Family Influence

Skill Formation and Investment Decisions

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Econ 350, Winter 2021

Introduction

Introduction

In the previous module, we have discussed neighborhood effects on the child outcomes. Another natural dimension to study is the influence of the family on child outcomes.

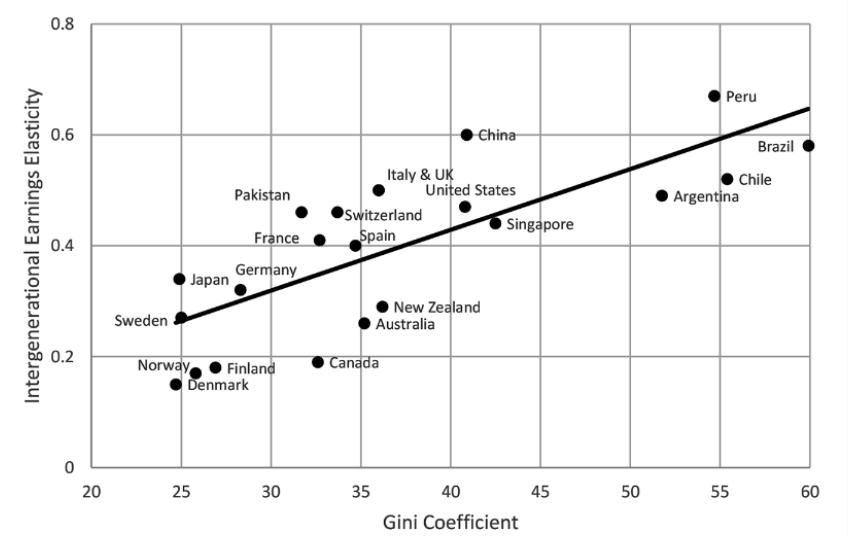
Family may influence children in many ways

- provide genes and epigenetics
- provide direct cognitive and noncognitive learning and instructions
  $\Rightarrow$ *timing* of the investment plays a vital role
- provide money to meet needs and learning opportunities
- influence information sets
- influence preferences
- influence peers and networks

We will focus on the influence of the family in human capital and skill formation

Modeling the impact: skill formation & investment decisions

Modeling impact: human capital and market dynamics (Becker-Kominers-Murphy-Spenkuch, 2018)



Modeling impact: human capital and market dynamics (Becker-Kominers-Murphy-Spenkuch, 2018)

- We have previously discussed the ”Great Gatsby curve”.
- Krueger suggested that it could be due to credit constrains and Durlauf and Sheshadri (2018) showed that neighborhood effects could explain it.
- Becker, Kominers, Murphy, and Spenkuch (2018) show that this relationship could be due to the family influence in human capital accumulation.

Modeling impact: human capital and market dynamics (Becker-Kominers-Murphy-Spenkuch, 2018)

- Study relation of the distribution of human capital and income and intergenerational mobility.
- They argue that complementarity between human capital of parent and parental investment in the production of children human capital is key
  - Means that at a given level of parental investment, its marginal productivity is higher for parents with higher human capital.
  - Wealthy parents on average invest more in their children than poorer ones, which leads to persistence.
- BKMS (2018) show that even without differences in initial ability and perfect credit market there could be persistence in economic position.
  - This result departs from Becker and Tomes (1986), which show that efficient investment under perfect credit markets lead to perfect intergenerational mobility.

Becker-Kominers-Murphy-Spenkuch (2018)

- Two periods: childhood and adulthood. Each parent has one child at the beginning of adulthood.
- Adults use human capital accumulated as children for generating income.
- Adults gain utility from own consumption (z), but also from future utility of their children. They maximize children utility by increasing their expected resources ($\bar{I}_c$), and it will depend in their degree of parental altruism ($\delta \in (0, 1)$):

$$V(I_p) = u(z) + \delta U_c(\bar{I}_c) \tag{1}$$

Becker-Kominers-Murphy-Spenkuch (2018)

They assume an isoelastic relationship of earnings and human capital:

$$E = rH^{\sigma}\epsilon \tag{2}$$

- $r > 0$: price human capital. Increase in r induce higher inequality but do not affect skewness.
- $\sigma > 0$ individual-level elasticity between human capital and earnings. Increase in σ increase inequality and skewness of distribution.
- ϵ error term, distributed independently of H and with mean 0.

They only assume one dimension of human capital (not different skills) and don't consider different tasks in the labor market.

Becker-Kominers-Murphy-Spenkuch (2018)

The production of human capital depend in the ability of children (A_c), parental human capital (H_p) and parental investment (γ):

$$H_c = A_c \gamma^{\alpha} H_p^{\beta} \tag{3}$$

They assume:

- $0 < \alpha < 1$: human capital investment increase human capital of child but with diminishing returns.
- $0 < \beta$: human capital of parent increase human capital of child.
- $A_c = 1$: everyone is equally able.

Becker-Kominers-Murphy-Spenkuch (2018)

This functional form and values of parameters leads to the key assumption: complementarity between parental investment (γ) and human capital of parent (H_p).

- Means that human capital raise productivity in marketplace but also in household production (Becker 1965).
- Means that parents with higher human capital invest more in their children.

They also assume perfect capital markets: parents can borrow as much as they want at exogenous rate $R_k > 0$, and leave negative bequest (so children pay debt with their adult earnings).

- Then all parents that care about their children ($\delta > 0$) would invest the optimal ammount (until the return to investment equals the interest rate).

Becker-Kominers-Murphy-Spenkuch (2018)

Parent choose consumption level z, investment γ, and bequest b_c, tomaximize V subject to the production function of human capital (3) the determinants of earning (2) and budget constrain:

$$z + \frac{b_c}{R_k} + \gamma = I_p \equiv E_p + b_p \tag{4}$$

Becker-Kominers-Murphy-Spenkuch (2018)

The optimal investment and technology of human capital formation would lead to the following relation between the human capital of parent and children:

$$H_c = \left(\frac{r\alpha\sigma}{R_k}\right)^{\alpha/(1-\alpha\sigma)} H_p^{\beta/(1-\alpha\sigma)} \tag{5}$$

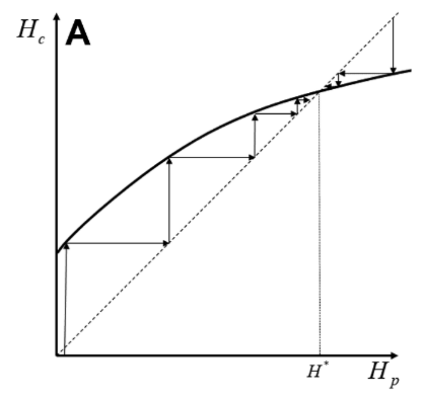
Then even if $\alpha + \beta < 1$ (decreasing returns to scale) the equilibrium relationship between the human capital of parents and children will be convex when $\alpha\sigma + \beta > 1$. In that case a necessary condition for convexity is $\sigma > 1$ (increase in human capital have disproportionate rewards). Could be result of:

- Superstar economy (Rosen, 1981)
- Positive assortative matching between workers and firms (Sattinger, 1979)

The next figure plot the relationship of parent and children human capital for different shapes of the transmission function:

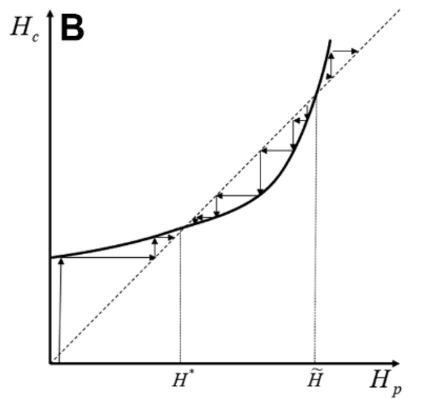
Becker-Kominers-Murphy-Spenkuch (2018)

When the transmission function is concave ($\alpha\sigma + \beta \leq 1$): all dynasties would converge to the same equilibrium.



Becker-Kominers-Murphy-Spenkuch (2018)

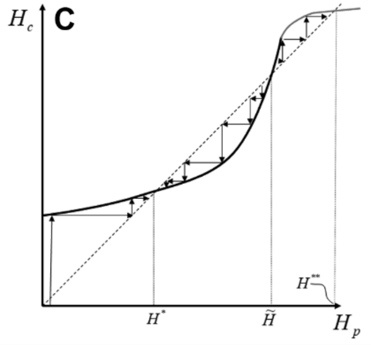
When the transmission function is convex ($\alpha\sigma + \beta > 1$): children of adults with human capital level over $\tilde{H}$ would diverge. Not all individuals tend to same equilibrium



Becker-Kominers-Murphy-Spenkuch (2018)

One possibility is that the transmission function is convex but at some point becomes concave. That would correct the ever accelerating growth in human capital of certain dynasties. Then we would have two steady states: social classes.

A similar result was obtained by Durlauf (1996) modeling social interactions.



Becker-Kominers-Murphy-Spenkuch (2018)

Parents provide human capital but transformation to earnings depend on market forces.

Under the steady state ($\sigma_c = \sigma_p$) the intergenerational earning elasticity equals the intergenerational human capital elasticity:

$$\frac{dlog(E_c)}{dlog(E_p)} = \frac{dlog(H_c)}{dlog(H_p)} = \frac{\beta}{1 - \alpha\sigma} \tag{6}$$

So there is a direct relation of parents and children human capital with perfect capital market due to complementarity of production, differing from Becker and Tomes (1986).

(See appendix for details).

Becker-Kominers-Murphy-Spenkuch (2018)

- A proportional increase in the return of human capital ($r \uparrow$) won't affect IGE , because it makes all families to proportionally increase investment in children:

$$\frac{d}{dr}\left(\frac{dlog(E_c)}{dlog(E_p)}\right) = 0 \tag{7}$$

- More than proportional increase in return of human capital ($\sigma \uparrow$) would increase inequality and persistence. Due complementarity in production high human capital families would have a more than proportional increase in investment:

$$\frac{d}{d\sigma}\left(\frac{dlog(E_c)}{dlog(E_p)}\right) > 0 \tag{8}$$

Modeling impact: human capital formation

One relevant question is how the skill formation happens.

Cunha and Heckman (2007) and Heckman and Mosso (2014) provide a model of multiperiod life of children so we can focus on timing of investment and life cycle credit constraints:

- Could appear relevant dynamic complementarities.
- Show implications of three different types of credit constraints:
  - Inability of selecting parents
  - Parents cannot borrow against future income of children
  - Inability of parents to borrow against own future income.
- Shed light on the equity-efficiency tradeoffs for late investment, but not for early investment.

Cunha and Heckman (2007); Heckman and Mosso (2014)

The technology of skill formation will be defined by the following equation:

$$\boldsymbol{\theta}_{t+1} = \mathbf{f}^{(t)}\left(\boldsymbol{\theta}_t, \mathbf{I}_t, \boldsymbol{\theta}_{P,t}\right) \tag{9}$$

where $\boldsymbol{\theta}_t$ is a vector of multiple dimensions of skills (e.g. cognitive, noncognitive, health), $\mathbf{I}_t$ is the vector of parental investment in period t and $\boldsymbol{\theta}_{P,t}$ the vector of parental abilities at period t. $\mathbf{f}^{(t)}$ is twice continuously differentiable, increasing in all arguments and concave in $\mathbf{I}_t$

The fact that this technology is increasing in the skill level ($\boldsymbol{\theta}_t$) captures 2 ideas:

- This technology has self-productivity as higher stock of a particular skill in one period leads to higher stock of skill in later period. This capture the idea that the skill level do not fully depreciate in one period.
- Cross partials could be positive between different skills. For example, higher stock of noncognitive skills could lead to higher level of cognitive skills.

Cunha and Heckman (2007); Heckman and Mosso (2014)

- This technology will have complementarity between skills and investments at later stages ($t > t^*$) of childhood:

$$\frac{\partial^2 \boldsymbol{\theta}_{t+1}}{\partial \boldsymbol{\theta}_t \partial \mathbf{I}'_t} > 0, \quad t > t^*$$

- It means that stock of skills obtained by period t (i.e. θ_t) makes investment in period t more productive.

Cunha and Heckman (2007); Heckman and Mosso (2014)

- Empirical evidence is consistent with the notion that investments and endowments are direct substitutes (or at least weak complements) at early ages

$$\frac{\partial^2 \boldsymbol{\theta}_{t+1}}{\partial \boldsymbol{\theta}_t \partial \mathbf{I}_t'} \leq 0, \quad t < t^* \quad \left(\text{or } \epsilon > \frac{\partial^2 \boldsymbol{\theta}_{t+1}}{\partial \boldsymbol{\theta}_t \partial \mathbf{I}_t'} > 0, \text{ for small } \epsilon \right)$$

- Complementarity increases with age (Cunha 2007, Cunha and Heckman 2008, Cunha et al. 2010):

$$\frac{\partial^2 \boldsymbol{\theta}_{t+1}}{\partial \boldsymbol{\theta}_t \partial \mathbf{I}_t'} \uparrow t \uparrow$$

Cunha and Heckman (2007); Heckman and Mosso (2014)

Due to self-productivity and complementarity of investment and skill level, this technology of skill formation will also have dynamic complementarity:

$$\frac{\partial^2 \boldsymbol{\theta}_{t+s+1}}{\partial \boldsymbol{I}_t \partial \boldsymbol{I}'_{t+s}} > 0, \quad s \geq 1 \tag{10}$$

Then investment in a particular period is always complement with previous investment.

Cunha and Heckman (2007); Heckman and Mosso (2014)

Definition
Period t^* is a critical period if it is the only period effective at producing a skill or ability.

Definition
Sensitive periods are the ones that are more effective in producing certain skills.

For example, t is a sensitive period in relation to t' if given the same endowment of skills and parental investment, investment in period t is more productive than in period t'.

Cunha and Heckman (2007); Heckman and Mosso (2014)

Let's assume that T=2 and that $\theta_1, I_1, I_2, \theta_P$ are scalars. Then given the recursive form of the equation of skill formation, the adult stock of skills is:

$$\theta_3 = f^{(2)}\left(\theta_1, I_1, I_2, \theta_P\right) \tag{11}$$

Early work from Becker and Thomes (1979, 1986) and subsequent research often assumes that there is one period in childhood which is equivalent to perfect substitution of early investment.

It is a special case of:

$$\theta_3 = f^{(2)}\left(\theta_1, \gamma I_1 + (1 - \gamma_1) I_2, \theta_P\right) \tag{12}$$

with $\gamma = 0.5$. Here the timing of investment is irrelevant and there is no critical period.

Cunha and Heckman (2007); Heckman and Mosso (2014)

The opposite would be perfect complementarity:

$$\theta_3 = f^{(2)}\left(\theta_1, \min\{I_1, I_2\}, \theta_P\right) \tag{13}$$

Here how investment is distributed is critical. If $I_1 = 0$ then there is no payoff of $I_2 > 0$. Complementarity implies:

- It's essential to invest early to get high adult outcomes.
- It's essential to invest late to get the fruits of early investment.

Cunha and Heckman (2007); Heckman and Mosso (2014)

More general framework: CES.

$$\theta_3 = f^{(2)}\left(\theta_1, [\gamma(I_1)^{\phi} + (1-\gamma)(I_2)^{\phi}]^{\frac{1}{\phi}}, \theta_P\right) \tag{14}$$

$\phi \leq 1, 0 \leq \gamma \leq 1.$

- ϕ measures the degree of complementarity (substitutability) between early and late investment.
    - $\phi \to -\infty$: converge to Leontief.
    - $\phi = 1$: perfect substitution.
    - $\phi = 0$: Cobb-Douglas ($f^{(2)}\left(\theta_1, I_1^{\gamma} I_2^{1-\gamma}, \theta_P\right)$)

Cunha and Heckman (2007); Heckman and Mosso (2014)

- At the beginning of adulthood a parent draws θ_1 (i.e. they have a baby) from distribution J, receive bequest b, and have parental skill θ_P. Note that all parents draw from the same distribution.
- Parents decide allocation of consumption and investment in time periods 1 and 2 and a bequest b' to leave their child.
- Budget constraint of parent is:

$$c_1 + I_1 + \frac{c_2 + I_2}{(1+r)} + \frac{b'}{(1+r)^2} = w\theta_P + \frac{w\theta_P}{(1+r)} + b \tag{15}$$

assuming human capital is a scalar, where:

- c_i consumption period i.
- w wage
- r interest rate

Cunha and Heckman (2007); Heckman and Mosso (2014)

The utility function of parent is denoted by u(.), β denotes the utility discount factor, and δ parental altruism. Then the problem of the parent is:

$$V(\theta_P, b, \theta_1) = \max\{u(c_1) + \beta u(c_2) + \beta^2 \delta E[V(\theta_3, b', \theta_1')]\} \tag{16}$$

Subject to the budget constrain (15) and CES technology (14).

Cunha and Heckman (2007); Heckman and Mosso (2014)

- If $\phi = 1$ perfect substitution of investment.
  - Early investment price is relatively higher (1 vs $\frac{1}{1+r}$), incentive to postpone investment.
  - One unit of I_1 produce γ, $(1 + r)$ units of I_2 produce $(1 + r)(1 - \gamma)$, higher γ incentivize early investment.
  - Parents invest early if: $\gamma > (1 + r)(1 - \gamma)$
- If $\phi \to -\infty$ converge to Leontief: optimally $I_1 = I_2$
  - Early investment is essential and should be followed with late investment for obtaining results.

Cunha and Heckman (2007); Heckman and Mosso (2014)

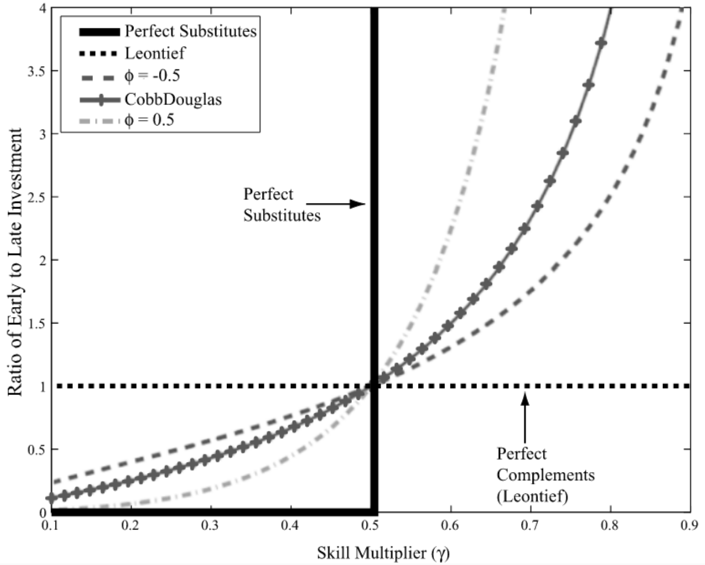
For $-\infty < \phi < 1$, FOC are necessary and sufficient for interior solution given concavity of production function. Then optimally:

$$\frac{I_1}{I_2} = \left[\frac{\gamma}{(1-\gamma)(1+r)}\right]^{\frac{1}{1-\phi}} \tag{17}$$

- $\phi \to -\infty$: ratio not sensitive to γ
- $\phi = 0$: close to 0 for low values of γ, $\to \infty$ for γ close to 1.

The next figure plots the ratio of early and late investment for different degrees of complementarity (ϕ) assuming r = 0.

Cunha and Heckman (2007); Heckman and Mosso (2014)



- Then, investing in early stages of more disadvantaged children is efficient as at the beginning they have low (θ_1) which makes investment in them relatively more productive than for children with higher θ_1 in the short run, but also all their future investment more productive.
- In the case of late investment, due to complementarity of investment and level of skill would be more efficient to invest in more advantages children: appears a equity-efficiency tradeoff.

Cunha, Heckman, and Schennach (2010)

Cunha, Heckman, and Schennach (2010) provide empirical estimates of a CES technology of skill formation:

- Self productivity of skills higher in second stage than first stage.
- Noncognitive skills are cross-productive with cognitive skills on first stage. No evidence cross-productivity of cognitive skills on noncognitive ones.
- Elasticity of substitution between investment and current endowment for cognitive skill is lower in the second term – it is more difficult to compensate for adverse environment at later age than earlier age.
- For cognitive skills, parental investment is relatively more important in the first stage.
- For noncognitive skills, parental investment has similar effect in both stages.

(See appendix for details).

Cunha, Heckman, and Schennach (2010)

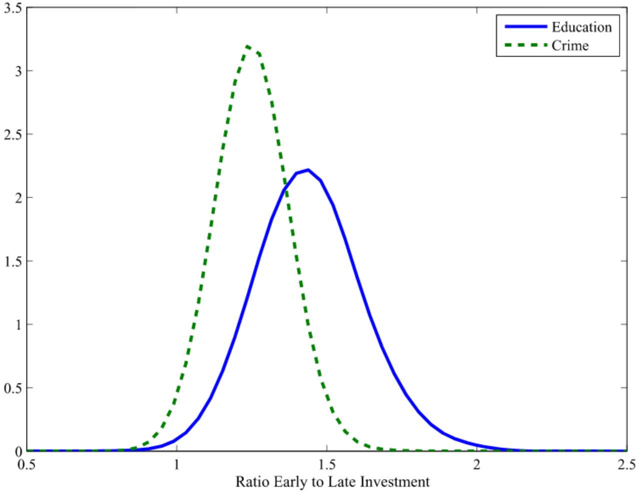


FIGURE 6.—Densities of ratio of early to late investments maximizing aggregate education versus minimizing aggregate crime.

The simulations suggest:

- For both outcomes is optimal to invest relatively more in the first period.
- As education is relatively more intensive in cognitive skill and as the compensation cost for cognitive skill is high in the second period, it's efficient to have more investment in the first period.
- Crime depends more in noncognitive skills than education, and as the compensation cost is relatively smaller for that skill, optimal policy is relatively more intensive in second period investment.

The Effect of Borrowing Constraints

Preview: Timing of income, dynamic complementarity and credit constraint

- Revisit credit constraints implications in Cunha and Heckman (2007)
- Dahl and Lochner (2012) — *study the role of parental income*
  - Study the effect of credit constraints on test scores of children in early adolescence
- Akee et al. (2010, 2018) — *study the role of parental income*
  - Study the effect of unconditional income transfer on academic achievement and child personality traits and behaviors
- Caucutt and Lochner (2020) — *directly test the presence of credit constraints*
  - Extend the dynamic human capital investment framework with a richer setting (e.g., adding earnings uncertainty and government policies) to facilitate a realistic quantitative analysis.
  - Quantify the extend of life-cycle borrowing constraints, their interactions with dynamic complementarity, and the resulting implications for intertemporal investment behavior.

Cunha and Heckman (2007): Three typs of credit constraints

1. Inability of a child to choose his or her parents
2. Non-negative parental bequests ($b' \geq 0$) — parents cannot leave debts to children
3. Parents are subject to lifetime liquidity constraints and constraints that prevent them from borrowing against their own future labor income — prevents the finance of investment in the early years.

Cunha and Heckman (2007): Inability of a child to choose parents

1. Inability of a child to choose his or her parents
    - In a "perfect" credit market, optimal investment levels are not affected by parental wages, endowments, or parameters that characterize the utility function.
    - But this contradicts findings that parental investments may depend on parental skills h as it affects the return to investment.
    - This is a still "market failure" in Child's POV — children would like to choose the optimal amount of h to complement their initial endowment θ_1.

Cunha and Heckman (2007): Non-negative parental bequests

1. Inability of a child to choose his or her parents
2. Non-negative parental bequests ($b' \geq 0$) — parents cannot leave debts to children
   - If $b' \geq 0$ binds, the early investment under lifetime liquidity constraints $\hat{I}_1$ is lower than the early investment under the perfect credit market model I_1^*.
   - Similar conclusion for late investment — $\hat{I}_2 < I_2^*$
   - Under-investment in skills starts from the early ages and continues throughout the life cycle — provide explanations why skill gaps open up early and are perpetuated
   - The effects of government intervention may depend on the complementarity between early and late investment.

Cunha and Heckman (2007): Lifetime liquidity constraints and borrowing constraints

1. Inability of a child to choose his or her parents
2. Non-negative parental bequests ($b' \geq 0$) — parents cannot leave debts to children
3. Parents are subject to lifetime liquidity constraints and constraints that prevent them from borrowing against their own future labor income
   - When both savings $s \geq 0$ and parental bequests $b' \geq 0$ bind, and investments at different time are not perfect substitutes, the timing of the investment matters.
   - To see this, let $u(c) = (c^{\sigma} - 1)/\sigma$, the ratio of early to late investment is

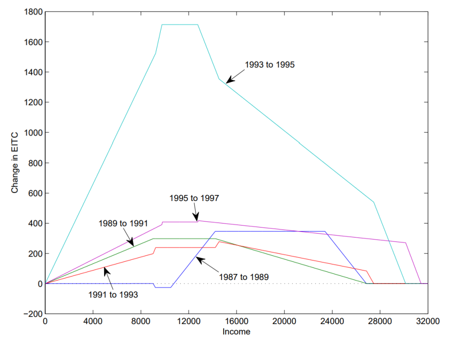
$$\frac{I_1}{I_2} = \left[\frac{\gamma}{(1-\gamma)(1+r)}\right]^{\frac{1}{1-\phi}} \times \left[\frac{(wh + b - I_1)}{\beta\left((1+\alpha)wh - I_2\right)}\right]^{\frac{1-\alpha}{1-\beta}}$$

If early income is low with respect to late income, the ratio I_1/I_2 will be lower than the optimal ratio. The deviation from the optimal ratio will he larger the lower the elasticity of intertemporal substitution of consumption (captured by σ).

(See appendix for details).

Dahl and Lochner (2012): EITC and Test Scores

- Empirical challenge: the endogenous income makes it difficult to estiamte the causal effect of income on children outcomes
- Use large, nonlinear changes in the Earned Income Tax Credit policy (up to 20% of family income) as instruments to estimate the causal effect of income on children's math and reading achievement



Dahl and Lochner (2012): EITC and Test Scores

Empirical challenge: the endogenous income makes it difficult to estiamte the causal effect of income on children outcomes

- Use large, nonlinear changes in the Earned Income Tax Credit policy (up to 20% of family income) as instruments to estimate the causal effect of income on children's math and reading achievement.
- Baseline estimates imply that a $1,000 increase in income raises combined math and reading PIAT test scores by a modest 6 percent of a standard deviation in the short run.
- Test gains are larger for children from disadvantaged families and robust to alternative specifications.

Dahl and Lochner (2012): Income effect and endogenity of labor supply

Are Dahl and Lochner (2012) estimating a pure income effect? Probably not.

- The amount of EITC benefit depends on a recipient's income and number of children. Heckman-Lochner-Cossa (2003) find that EITC may induce more employment, but workers may reduce their working hours.
- Evidence suggests that maternal working time has substaintial effects on child test scores. Dahl and Lochner (2012) attempt to control for the time-allocation effects.
- In particular, to account for the endogenity of the decisions of labor supply of the families or for the parental investments, authors control for the changes in maternal LFP and hours worked.
  - ▷ Most studies find very small negative effects of the EITC expansion on hours worked by women who were already working.
  - ▷ Controlling labor supply can identify the effect of total income changes.

Dahl and Lochner (2012): Income effect and endogenity of labor supply

How should we think about the modest effect on children's scholastic achievement related to the credit constraints?

- Credit-constrained parents may have already made their decisions on early investment at the time of policy change.
- If there is a greater complementarity between the early and late investments, the short-run impact of an unanticipated policy aiming the late investment will be smaller.

Akee et al. (2010): Unconditional cash transfer and education attainment

- Use opening of Casino as income shock (an average of $4000 per person per year) that affected Eastern Cherokee households but not non-Cherokee.
- The Great Smoky Mountains Study of Youth (GSMS) were conducted in three cohorts: Age 9 Cohort (C1), Age 11 Cohort (C2), and Age 13 Cohort (C3).
  - Age 9 and age 11 at survey intake residing in households that receive the unconditional transfers for 4 and 2 years
  - Age 13 cohort who were 13 years old at survey intake were not exposed to unconditional transfers by age 16.

Akee et al. (2010): Unconditional cash transfer and education attainment

- Use opening of Casino as income shock (an average of $4000 per person per year) that affected Eastern Cherokee households but not non-Cherokee.

Wave	1	2	3	4		5	6	7	8	9	10	11	12	13	14	15	16	17
Year	1993	1994	1995	1996		1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Age																		
9	C1				Casino opening													
10		C1																
11	C2		C1															
12		C2		C1														
13	**C3**		C2															
14		**C3**		C2			C1											
15			**C3**			C2		C1										
16				**C3**			C2		C1									
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18																		
19								**C3**		C2		C1						
20																		
21										**C3**		C2		C1				
22																		
23																		
24													**C3**		C2		C1	
25														**C3**		C2		C1

Akee et al. (2010): Unconditional cash transfer and education attainment

- Use opening of Casino as income shock (an average of $4000 per person per year) that affected Eastern Cherokee households but not non-Cherokee.
- Authors observe three age cohorts:
  - Age 9 and age 11 at survey intake residing in households that receive the unconditional transfers for 4 and 2 years
  - Age 13 cohort who were 13 years old at survey intake were not exposed to unconditional transfers by age 16.
- Effects strongest for children in lowest income households.
- For household in poverty prior to transfer, authors find a $4000 per year for the poorest households increases 15% high school graduation rate, one additional year of education by age 21, and 22% reduciton in probability of being arrested.

Akee et al. (2010): Unconditional cash transfer and education attainment

Connect the results with the credit constraints in Cunha and Heckman (2007):

- Comparison between the age 9 and age 13 cohorts provides the counterfactual observations of a household in which incomes were unchanged for a shorter period of time — 6 years vs. 2 years.
- If the credit-constrained parents already made their decisions on early investment prior to the casino opening, we would expect a weaker effect due to the income shock if early and late investments are complementary.

Akee et al. (2010): Unconditional cash transfer and education attainment

Connect the results with the credit constraints implications:

- Comparison between the age 9 and age 13 cohorts provides the counterfactual observations of a household in which incomes were unchanged for a shorter period of time — 6 years vs. 2 years.
- If the credit-constrained parents already made their decisions on early investment prior to the casino opening, the effect of the income shock is weaker for the older cohort.

	Years of education, age 21	Probability of HS graduate, age 19	Probability of HS graduate/ GED, age 19
Independent variables	Coefficient	Marg. eff.	Marg. eff.
Interaction 1: age cohort 1 × number of American Indian parents	0.379 (0.447)	0.156** (0.073)	0.086 (0.054)
Interaction 2: age cohort 2 × number of American Indian parents	0.117 (0.304)	0.042 (0.066)	0.033 (0.044)

Akee et al. (2010): Unconditional cash transfer and education attainment

Is there any other confounding factor? Footnote 9 seems to be suspicious.

- Recall that all adult tribal members received per capita disbursement.
- *"All enrolled American Indian children were eligible for the casino disbursement at age 18 if they completed high school. If they did not complete high school, they would receive the casino transfers at age 21."*
- An alternative interpretation of the results is that children were responding to the short-term monetary incentive for graduating high school on time, instead of a positive long-term effect induced by the increased parental income.

Akee et al. (2018): Child personality traits and behaviors outcomes

Go beyond the conventional measures of test scores and education attainment.

- Authors carefully construct personality traits and behaviors measures
- Find large beneficial effects on children's emotional and behavioral health and personality traits during adolescence
- Effects are most pronounced for children who start out with the lowest initial endowments.
- Results in improvements in parental relationships — a potential mechanism behind these findings

Caucutt and Lochner (2020): Overview

- Provide rich calibration results and extend the economic environment with more realistic setting by adding earnings uncertainty, government policies, etc.
- Unanticipated changes in income for parents of college-age children have modest effects on their college-going behavior and future wage.
- If parents anticipate the future income change when their children are young, the impacts on college attendance are more than twice as large and the impacts on post-school earnings are more than six time as large (as a result of early and late investment combination).
- Combining these two facts, the estimates on wealth and income shocks from quasi-experiments to families of adolescent children could substantially underestimate the long-run impact.

Caucutt and Lochner (2020): Policy analysis

1. Consider different loan policies to evaluate the importance of borrowing constraints at different stages of child development.
2. Consider fiscally equivalent early- and late-investment subsidy policies.
3. Consider the effects of a fiscally equivalent increase in the level of early public investment.

Caucutt and Lochner (2020): Policy analysis — borrowing constraints

1. Increasing borrowing limits by $2500 for young and old parents[1] — investigate the complementarity between early and late investments quantitatively
    - Relaxing borrowing constraints on young parents would lead to modest increases in investment in the short run.
    - Increases in early investment are met with increases in late investment, especially in college attendance, due to dynamic complementarity.

Parental Education	Short-Run Effects (% Change)				
	Average i_1	Average i_2	High School or More	Some College or More	Average W_3
All levels	2.6	1.9	.5	4.8	.4
High school dropout	3.2	3.2	3.9	5.4	.3
High school graduate	5.8	3.2	.1	8.2	.5
Some college	4.7	3.7	−.4	7.9	.8
College graduate	.5	.1	.0	.4	.1

[1] People live through six periods in their lives: young and old childhood (periods 1 and 2), young and old parenthood (periods 3 and 4), postparenthood (period 5), and retirement (period 6).

Caucutt and Lochner (2020): Policy analysis — borrowing constraints

- In the short run, relaxing borrowing constraints on young parents would lead to modest increases in investment.
- Increases in early investment are met with increases in late investment, especially in college attendance, due to dynamic complementarity.
- Long-run changes incorporate the fact that some young parents borrow more and accumulate more debt as old parents — parents transfer less to their children.
- Asset level declines lead to lower overall investment levels and negligible long-run effects on average wages.

Long-Run Effects (% Change)

Average i_1	Average i_2	High School or More	Some College or More	Average W_3
−.6	−.6	.1	1.5	−.1
1.4	1.7	3.1	2.3	.1
−.8	−1.1	−.6	.5	−.1
−.8	−.5	−.8	3.1	−.1
.5	.1	.0	.4	.1

Caucutt and Lochner (2020): Policy analysis — borrowing constraints

- Relaxing borrowing constraints is not a panacea!
- In the short run, investment and debt increase among constrained families, leading to reductions in intergenerational transfers.
- Takeaway: some policies may have important indirect effects on asset accumulation if future generations are affected. Such a policy may cause current generations to respond even if they themselves are not directly affected by the policy.

Caucutt and Lochner (2020): Policy analysis — borrowing constraints

1. Increasing borrowing limits by $2500 for young and old parents — investigate the complementarity between early and late investments quantitatively

    - Relaxing borrowing constraints on young parents would lead to modest increases in investment in the short run.
    - Relaxing borrowing constraints on older parents has even greater impacts on investments in children.
        - Early investment increases by 10.9% in the short run.
        - College attendance rate increases by 5%.
        - Average earnings rise by 1.8%, with the largest increases among youth whose parents went to college.

Caucutt and Lochner (2020): Policy analysis — borrowing constraints

1. Increasing borrowing limits by $2500 for young and old parents — investigate the complementarity between early and late investments quantitatively

    - Calibraton implies no effect of expanding student loan opportunities for old children, while increasing borrowing limits on either young or old parents one at a time has only modest impacts on investment behavior.
    - Does this suggest that the credit market limits only play a modest role?
        - Eliminating all life-cycle borrowing constraints simultaneously allows more than quadruple borrowing opportunities at each stage of life and generates substantial increases investments and earnings and shrink the intergenerational correlation in human capital by one-quarter.

Caucutt and Lochner (2020): Policy analysis — subsidizing investments

2. What are the consequences of increasing subsidy rates for early and late human capital investments?

    - Complementarity implies that if children do not receive adequate early investments, it may not be worthwhile for parents to make later investments, even if they are heavily subsidized.

    - Early-investment subsidies enable families to increase investments in their young children without having to sacrifice current consumption or borrow more.

Caucutt and Lochner (2020): Policy analysis — subsidizing investments

How should we think about policies targeted at college-age students from low-income families?

- Late subsidies have weaker impacts on college completion than early subsidies.
- The interaction between credit constraints and dynamic complementarity matters! If we fail to account for adjustments in early investment, the impact may be underestimated.

Caucutt and Lochner (2020): Policy analysis — public provision of early investment

3. What are the impact of increasing the amount of publicly provided early investment?
   - An increase of $880 in early public provision crowds out $344 of early private investment (39 percent).
   - High school completion rates increase by 16%, and the fraction that attends some college (or higher) increases by 20%.
   - Small effect on college completion rates of 5%, and a 2.8% increase in average wages increase — one-third of the response to an increase in early subsidy rates.

Caucutt and Lochner (2020): Policy analysis — public provision of early investment

The increase in early public investments and early subsidies affect educational outcomes at opposite ends of the distribution.

- A modest increase in early public provision does not raise early investments enough to make college completion worthwhile for those who were investing little to begin with.
- An increase in early subsidy encourages those who were already making investments to invest more, and thus, push more students to complete the college.

Putting the humans back in the family

Family may influence children in many ways

- provide genes and epigenetics
- provide direct cognitive and noncognitive learning and instructions
  $\Rightarrow$ *timing* of the investment plays a vital role
- provide money to meet needs and learning opportunities
- influence information sets
- influence preferences
- influence peers and networks

Putting the humans back in the family

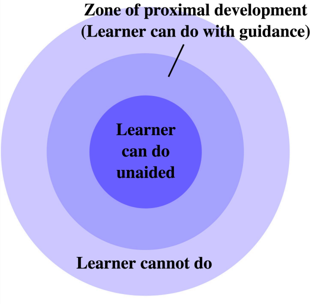
- So far, we've focused on the technology of skill formation and human capital transmission through money investments.
- In this next section, we will place the technology into a household decision problem. (Del Boca, Flinn and Wiswall 2014)
- Then, we consider how we could model the parent-child relationship as it relates to the technology. (Lizzeri and Siniscalchi 2008 and Del Boca, et al 2019).
- Doing so may change the policy prescriptions we consider.

Parenting is more than a series of investment decisions

- We have mainly treated the parent-child relationship as a money investment problem, but we have the sense that parents are doing more.
- For two, let's consider affection and teaching.
- Psychology/neuroscience literature on attachment shows profound effects of affection from and proximity to parent early in life (Bowlby, 1969; see Tough, 2013 for pop science)
- Policy example: Heckman et al (2017) investigate Nurse Family Partnership, a nonprofit that coaches young parents at parenting. After the nurse visits, children show growth in cognitive ability which persists for boys.

Parenting

- We also have the idea that children learn better when the inputs are well targeted.
  - Vygotsky (1934) provides an idea of scaffolded learning. There's a "zone of proximal development" where the learner can expand their skill set.
  - Some parents might be better at it.



- So, perhaps parenting is a bundle of skills (love can be a skill, right?) which can be learned and may correlate with traditional measures of human capital.

DWF (2014) Adding time to the household decision process

- Del Boca, Flinn and Wiswall (2014) put a CH-type technology of skill formation into a household decision process.
- **What's new?:** "corrects" for the endogeneity of inputs (particularly time) and estimate the household preferences that lead to the input decisions (under explicit functional form assumptions).
    - The household problem balances parental time constraints and household budget constraints with the desire to have a smart child, and for leisure and consumption.
- Under their functional form assumption and estimation strategy, they find that time inputs are "extremely important in the cognitive development process" and that money inputs are less important in early childhood.

DFW (2014) The model

- Household utility is log linear in leisure of each parent ($l_{i,t}$), household consumption (c_t) and child quality (k_t).
- In each period, the household makes seven choices:
  - hours of work (two choices) h_{it}
  - active and passive parenting time (four choices) $\tau_{it}(a)$ and $\tau_{it}(p)$
  - expenditure on child good e_t
- In each period there's a constraint on time for each parent

$$\tau_{it}(a) + \tau_{it}(p) + h_{it} + l_{it} = T$$

  and a budget constraint (no saving/borrowing). (I_t is exogenous non-labor income)

$$w_{1t}h_{1t} + w_{2t}h_{2t} + I_t = e_t + c_t$$

- Wages are determined endogenously based on parent characteristics (Mincer model) with i.i.d. wage shocks. (all growth comes from experience).

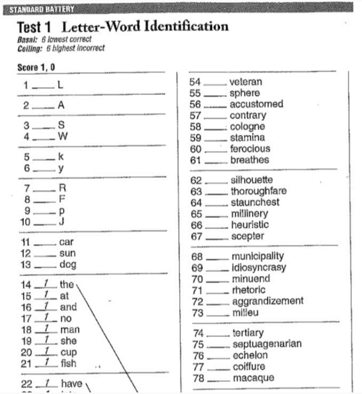
DFW (2014) Technology

- Technology of "child quality" production (k_{t+1}) is Cobb-Douglas in parent time, both passive and active, child goods, and previous quality (k_t) with a TFP term.

$$k_{t+1} = R_t e_t^{\delta_{3,t}} k_t^{\delta_{4,t}} \prod_{i \in \{1,2\}} \prod_{j \in \{a,p\}} \tau_{i,t}(j)^{\delta_{i,t}(j)}$$

- Following child dev literature, they allow factor productivity to vary over the life course. (i.e. $\delta_{\xi,t}$ changes over time for all 6 inputs).
- Skill formation is the "dynamic" part of this model.
- Parents have full knowledge of the technology. (See appendix for details from 2019 model).

DFW (2014) Measurement: what is child quality?



STANDARD BATTERY

Test 1 Letter-Word Identification

Basal: 6 lowest correct
Ceiling: 6 highest incorrect

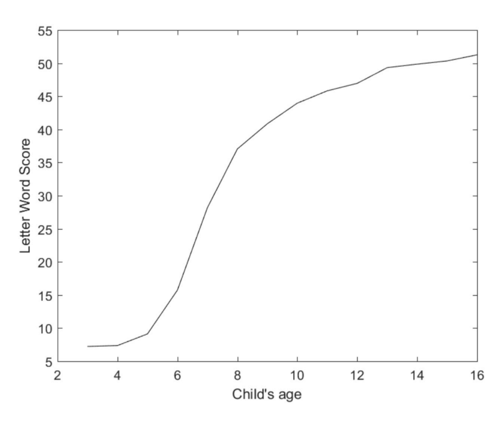
Score 1, 0

1 ___ L
2 ___ A
3 ___ S
4 ___ W
5 ___ k
6 ___ y
7 ___ R
8 ___ F
9 ___ p
10 ___ J
11 ___ car
12 ___ sun
13 ___ dog
14 _1_ the
15 _1_ at
16 _1_ and
17 _1_ no
18 _1_ man
19 _1_ she
20 _1_ cup
21 _1_ fish
22 _1_ have
54 ___ veteran
55 ___ sphere
56 ___ accustomed
57 ___ contrary
58 ___ cologne
59 ___ stamina
60 ___ ferocious
61 ___ breathes
62 ___ silhouette
63 ___ thoroughfare
64 ___ staunchest
65 ___ millinery
66 ___ heuristic
67 ___ scepter
68 ___ municipality
69 ___ idiosyncrasy
70 ___ minuend
71 ___ rhetoric
72 ___ aggrandizement
73 ___ milieu
74 ___ tertiary
75 ___ septuagenarian
76 ___ echelon
77 ___ coiffure
78 ___ macaque

Excerpt from Woodcock Johnson Letter Words Identification test – (note this version of the test has more than 57 words)

- Scores k^* taken as noisy measure of underlying latent variable k following psychometrics literature.
- k^* drawn from Binomial$(NQ, p(k))$
- NQ is number of questions and $p(k) = \frac{k}{1+k}$ likelihood of answering a given question under functional form assumption. Then, $k = \frac{\tilde{p}}{1-\tilde{p}}$.
- They put a uniform prior Beta(1,1) on k
- Finally, they draw $\tilde{p}$ from the posterior which is Beta $(1 + k_t^*, (NQ - k_t^*) + 1)$.

DFW (2014) Measurement: what is child quality?



Average Raw Letter Words Score by age (from Del Boca et al (2019). Data combined from CDS 1997, 2002 and 2007.

- DFW argue this works because a) we account for noise, b) it maps a discrete scale to $(0, \infty)$
- Is this the natural scale of child quality?
- CHS (2010) anchor ability measures to outcomes. Notably, the outcomes chosen affect our assessment of the technology of skill production.
- See Agostinelli and Wiswall (2020) for discussion of measurement issues.

DFW (2014):Results

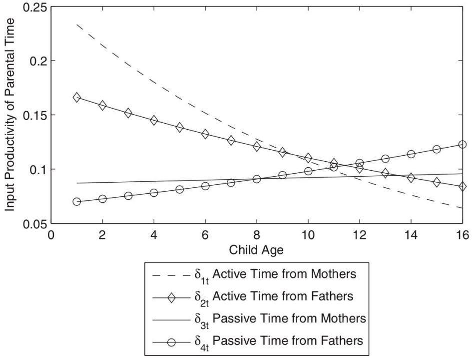


FIGURE 2

Estimated child development parameters by child age (one-child model).

Notes: This graph plots the function estimate by child age (from Table 6)

- The model doesn't say much without parameter estimates.
- They fit each $\delta_\xi(t) = \exp(\gamma_{0\xi} + \gamma_{1\xi} t)$, where ξ standing in for any technology input.
- The figure shows that active time inputs are highly productive in the early years. (see next graph for comparison to money inputs)

DFW (2014):Results

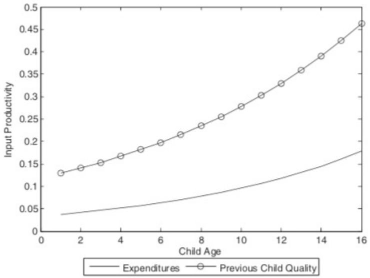


FIGURE 3
Estimated child development parameters by child age (one-child model).

- We see the productivity of previous skills increasing over time and a similar pattern for money inputs.
- Compare to CHS (2010); both find early investment important. But here they stress the impact of parental time.
- A policy experiment furthers this argument. They say the fungibility of dollars means transfers are split between leisure, consumption and the child's development.
    - So money is less productive and cash transfers are not well targeted in their analysis.

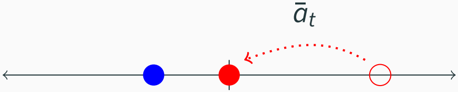
Allowing the child to act

Lizzeri and Siniscalchi (2008) Parental Guidance and Supervised Learning.

In each period, there's a goal for the child drawn at random from some distribution.

The child makes a guess at the goal

The parent makes an adjustment and the payoff occurs.



The child takes loss $l_t = (● - ●)^2$

The model: How fast can the child learn what is “right” (M)

- In each period of life, there is a goal X_t drawn from some distribution centered at M.
- **Child** picks a number $\bar{b}_t$ in each period 1 to L.
- The child has a prior on M with prior $\mathcal{N}(M_0, \frac{1}{p_0})$,
- **Parent** choose $\bar{a}_t$ in periods 1 to $T < L$. They known M and M_0 and observe X_t at the end of the period
- The period t loss is $(X_t + \bar{a}_t - \bar{b}_t)^2$.

The model

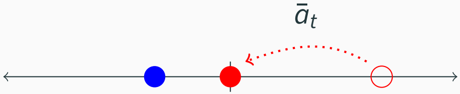
- Then the goal for both parent and child is

$$\min \sum^{L} \delta^{t-1}(X_t + \bar{a}_t(t \leq T) - \bar{b}_t)^2$$

- The parent and child may have distinct discount rates δ. It turns out that the parent's discount rate is what matters.

LS (2008) Bayesian Learning

- Parent's point of view: "This child needs help."



- Child's point of view: "My guess wasn't too bad!"

LS (2008) Bayesian Learning

The child uses Bayes rule to update the prior.

- Absent a parent, learning is unbiased

$$E[X_t|X_1 = x_1, ..., X_{t-1} = x_{t-1}] = \frac{p_0 M_0 + p_X \sum_{s=1}^{t-1} x_s}{p_0 + (t-1)p_X}$$

- With parent, assuming child unaware of parental intervention, learning is biased.

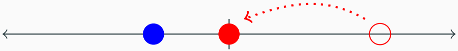
$$M_t^a \equiv E[X_t|X_1 = x_1 + \bar{a}_1, ..., X_{t-1} = x_{t-1} + \bar{a}_{t-1}] = \frac{p_0 M_0 + p_X \sum_{s=1}^{t-1} x_s + \bar{a}_s}{p_0 + (t-1)p_X}$$

- We see that the parental intervention biases learning.
- On the other hand, the intervention directly affects the child per-period payoff.

$$E[(X_t + a_t - M_{t-1}^a)^2|X_1, ..., X_{t-1}, M] = \underbrace{p_X^{-1}}_{Variance} + \underbrace{(M + a_t - M_{t-1}^a)^2}_{Bias^2}$$

LS (2008) What is the optimal parenting strategy in this world?

- What teaching policy should the parent choose $\{b_t\}_{1:T}$ so that the child learns M?
  - **Sheltering**: always push child to M. $a_t = (M^a_{t-1} - M)$ to minimize period $t < T$ pain. (For later, let $1 = \gamma^{FS}$

    Sheltering leads to minimal loss during childhood, but the child develops strong biased beliefs about how to live in the world.
  - **Learn by doing**: pick $a_t = 0$ for unbiased learning.

    The child learns M, but at the cost of high loss in the early period (where we have the smallest discount).

LS (2008) What is the optimal parenting strategy in this world?

- **Bootcamp:**[2] $a_t = (M - M^a_{t-1})\frac{p_0+(t-1)p_X}{p_X}$. My calculation of ϕ assumes the child plays rationally ($b_t = M^a_{t-1}$)

- This method exacerbates loss.

$$E[(M + a_t - b_t)^2] = \phi E[(M - b_t)^2]$$

where $\phi = 1 + \frac{p_0+(t-1)p_X}{p_X}$. (For later, call $\gamma^{BC} = -\frac{p_0+(t-1)p_X}{p_X}$)

- The benefit is that in expectation the child learns M in one period.
- The loss multiplier grows over time since the child's posterior becomes more precise over time and it's harder to "convince" them they're wrong.

[2] Recall, M^a_{t-1} is the child's belief about M at time $t-1$ given a. p_0 is the precision of the prior and p_X is the precision of M.

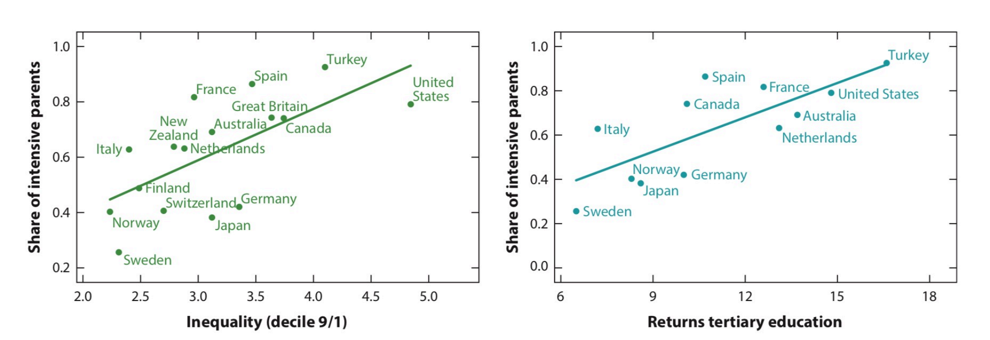
LS (2008) The findings

- Optimal parenting involves *partial sheltering*. It takes the form $a_t = \gamma_t(M^a_{t-1} - M)$ where γ_t is a weighted average of full sheltering and boot camp. $\gamma_t = \mu_t \gamma^{FS} + (1 - \mu_t)\gamma^{BC}$
- Parenting intensity μ_t depends on discount rates, the amount of time the child lives beyond the parent, the strength of conviction (p^a_{t-1}) and easy of learning (p_X).
- μ_t will be higher (closer to 1) when the cost of mistakes today are high relative to the following periods.
- They take the "partial sheltering" result as a win for Vygotsky and a loss for helicopter parents.
- $(M^a_{t-1} - M)$ implies parents will be more or less responsive depending on their child's type.

Adding parent human capital

- We discard the assumption the parent knows the true M.
- Parents now have beliefs that M is distributed $\mathcal{N}(Z_0, \frac{1}{p_{Z_0}})$. We can think of Z_0 being the parents prior and learning from their childhood (i.e. human capital).
  - For logical consistency, we limit the model to two-periods.
- Now the parent's optimal strategy is $a_1 = \gamma_1(M_0 - Z_0)$.
- The implications are:
  - The parent shelters more when they're more different from the child (i.e. $M_0 - Z_0$ gets larger.)
  - The parents who learned Z_0 close to M can transmit that information.
- Interesting unexplored implications about cultural heterogeneity. (Who gets to say what M is?) (Saéz: "Observational learning and parental influence" may cover this.)

Doepke, Sorrenti and Zilibotti (2019) Avg national parenting styles correlate with macro-climate



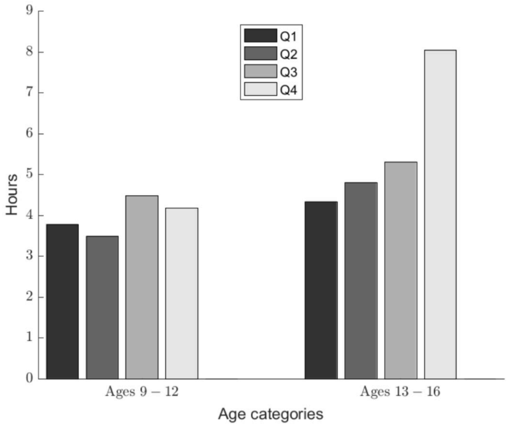
The correct way to live in the world M (and/or Z_0) could be a product of the environment. Here we have associations between intensive parenting styles and the macro-climate related to inequality.

Discussion

- This bare bones model gives us a flavor what is required to model parent-child interactions in a technology of skill formation setting.
    - First, we need two optimizing agents. The parents typically are altruistic but not always.
    - The model is easier to solve when the parent is able to manipulate outcomes of the child.
    - Finally, add heterogeneity.
- The next model will take these features and marry them with a household decision problem to get a model that can be brought to data. .

Del Boca et al (2019) Actors in the Child Development Process

Figure 1: Child Self-investment Time by Income Quartile and Child Age



Del Boca et al (2019) Actors in the Child Development Process

- Takes DFW (2014) and adds parent-child relationship.
- Tries to explain how parents influence their children's behavior through "time leadership" (in a Stackelberg type game) and with incentives (with a principal-agent model with altruism).
- The model "emphasizes the advantages that better-educated and wealthier parents have in producing child quality directly and indirectly through the design of incentives offered to the child to increase her study time".
- Operationalize "Parenting styles" (Doepke et al 2019) through parenting time use and (costly) monitoring of child behavior

Del Boca et al (2019) The Model

- Similar model as DFW (2014), but now child has utility based on leisure / play, consumption and their own cognitive capability.
- The parents utility is a mixture of own private utility and child's: $\tilde{u}_{p,t} = (1-\phi)u_{p,t} + \phi u_{c,t}$
  - ϕ is an altruism parameter
- All utilities are Cobb-Douglas and again there is no credit market. (See appendix for details).

Del Boca et al (2019) Stackleberg game

- In the Stackleberg game, the altruistic parents move first and decide how much (active and productive) time to spend with their child.
- The child chooses time for self-investment in response to the parent time decisions (and time mandated to be in school s_t).

$$\tau^*_{c,t}(\tau_{p,t}) = \gamma_t(T - \text{school time} - \tau_{p,t})$$

- We'll explain where γ_t comes from next slide.
- The reaction function is a best response. Since $\tau_{c,t}$ is the only choice parameter for the child, we can reformulate the decisions into a single problem.

Del Boca et al (2019) Child self-investment

- $\gamma_t \in [0, 1]$ is an endogenous parameter

$$\gamma_t = \frac{\Delta_{c,t}}{\lambda_1 + \Delta_{c,t}}$$

where

$$\Delta_{c,t} = \beta_{c,t}\delta_{5,t}\frac{\partial EV_{c,t+1}}{\partial \ln k_{t+1}} = \beta_{c,t}\frac{\partial \ln k_{t+1}}{\partial \ln \tau_{c,t}}\frac{\partial EV_{c,t+1}}{\partial \ln k_{t+1}}$$

- $\Delta_{c,t}$ is the future utility flow of the time investment discounted with current discount rate.
  - Cognitive function develops later in life, so in general $\beta_t > \beta_{t-1}$ (Steinberg et al 2009)
- **Takeaway**: In the Stackleberg game, the child is going to choose to spend time developing skills / studying based on how much they care about the future, the skill-complementarity and how much the enjoy play time.

Del Boca et al (2019) Internal Conditional Cash Transfers (ICCT)

- Next, they give parents the ability to provide incentives.
- For a monitoring cost, the parent can induce the child to study more by withholding some of their consumption x_t.
- They can make a contract in the style of principal-agent.
  - Recall in a principal-agent contract, the principal maximizes their own objective while ensuring the agent gets enough utility to meet their participation constraint.
- The parent can offer a contract in the form of a two-part tariff (and so extract all the surplus).

$$x_t = exp(b_t + r_t \log \tau_{c,t})$$

- Conceptually, the parent can pick r_t to induce any $\tau_{c,t}$ and then can set b_t to get a desired x_t.

Del Boca et al (2019) Internal Conditional Cash Transfers (ICCT)

- Now the best response is

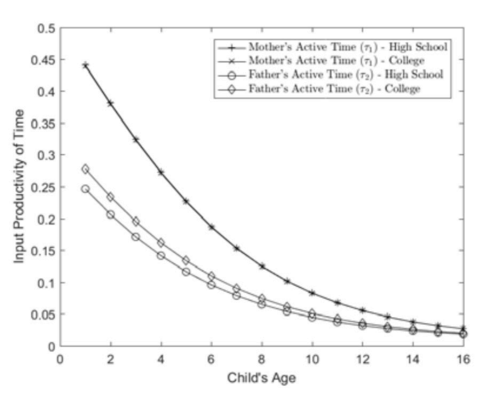
$$\tau^*_{c,t}(\tau_{p,t}, r_t) = \frac{\lambda_2 r_t + \Delta_{c,t}}{\lambda_1 + \lambda_2 r_t + \Delta_{c,t}}(T - \text{school time} - \tau_{p,t})$$

  where λ_2 is the utility parameter on child's consumption. As before the parameter is a ratio of the benefit to using time to study to the total utility from time use.
- Before adding a monitoring cost, the parent is strictly better off using incentives.
- Since child quality is a public good, the resulting k_M from the cooperative game will generally be higher than in the noncooperative game.

Del Boca et al (2019) Comments on ICCT

- The monitoring costs provide a source of heterogeneity in households choice. Otherwise all would do it.
    - We could imagine this reflects parent human capital.
    - The choice of monitoring aligns with the idea of "parenting styles".
- Unlike an external CCT, parents can payout based on effort in a timely manner and understand their child's preferences better than a social planner.
    - All else equal, the social planner (who wants to increase k in population) wants parents to use ICCTs.
- In the dynamic game, you may be concerned with sub-game perfection. They give the parents a second altruism parameter that is a cost to breaking a promise with the child. This makes parent reliability a theoretic (but not explored) point of heterogeneity.

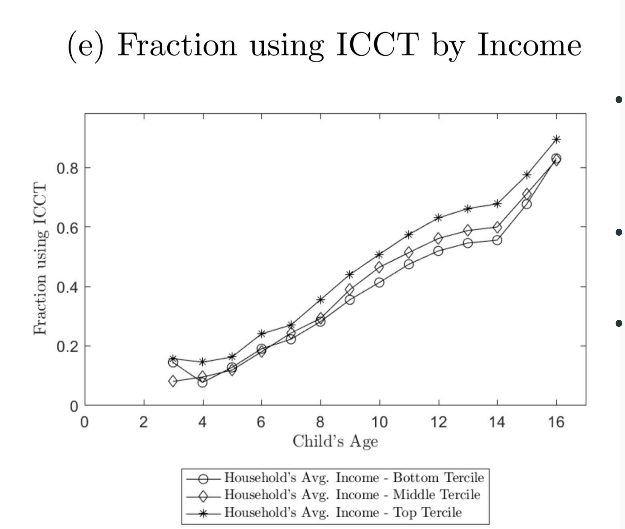
Del Boca et al (2019) Parent education leads to small differences in productivity



Productivity of parent time inputs by education

- We cannot distinguish estimates of mothers' time productivity based on education.
- More educated fathers have slight time productivity advantages that diminish over time.

Del Boca et al (2019) Wealthier families are slightly more likely to use ICCTs



- As children get older families at each income level are more likely to use ICCT.
- Wealthier families use ICCT slightly more (sometimes up to 12% more)
- The evidence is consistent with wealthier families having more money for pecuniary incentives (a la Weinberg 2001) though not overwhelmingly.

Conclusion

- We started with a Becker type model and noticed that a one-period childhood model masks complementarities and the impact of timing of investments.
- Cunha and Heckman (2007) provide a framework which allows us to think about the timing and complementarity as well as different policies.
- Absent perfect credit markets, we looked at how cash transfers might help families overcome credit constraints.
- Finally, we saw three models that brought in household relationship dynamics and preferences, which provided insights as to how time and money investments might be mediate through parent preferences and parenting styles.
- Throughout, we saw that parents endowments could allow them to parent more "optimally" – but that these were not one dimensional.
    - Further, parents goals and preferences need not align with the "social planners".

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Appendix

Del Boca et al (2019) The Model: Child Preferences

- Child's utility

$$u_{c,t} = u_c(l_{c,t}, x_t, k_t)$$
$$= \lambda_1 \ln l_{c,t} + \lambda_2 \ln x_t + \lambda_3 \ln k_t$$

$l_{\cdot,t}$ = leisure/play, x_t = child's private good consumption, k_t = child's cognitive capability

back to 2019 back to 2014

Del Boca et al (2019) The Model: Parent Preferences

- parent private utility (treated as a single agent).

$$
\begin{aligned}
u_{p,t} &= u_p(l_{1,t}, l_{2,t}, c_t, k_t) \\
&= \alpha_1 \ln l_{1,t} + \alpha_2 \ln l_{2,t} + \alpha_3 \ln c_t + \alpha_4 \ln k_t
\end{aligned}
$$

  $c_t =$ parents' private good consumption; all else as before.
- Parent total utility

$$
\tilde{u}_{p,t} = (1 - \phi)u_{p,t} + \phi u_{c,t}
$$

- Parents are altruistic as determined by $\phi > 0$.
- Note the child's play, consumption and cognitive ability are "public goods".

back to 2019 back to 2014

Del Boca et al (2019) Child development technology

$$\ln k_{t+1} = \ln R_t + \delta_{1,t} \ln \tau_{1,t} + \delta_{2,t} \ln \tau_{2,t} + \delta_{3,t} \ln \tau_{12,t} + \delta_{4,t} \ln e_t + \delta_{5,t} \ln \tau_{c,t} + \delta_{6,t} \ln k,$$

- $\tau_{\cdot,t}$ is time spent on child investment by parents separately 1 or 2, jointly 12 or by the child c.
- e_t is time spent in school.
- R_t is total factor productivity (TFP) which is allowed to be a time varying function of household characteristics (e.g. parent education).

back

Del Boca et al (2019) Bellman's

- Assume that this part of childhood is M periods. Define terminal utility on k_{M+1}

$$\psi_{c,M+1} = \xi_c \gamma_3$$
$$\psi_{p,M+1} = \xi_p \alpha_4$$

- Then the state variables are $\Gamma_t = (w_{1,t}, w_{2,t}, I_t, k_t)$
- The parents problem is then

$$V_{p,t}(\Gamma_t) = \max_{a_{p,t}|\tau^*_{c,t}(a_{p,t}),c_{p,t}} \tilde{u}_p(l_{1,t}, l_{2,t}, l_{c,t}, c_t, x_t, k_t) + \beta_p E V_{p,t+1}(\Gamma_t | a_{p,t})$$

  where $a_{p,t}$ is the parents action at time t.
- The child's problem is defined similarly, but is conditional on parent actions $a_{p,t}$ and with discount rates that change over time $\beta_{c,t}$.

back to 2019 back to 2014

Del Boca et al (2019) Crowding out

- γ_t is the ratio of the future utility flow from time investment to the total utility from time use.
  - Notice if the kid has very low β_t, then they won't devote time to studying.
  - Also notice that the child's time use is deterministic response to the parents' choices.
- Then, the extent to which there is crowing out depends on the child's preferences and impatience.

$$\frac{\partial \tau^*_{c,t}(\tau_{p,t})}{\tau_{p,t}} = -\gamma_t$$

Parent choice variable: Labor force participation

Table 2: Parental Labor Supply by Child Age

	Fraction Working > 0 Hours				
Child Age	**3**	**4-5**	**6-8**	**9-11**	**12-15**
Mothers working	0.796	0.766	0.787	0.816	0.846
Fathers working	0.990	0.985	0.982	0.985	0.961
Both working	0.781	0.746	0.758	0.794	0.804
	Avg. Hours Working (> 0 Hours)				
Child Age	**3**	**4-5**	**6-8**	**9-11**	**12-15**
Mothers	27.77	28.87	28.97	29.99	31.77
Fathers	43.57	43.14	44.56	43.33	43.76

Notes: Upper half of the table shows labor force participation rates. Bottom half shows average labor hours conditional on working positive hours.
Source: PSID-CDS combined sample from 1997, 2002 and 2007 interviews and PSID core data between 1986 and 2010.

Choice variable: Time use

Table 3: Time Allocation by Child Age (Average Hours per Week)

Child Age	**3**	**4-5**	**6-8**	**9-11**	**12-15**
Mother's Work Hours	22.10	22.12	22.81	24.47	26.89
Father's Work Hours	43.13	42.49	43.78	42.70	42.03
Mother's Active Time	26.12	20.41	14.32	11.18	6.88
Father's Active Time	13.68	7.92	5.28	4.95	4.18
Joint Parental Time	7.17	10.59	9.59	10.10	7.88
Child's Self-Investment Time	0.00	0.67	1.42	3.37	6.01
School Time	11.25	11.89	27.77	31.43	35.17

Notes: Parental work hours, wages and non-labor income statistics are averaged over all years where the child is between 0 and 16 years old, ranging from 1986 to 2010.
Source: PSID-CDS combined sample from 1997, 2002 and 2007 interviews and PSID core data between 1986 and 2010.

Cunha, Heckman, and Schennach (2010): appendix

The specification they use is:

$$\theta_{k,t+1} = [\gamma_{s,k,1}\theta_{C,t}^{\phi_{s,k}} + \gamma_{s,k,2}\theta_{N,t}^{\phi_{s,k}} + \gamma_{s,k,3}I_t^{\phi_{s,k}} + \gamma_{s,k,4}\theta_{C,P}^{\phi_{s,k}} + \gamma_{s,k,5}\theta_{N,P}^{\phi_{s,k}}]^{1/\phi_{s,k}} e^{\eta_{k,t+1}} \quad (18)$$

where $\gamma_{s,k,l} \geq 0, \sum_{l=1}^{5} \gamma_{s,k,l} = 1, k \in \{C, N\}, t \in \{1, 2\}, s \in \{1, 2\}$.

- They assume $\eta_{k,t} \sim N(0, \delta_{\eta,s}^2)$ and that $I_{C,t} = I_{N,t}$.
- They divide the periods in two stages of development: from 0 to 5-6 years old, and from 5-6 to 13-14 years old. They have data of children every two years.
- The sample includes 2207 firstborn white children between 0 and 14 years old from Children of the NLSY/79 (CNLSY/79) sample.

Cunha, Heckman, and Schennach (2010): appendix

TABLE IV

THE TECHNOLOGY FOR COGNITIVE AND NONCOGNITIVE SKILL FORMATION:
LINEAR ANCHORING ON EDUCATIONAL ATTAINMENT (YEARS OF SCHOOLING);
ALLOWING FOR UNOBSERVED HETEROGENEITY (π);
FACTORS NORMALLY DISTRIBUTED[a]

		First Stage Parameters		Second Stage Parameters
Panel A: Technology of Cognitive Skill Formation (Next Period Cognitive Skills)				
Current Period Cognitive Skills (Self-Productivity)	$\gamma_{1,C,1}$	0.479 (0.026)	$\gamma_{2,C,1}$	0.831 (0.011)
Current Period Noncognitive Skills (Cross-Productivity)	$\gamma_{1,C,2}$	0.070 (0.024)	$\gamma_{2,C,2}$	0.001 (0.005)
Current Period Investments	$\gamma_{1,C,3}$	0.161 (0.015)	$\gamma_{2,C,3}$	0.044 (0.006)
Parental Cognitive Skills	$\gamma_{1,C,4}$	0.031 (0.013)	$\gamma_{2,C,4}$	0.073 (0.008)
Parental Noncognitive Skills	$\gamma_{1,C,5}$	0.258 (0.029)	$\gamma_{2,C,5}$	0.051 (0.014)
Complementarity Parameter	$\phi_{1,C}$	0.313 (0.134)	$\phi_{2,C}$	−1.243 (0.125)
Implied Elasticity of Substitution	$1/(1-\phi_{1,C})$	1.457	$1/(1-\phi_{2,C})$	0.446
Variance of Shocks $\eta_{C,t}$	$\delta^2_{1,C}$	0.176 (0.007)	$\delta^2_{2,C}$	0.087 (0.003)

(Back)

Cunha, Heckman, and Schennach (2010): appendix

Panel B: Technology of Noncognitive Skill Formation (Next Period Noncognitive Skills)				
Current Period Cognitive Skills (Cross-Productivity)	$\gamma_{1,N,1}$	0.000 (0.026)	$\gamma_{2,N,1}$	0.000 (0.010)
Current Period Noncognitive Skills (Self-Productivity)	$\gamma_{1,N,2}$	0.585 (0.032)	$\gamma_{2,N,2}$	0.816 (0.013)
Current Period Investments	$\gamma_{1,N,3}$	0.065 (0.021)	$\gamma_{2,N,3}$	0.051 (0.006)
Parental Cognitive Skills	$\gamma_{1,N,4}$	0.017 (0.013)	$\gamma_{2,N,4}$	0.000 (0.008)
Parental Noncognitive Skills	$\gamma_{1,N,5}$	0.333 (0.034)	$\gamma_{2,N,5}$	0.133 (0.017)
Complementarity Parameter	$\phi_{1,N}$	−0.610 (0.215)	$\phi_{2,N}$	−0.551 (0.169)
Implied Elasticity of Substitution	$1/(1-\phi_{1,N})$	0.621	$1/(1-\phi_{2,N})$	0.645
Variance of Shocks $\eta_{N,t}$	$\delta^2_{1,N}$	0.222 (0.013)	$\delta^2_{2,N}$	0.101 (0.004)

(Back)

Becker-Kominers-Murphy-Spenkuch (2018): appendix

Combining the earning function and the function of transmission of human capital we can show that the intergenerational transmission of earning is defined by:

$$log(E_c) = \mu + \frac{\beta}{1-\alpha\sigma}\frac{\sigma_c}{\sigma_p}log(E_p) + \tilde{\epsilon} \tag{19}$$

where:

$$\mu = \frac{1}{1-\alpha\sigma}log(r_c) - \frac{\beta}{1-\alpha\sigma}\frac{\sigma_c}{\sigma_c}log(r_p) + \frac{\alpha\sigma_c}{1-\alpha\sigma_c}log(\frac{\alpha\sigma_c}{R_c}) \tag{20}$$

and

$$\tilde{\epsilon} = log(\epsilon_c) - \frac{\beta}{1-\alpha\sigma_c}\frac{\sigma_c}{\sigma_p}log(\epsilon_p) \tag{21}$$

Steady state implies $\sigma_c = \sigma_p$.

(back).

Appendix: Cunha and Heckman (2007): another market failure

- Parents may not be allowed to borrow against their future income.
- Let's define α as the rate that parental productivity growths. Then the budget constrain depends on the period:

$$
\begin{aligned}
c_1 + l_1 + \frac{s}{1+r} &= \quad wh + b \qquad &\text{(first stage)} \\
c_2 + l_2 + \frac{b'}{1+r} &= w(1+\alpha)h + s \qquad &\text{(second stage)}
\end{aligned}
$$

  - $s \geq 0$: parents cannot borrow against income of period 2
  - $b' \geq 0$

(back).

Appendix: Cunha and Heckman (2007): another market failure

- If both restrictions bind and investment in different periods are not perfect substitutes, the timing of income would affect the human capital accumulation of children.
- If $u(c) = (c^\sigma - 1)/\sigma$ the ratio of investment would be:

$$\frac{I_1}{I_2} = \left[\frac{\gamma}{(1-\gamma)(1+r)}\right]^{\frac{1}{1-\phi}} * \left[\frac{wh + b - I_1}{\beta((1+\alpha)wh - I_2)}\right]^{\frac{1-\sigma}{1-\phi}}$$

- If early income is relatively low, then the ratio would be lower than the optimum.
- This deviation would be larger the lower is the elasticity of intertemporal substitution σ.
- Ratio of incomes would be irrelevant if consumption of different stages are perfect substitution $\sigma = 1$.

(back).