Derivation of Other Weights

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a. MTE needs no introduction
b. $ATE = \int_0^1 E(\Delta | X = x, U_D = u)\, du$
c. $TT(X, P(Z), D = 1) = \frac{1}{P} \int_0^{P(Z)} E(\Delta | X, U_D = u)\, du$
d. $LATE = \dfrac{\int_{P(Z')}^{P(Z)} E(\Delta | X, U_D = u)\, du}{P(Z) - P(Z')}$

(Note it is instrument dependent–depends on Z and the particular support of $P(Z)$).

$$\Delta^{TT}(X, D = 1) = \int_0^1 \left[TT(X, P(Z), D = 1) \right] dF(P | X, D = 1)$$

Bayes' Rule derivations

$$dF\left(p|X=x,D=1\right)\Pr\left(D=1|X=x\right)=\Pr\left(D=1|X=x,P\left(Z\right)=p\right)dF_P\left(p|X\right)$$

$$\therefore \frac{1}{\Pr\left(D=1|X\right)}\int_0^1\left(\frac{1}{P}\right)\left[\int_0^{P(Z)}E\left(\Delta|X,U_D=u\right)du\right]PdF_P\left(P|X\right)$$

$$=\frac{1}{\Pr\left(D=1|X\right)}\int_0^1\int_0^{P(Z)}E\left(\Delta|X,U_D=u\right)dudF_P\left(P|X\right)$$

From integration by parts

$$\begin{aligned}\Pr(D=1|X=x) &= E(P|X=x)\\ &= \int_0^1 (1-F_P(t))\,dt\end{aligned}$$

$$\int_0^1 PF(P|X)\,dP = \underbrace{(1-F(P|X))\,P|_0^1}_{=0} + \int_0^1 [1-F(t)]\,dt$$

$$\int_0^1 \int_0^{P(Z)} E(\Delta|X, U_D = u)\,dudF_P(P|X)$$
$$= \int_0^1 \int_0^1 E(\Delta|X=x, U_D=u)\,\mathbf{1}(u \leq P)\,dudF_P(P|X)\,dP$$

Reverse order of integration:

$$= \int_0^1 E\left(\Delta | X = x, U_D = u\right)\left(1 - F_P\left(u|x\right)\right) du$$

$\therefore$weight is:

$$\frac{1 - F_P\left(u|x\right)}{\int \left[1 - F_P\left(u|x\right)\right] du} > 0$$

Lots of weight placed on low u people: low u people are the ones likelyto get treatment.

The Policy Relevant Treatment Effects

$$
\begin{aligned}
E(V(Y \mid baseline) &= \int_0^1 E(Y \mid P(z) = p) dF(P) \\
&= \int_0^1 \left[\int_0^1 E(V(Y_1) \mid U = u) \mathbf{1}(0 \le P \le U) \right. \\
&\quad + \left. E(V(Y_0 \mid U = u) \mathbf{1}(U \le P \le 1) \right] du dF(P)
\end{aligned}
$$

$U \le P$

Reverse the limits

$$
= \int_0^1 \left[E(V(Y_1) \mid U_D = u) F_P(U) + E(V(Y_0 \mid U_D = u)(1 - F_P(U)) \right] du
$$

Change in utility: $F_P \quad F_{P*}$

$$E(V(Y \mid \mathit{baseline}) - E(V(Y) \mid *) = \int_0^1 E(V(Y_1) - V(Y_0) \mid U) \left[F_P(u) - F_{P*}(u)\right] du$$

We can normalize to get per capita magnitudes.