

# Gittins Index, Pandora's Box, and Miller's Model of Learning and Labor Market Turnover (Excerpt)

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Econ 350, Spring 2022

## **Applications:** Job Search, Pandora's Box and Miller **Job Search:**

(One Spell Model)

- Jobs last forever, no learning.
- Only issue is when to stop.
- This is a degenerate, single arm bandit.
- $c$  = cost of playing machine.
- $X_t$  = reward on  $t^{\text{th}}$  trial.
- No learning implies  $X_t$  is independent and identically distributed with cdf  $F(x)$  known to agent.

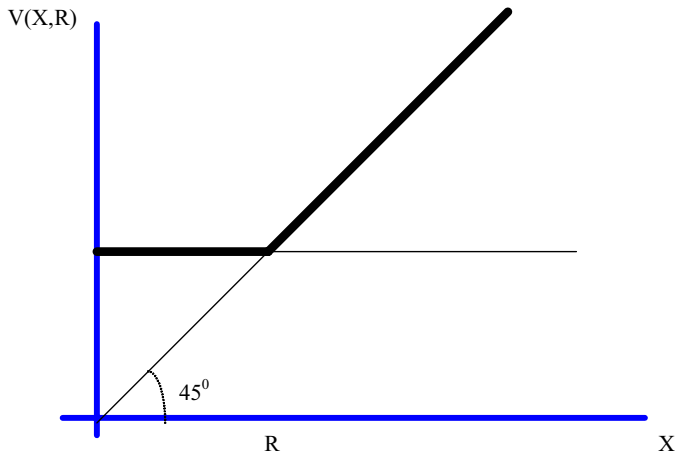
$$V(X) = \max \left[ X, -c + \beta \int_0^{\infty} V(y) dF(y) \right] \quad (1)$$

**Optimal policy:** Search if  $X \in [0, R]$  reservation price is  $R$  :

$$V(X) = \begin{cases} R = -c + \beta \int_0^{\infty} V(y) dF(y) & \text{if } X < R \\ X & \text{if } X \geq R \end{cases} \quad (2)$$

$R$  is the value that makes a person indifferent between stopping and continuing.

This figure graphs the functional equation (1) and it reveals that the optimal solution is of the form of (2) (see e.g. Sargent's textbook Ch. 6):



## Solving for reservation wage:

using (2) we convert (1) into an ordinary equation in the reservation wage:

$$R = -c + \beta \left[ \int_0^R R dF(x) + \int_R^\infty X dF(x) \right]$$
$$R \left( \int_0^R dF(x) + \int_R^\infty dF(x) \right) = -c + \beta \left[ \int_0^R R dF(x) + \int_R^\infty X dF(x) \right]$$

or

$$(1 - \beta)R \int_0^R dF(x) = -c + \int_R^\infty (\beta X - R) dF(x)$$

## Solving for reservation wage:

adding  $(1 - \beta)R \int_R^\infty dF(x)$  to both sides we have

$$R = \frac{-c}{1 - \beta} + \frac{\beta}{1 - \beta} \int_R^\infty (X - R) dF(x) \quad (3)$$

with unique solution for  $R$ . To see this, define

$$g(R) = \frac{-c}{1 - \beta} + \frac{\beta}{1 - \beta} \int_R^\infty (X - R) dF(x)$$

## Solving for reservation wage:

with

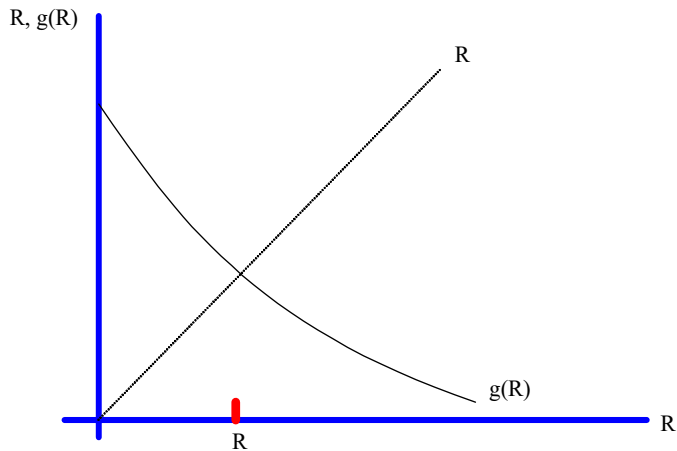
$$g'(R) = -\frac{\beta}{1-\beta} [1 - F(R)] < 0$$

$$g''(R) = \frac{\beta}{1-\beta} F'(R) > 0$$

the optimal reservation wage satisfies, from (3)

$$R^* = g(R^*)$$

## Solving for reservation wage:





## **Pandora** (Weitzmann, *Econometrica*, May, 1979)

- $N$  different occupations (or  $N$  college majors) each yielding an unknown reward
- Each occupation has its own search cost  $c_i$  and independent probability distribution  $F_i$  for the reward  $X_i$
- Occupations are sampled sequentially, in whatever order is desired. When it has been decided to stop searching, only one occupation is accepted, the maximum sampled reward.
- Under this formulation, what sequential search strategy maximizes expected present discounted value?

- **Pandora's problem:** At each state Pandora must decide whether or not to open a box. If she chooses to stop searching, Pandora collects at that time the maximum reward she has thus far uncovered. Should Pandora wish to continue sampling, she must select the next box to be opened and pay  $c_i$
- **Solution:**
  - Compute Reservation Price for each box
  - Try jobs with highest  $R_i^*$ 's
  - Keep trying until one gets  $X_t(i^*) \geq \max_i \{R_i^*\}$   
(i.e. keep going until you get a realization  $\geq$  sampled values).

**Proof:** Compute reservation prices for each occupation  $R_i(x_i)$ . Consider occupation  $i$ . If occupation  $i$  is tried, then  $X_i$  is known. For an  $i$ -th occupation with known trial,

$$V_i(X_i, R) = \max [R, X_i] .$$

Therefore,  $R_i(X_i) = X_i$ . Thus, the reservation price for a sampled occupation is just the wage realized.

For an untried occupation,

$$\begin{aligned}\tilde{V}_i(R) &= V_i(-c_i, R) \\ &= \max \left[ R, -c_i + \beta \int_0^{\infty} V_i(y, R) dF_i(y) \right] \\ &= \max \left[ R, -c_i + \beta \left( F_i(R)R + \int_R^{\infty} y dF_i(y) \right) \right]\end{aligned}$$

Reservation wage is the smallest value of  $R_i$  such that

$$R_i = -c_i + \beta R_i F_i(R_i) + \beta \int_R^\infty y dF_i(y)$$
$$\therefore R_i = -\frac{c_i}{1-\beta} + \frac{\beta}{1-\beta} \int_{R_i}^\infty (X - R_i) dF_i(y)$$

as in the above search model.

Consider a binomial case with  $\beta = 1$  (no discounting):

$$X_i = 0 \quad \text{wp.} \quad (1 - p_i)$$

$$X_i = r_i \quad \text{wp.} \quad p_i$$

and  $R_i$  satisfies

$$c_i = \int_{R_i}^{\infty} (X - R_i) dF_i(X)$$

2 cases:

- If  $R_i = 0$ , then

$$c_i = \int_0^{\infty} X dF_i(X) = p_i r_i.$$

No reason for this to be satisfied in general. Therefore,

- We assume  $R_i \in (0, r_i]$

$$c_i = \int_{R_i}^{\infty} (X - R_i) dF_i(X) = p_i r_i - R_i p_i$$

$$\therefore R_i = \frac{p_i r_i - c_i}{p_i}$$

- Thus, take two projects with equal expected value ( $p_i r_i - c_i$ ). The project with lower expected probability of reward is the one to try (go for riskier project). Why? Suppose costs are the same. Then  $r_i$  is higher, since  $p_i$  is lower.
- Suppose that rewards are the same. But costs being lower implies that  $p_i$  is lower (trying less costly combo).
- No further learning.