Economics 312

Problem Set 2

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(1) Consider the distribution of random variable Q as a function of parameter θ: $F(q \mid \theta)$. Define identification of θ: consider both point and set identification.

(2) Consider a Generalized Roy model of schooling:

$$
\begin{aligned}
&Y_1 = \text{earnings (PV) if go to college} \\
&Y_0 = \text{earnings (PV) if go to high school} \\
&Y_1 = \mu_1(X) + U_1 \\
&Y_0 = \mu_0(X) + U_0 \\
&C = \phi(Z) + U_c \quad \text{“Tuition costs”} \\
&S = 1\big(Y_1 - Y_0 - C > 0\big)
\end{aligned}
$$

$$(X, Z) \perp\!\!\!\perp (U_1, U_0, U_c)$$

$\mu_1(X)$, $\mu_0(X)$ are nondegenerate functions of X; $\phi(Z)$ is a nondegenerate function of Z.

Assume:

$$(U_1, U_0, U_c) \sim N(0, \Sigma)$$

where

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{10} & \sigma_{1c} \\ \sigma_{01} & \sigma_{00} & \sigma_{0c} \\ \sigma_{c1} & \sigma_{c0} & \sigma_{cc} \end{pmatrix}; \quad \text{positive definite.}$$

Compute the following objects:

(a) $\Pr(S = 1 \mid X, Z)$; (How does this relate to the propensity score?)

(b) $E\Big(Y_j \mid S = 1, X, Z\Big), \quad j = \{0, 1\}$

(c) $\text{Var}\left(Y_j \mid S = 1, X, Z\right) \quad j \in \{0, 1\}$

(d) $\text{Cov}\left(Y_1, Y_0 \mid S = 1, X, Z\right)$

(e) $\psi\Big(Y_1 - Y_0 \mid S = 1\Big)$

where ψ is the quantile function.

(f)

$$\begin{array}{ll} f\Big(Y_1 \mid S = 1, X, Z\Big) & f\Big(Y_1 \mid S = 0, X, Z\Big) \\ f\Big(Y_0 \mid S = 0, X, Z\Big) & f\Big(Y_0 \mid S = 1, X, Z\Big) \end{array}$$

(g) MTE function: How does this relate to the willingness to pay?

(h) How do (2)b, (2)c and (2)e depend on $\Pr(S = 1 \mid X, Z)$?

(i) Compute LATE, ATE, TOT, TUT, PRTE and the subjective treatment effect for this model.

(j) How would you assess if an economy governed by this model is meritocratic or not?

(3) Define $Y = SY + (1-S)Y_0$ for a sample of iid observations on Y, S, X, Z.

(a) Prove identification or nonidentification of the following parameters:

$$\mu_0(X),\ \mu_1(X),\ \mu_\phi(Z),\ \Sigma.$$

Give conditions for identification.

(b) Express LATE, MTE, $E(Y_1|S=0,X,Z), E(Y_1-Y_0|S=1,X,Z)$, ATE as functions of $\Pr(S=1 \mid X,Z)$.

(c) What is the relationship between MTE and LATE?

(4) Modify your answer to (2) for the special case of $U_C \equiv 0$,

(This called the Extended Roy model X. See Heckman and Vytlacil, 2007a,b.)

(5) Suppose that

$$Y_j^* = Y_j + \varepsilon_j$$
$$C_j^* = C + \varepsilon_C$$

where $(\varepsilon_1, \varepsilon_0, \varepsilon_C)$ are shocks realized after schooling decision S is made and

$$(\varepsilon_1, \varepsilon_0, \varepsilon_C) \perp\!\!\!\perp (X, Z, U_0, U_1, U_C).$$

(a) Answer questions (2), (3) and (4) for this model but also consider the variances of ε_1, ε_0 and ε_C in your answer. Define the *ex ante*

and *ex post* treatment effect for ATE, LATE, MTE, TOT, TUT, the subjective treatment effect, PRTE, $E(Y_0|S=1,X,Z)$.

(b) How can you test for information updating (i.e., do agents update their information sets)?

(c) As the variance of $\varepsilon_1, \varepsilon_0, \varepsilon_C \to \infty$ what are the properties of matching estimators for the treatment parameters listed in (2).

(6) Estimate the parameters of these models and MTE defined above for the three datasets posted on the Canvas page.

References

Heckman, J. J. and E. J. Vytlacil (2007a). Econometric evaluation of social programs, part I: Causal models, structural models and econometric policy evaluation. In J. J. Heckman and E. E. Leamer (Eds.), *Handbook of Econometrics*, Volume 6B, Chapter 70, pp. 4779–4874. Amsterdam: Elsevier B. V.

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