j; h_p^0) \quad (5)$$

$$h_c = H(I^{J_c-1}, \tau_I^{J_c-1}, h_p, z^{J_c-1}) \quad (6)$$

$$a_{J+1}^i = 0 \quad (7)$$

$$(8)$$

- Superscripts j denote the history of a variable up until that age, while subscripts denote a variable at that period.
- $U(c, n)$ is the agent's lifetime utility from consumption and leisure.
- b_c and h_c are the bequests and the child's initial human capital, δ captures altruism and $V(b_c, h_c)$ is the child's value function.
- a_j^i and R_j^i are the stocks and returns to asset i .
- B_j captures either bequests or pecuniary investments in the child's human capital, depending on the age period. J_f marks the beginning of child-rearing and J_c the year the child becomes an adult. Earnings e_j are determined by the function G_j , which, for flexibility, depends on the history of choices y^j , n^j , as well as shocks z^j and initial human capital h_p .

- y denotes other choices, such as time spent investing in one's own human capital, while τ_I is time investments in human capital production for the child.
- The child's human capital is determined by the history of pecuniary and time investments, other parent choices y , and initial parent human capital as well as the history of shocks.
- The dependence on other choices and initial parent human capital can reflect the direct affect of genetics and other training (including formal schooling) of the parents on the child.

- As in Huggett (2012), we consider a restricted version of this problem.
- Let earnings be exogenously given at the optimal values, i.e., by e^* . Further, assume that leisure n be no more than n^* , and time investments be no more than τ_l^* .
- Note that if $(c^*, n^*, e^*, y^*, a^*, l^*, \tau_l^*)$ is a solution to the unrestricted problem then $(c^*, n^*, \tau_l^*, l^*, b_c^*, a^*)$ solves the restricted problem.

- If we write the Lagrangian, and substitute the expression for child's human capital, to this restricted problem we get

$$L = U(c, n) + \delta V(b_c, H(I^{J_c-1}, \tau_I^{J_c-1}, h_p, z^{J_c-1})) \quad (9)$$

$$+ \sum_j \sum_{z^j} \lambda(z^j) \left[\sum_i a^i(z^{j-1}) R^i(z^j) \right. \quad (10)$$

$$\left. + e^*(z^j) - c(z^j) - B(z^j) - \sum_i a^i(z^j) \right] + \quad (11)$$

$$+ \sum_j \sum_{z^j} \gamma(z^j) [n(z^j) - 0] + \sum_j \sum_{z^j} \rho(z^j) [n^*(z^j) - n(z^j)] \quad (12)$$

$$+ \sum_j \sum_{z^j} \nu(z^j) [\tau_I(z^j) - 0] + \sum_j \sum_{z^j} \mu(z^j) [\tau_I^*(z^j) - \tau_I(z^j)] \quad (13)$$

for which we can take the following first order conditions

$$\text{P1-1: } dU(c, n)/dc(z^j) - \lambda(z^j) = 0 \quad (14)$$

$$\text{P1-2: } dU(c, n)/dn(z^j) + \gamma(z^j) - \rho(z^j) = 0 \quad (15)$$

$$\text{P1-3: } -\lambda(z^j) + \sum_{z_{j+1}} \lambda(z^j, z_{j+1}) R^i(z^j, z_{j+1}) = 0, \quad \forall i \quad (16)$$

$$\text{P1-4: } \delta(dV(b_c, h_c)/dh_c)dH/d\tau_l(z^j) + \nu(z^j) - \mu(z^j) = 0, \quad (17)$$

$$j \in \{J_f, \dots, J_c - 1\} \quad (18)$$

$$\text{P1-5: } \delta(dV(b_c, h_c)/dh_c)dH/dl(z^j) - \lambda(z^j) = 0, \quad j = J_c \quad (19)$$

Modified Problem (P2)

- Now consider the problem with the hypothetical firm that follows the agent's human capital decisions.
- The problem looks like the following.

$$\max_{c,n,a,l,\tau_l} U(c, n) + \delta V(b_c, h_c) \quad (20)$$

$$s.t. \quad c_j + B_j + \sum_i a_{j+1}^i + s_{j+1}v_j + p_j n_j \quad (21)$$

$$= \sum_i a_j^i R_j^i + s_j(v_j + d_j) \quad (22)$$

$$B_j = \begin{cases} l_j & \text{if } j \in \{J_f, \dots, J_c - 1\} \\ b_c^0 & \text{if } j = J_c \\ 0 & \text{if } j < J_f \text{ or } j > J_c \end{cases} \quad (23)$$

$$h_c = H(l^{J_c-1}, \tau_l^{J_c-1}, h_p, z^{J_c-1}) \quad (24)$$

$$a_{J+1}^i = 0 \quad (25)$$

$$d_j \equiv e_j^* + p_j n_j^* \quad (26)$$

$$p_j \equiv \frac{dU(c^*, n^*)/dn_j}{dU(c^*, n^*)/dc_j} \quad (27)$$

- We can show that the necessary conditions of the original problem evaluated at $(c^*, n^*, a^*, l^*, \tau_l^*)$ imply the necessary (and sufficient, under concavity) conditions for the problem above at $(c^*, n^*, a^*, l^*, \tau_l^*, s^*)$ when $s_j^* = 1$.
- First, we write the Lagrangian.

$$L = U(c, n) + \delta V(b_c, H(l^{J_c-1}, \tau_l^{J_c-1}, h_p, z^{J_c-1})) \quad (30)$$

$$+ \sum_j \sum_{z^j} \lambda(z^j) \left[s(z^{j-1}) [v(z^j) + d(z^j)] \right] \quad (31)$$

$$+ \sum_i a^i(z^{j-1}) R^i(z^j) - c(z^j) - B(z^j) - \sum_i a^i(z^j) \quad (32)$$

$$\left. - s(z^j)v(z^j) - p(z^j)n(z^j) \right] \quad (33)$$

$$+ \sum_j \sum_{z^j} \gamma(z^j) [n(z^j) - 0] + \sum_j \sum_{z^j} \delta(z^j) [1 - n(z^j)] \quad (34)$$

$$+ \sum_j \sum_{z^j} \nu(z^j) [\tau_l(z^j) - 0] + \sum_j \sum_{z^j} \omega(z^j) [1 - \tau_l(z^j)] \quad (35)$$

- The first order conditions are as follows.

$$\text{P2-1: } dU(c, n)/dc(z^j) - \lambda(z^j) = 0 \quad (36)$$

$$\text{P2-2: } dU(c, n)/dn(z^j) - \lambda(z^j)p(z^j) + \gamma(z^j) - \delta(z^j) = 0 \quad (37)$$

$$\text{P2-3: } -\lambda(z^j) + \sum_{z_{j+1}} \lambda(z^j, z_{j+1})R^i(z^j, z_{j+1}) = 0, \quad \forall i \quad (38)$$

$$\text{P2-4: } \delta(dV(b_c, h_c)/dh_c)dH/d\tau_l(z^j) + \nu(z^j) - \omega(z^j) = 0, \quad (39)$$

$$j \in \{J_f, \dots, J_c - 1\} \quad (40)$$

$$\text{P2-5: } \delta(dV(b_c, h_c)/dh_c)dH/dl(z^j) - \lambda(z^j) = 0, \quad j = J_c \quad (41)$$

$$\text{P2-6: } \nu(z^j) + \sum_{z_{j+1}} \frac{\lambda(z^j, z_{j+1})}{\lambda(z^j)} (\nu(z^j, z_{j+1}) + d(z^j, z_{j+1})) = 0 \quad (42)$$

- Note that P1-1 implies P2-1. P2-1, the definition of $p(z^j)$ and setting $\gamma(z^j) = \delta(z^j) = 0$ implies P2-2. P1-3 implies P2-3.

- Then, substituting P2-1 into P2-6, we see that P2-6 holds by the definition of ν . P1-4 implies P2-4 setting $\omega(z^j) = \mu(z^j) = 0$.
- P1-5 implies P2-5. Now we must check to see all constraints and complementary slackness conditions of P2 hold.
- The budget constraint in P2 holds by the budget constraint in the restricted version of P1 at $(c^*, n^*, a^*, \tau_l^*, l^*)$, by $s_j^* = 1$ and by the definition of dividends.
- The leisure and time investment restrictions hold at n^* and τ_l^* , and the associated complementary slackness conditions hold by setting $\gamma(z^j) = \delta(z^j) = 0$ and $\nu(z^j) = \omega(z^j) = 0$.

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