Automation and Jobs

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Acemoglu and Restrepo, 2018,2019

- The implications of technological change on employment and wages are not clear
  - Some fear that automation can lead to wide spread joblessness
  - On the other hand, Technological improvements increase productivity and increase demand for labor
- The canonical approach does not capture these implications, as it assumes that the production function takes the form of $Y = F(A_k K, A_L L)$, which imposes that all technological changes takes only factor augmenting form. This approach implies
  - advancement in robotics, do not make capital or labor more productive, but expand the set of tasks that can be produced by capital
  - Capital-augmenting technological change, or labor augmenting technological change makes all the relevant factors uniformly more productive

Acemoglu and Restrepo, 2018,2019

- This series of papers consider a task based approach to think how automation is different then simple factor augmenting technological change
  - **Automation**, which corresponds to development and adoption of new technologies that enable capital to be substituted for labor, in a range of tasks
  - Other technologies, which create **new tasks**, wherein labor has some comparative advantage
  - **Factor-augmenting technological change**

Acemoglu and Restrepo, 2018,2019

Model

- The production function

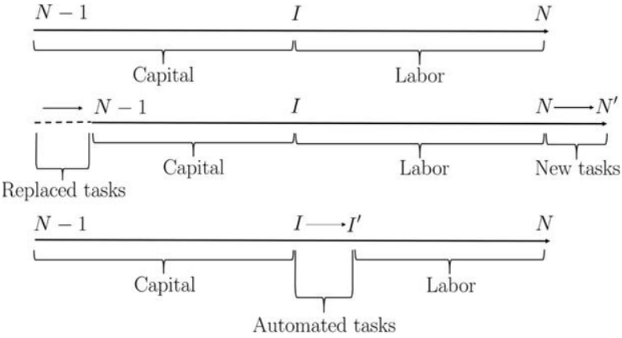
$$Y = F(Task) = \left(\int_{N-1}^{N} Y(z)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

- The output of task z is given by

$$Y(z) = f(l, K) = \begin{cases} A^L \gamma^L(z) l(z) + A^K \gamma^K(z) k(z) & \text{if } z \in [N-1, I] \\ A^L \gamma^L(z) l(z) & \text{if } z \in (I, N] \end{cases}$$

- The tasks dynamics is governed by the technology parameters, N and I
- Capital, K, and Labour L are fixed
- For simplicity, we assume that tasks which can be automated, are produced by capital

Acemoglu and Restrepo, 2018,2019



Acemoglu and Restrepo, 2018,2019

Model

- Under perfect competition, output is given by

$$Y = \Pi(I,N)\left(\Gamma(I,N)^{\frac{1}{\sigma}}\left(A^L L\right)^{\frac{\sigma-1}{\sigma}} + (1-\Gamma(I,N))^{\frac{1}{\sigma}}\left(A^K K\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma-1}{\sigma-1}}$$

- where $\Gamma(I,N)$ is the task content

$$\Gamma(I,N) = \frac{\int_I^N \gamma^L(z)^{\sigma-1}}{\int_{N-1}^I \gamma^K(z)^{\sigma-1}dz + \int_I^N \gamma^L(z)^{\sigma-1}}$$

- and the TFP is given by

$$\Pi(I,N) = \left(\int_{N-1}^I \gamma^K(z)^{\sigma-1}dz + \int_I^N \gamma^L(z)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$$

Acemoglu and Restrepo, 2018,2019

- Similar to the canonical model, A^K and A^L increase task productivity uniformly.
- σ is the elasticity of substitution between tasks, but, in this case, also between capital and labor
- The share parameters of the CES function, depends on the share of tasks performed by labor relative to capital.
  - $\Gamma(I, N)$ is an increasing function of N, and a decreasing function in I.
  - This implies that an increase in I, shifts the task content of production away from labor, as it entails capital taking over tasks, previously performed by labor.
- An increase in either N or I generates a productivity increase, through the TFP.

Acemoglu and Restrepo, 2018,2019

- The effect of automation on wages/labor demand

$$\begin{aligned}\frac{\partial \ln W^d(L,K;\theta)}{\partial I} =& \frac{\partial \ln Y(L,K;\theta)}{\partial I} \quad \text{(Productivity effect)}\\ &+ \frac{1}{\sigma}\frac{1-s^L(L,K;\theta)}{1-\Gamma(I,N)}\frac{\partial \ln \Gamma(I,N)}{\partial I} \quad \text{(Displacement effect)}\end{aligned}$$

- The effect of new technology

$$\begin{aligned}\frac{\partial \ln W^d(L,K;\theta)}{\partial N} =& \frac{\partial \ln Y(L,K;\theta)}{\partial N} \quad \text{(Productivity effect)}\\ &+ \frac{1}{\sigma}\frac{1-s^L(L,K;\theta)}{1-\Gamma(I,N)}\frac{\partial \ln \Gamma(I,N)}{\partial N} \quad \text{(Reinstatement effect)}\end{aligned}$$

Acemoglu and Restrepo, 2018,2019

- The productivity effect arises from the fact that automation increases value added, and this raises the demand for labor from non-automated tasks.
- displacement effect— cause labor to shift from the tasks previously allocated to it—which shifts the task content of production against labor and always reduces the labor share.
- Automation therefore increases the size of the pie, but labor gets a smaller slice. There is no guarantee that the productivity effect is greater than the displacement effect;
- Contrary to a common presumption in popular debates, it is not the "brilliant" automation technologies that threaten employment and wages, but "so-so technologies" that generate small productivity improvements. This is because the positive productivity effect of so-so technologies is not sufficient to offset the decline in labor demand due to displacement.

Acemoglu and Restrepo, 2018,2019

- Different technologies are accompanied by productivity effects of varying magnitudes, and hence, we cannot presume that one set of automation technologies will impact labor demand in the same way as others.
- the productivity gains of automation depend on the wage, the net impact of automation on labor demand will depend on the broader labor market context
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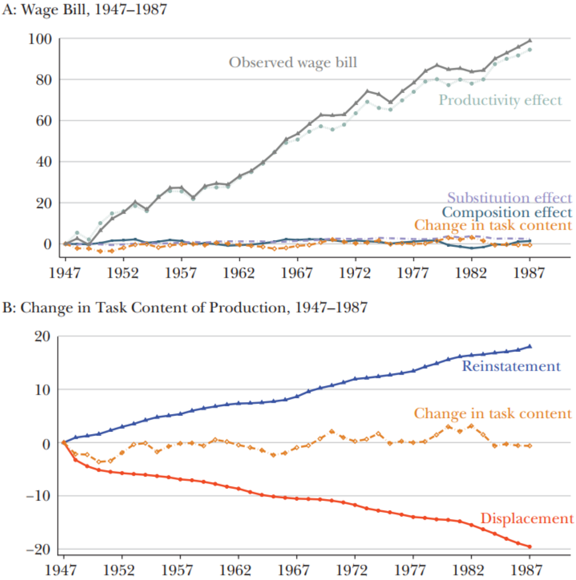
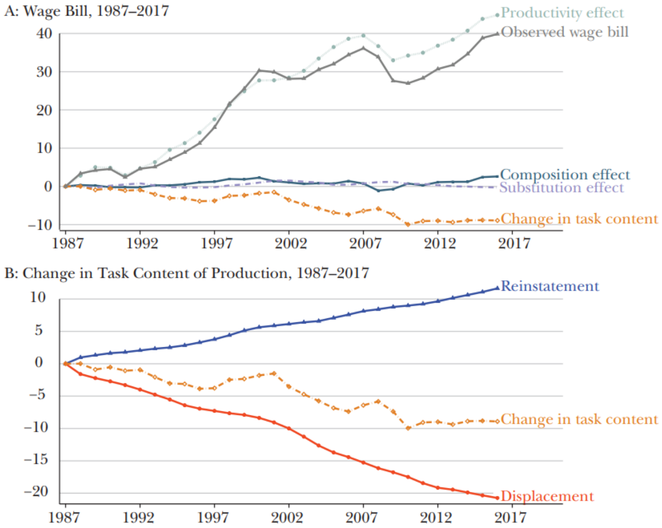
Acemoglu and Restrepo, 2019



Acemoglu and Restrepo, 2019



Acemoglu and Restrepo, 2020 - Unpacking Skill Bias

- Now consider that tasks can be preformed either by capital, or by high skilled or low skilled workers

$$Y(z) = \psi_L(z)\ell(z) + \psi_H(z)h(z) + \psi_K(z)k(z)$$

- It can be shown that the change in skill premium is given by

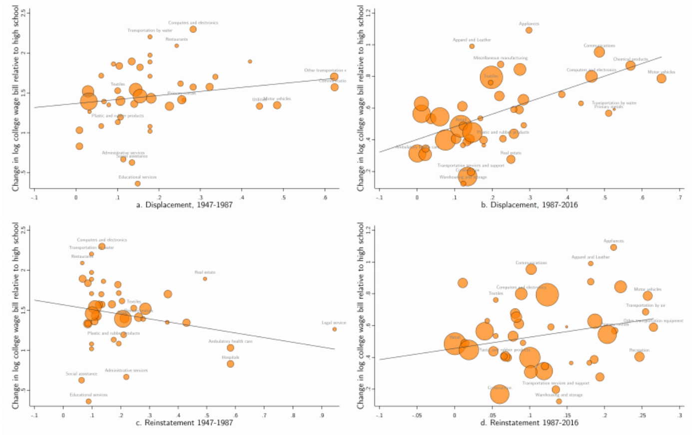
$$d\ln\left(\frac{w_H}{w_L}\right) = -\frac{1}{\phi}d\ln\left(\frac{H}{L}\right) + \frac{\phi-1}{\phi}d\ln\left(\frac{A_H}{A_L}\right) + \frac{1}{\sigma}d\ln\left(\frac{\Gamma_H}{\Gamma_L}\right)\Bigg|_{\frac{A_H H}{A_L L}}$$

- where $\Gamma_j = \dfrac{\frac{1}{M}\int_{\mathcal{T}_j}\gamma_j(x)^{\lambda-1}dx}{1-\frac{1}{M}\int_{\mathcal{T}_K}\left(\frac{\psi_K(x)}{q(x)}\right)^{\lambda-1}dx}$ for $j \in \{L, H\}$
- and $\phi = \sigma/\left(1 - \frac{\partial\ln(\Gamma_H/\Gamma_L)}{\partial\ln(A_H H/A_L L)}\right) \geq \sigma$

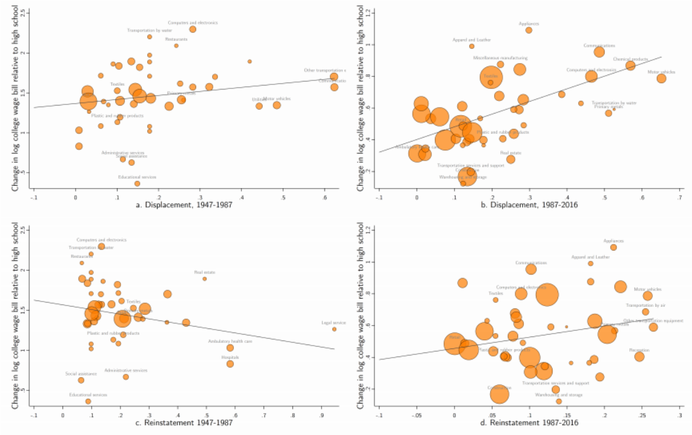
Acemoglu and Restrepo, 2020 - Skill Bias

- ϕ is the derived elasticity of substitution between skilled and unskilled labor, it reflects:
  - substitution between tasks
  - substitution in allocation between labor and capital
- The last term is the new component, which captures the effect of changes in the allocation of tasks to factors on skill premium.
  - Automation in technologies of low skill workers would increase the wage premium
  - Increasing the number of tasks, that are preformed by skilled workers increases the skill premium

Acemoglu and Restrepo, 2020 - Skill Bias



Acemoglu and Restrepo, 2020 - Skill Bias



Overview: Technical Change and Labor Market Outcomes

How to rationalize the relation between technical change and wages?

- Skill-Biased Technical Change (Katz and Murphy, 1992)
  - Is inconsistent with wage polarization
  - Is inconsistent with employment polarization
  - Is inconsistent with decline in employment
- Routine-Task Biased Technical Change (Acemoglu and Autor, 2011; Beaudry et al., 2016)
  - Cannot explain patterns shared by some routine and non-routine tasks
- Complex-Task Biased Technical Change (Caines et al., 2017)
  - Models task complexity as the key complement to technical progress
- Automation Vs. New-Task technology (Acemoglu and Restrepo, 2018,2019,2020)
  - Examine the effect of different types of technological advances

Coming up: Our Work

The China Syndrome: Local Labor Market Effects of Import Competition in the United States

David Autor, David Dorn, and Gordon Hanson

Trade and the "China Shock" - Motivation

- Between 2000-2007 China has experienced a spectacular economic growth and increase in exports. In 2000, the lowincome-country share of US imports reached 15 percent and climbed to 28 percent by 2007, with China accounting for 89 percent of this growth. The share of total US spending on Chinese goods rose from 0.6 percent in 1991 to 4.6 percent in 2007
- Over the same period, the fraction of US working-age population employed in manufacturing fell by a third, from 12.6 percent to 8.4 percent.

Trade and the "China Shock" - Motivation

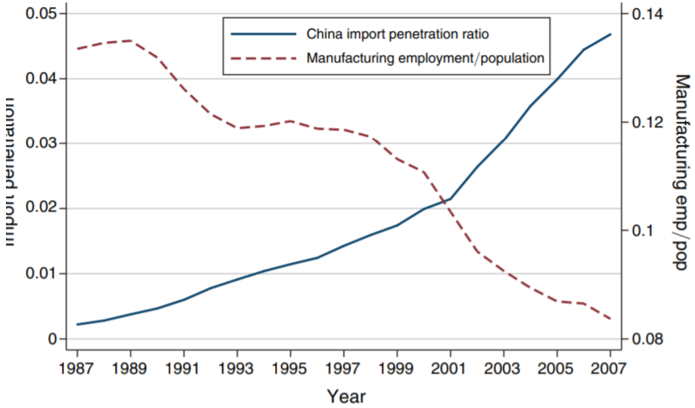


FIGURE 1. IMPORT PENETRATION RATIO FOR US IMPORTS FROM CHINA (*left scale*), AND SHARE OF US WORKING-AGE POPULATION EMPLOYED IN MANUFACTURING (*right scale*)

Figure: Caption

Trade and the "China Shock" - from David Card's Notes

- The idea behind the paper is captured by a model which includes a local labor market with 3 industries: 2 traded goods and one not traded (can be thought of as the "home sector")
- Within each of the two traded sectors there are multiple differentiated products (one per firm).
- Consumers spend a share $1-\gamma$ of income on the non traded good, and shares $\frac{\gamma}{2}$ on each of the traded goods so their utility function is a nested form, with Cobb Douglas at the top, and 2 CES sub-utilities for the traded good aggregates.
- Labor is freely mobile across sectors within a local labor market so there is only one wage per market (CZ).
- The non-traded sector has decreasing returns to scale, so if labor is "shed" from the traded sectors, wages have to fall.

Trade and the "China Shock" - from David Card's Notes

- Firms in the traded sectors have a very simple labor demand functions of the form

$$l = \alpha_{i,j} + \beta_{i,j} x_{i,j}$$

  where l is units of labor demanded and x is output of the firm, and use markup pricing.
- There's free entry and the number of firms is set such that each firm has profit zero
- The key feature of this model is that trade impacts are mediated through the number of varieties produced by a trade competitor i.e., the "quantity" of trade (compare to prices).

Trade and the "China Shock" - from David Card's Notes

- The model ending up giving us the following equations

$$\hat{W}_i = \sum_j c_{ij} \frac{L_{ij}}{L_{Ni}} \left[\theta_{ijC} \hat{E}_{Cj} - \sum_k \theta_{ijk} \phi_{Cjk} \hat{A}_{Cj} \right]$$
$$\hat{L}_{Ti} = \rho_i \sum_j c_{ij} \frac{L_{ij}}{L_{Ti}} \left[\theta_{ijC} \hat{E}_{Cj} - \sum_k \theta_{ijk} \phi_{Cjk} \hat{A}_{Cj} \right]$$
$$\hat{L}_{Ni} = \rho_i \sum_j c_{ij} \frac{L_{ij}}{L_{Ni}} \left[-\theta_{ijC} \hat{E}_{Cj} + \sum_k \theta_{ijk} \phi_{Cjk} \hat{A}_{Cj} \right]$$

- Where $\hat{W}_i$ is the change in wages in region i, $L_{T,i}$ and $\hat{L}_{N,i}$ are the changes in employment for traded and non traded goods, $A_{C,j}$ is the change in China's productivity at sector j, $E_{C,j}$ is China's expenditure on sector j (Both exogenous)

Trade and the "China Shock" - from David Card's Notes

- Wage and employment outcomes are the sum of
  - demand for region i's exports to China, given by the change in expenditure in China times the initial share of output by region i that is shipped to China ($\theta_{ijc} = XijC/X_{ij}$)
  - the decrease in demand for region i's shipments to all markets in which it competes with China. $(A_{C,j}, (\theta_{ijk} \equiv X_{ijk}/X_{ij}), (\phi_{Cjk} \equiv M_{kjC}/E_{kj})$
- ρ captures the trade deficit of the US. If $\rho = 0$, then reduced labor demand in US regions relatively exposed to import competition from China would be offset by labor demand growth in US regions enjoying expanded export production for China, such that for the aggregate US economy labor demand may be unchanged.
- With imbalanced trade, this is not the case, as the demand shock in China is a function of growth in its expenditure, not income.

Trade and the "China Shock" - from David Card's Notes

- Following the model, they can approximate the exposure to import competition is the change in Chinese import exposure per worker in a region as

$$\Delta IPW_{uit} = \sum_j \frac{L_{ijt}}{L_{ujt}} \frac{\Delta M_{ucjt}}{L_{it}}$$

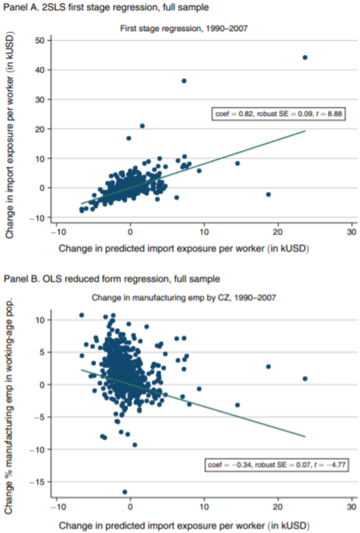
- where $L_{i,t}$ is the start of period employment in region i, ΔM_{ucjt} is the observed change in US imports from China in industry j between the start and end of the period.
- This variable might be correlated with demand shocks, which would bias the OLS estimate downwatds. Therefore they use the following instrument

$$\Delta IPW_{oit} = \sum_j \frac{L_{ijt-1}}{L_{ujt-1}} \frac{\Delta M_{ocjt}}{L_{it-1}}$$

Trade and the "China Shock" - from David Card's Notes

- This instrument would identify the causal effect if the changes in Chinese exports are correlated between high-income countries, but are not
  - Correlated through demand shocks
  - Correlated through technological changes which common in high-income countries

Trade and the "China Shock" - Results



Trade and the "China Shock" - Results

TABLE 2—IMPORTS FROM CHINA AND CHANGE OF MANUFACTURING EMPLOYMENT IN CZS, 1970–2007: 2SLS ESTIMATES

Dependent variable: 10 × annual change in manufacturing emp/working-age pop (in % pts)

	I. 1990–2007			II. 1970–1990 (pre-exposure)		
	1990–2000	2000–2007	1990–2007	1970–1980	1980–1990	1970–1990
	(1)	(2)	(3)	(4)	(5)	(6)
(Δ current period imports from China to US)/worker	−0.89***	−0.72***	−0.75***			
	(0.18)	(0.06)	(0.07)			
(Δ future period imports from China to US)/worker				0.43***	−0.13	0.15
				(0.15)	(0.13)	(0.09)

Notes: $N = 722$, except $N = 1{,}444$ in stacked first difference models of columns 3 and 6. The variable "future period imports" is defined as the average of the growth of a CZ's import exposure during the periods 1990–2000 and 2000–2007. All regressions include a constant and the models in columns 3 and 6 include a time dummy. Robust standard errors in parentheses are clustered on state. Models are weighted by start of period CZ share of national population.

***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

Trade and the "China Shock" - Results

Table 3—Imports from China and Change of Manufacturing Employment in CZs, 1990–2007: 2SLS Estimates

Dependent variable: 10 × annual change in manufacturing emp/working-age pop (in % pts)

	I. 1990–2007 stacked first differences					
	(1)	(2)	(3)	(4)	(5)	(6)
(Δ imports from China to US)/worker	−0.746***	−0.610***	−0.538***	−0.508***	−0.562***	−0.596***
	(0.068)	(0.094)	(0.091)	(0.081)	(0.096)	(0.099)
Percentage of employment in manufacturing$_{-1}$		−0.035	−0.052***	−0.061***	−0.056***	−0.040***
		(0.022)	(0.020)	(0.017)	(0.016)	(0.013)
Percentage of college-educated population$_{-1}$				−0.008		0.013
				(0.016)		(0.012)
Percentage of foreign-born population$_{-1}$				−0.007		0.030***
				(0.008)		(0.011)
Percentage of employment among women$_{-1}$				−0.054**		−0.006
				(0.025)		(0.024)
Percentage of employment in routine occupations$_{-1}$					−0.230***	−0.245***
					(0.063)	(0.064)
Average offshorability index of occupations$_{-1}$					0.244	−0.059
					(0.252)	(0.237)
Census division dummies	No	No	Yes	Yes	Yes	Yes
	II. 2SLS first stage estimates					
(Δ imports from China to OTH)/worker	0.792***	0.664***	0.652***	0.635***	0.638***	0.631***
	(0.079)	(0.086)	(0.090)	(0.090)	(0.087)	(0.087)
R^2	0.54	0.57	0.58	0.58	0.58	0.58

Notes: $N = 1{,}444$ (722 commuting zones × 2 time periods). All regressions include a constant and a dummy for the 2000–2007 period. First stage estimates in panel II also include the control variables that are indicated in the corresponding columns of panel I. Routine occupations are defined such that they account for 1/3 of US employment in 1980. The offshorability index variable is standardized to mean of 0 and standard deviation of 10 in 1980. Robust standard errors in parentheses are clustered on state. Models are weighted by start of period CZ share of national population.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

Trade and the "China Shock" - Results

- How should we interpret the size of these results?
- column 6 of Table 3 implies that a $\$1,000$ per worker increase in import exposure over a decade reduces manufacturing employment per working-age population by 0.596 percentage points.
- Chinese imports increased by 1140, per worker, between 1990 and 2000, and by 1839, between 2000 and 2007. This implies that rising Chinese import exposure reduced US manufacturing employment per population by **0.68** percentage points in the first decade of our sample and **1.10** percentage points in the second decade of our sample
- US manufacturing employment per population fell by 2.07 percentage points between 1990 and 2000 and by 2.00 percentage points between 2000 and 2007.
- Hence, rising exposure to Chinese import explains 33% of the US mfg employment decline between 1990 and 2000, 55% of the decline between 2000 and 2007, and 44% percent of the decline for the full 1990 through 2007 period.

Trade and the "China Shock" - Results

Table 5—Imports from China and Employment Status of Working-Age Population within CZs, 1990–2007: 2SLS Estimates

Dependent variables: Ten-year equivalent changes in log population counts and population shares by employment status

	Mfg emp (1)	Non-mfg emp (2)	Unemp (3)	NILF (4)	SSDI receipt (5)
Panel A. 100 × log change in population counts					
(Δ imports from China to US)/worker	−4.231***	−0.274	4.921***	2.058*	1.466***
	(1.047)	(0.651)	(1.128)	(1.080)	(0.557)
Panel B. Change in population shares					
All education levels					
(Δ imports from China to US)/worker	−0.596***	−0.178	0.221***	0.553***	0.076***
	(0.099)	(0.137)	(0.058)	(0.150)	(0.028)
College education					
(Δ imports from China to US)/worker	−0.592***	0.168	0.119***	0.304***	—
	(0.125)	(0.122)	(0.039)	(0.113)	
No college education					
(Δ imports from China to US)/worker	−0.581***	−0.531***	0.282***	0.831***	—
	(0.095)	(0.203)	(0.085)	(0.211)	

Notes: N = 1,444 (722 CZs × two time periods). All statistics are based on working age individuals (age 16 to 64). The effect of import exposure on the overall employment/population ratio can be computed as the sum of the coefficients for manufacturing and nonmanufacturing employment; this effect is highly statistically significant ($p \leq 0.01$) in the full sample and in all reported subsamples. All regressions include the full vector of control variables from column 6 of Table 3. Robust standard errors in parentheses are clustered on state. Models are weighted by start of period CZ share of national population.

*** Significant at the 1 percent level.
** Significant at the 5 percent level.
* Significant at the 10 percent level.

Trade and the "China Shock" - Results

TABLE 6—IMPORTS FROM CHINA AND WAGE CHANGES WITHIN CZs, 1990–2007: 2SLS ESTIMATES

Dependent variable: Ten-year equivalent change in average log weekly wage (in log pts)

	All workers (1)	Males (2)	Females (3)
Panel A. All education levels			
(Δ imports from China to US)/worker	−0.759***	−0.892***	−0.614***
	(0.253)	(0.294)	(0.237)
R^2	0.56	0.44	0.69
Panel B. College education			
(Δ imports from China to US)/worker	−0.757**	−0.991***	−0.525*
	(0.308)	(0.374)	(0.279)
R^2	0.52	0.39	0.63
Panel C. No college education			
(Δ imports from China to US)/worker	−0.814***	−0.703***	−1.116***
	(0.236)	(0.250)	(0.278)
R^2	0.52	0.45	0.59

Notes: $N = 1{,}444$ (722 CZs × two time periods). All regressions include the full vector of control variables from column 6 of Table 3. Robust standard errors in parentheses are clustered on state. Models are weighted by start of period CZ share of national population.

*** Significant at the 1 percent level.
** Significant at the 5 percent level.
* Significant at the 10 percent level.

Trade and the "China Shock" - Results

TABLE 8—IMPORTS FROM CHINA AND CHANGE OF GOVERNMENT TRANSFER RECEIPTS IN CZs, 1990–2007: 2SLS ESTIMATES

Dep vars: Ten-year equivalent log and dollar change of annual transfer receipts per capita (in log pts and US$)

	Total individual transfers (1)	TAA benefits (2)	Unemployment benefits (3)	SSA retirement benefits (4)	SSA disability benefits (5)	Medical benefits (6)	Federal income assist (7)	Educ/training assist (8)
Panel A. Log change of transfer receipts per capita								
(Δ imports from China to US)/worker	1.01***	14.41*	3.46*	0.72*	1.96***	0.54	3.04***	2.78**
	(0.33)	(7.59)	(1.87)	(0.38)	(0.69)	(0.49)	(0.96)	(1.32)
R^2	0.57	0.28	0.48	0.36	0.32	0.27	0.54	0.33
Panel B. Dollar change of transfer receipts per capita								
(Δ imports from China to US)/worker	57.73***	0.23	3.42	10.00*	8.40***	18.27	7.20***	3.71***
	(18.41)	(0.17)	(2.26)	(5.45)	(2.21)	(11.84)	(2.35)	(1.44)
R^2	0.75	0.28	0.41	0.47	0.63	0.66	0.53	0.37

Notes: $N = 1{,}444$ (722 CZs × two time periods), except $N = 1{,}436$ in column 2, panel A. Results for TAA benefits in column 2 are based on state-level data that is allocated to CZs in proportion to unemployment benefits. Unemployment benefits in column 3 include state benefits and federal unemployment benefits for civilian federal employees, railroad employees, and veterans. Medical benefits in column 6 consist mainly of Medicare and Medicaid. Federal income assistance in column 7 comprises the SSI, AFDC/TANF, and SNAP programs while education and training assistance in column 8 includes such benefits as interest payments on guaranteed student loans, Pell grants, and Job Corps benefits. The transfer categories displayed in columns 2 to 8 account for over 85 percent of total individual transfer receipts. All regressions include the full vector of control variables from column 6 of Table 3. Robust standard errors in parentheses are clustered on state. Models are weighted by start of period CZ share of national population.

*** Significant at the 1 percent level.
** Significant at the 5 percent level.
* Significant at the 10 percent level.

Appendix

Decomposing the wage bill

- The economy-wide wage bill is the sum of wage bills across sectors

$$\ln\left(W_t L_t\right) = \ln\left(Y_t \sum_i \chi_{i,t} s^L_{i,t}\right)$$

$$\ln\left(W_{t_0} L_{t_0}\right) = \ln\left(Y_{t_0} \sum_i \chi_{i,t_0} s^L_{i,t_0}\right)$$

- We can then write the normalized wage bill as

$$\begin{aligned}\ln\left(\frac{W_t L_t}{N_t}\right) - \ln\left(\frac{W_{t_0} L_{t_0}}{N_{t_0}}\right) &= \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t_0}}{N_{t_0}}\right) \\ &+ \ln\left(\sum_i \chi_{i,t} s^L_{i,t}\right) - \ln\left(\sum_i \chi_{i,t_0} s^L_{i,t}\right) \\ &+ \ln\left(\sum_i \chi_{i,t_0} s^L_{i,t}\right) - \ln\left(\sum_i \chi_{i,t_0} s^L_{i,t_0}\right)\end{aligned}$$

- where $\chi_{i,t} = \frac{P_i Y_i}{Y}$ is he share of sector i's in total value added

Decomposing the wage bill

- Notice that

$$\text{Productivity effect }_{t_0,t} = \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t_0}}{N_{t_0}}\right)$$

$$\text{Composition effect }_{t_0,t} = \ln\left(\sum_i \chi_{i,t} s^L_{i,t}\right) - \ln\left(\sum_i \chi_{i,t_0} s^L_{i,t}\right)$$

Decomposing the wage bill

- Last, taking a Taylor expansion of the last expression we get

$$\ln\left(\sum_i \chi_{i,t_0} s^L_{i,t}\right) - \ln\left(\sum_i \chi_{i,t_0} s^L_{i,t_0}\right) \approx \sum_i \frac{\partial \ln\left(\sum_j \chi_{j,t_0} s^L_{j,t_0}\right)}{\partial \ln s^L_{i,t_0}} \cdot \left(\ln s^L_{i,t} - \ln s^L_{i,t_0}\right)$$

$$= \sum_i \frac{\chi_{i,t_0} s^L_{i,t_0}}{\sum_j \chi_{j,t_0} s^L_{j,t_0}} \cdot \left(\ln s^L_{i,t} - \ln s^L_{i,t_0}\right)$$

$$= \sum_i \ell_{i,t_0} \left(\ln s^L_{i,t} - \ln s^L_{i,t_0}\right)$$

- where $\ell_{i,t_0} = \frac{W_{i,t_0} L_{i,t_0}}{\sum_j W_{j,t_0} L_{j,t_0}}$. using the fact that we can express the labor share as

$$s^L = \frac{1}{1 + \frac{1-\Gamma(I,N)}{\Gamma(I,N)} \left(\frac{A^L}{W}\frac{R}{A^K}\right)^{1-\sigma}} = s^L = \frac{1}{1 + \frac{1-\Gamma(I,N)}{\Gamma(I,N)} \rho^{1-\sigma}}$$

- we can take another taylor expnession of $ln(s^L_i)$

$$\ln s^L_{i,t} - \ln s^L_{i,t_0} \approx (1-\sigma)\left(1 - s^L_{i,t_0}\right)\left(\ln \frac{W_{i,t}}{W_{i,t_0}} - \ln \frac{R_{i,t}}{R_{i,t_0}} - g^A_{i,t_0,t}\right)$$

$$+ \frac{\left(1 - s^L_{i,t_0}\right)}{1 - \Gamma_{i,t}} \left(\ln \Gamma_{i,t} - \ln \Gamma_{i,t_0}\right)$$

Decomposing the wage bill

- Then we have

$$\text{Substitution effect}_{i,t_0,t} = (1-\sigma)\left(1 - s^L_{i,t_0}\right)\left(\ln \frac{W_{i,t}}{W_{i,t_0}} - \ln \frac{R_{i,t}}{R_{i,t_0}} - g^A_{i,t_0,}\right.$$

- where $g^A_{i,t_0,t}$ is the growth rate of A^L_t / A^K_t
- The residual is the change in task content

$$\text{Change task content}_{i,t_0,t} = \ln s^L_{i,t} - \ln s^L_{i,t_0} - (1-\sigma)\left(1 - s^L_{i,t_0}\right)\left(\ln \frac{W_{i,t}}{W_{i,t_0}}\right.$$

Decomposing the wage bill

- Last, under the assumption that sector can either experience Displacement or Reinstatement, we have that

$$\text{Displacement }_{t-1,t} = \sum_{i \in \mathcal{I}} \ell_{i,t_0} \min \left\{ 0, \frac{1}{5} \sum_{\tau=t-2}^{t+2} \text{ Change task content }_{i,\tau-1,\tau} \right\}$$

$$\text{Reinstatement }_{t-1,t} = \sum_{i \in \mathcal{I}} \ell_{i,t_0} \max \left\{ 0, \frac{1}{5} \sum_{\tau=t-2}^{t+2} \text{ Change task content }_{i,\tau-1,\tau} \right\}$$