Selection and Wages - TA Session

[redacted]

January 26, 2021

Outline

- Some empirical application of the normal Roy Model
- Go a bit further into the identification of the general Roy model
- Flip the question and ask how occupations/tasks are being formed
- Look at the demand side (Acemoglu and Autor, 2011)

Heckman and Sedlacek, 1985

- Main Goal: Using the Roy model in order to identify tasks demand function and the task production function
- there's a distribution $g(S|\theta)$ of skills
- Let $t_i(s)$ be a non negative function that expresses the amount of sector i specific task a worker with skill endowment s can perform.
- The sector's outcome is given by

$$Y_i = F^{(i)}\left(T_i, \mathrm{A}_i\right), \quad i = 1, 2$$

- Sector's prices are given by

$$\pi_i = P_i \frac{\partial F^{(i)}}{\partial T_i}, \quad i = 1, 2$$

- Selection is given by

$$\pi_i t_i(\mathrm{s}) \geq \pi_j t_j(\mathrm{s}), \quad i \neq j, i, j = 1, 2$$

Heckman and Sedlacek, 1985

- Assume a functional form of $t_i = C_i S$
- which can be rewritten as $\ln t_i = \beta_i x + u_i, \quad i = 1, 2$
- Wages are given as

$$\ln w_i = \ln \pi_i + \ln t_i = \ln \pi_i + \beta_i x + u_i, \quad \text{i} = 1, 2$$

β

- Demand for skill is given by

$$\ln T_{il} = \delta_{0i} + \delta_{1i} \ln \left(\frac{\pi_{il}}{P_{il}} \right) + \boldsymbol{\delta}_{2i} \ln \left(\frac{\mathrm{P}_{Al}}{P_{il}} \right) + e_{il}$$
$$l = 1, \ldots, L$$

- Can be identify from the wage bill, if we can identify π_i

$$\ln \left(\frac{WB_{il}}{P_{il}} \right) = [\delta_{0i} - \beta_{0i} (\delta_{1i} + 1)] + (\delta_{1i} + 1)(\ln \pi_i - \ln P_{il})$$

$$+ \delta_{2i} \ln$$

Heckman and Sedlacek, 1985

- The paper try to estimatre the model paramters for two Sectors - Manufacturing and non-manufacturing
- In practice: The basic Roy's Model is rejected by the data.
- They then add the following modifications
  1. Allow workers to maximize utility and not only wage

$$V_i > V_j, \quad i \neq j, i, j = 1, 2, 3, \ln V_i = \gamma_i \mathsf{f} + v_i, \quad i = 1, 2, 3$$

  2. Decompose earnings to hoursly ratre and hours work
  3. Developing a general nonnormal model for unmeausred skills

$$\frac{t_i^{\lambda_i} - 1}{\lambda_i} = \beta_i x + u_i, \quad i = 1, 2$$

  4. Add a "home sector" to the two sectors,

Heckman and Sedlacek, 1985

TABLE 1

ESTIMATES OF THE MODEL PARAMETERS

	Estimated Coefficient	Standard Error*	Normal Statistic†
Utility function in the nonmanufacturing sector (γ_1):			
Intercept	4.238367	.469394	9.029442
Education	.338785	.042739	7.926800
Experience	.224682	.028620	7.850411
Experience squared/100	−.333751	.071232	−4.685396
South dummy	.282627	.136377	2.072390
Predicted nonlabor income/100	.242310	.033105	7.319353
1980 intercept (γ_{01t} for 1980)	.113196	.094107	1.202837
Utility function in the manufacturing sector (γ_2):			
Intercept	3.103701	.565689	5.486586
Education	.285896	.053022	5.392017
Experience	.163867	.036530	4.485828
Experience squared/100	−.257929	.072256	−3.569655
South dummy	.019389	.106355	.182301
Predicted nonlabor income/100	.172409	.036337	4.744774
1980 intercept (γ_{02t} for 1980)	.017729	.074623	.237583
Correlation coefficient between v_1 and v_2:			
correl(v_1, v_2)	.296560	.147650	2.008529
Standard deviation of v_2:			
$[\text{var}(v_2)]^{1/2}$	.850640	.117044	7.267723
Parameters of the mapping of the observed skills to the nonmanufacturing task (β_1):			
Intercept	−.112678	.101883	−1.105953
Education	.040472	.007908	5.117798
Experience	.005979	.008301	.720287

Heckman and Sedlacek, 1985

TABLE 1 (*Continued*)

	Estimated Coefficient	Standard Error*	Normal Statistic†
Experience squared/100	.019015	.018805	1.011173
South dummy	.016770	.042527	.394325
1980 intercept (β_{01t} for 1980)	−.312877	.356679	−.877195
Parameters of the mapping of the observed skills to the manufacturing sector task ($\boldsymbol{\beta}_2$):			
Intercept	−.331493	.299324	−1.107471
Education	.082424	.010596	7.778808
Experience	.027506	.012970	2.120790
Experience squared/100	−.027446	.028786	−.953469
South dummy	−.102184	.060104	−1.700135
1980 intercept (β_{02t} for 1980)	.038270	1.152317	.033212
Covariance structure of the latent task distribution:			
$(\sigma_{11}^{1/2}) = [\mathrm{var}(u_1^*)]^{1/2}$	.574169	.006098	94.159852
$(\sigma_{22}^{1/2}) = [\mathrm{var}(u_2^*)]^{1/2}$	.486769	.081631	5.963048
$\rho_{12}^* = \mathrm{correl}(u_1^*, v_2 - v_1)$	.241512	.029820	8.351013
$\rho_{11}^* = \mathrm{correl}(u_1^*, v_1)$	.454436	.029116	15.607939
$\rho_{21}^* = \mathrm{correl}(u_2^*, v_2 - v_1)$	.235583	.009276	25.397051
$\rho_{22}^* = \mathrm{correl}(u_2^*, v_2)$	.159303	.004145	38.435299
1980 estimated log task price change where $\pi_1(1976) = \pi_2(1976) = 1$:			
Nonmanufacturing sector ($\widehat{\ln \pi_{1t}}$ for 1980)	.216560	.003588	60.358733
Manufacturing sector ($\widehat{\ln \pi_{2t}}$ for 1980)	−.225510	.005036	−44.777223

Heckman and Sedlacek, 1985

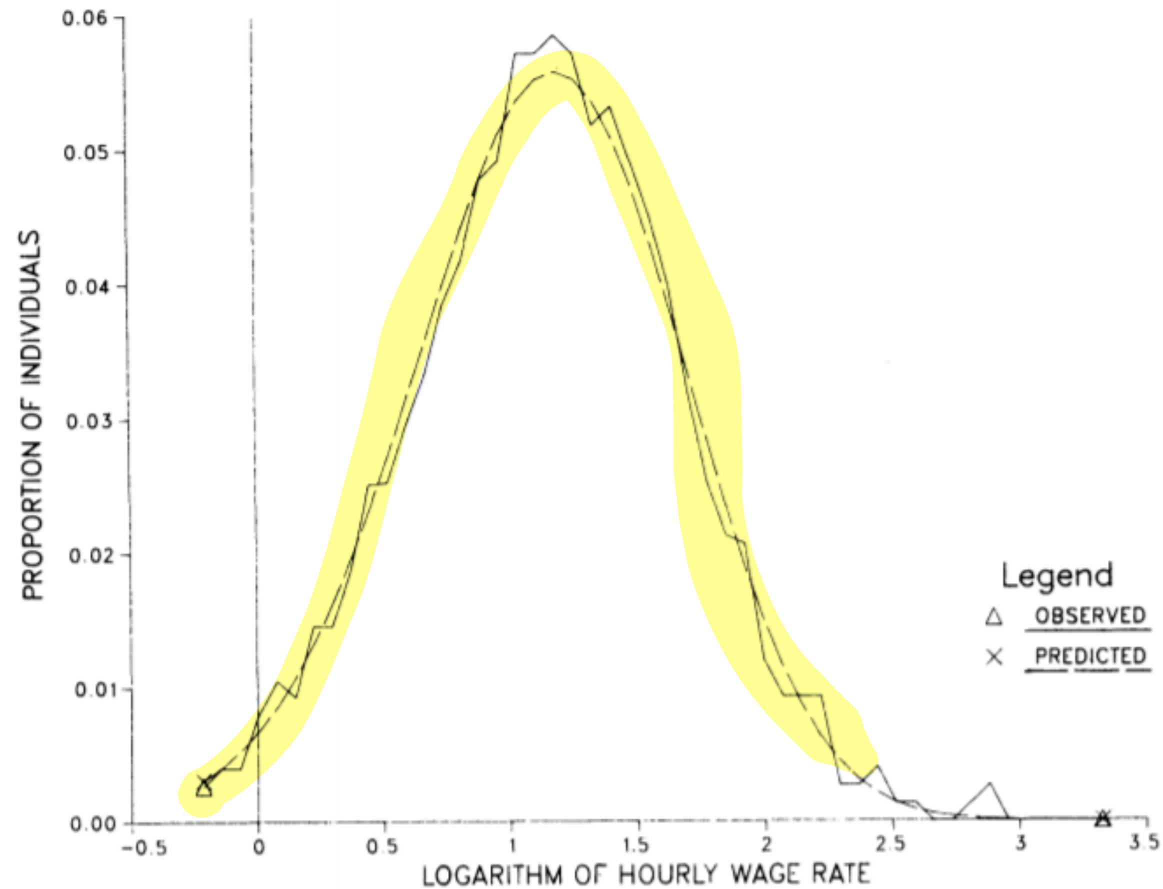


FIG. 4.—Manufacturing sector: predicted versus observed log wage distribution

Heckman and Sedlacek, 1985

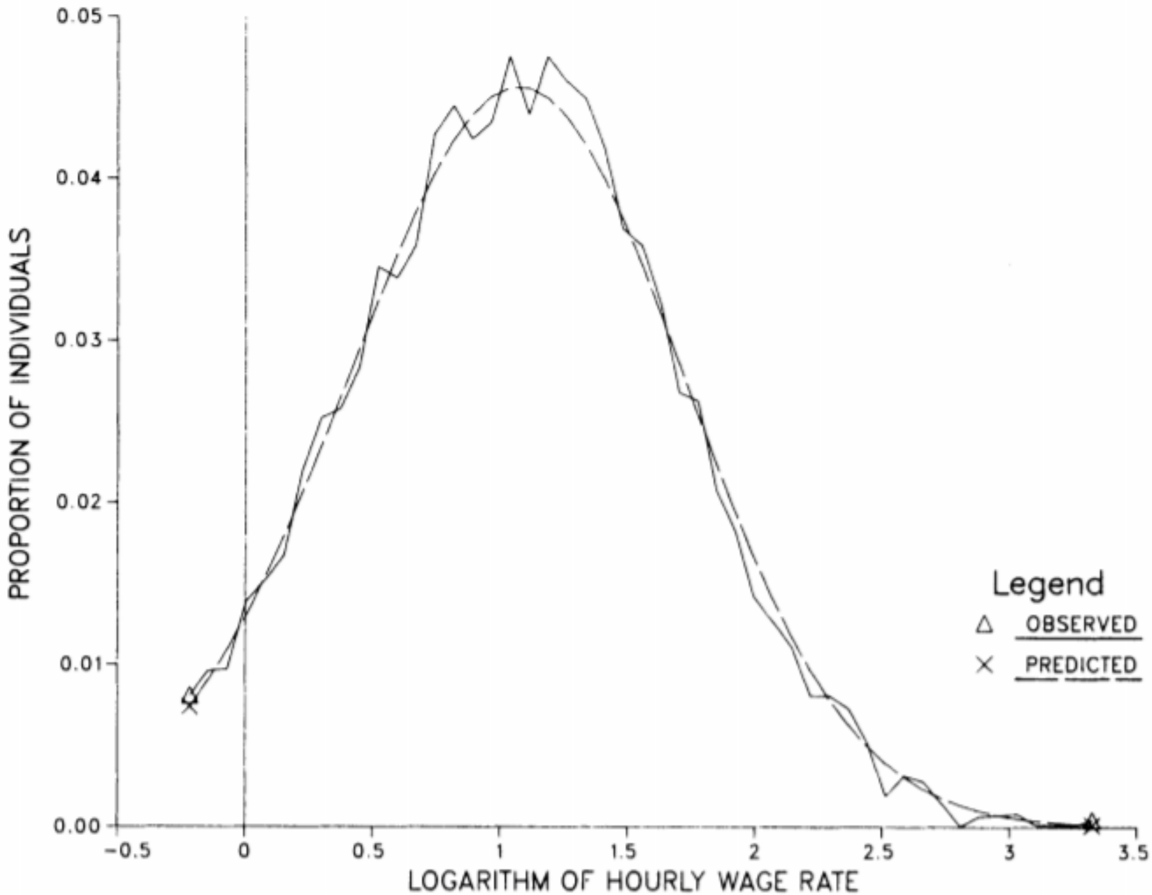


FIG. 3.—Nonmanufacturing sector: predicted versus observed log wage distribution

Tasks

The Canonical Model: Skill-Biased Technical Change

Katz and Murphy, 1992

1. Output depends on high skill workers H and low-skill workers L:

$$Y = \left[(A_L L)^{\frac{\sigma-1}{\sigma}} + (A_H H)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

2. Perfect competition implies that the equilibrium wage rates are

$$W_H = \frac{\partial Y}{\partial H} = A_H^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} (H/L)^{-\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

$$W_L = \frac{\partial Y}{\partial L} = A_L^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} (H/L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

$\Longrightarrow$ wages should increase with improvements in technology

3. We can also derive the log-wage premium:

$$\log\left(\frac{W_H}{W_L}\right) = \frac{\sigma-1}{\sigma} \log\left(\frac{A_H}{A_L}\right) - \frac{1}{\sigma} \log\left(\frac{H}{L}\right)$$

Acemoglu and Autor,2011, The Canonical Model

Model predictions

- Skill premium *decreasing* in relative stock of workers with high skills:
$$\frac{\partial \ln(W_H/W_L)}{\partial \ln(H/L)} = -\frac{1}{\sigma} < 0$$
- Skill premium *increasing* in relative stock of high skill technology:
$$\frac{\partial \ln(W_H/W_L)}{\partial \ln(A_H/A_L)} = \frac{-1}{\sigma} > 0 \quad when \quad \sigma > 1$$

Some things to keep in mind

- Technologies, in this framework, are factor-augmenting, which implies that changes can increase the productivity of either low or high skill workers. This implies that any technological improvements lead to higher wages and higher employment for both skill groups
- Technology does not "replace" skill
- There are other issues as well, which I would not address (no selection, unbundeling of skill, wages are not "total compensation")

Introducing the Task-Based Approach

$t(s,h,L) = y_i$

- A **task** is a unit of work activity that produces output as function of inputs.
- We can think of occupations are bundles of tasks, but the task composition of occupations is equally subject to changes over time.
- Tasks are more likely to be **a stable unit of analysis**.
  - Example of task: ensuring that words are spelled correctly.
  - How does the **skill content** of this task change over time?
  - In 1960: a typist personally checks the spelling.
  - In 2020: spell-check is performed by software (capital), which is complementary with developers and programmers (skilled labor).

A bird's eye view of task models

Production

- Firm output depends directly on *tasks*
- Firms employ capital and workers' *skills* in the production of these tasks
- Production of some of these tasks may exhibit *dynamics*

Model

1. $Y = F(Tasks)$
2. $Task_{i,t} = f_{i,t}(skills, capital, \theta_t)$
3. $\theta_t = g_j(Task_{i,t-1}, skills, capital)$

Labour supply

1. Labour supply: modelling the stock of workers with skills which can be employed in the production of tasks

SBTC as a special case

Production

1. Output depends on two tasks Y_H and tasks Y_L:

$$Y = [(A_L Y_L)^{\frac{\sigma-1}{\sigma}} + (A_H Y_H)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

2. Linear production function with static mapping from skills into tasks

$$Y_H = L_H \quad Y_L = L_L$$

3. No dynamics in task production

Labor Supply

1. Workers choose whether to obtain "high skills"
2. Workers are then employed in tasks matching their skill-set

Acemoglu & Autor (2011)

Production

1. Output is a function of infinitely many tasks, represented by the unit interval:

$$Y = \exp\left(\int_0^1 \ln y(i)di\right)$$

2. Each task can be produced by static technology over three skills and capital:

$$y(i) = A_L\alpha_L(i)l(i) + A_M\alpha_M(i)m(i) + A_H\alpha_H(i)h(i) + A_k\alpha_k(i)k(i)$$

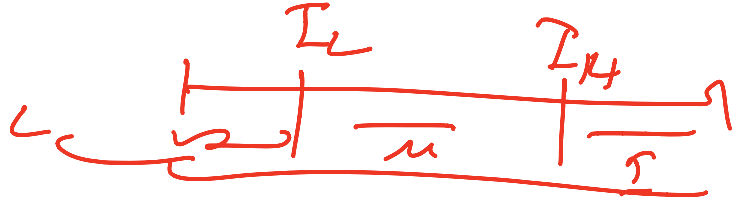
3. No dynamics in production of intermediate tasks

Labor supply

1. Fixed measures of low, medium and high skilled workers
2. Workers choose the task in which they wish to supply their skills

Acemoglu & Autor (2011)

Equilibrium



- In any equilibrium there exist I_L and I_H such that $0 < I_L < I_H < 1$ and for any $i < I_L$, $m(i) = h(i) = 0$, for any $i \in (I_L, I_H)$, $\mathrm{l(i)} = \mathrm{h\ (i)} = 0$, and for any $i > I_H$, $l(i) = m(i) = 0$.
- Implies law of one price within types

$$
\begin{aligned}
w_L &= p(i)A_L\alpha_L(i) \quad i \leq I_L \\
w_M &= p(i)A_M\alpha_M(i) \quad i_L < i < I_H \\
w_H &= p(i)A_H\alpha_H(i) \quad i \geq I_H \\
\implies P_j &= p(i)\alpha_j(i) \quad j \in \{L, M, H\}
\end{aligned}
$$

- Can use this to describe wages by type:

$$w_j = P_j A_j \quad j \in \{L, M, H\}$$

- The share of workers at each task is the same, i.e. $l(i) = \frac{L}{I_L}$

Acemoglu & Autor (2011)

- Using the fact the demand for each task is the same, we get

$$p(i)A_M\alpha_M(i)m(i) = p(i')A_H\alpha_H(i')h(i') \implies$$

$$\frac{P_H}{P_M} = \left(\frac{A_H H}{1-I_H}\right)^{-1}\left(\frac{A_M M}{I_H - I_L}\right)$$

and similarly

$$\frac{P_M}{PL} = \left(\frac{A_M M}{I_H - I_L}\right)^{-1}\left(\frac{A_L L}{I_L}\right)$$

- Last, at the boundary, the firm is indifferent between using different type of workers, then

$$\frac{A_M\alpha_M(I_H)M}{I_H - I_L} = \frac{A_H\alpha_H(I_H)H}{1-I_H}$$

$$\frac{A_L\alpha_L(I_L)L}{I_L} = \frac{A_M\alpha_M(I_L)M}{I_H - I_L}$$

Acemoglu & Autor (2011)

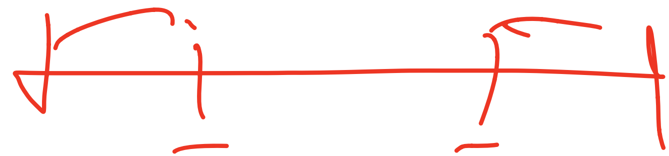
- Last, we can characterize the wage premium as

$$\frac{w_H}{w_M} = \frac{P_H A_H}{P_M A_M} = \left(\frac{1 - I_H}{I_H - I_L}\right)\left(\frac{H}{M}\right)^{-1}$$

$$\frac{w_M}{w_L} = \left(\frac{I_H - I_L}{I_L}\right)\left(\frac{M}{L}\right)^{-1}$$

- Proposition 1. There exists a unique equilibrium summarized by

Acemoglu & Autor (2011)



Comparative statics

- Re-writing the no-arbitrage equations as

$$\begin{aligned} \ln A_M - \ln A_H + \beta_H\left(I_H\right) + \ln M - \ln H - \ln\left(I_H - I_L\right) + \ln\left(1 - I_H\right) = 0 \\ \ln A_L - \ln A_M + \beta_L\left(I_L\right) + \ln L - \ln M + \ln\left(I_H - I_L\right) - \ln\left(I_L\right) = 0 \end{aligned} \tag{1}$$

where

$$\beta_H(I) \equiv \ln \alpha_M(I) - \ln \alpha_H(I) \quad \text{and} \quad \beta_L(I) \equiv \ln \alpha_L(I) - \ln \alpha_M(I)$$

- Both curves are increasing on the I_L, I_H space

Acemoglu and Autor, 2011

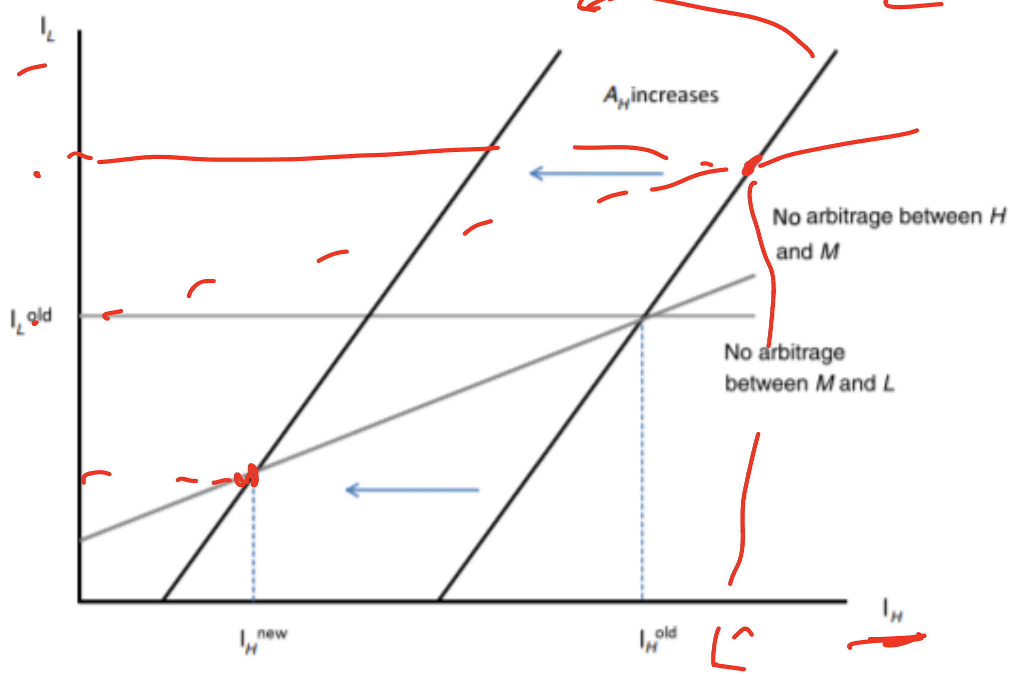


Figure 25 *Comparative statics.*

Figure: Caption

Acemoglu and Autor, 2011

It can be shown that

3. *(The response of wages to factor-augmenting technologies):*

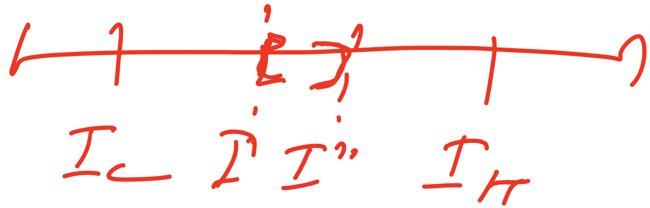
$$\frac{d\ln(w_H/w_L)}{d\ln A_H} > 0, \qquad \frac{d\ln(w_M/w_L)}{d\ln A_H} < 0, \qquad \frac{d\ln(w_H/w_M)}{d\ln A_H} > 0;$$

$$\frac{d\ln(w_H/w_L)}{d\ln A_L} < 0, \qquad \frac{d\ln(w_M/w_L)}{d\ln A_L} < 0, \qquad \frac{d\ln(w_H/w_M)}{d\ln A_L} > 0;$$

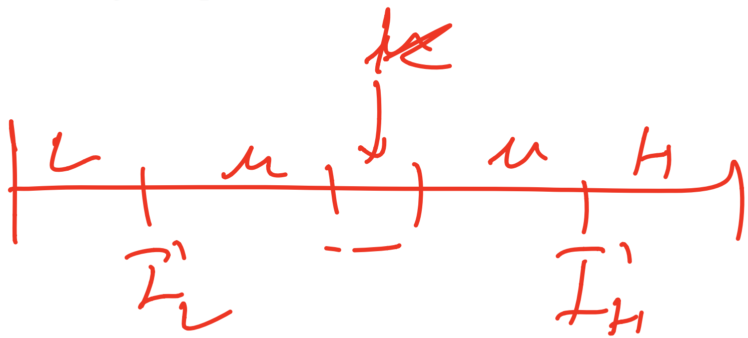
$$\frac{d\ln(w_H/w_M)}{d\ln A_M} < 0, \qquad \frac{d\ln(w_M/w_L)}{d\ln A_M} > 0, \quad \textit{and}$$

$$\frac{d\ln(w_H/w_L)}{d\ln A_M} \lesseqqgtr 0 \quad \textit{if and only if} \quad \left|\beta_L'(I_L)\,I_L\right| \gtreqqless \left|\beta_H'(I_H)\,(1-I_H)\right|.$$

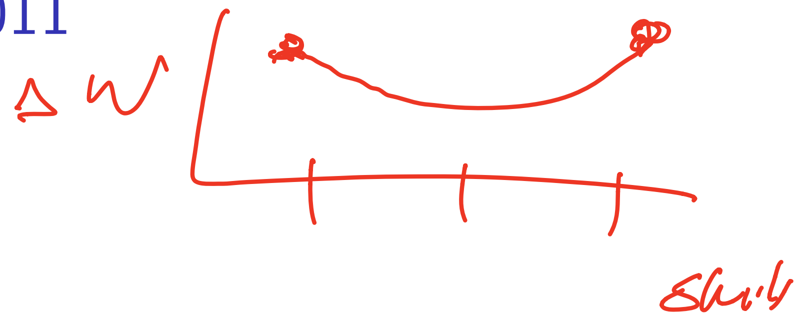
Acemoglu and Autor, 2011



Proposition 3. *Suppose we start with an equilibrium characterized by thresholds* $[I_L, I_H]$ *and technical change implies that the tasks in the range* $[I', I''] \subset [I_L, I_H]$ *are now performed by machines. Then after the introduction of machines, there exists new unique equilibrium characterized by new thresholds* $\hat{I}_L$ *and* $\hat{I}_H$ *such that* $0 < \hat{I}_L < I' < I'' < \hat{I}_H < 1$ *and for any* $i < \hat{I}_L$, $m(i) = h(i) = 0$ *and* $l(i) = L/\hat{I}_L$; *for any* $i \in (\hat{I}_L, I') \cup (I'', \hat{I}_H)$, $l(i) = h(i) = 0$ *and* $m(i) = M/(\hat{I}_H - I'' + I' - \hat{I}_L)$; *for any* $i \in (I', I'')$, $l(i) = m(i) = h(i) = 0$; *and for any* $i > \hat{I}_H$, $l(i) = m(i) = 0$ *and* $h(i) = H/(1 - \hat{I}_H)$.



Acemoglu and Autor, 2011



Proposition 4. *Suppose we start with an equilibrium characterized by thresholds* $[I_L, I_H]$ *and technical change implies that the tasks in the range* $[I', I''] \subset [I_L, I_H]$ *are now performed by machines. Then:*

1. w_H / w_M *increases;*
2. w_M / w_L *decreases;*
3. w_H / w_L *increases if* $\left|\beta_L'(I_L)\, I_L\right| < \left|\beta_H'(I_H)\,(1 - I_H)\right|$ *and* w_H / w_L *decreases if* $\left|\beta_L'(I_L)\, I_L\right| > \left|\beta_H'(I_H)\,(1 - I_H)\right|$.