Using A Cross Section to Estimate Life Cycle Labor Supply

James J. Heckman

Econ 350, Spring 2022

- Lifetime Utility for:
- $U = U(X, L, \varepsilon)$ quasiconcave in (X, L). ε is unobservable (to economist/econometrician).
- Price of goods is 1: Permanent wage per unit time is w.

$$0 \leq L \leq 1 : \quad \text{one unit of lifetime}$$

- $W =$ maximum amount person can earn. Real Assests $= A$. Assets taken as exogenous to the process.
- Assume $r = 0$. Time is a perfect substitute so no time preference.

Question 1

- Does A Person Ever Work?

$$\frac{U_2}{U_1} = \underbrace{\frac{U_2(A,1,\varepsilon)}{U_1(A,1,\varepsilon)}}_{\text{reservation wage}} \geq W$$

if "$\geq$" then no.

THE UNIVERSITY OF CHICAGO

- Define an index for

$$Z_1 = \frac{U_2}{U_1} - W,$$

- difference between reservation wage and market wage, if $Z_1 \geq 0$, $D = 0$ person doesn't work if $Z_1 < 0$, $D = 1$ person works $\left(\frac{\partial}{\partial A}\left[\frac{\partial U_2}{\partial U_1}\right] \geq 0\right)$.
- Assume $\varepsilon W, A$. This may be questionable in some contexts—less so in the current one.

- $\mathcal{E}$ is support of ε, domain of *cdf* of ε.
- The restriction $Z_1 \geq 0$ implies (for given W, A) a partition in support of ε so that for $\varepsilon \in \mathcal{E}_D \quad , \quad Z_1 \geq 0 \qquad$ (given W, ε)
- $\mathcal{E}_D = \left\{ \varepsilon \,\middle|\, Z_1 = \frac{U_2(A,1,\varepsilon)}{U_1(A,1,\varepsilon)} - W \geq 0 \right\}$
- $\varepsilon_1 = \varepsilon - \varepsilon_0$ (support of the original set) may be disconnected regions of the real line.

- Fraction of population not working $=$
- $\Pr(D = 0 \mid W, A) = \int_{\mathcal{E}_D} dF(\varepsilon)$ when F is *cdf*.
- As an example, let

$$\frac{U_2}{U_1}(A, 1, \varepsilon) = \alpha_0 + \alpha_1 A + \varepsilon$$

Suppose higher $\varepsilon \implies$ higher value on time.

- Normalize.
- $\alpha_1 > 0$ if leisure is normal (demand price for leisure goes up).

$$\begin{aligned} \Pr(D &= 0 \mid W, A) = \Pr(\alpha_0 + \alpha_1 A + \varepsilon \geq W | W, A) \\ &= \Pr(\varepsilon \geq W - \alpha_0 - \alpha_1 A | W, A) \end{aligned}$$

- Partition of $\mathcal{E}$ is into two exhaustive regions:

- $W - \alpha_0 - \alpha_1 A$ $\mathcal{E}$
- Suppose ε is uniform

$$K > \varepsilon > 0$$

$$f(\varepsilon) = \frac{1}{K} d\varepsilon \qquad \text{(Lebesgue measure)}$$

- Prob of non participation given W, A is

$$\Pr(D = 0|W, A) = \int_{W-\alpha_0-\alpha_1 A}^{K} \frac{1}{K} d\varepsilon$$

(implicit restriction: $0 \leq W - \alpha_0 - \alpha_1 A \leq K$).

- Support of W lies in region given by inequality

$$\Pr(D = 1 \mid W, A) = \frac{K - (W - \alpha_0 - \alpha_1 A)}{K} = 1 + \frac{\alpha_0 + \alpha_1 A - W}{K}.$$

- How sensitive, how defensible, *etc.*
- Regression of micro data (Recorded at morgues or taken from obituaries - Does the person work?)

$$D_i, \qquad i = 1, \ldots$$

- Assume *iid* corpses (given W, A) - use subscripts
- For persons:

$$E(D_i|W_i, A_i) = \frac{W_i - \alpha_0 - \alpha_1 A_i}{K}.$$

- Regression of participation (ever) on W_i, A_i lets us estimate K, α_0, α_1.

- Interpretation of wage effect: As taste parameter -
- Also: Can estimate dist of lifetime reservation wages (given A)

$$R = \alpha_0 + \alpha_1 A + \varepsilon \quad \text{Reservation wages}$$

$$0 \leq \varepsilon \leq K$$

$$f(R) = \frac{1}{K}$$

where $\alpha_0 + \alpha_1 A \leq R \leq K + \alpha_0 + \alpha_1 A$ (translated uniform)

- Therefore, LFPR issue silent on question of hours of work for working women.

Question 2

- Suppose the person works **sometime** in her lifetime.
- Interior solve case: we solve out a labor supply for

$$\frac{U_2}{U_1}(A + W(1 - L), L, \varepsilon) = W$$

only valid for $\varepsilon \in \mathcal{E}_1$(subset of support) for given W, A, $\Lambda_1 \in (\mathbf{W}, \mathbf{A}, \varepsilon)$, set of values so that $D_{i=1}$ (defines an implicit restriction on those values).

- Linearize to reach.
- Fraction of lifetime that a person works:
  $(1 - L) = h = H(W, A, \varepsilon)$.
- Linearization (*) $H = \beta_0 + \beta_1 w + \beta_2 A + \beta_3 \varepsilon$

$$(\text{For } W, A, \varepsilon \in \Lambda_1).$$

- The effect β_1, β_2 (Hicks-Slutsky income-subs effective.
- Note: R is that variable W so that $H = 0$

$$0 = \beta_0 + \beta_1 R + \beta_2 A + \beta_3 \varepsilon$$

- Therefore

$$R = -\frac{\beta_0}{\beta_1} - \frac{\beta_2}{\beta_1} A - \frac{\beta_3}{\beta_1} \varepsilon$$

$$\alpha_0, \qquad \alpha_1 = -\frac{\beta_2}{\beta_1}, \qquad \beta_3 = -\beta_1.$$

- Thus in this specification we can conclude that

$$h = \beta_1 (w - R) = -\beta_1 Z_1$$

hours of work are given by difference between market wage and reservation wage.

- What is the compensated substitution effect in this model? (uncompensated effects are constant).

$$
\begin{array}{ll}
\beta_1 = S + (1 - L)\beta_2 & \beta_2 < 0 \\
S = \beta_1 - (1 - L)\beta_2 & \text{as } \uparrow \text{, subs effect} \\
\beta_1 = S \text{ at } h = 0 & \text{becomes more}
\end{array}
$$

- **Suppose** that all women work (This is a restriction on the support of w, A, ε

$$\mathcal{E}_1 = \mathcal{E},\ \mathcal{E}_0 = \emptyset)$$

(we assume we can/is more).

- From labor force participation, we have fraction of lifetime that a woman can work.
- Regression estimates of $(*)$ are consistent - estimate labor supply of women - β_1 β_2 (income subs. effects).
- Suppose we live in a steady state environment.
- If $r > 0$, women will bunch all their work at beginning of life.
- Fraction of all women (with W, A) in L.F. is h.
- Therefore from **participation data**, we can estimate income and subs effects.

- DeFinetti type argument. See Lindley (1977).
- Therefore we have an interpretation of the estimates –
- Note: If all women work, we observe wage rate for all women – no censoring, **etc.**

Question 3

- Suppose some women never work –
- Censored Sample Problem: We observe some women only if they pass through filter:

$$Z_1 < 0.$$

- Hours of work for working is given by

$$E(h|w, A, h > 0) = E(h|w, A, Z_1 < 0).$$

- For special example:

$$= \beta_0 + \beta_1 w + \beta_2 A + \beta_3 E(\epsilon | \epsilon < w - \alpha_0 - \alpha_1 .$$

- Note:

$$\begin{aligned} F(\epsilon | \epsilon < w - \alpha_0 - \alpha_1 A, w, A) &= \frac{\left(\frac{1}{K}\right)}{\left(\frac{w - \alpha_1 - \alpha_1 A}{K}\right)} \quad \Pr(\epsilon < w - \alpha_0 - \alpha_1 A | w, A) \\ E(\epsilon | \epsilon < w - \alpha_0 - \alpha_1 A) &= \frac{\int_0^{w - \alpha_0 - \alpha_1 A} \frac{\epsilon}{K} d\epsilon}{\left(\frac{w_0 - \alpha_0 - d, A}{K}\right)} \\ &= \frac{1}{2}(w - \alpha_0 - \alpha_1 A). \end{aligned}$$

- Suppose we fit regressions on a subsample of working women.

- $E(h|w, A, h > 0) = \beta_0 + \beta_1 w + \beta_2 A + \frac{\beta_3}{2}(w - \alpha_0 - \alpha_1 A)$.
- Collect terms

$$
\begin{aligned}
&= \left(\beta_0 - \frac{\beta_3\alpha_0}{2}\right) + \left(\beta_1 + \frac{\beta_3}{2}\right) w + \left(\beta_2 - \frac{\beta_3}{2}\alpha_1\right) A \\
&= \beta_0 - \frac{\beta_3}{2}\left(-\frac{\beta_0}{\beta_1}\right) + \frac{\beta_1}{2} w + \beta_2 + \frac{\beta_1}{2}\left(-\frac{\beta_2}{\beta}\right) \\
&= \frac{\beta_0}{2} + \frac{\beta_1}{2} w + \frac{\beta_2}{2} A
\end{aligned}
$$

- Bids towards zero in absolute value.

- Example of censoring: Raise wage
- Change within sample dist. of $\mathcal{E}$ (raises mean) but $\beta_3 < 0$.
- Therefore $\widehat{\beta}_1$ down bias.
- Two effects:

- The first is Hicks-Slutsky effect: β_1 (holds sample composition constant); and consumed effect sample: $\beta_3 \frac{\partial}{\alpha w} E(\epsilon | \epsilon < w - \alpha_0 - \alpha_1 A)$ (effect on conditional mean).
- Similar effects for A. // $\beta_2 < 0$.
- Don't estimate a Hicks Slutsky subs. effect in this sample using only workers.

- (This is sample: People who over worked). More generally: we estimate projection of $E(\epsilon|\epsilon < w - \alpha_0 - \alpha_1 A)$ onto w, A.
- No structural interpretation (in terms of Hicks-Slutsky), but perfectly valid prediction equations - even out of sample.

Question 4

- What is estimated in A Cross Section. Regress LFPR on w, A
- Assume these data are available for everyone.

$$
\begin{aligned}
E(h|w,A) &= E(h|w,A,h>0)\Pr(h>0|w,A) \\
&= \frac{(\beta_0+\beta_1 w+\beta_2 A)}{2}\left[\frac{1}{K}\left(w+\frac{\beta_0}{\beta_1}+\frac{\beta_2}{\beta_1}A\right)\right]
\end{aligned}
$$

- Spec. nonlinear: estimate neither Hicks-Slutsky nor other effects.

- This function is the agg. labor supply curve $=$ fraction of population (with $w\&A$) at work in any period.
- $=$ prob. of finding a woman at work in a "random" draw (random sample of population).
- His lessons for economist so called M in his work assumes that $\Pr(h > 0) = 1$.
- Therefore estimates from cross section LFPR are income and substitution effects.
- Since $E(h|w, A, h > 0) = E(h|w, A)$ in this case.

- Ben Porath: Assumes a 2 point distribution
- Some people work all the time.
- Some people never work

$$E(h|w, A, h > 0) = 1.$$

- Therefore, in cross section we estimate part

$$\Pr(h > 0).$$

- This contrast recurs throughout literature.

- We can change the above analysis so that U refers to a single period (this is the "2nd generation approach" in Killingsworth) – this analysis allows one to contrast participation **% US** hours of work – then if "period" is an "instant", then we have that labor force participation regressions do not estimate Hicks-Slutsky parameters.