Yitzhaki Derived the Weights Used by the Proponents of LATE but Without Citation

The Weights Have a Lot of Intuition

James J. Heckman
THE UNIVERSITY OF CHICAGO

Econ 312, Spring 2021

5.5 Yitzhaki Weights as a Version of Theil Weights (1950)

- Another intuition behind the Yitzhaki weights (Yitzhaki, 1989).
- Sample Size I (finite sample)
- Take Case $P(Z) = J(Z)$ (propensity score is the instrument).

- Recall the Theil (1950) formula for *OLS*

$$Y = X\beta + \varepsilon \qquad E(\varepsilon \mid X) = 0$$

- *OLS* is weighted average of all pairwise *OLS* slopes.

$$\hat{\beta}_{OLS} = \frac{\sum_{1 \le i \le j \le I} (X_j - X_i)(Y_j - Y_i)}{\sum_{1 \le i \le j \le I} (X_j - X_i)^2}$$

- Form pairwise slopes (Theil)

$$b_{ji} = \frac{Y_j - Y_i}{X_j - X_i} \mathbf{1}[X_j \neq X_i]$$

$$\hat{\beta}_{OLS} = \sum_{1 \le i \le j \le I} b_{ji}\omega_{ji} \qquad \omega_{ji} = \frac{(X_i - X_j)^2}{N\sigma_X^2}$$

- Weights are obviously positive on each b_{ji} if $X_i \neq X_j$
- Yitzhaki orders the X and produces a pairwise representation of OLS
- $X_1 < X_2 < \cdots < X_I$ (neglect ties)
- Concomitants $Y_1, \ldots, Y_I$

- Slopes for ordered data

$$b_i = \frac{Y_i - Y_{i-1}}{X_i - X_{i-1}} \mathbf{1}\left[X_i \neq X_{i-1}\right]$$

- Substitute into formula for OLS and collect terms on the b_i

$$\hat{\beta}_{OLS} = \sum_{i=1}^{I} b_i \omega_i \qquad \omega_i = \left(\frac{N-i}{N}\right) \frac{E\left(X - \bar{X} \mid X > x_i\right.}{\sigma_X^2}$$

- $(N-i)/N$ is proportion of X bigger than x_i
- Obviously weights are positive.
- They place more weight on the center of the distribution of the X.