Understanding the Heterogeneity of Intergenerational Mobility across Neighborhoods:

A Case Study of Denmark

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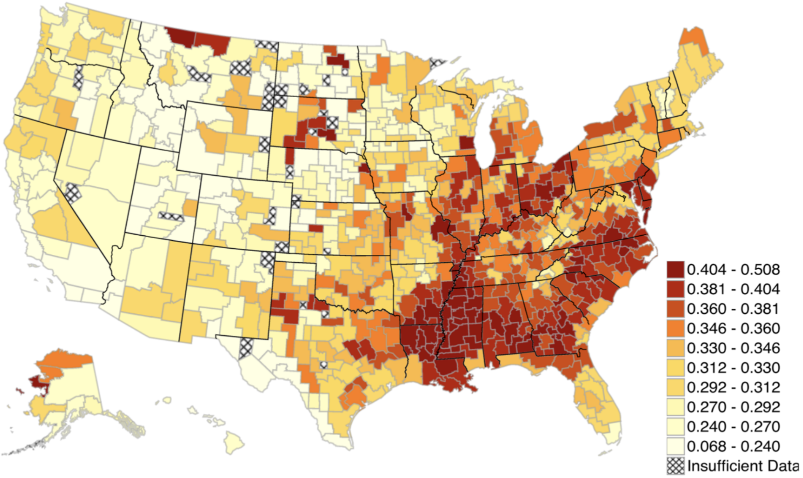
Background: Literature

- Recent studies has found large heterogeneity in integenerational mobility across geographic regions (e.g., Cooper et al. 1992, Hertz 2008, Davis and Mazumder 2018) and more granular neighborhoods (Chetty et al. 2014, 2018)
- **Major focus:** The intergenerational elasticity (IGE) between child income and parent income across neighborhoods, $\{\beta_n \mid n \in \mathcal{N}\}$, from

$$Y_{in}^{c} = \alpha_n^{IGE} + \beta_n^{IGE} Y_{in}^{p} + \varepsilon_{in}$$

"Where is the Land of Opportunity?" (Chetty et al. 2014)



Background: Literature

Less focus on...

- The intercepts α_n^{IGE} and their relationship with β_n^{IGE}
- Statistical tests accounting for multiplicity of hypotheses and equality of coefficient estimates
- Statistical analyses decomposing true heterogeneity from sampling variance
- Residualizing these coefficients with various other family characteristics that vary across neighborhoods

Background: Policy Prescriptions

- Complementary work has found causal effects of neighborhood exposure on later-life outcomes (Wodtke, Harding, and Elwert 2011; Chetty, Hendren, and Katz 2016; Chetty and Hendren 2018)
- This suggests the **“power of place”** in determining child outcomes and a set of interesting policy proposals:
  - Housing vouchers for disadvantaged families?
  - Re-distributing resources from low- to high-poverty neighborhoods?
- **Considerations for implementing these policies at scale?**

Background: Denmark as a Case Study

- Denmark is one of the most generous welfare states in the world, with generous means-tested social assistance, universal insurance programs, and free college tuition and fees
  - High marginal tax rates for upper income brackets; high transfers to lower income brackets
  - Strong place-based redistribution policies across neighborhoods in school expenditure and local public goods
- A natural case study of **scaling up** previously mentioned policy proposals

Motivation

- But this equalization may induce strong **neighborhood sorting** among families in. . .
  - Income
  - Education
  - Race and immigrant status
  - Crime

  . . . which may result in inequality across neighborhoods
- A neighborhood's *composition of family characteristics* may vary across neighborhoods and drive differences in mobility, rather than the neighborhood's *location* per se
  - A phenomena combining aspects of Durlauf and Seshadri (2018) and Becker et al. (2018)?
- Sorting would affect the degree of heterogeneity and interpretation of neighborhood-level mobility statistics

Key Questions and Objectives

1. How heterogeneous is the IGE across different neighborhoods in Denmark? Predicted child income?
2. To what extent are neighborhood differences in mobility explained by family characteristics and sorting? Is there an irreducible "neighborhood effect" after controlling for various characteristics? Do "types" of neighborhoods emerge?
3. What role do individual transfers and taxes play in neighborhood heterogeneity? Local public goods?

Outline

Outline

Sample

Danish administrative records of the 1973-1983 birth cohorts and their parents

- **Units:** Parents defined as birth/adoptive parents from birth registers (need not live with child)
- **Income:** Gross income before transfers and taxes (2010 U.S. $) available between 1980-2018
  - Child income defined as log average income between ages 30-45 (whenever available)
  - Parent income defined as log average sum of father and mother income when the child is ages 0-17 (whenever available)
- **Neighborhoods:** Match children to the parish they lived in for the longest duration during childhood
- **Family characteristics:** Link to population, household, income, hospital, and crime registers

Sample

- Remove observations that...
  - Contain missing data
  - Lie in extreme 0.5% tails of child or parent income distribution or with missing data
  - Reside in parishes with fewer than 25 families in the sample
- **Final sample size:** 537,895 families

Table: Sample Sizes of Neighborhoods

N	Mean	S.D.	Min.	p10	p25	p50	p75	p90	Max.
1,923	279.7	338.3	25	41	69	148	352	698	3,343

This table reports summary statistics of the number of families matched with parishes in the main sample.

Outline

Outline

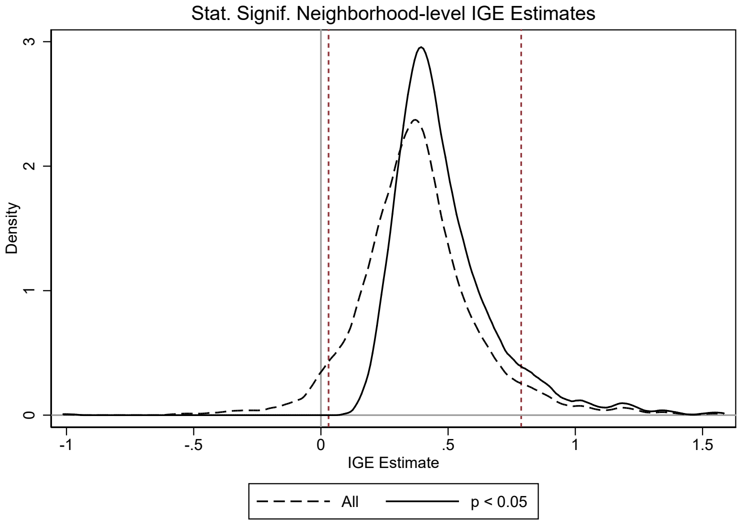
Estimating Neighborhood-level IGE and Intercepts

- $i \in \mathcal{I}$ indexes families; $n \in \mathcal{N}$ indexes neighborhoods
- Estimate neighborhood-level fixed effects (intercepts) and neighborhood-level fixed effects interacted with parent income (IGEs) for all ∼2,000 parishes in the regression

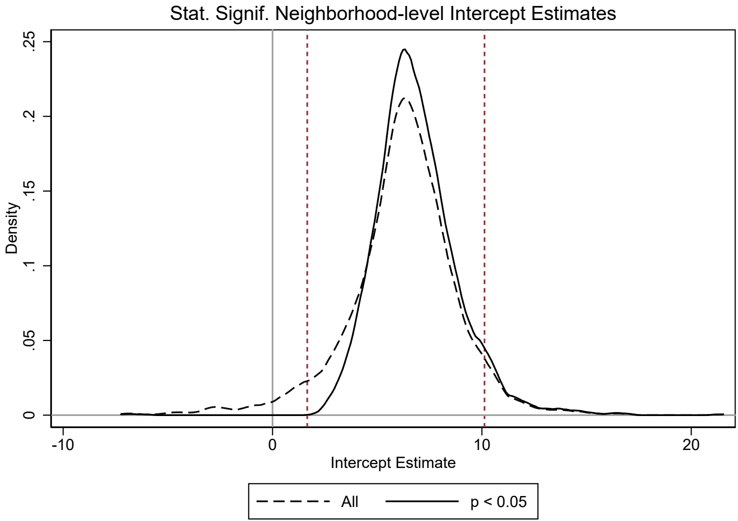
$$Y_{in}^{c} = \alpha_n^{IGE} + \beta_n^{IGE} Y_{in}^{p} + \varepsilon_{in}$$

  where ε_{in} is assumed to be i.i.d. across i and n
- Interested in the distribution functions $F(\alpha_n^{IGE})$, $F(\beta_n^{IGE})$, and $F(\alpha_n^{IGE}, \beta_n^{IGE})$

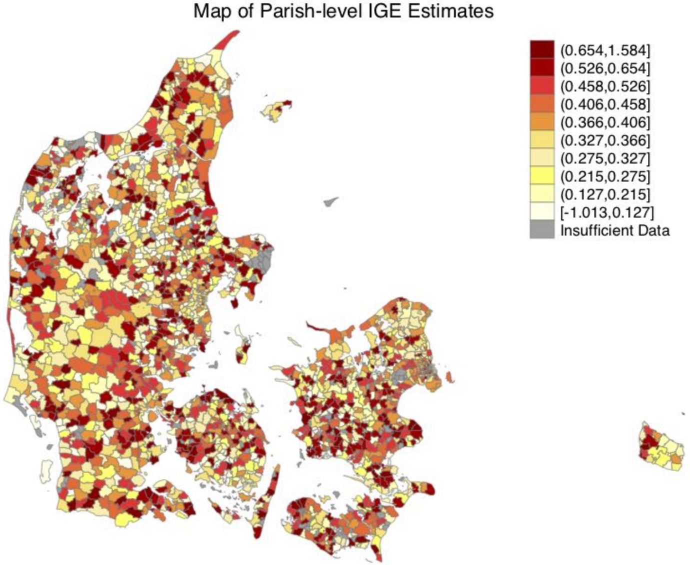
Distribution of Neighborhood-level IGEs β_n^{IGE}



Distribution of Neighborhood-level Intercepts α_n^{IGE}



Is Denmark *the* Land of Opportunity?



Statistical Significance of Neighborhood-level Coefficients

Null Hypothesis H_0	Test Result	
Panel A: Single Tests	$\lvert\{n \in \mathcal{N} \mid p_n < 0.05\}\rvert$	$\lvert\{n \in \mathcal{N} \mid p_n < 0.01\}\rvert$
$\alpha_n^{IGE} = 0$	1,653 (86.0%)	1,428 (74.3%)
$\beta_n^{IGE} = 0$	1,315 (68.4%)	1,010 (52.5%)
Panel B: Multiple Tests	$\lvert n \in \mathcal{N} \mid p_n < 0.05\rvert$	$\lvert\{n \in \mathcal{N} \mid p_n < 0.01\}\rvert$
$\{\alpha_n^{IGE} = 0 \mid n \in \mathcal{N}\}$	**1,635 (85.0%)**	1,384 (72.0%)
$\{\beta_n^{IGE} = 0 \mid n \in \mathcal{N}\}$	**1,219 (63.4%)**	893 (46.4%)
$\{\alpha_n^{IGE} = 0 \mid n \in \mathcal{N}\} \bigcup \{\beta_n^{IGE} = 0 \mid n \in \mathcal{N}\}$	α_n^{IGE}: 1,613 (83.4%)	α_n: 1,355 (70.5%)
$\{\alpha_n^{IGE} = 0 \mid n \in \mathcal{N}\} \bigcup \{\beta_n^{IGE} = 0 \mid n \in \mathcal{N}\}$	β_n^{IGE}: 1,247 (64.9%)	β_n: 923 (48.0%)

This table reports results from hypothesis tests on collections of neighborhood-level mobility coefficient estimates. Columns 2 and 3 report number of neighborhoods $n \in \mathcal{N}$ whose p-values, p_n, fall below 0.05 and 0.01, respectively.

Outline

Existence of Nbhd.-level Heterogeneity in Coefficients

Null Hypothesis H_0	F-statistics	p-value	Test Result
$\alpha_n^{IGE} = 0,\ \forall n \in \mathcal{N}$	$F_{(1923,534049)} = 28.086$	0.000	Reject
$\alpha_n^{IGE} = \alpha,\ \forall n \in \mathcal{N}$	$F_{(1922,534049)} = 1.663$	0.000	Reject
$\beta_n^{IGE} = 0,\ \forall n \in \mathcal{N}$	$F_{(1923,534049)} = 14.220$	0.000	Reject
$\beta_n^{IGE} = \beta,\ \forall n \in \mathcal{N}$	$F_{(1922,534049)} = 1.642$	0.000	Reject
$(\alpha_n^{IGE}, \beta_n^{IGE}) = (\alpha, \beta),\ \forall n \in \mathcal{N}$	$F_{(3844,534049)} = 1.863$	0.000	Reject

This table reports results from joint hypothesis tests on collections of neighborhood-level mobility coefficients.

There **exists** differences in neighborhood-level mobility coefficients across neighborhoods.

Magnitude of Neighborhood-level Heterogeneity

$$\text{Var}(\hat{\beta}_n^{IGE}) = \underbrace{\mathbb{E}\left[\hat{\sigma}^2(\hat{\beta}_n^{IGE})\right]}_{\text{within-neighborhood}} + \underbrace{\text{Var}\left(\beta_n^{IGE}\right)}_{\text{between-neighborhood}}$$

where $i \in n$ denotes family i is a member of neighborhood n. (Analogously for $\hat{\alpha}_n$.)

- The “within-neighborhood” component is the average estimation error of the estimator $\hat{\beta}_n$ across $n \in \mathcal{N}$
- The “between-neighborhood” component is consequently the remaining true variation in the neighborhood-level IGE estimates

Magnitude of Neighborhood-level Heterogeneity

Nbhd. Coefficient	Weighted	Total	Within (% Total)	Between (% Total)
α_n^{IGE}		6.991	4.994 (71.4%)	1.997 (28.6%)
α_n^{IGE}	✓	2.776	1.783 (64.2%)	0.993 (35.8%)
β_n^{IGE}		0.056	0.041 (72.8%)	0.015 (27.2%)
β_n^{IGE}	✓	0.022	0.015 (65.2%)	0.008 (34.8%)

This table reports variance decompositions of the neighborhood-level mobility estimates. The second column indicates whether the variance is weighted by the sample size of the neighborhoods.

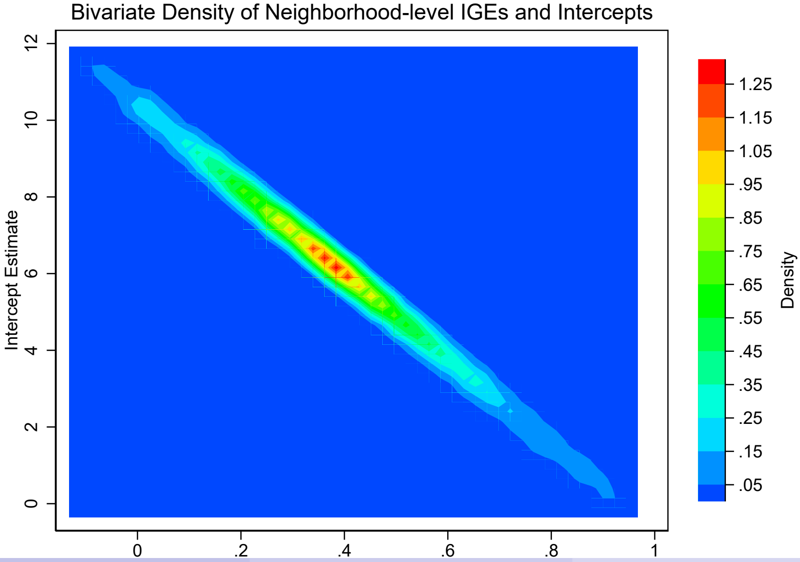
Magnitude of true neighborhood-level heterogeneity is **much smaller** than what is observed.
Eliminating noise $\Longrightarrow \sigma(\hat{\beta}_n^{IGE})$ falls from 0.237 to 0.122.

Outline

Predicted Child Income across Neighborhoods

- While mobility coefficient estimates are significant, do their joint mapping result in meaningful differences in child income across neighborhoods?
- Recall compressed population-level parent income distribution $\Longrightarrow$ do children generally fare similar outcomes?

Neighborhood-level IGEs and Intercepts are Collinear



Neighborhood-level IGEs and Intercepts are Collinear

Correlation coefficient = -0.999.
Partly due to use of log incomes.
Rank-rank and level-level estimates < -0.8.

- This means that

$$\alpha_n^{IGE} \approx \kappa_0 - \kappa_1 \beta_n^{IGE},$$

for $\kappa_0, \kappa_1 > 0$

- This implies that

$$\begin{aligned} \mathbb{E}[Y_i^c \mid Y_i^p = y^p, i \in n] &= \alpha_n^{IGE} + \beta_n^{IGE} y^p \\ &\approx \left(\kappa_0 - \kappa_1 \beta_n^{IGE}\right) + \beta_n^{IGE} y^p \\ &= \kappa_0 + \beta_n^{IGE} \cdot (y^p - \kappa_1), \end{aligned}$$

so $y^p \longrightarrow \kappa_1 \Longrightarrow \mathbb{E}[Y_i^c \mid Y_i^p = y^p, i \in n] \longrightarrow \kappa_0$, **irrespective of the neighborhood of residence**

Neighborhood-level IGEs and Intercepts are Collinear

- **Running the regression** $\hat{\alpha}_n^{IGE} = \kappa_0 - \kappa_1 \hat{\beta}_n^{IGE} + \varepsilon_n$**...**

	Unweighted	Weighted
$\hat{\beta}_n^{IGE}$	11.119 (0.009)	11.122 (0.001)
Constant	10.443 (0.004)	10.433 (0.000)
N	1,923	537,895
R^2	0.999	0.998

- $\kappa_1 = 11.119$ corresponds to the 50.03th percentile of the log parent income distribution

Testing Heterogeneity of Predicted Child Income

How many neighborhoods have same predicted child income given parent income level $Y_i^p = y^p \equiv med(Y_i^p) =$ **\$67,433?**

- *Note*: $y^p \in range\{Y_i^p \mid i \in n\}, \forall n \in \mathcal{N}$
- Rank neighborhoods by the magnitude of their residual from smallest to largest, $n_{(1)}, n_{(2)}, \ldots n_{(N)}$
- Denote $\mathcal{N}_{\bar{n}} \equiv \{n_{(1)}, n_{(2)}, \ldots, n_{(\bar{n})}\}$
- Use an F-test on the linear restriction

$$\alpha_n^{IGE} + \beta_n^{IGE} y^p = \alpha_{n'}^{IGE} + \beta_{n'}^{IGE} y^p, \quad n, n' \in \mathcal{N}_{\bar{n}} \subseteq \mathcal{N}.$$

  Index p-value of this F-statistic as $p_{\bar{n}}$
- Find $\sup\{\bar{n} \in \{1, \ldots, N\} \mid p_{\bar{n}} \geq 0.05\}$

Testing Heterogeneity of Predicted Child Income

$$\bar{n} = \mathbf{1{,}518}$$

This means that 79% of Denmark's neighborhoods have the same expected child income, conditional on parent income at the national median.

- Roughly 10% of neighborhoods have predicted child income below national predicted level
- Roughly 11% of neighborhoods have predicted child income above national predicted level

Outline

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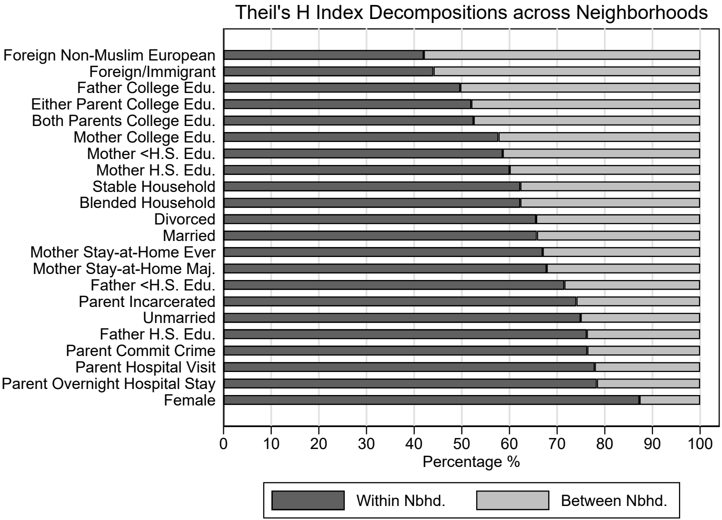
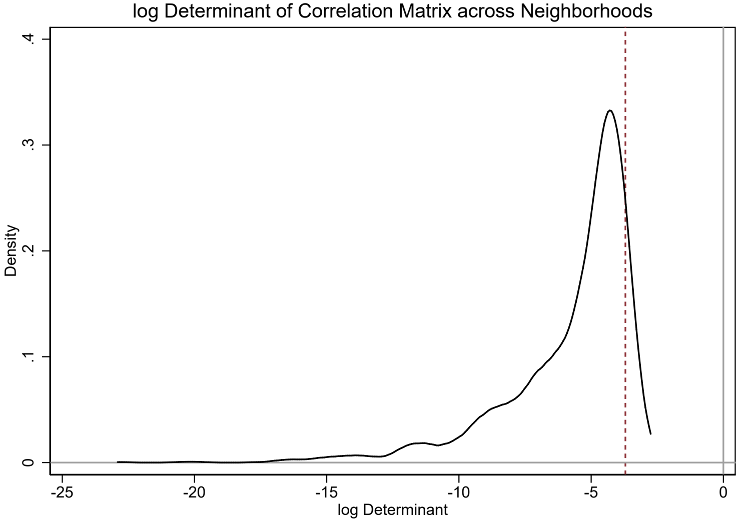
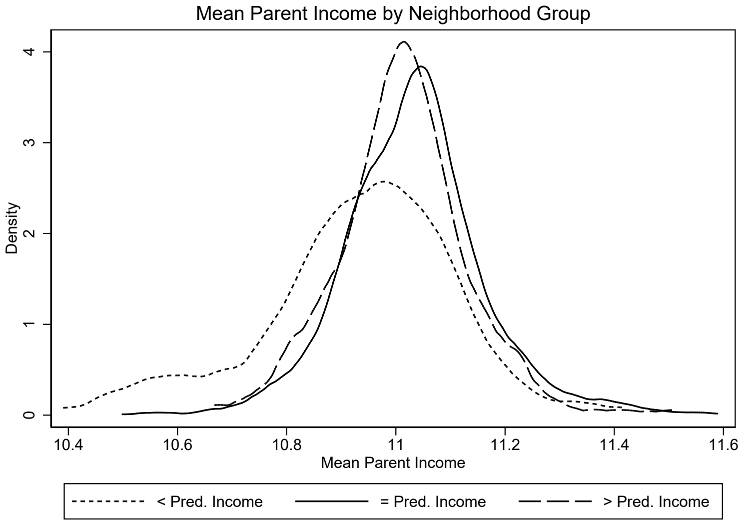
Families sort on income and other characteristics



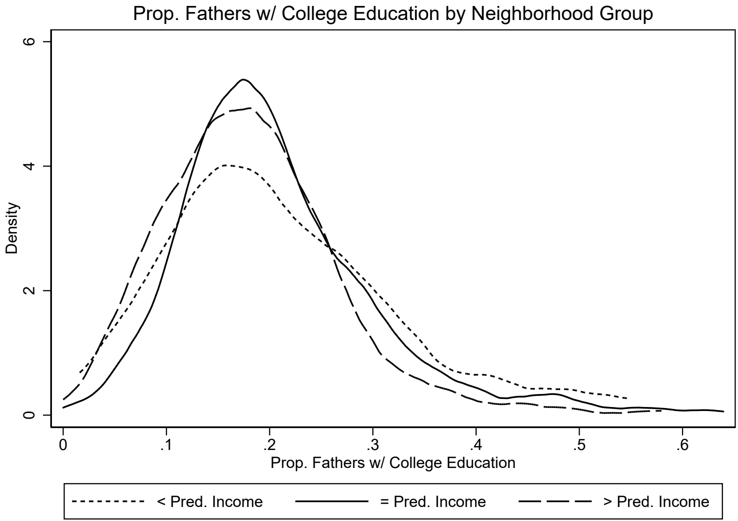
High self-similarity within neighborhoods



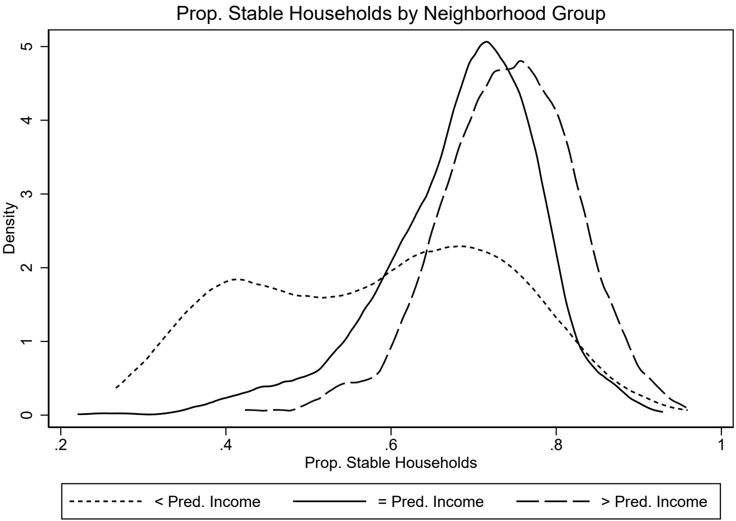
Nbhd. Mean Family Characteristics by Predicted Y^c_{in}



Nbhd. Mean Family Characteristics by Predicted Y_{in}^{c}



Nbhd. Mean Family Characteristics by Predicted Y_{in}^c



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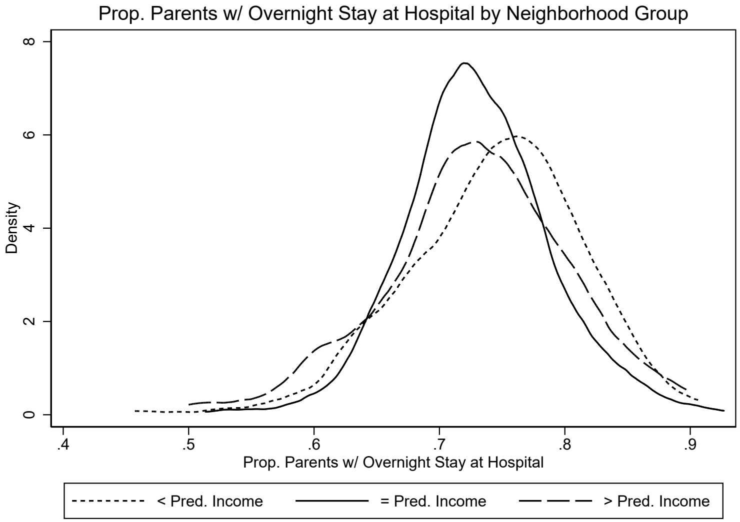

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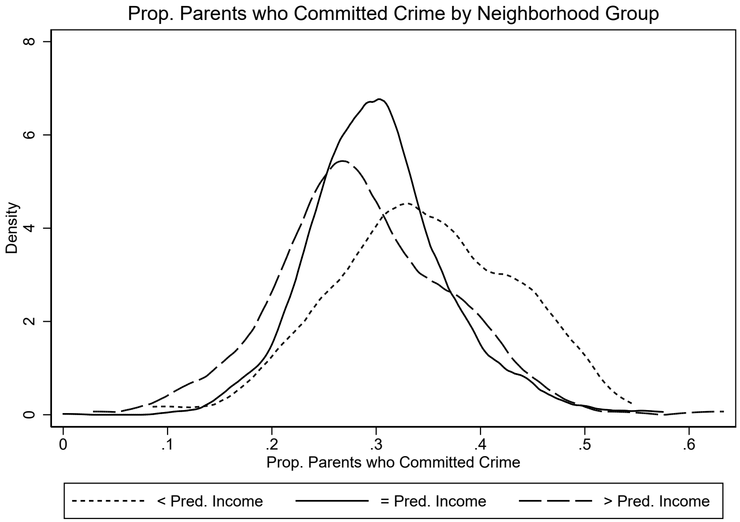


Motivating Questions

1. After conditioning on family characteristics, are the remaining neighborhood-level fixed effects different across neighborhoods?
   - *Individual-level* family characteristics $\longrightarrow$ productivity of family investments in skill technology (Heckman and Mosso 2014, Becker et al. 2018)
   - *Neighborhood-level* family characteristics $\longrightarrow$ social interactions in human capital and behavior (Durlauf and Seshadri 2018)
2. If differences remain, how much variation can be explained by pure *location* effects?
   - Account for sampling variance as before...
   - ...in addition to variation explained by family characteristics
3. How should one interpret irreducible variation of neighborhood's location effects?

Outline

A Micro-foundational Intergenerational Mobility Model

The "neighborhood-level" mobility model:

$$Y_{in}^{c} = \alpha_{n}^{IGE} + \beta_{n}^{IGE} Y_{in}^{p} + \varepsilon_{in}$$

The "micro-level" mobility model:

$$Y_{in}^{c} = \alpha\left(\boldsymbol{X}_{in},\, F_n(\boldsymbol{X}),\, \alpha_n\right) + \beta\left(\boldsymbol{X}_{in},\, F_n(\boldsymbol{X}),\, \beta_n\right) \cdot Y_{in}^{p} + \varepsilon_{in}$$

- Individual characteristics $\boldsymbol{X}_{in}$
- Neighborhood's distribution of family characteristics $F_n(\boldsymbol{X})$
- Neighborhood fixed effects (α_n, β_n), which have strong interpretation of the **"location effect"**

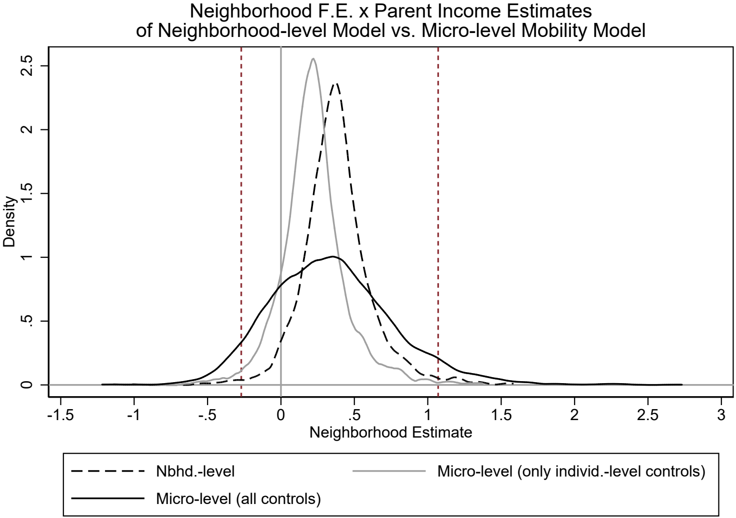
Specification of Micro-level Mobility Model

$$Y_{in} = \boldsymbol{\alpha}_0 \boldsymbol{\mathcal{X}}_{in} + \alpha_n + \boldsymbol{\beta}_0 \boldsymbol{\mathcal{X}}_{in} Y_{in}^p + \beta_n Y_{in}^p + \varepsilon_{in}$$

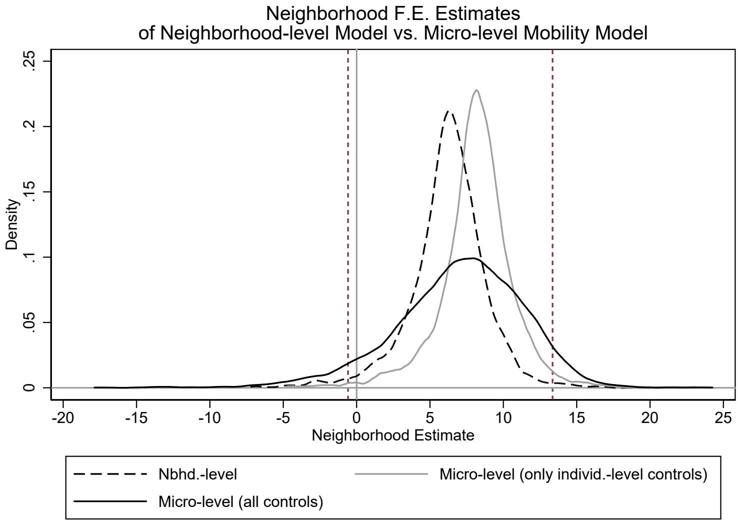
where $\boldsymbol{\mathcal{X}}_{in}$ includes:

1. Standardized **individual-level family characteristics $\boldsymbol{X}_{in}$** in age, education, assets, mother labor force participation, household size, household structure, marital status, hospitalizations, and crime
2. Principal components of standardized **"local population" family characteristics**
   - Average of other families: $\bar{X}^k_{-in} \equiv \frac{1}{I_n - 1} \sum_{j \neq i, j \in n} X^k_{in}$, for each $k = 1, \ldots, K$
   - Absolute social distance: $sgn(X^k_{in} - \bar{X}^k_{-in}) \cdot (X^k_{in} - \bar{X}^k_{-in})^2$, for each $k = 1, \ldots, K$

Effect of incl. All Family Chars. on $\hat{F}(\hat{\beta}_n)$



Effect of incl. All Family Chars. on $\hat{F}(\hat{\alpha}_n)$



Statistical Significance of Nbhd. F.E. Components

Table: Statistical Significance of Neighborhood F.E. Components of Micro-level Mobility Coefficients

Null Hypothesis H_0	$\lvert\{n \in \mathcal{N} \mid p_n < 0.05\}\rvert$	$\lvert\{n \in \mathcal{N} \mid p_n < 0.01\}\rvert$
Panel A: Single Hypothesis Tests		
$\alpha_n = 0$	1,282 (66.7%)	976 (50.8%)
$\beta_n = 0$	734 (38.2%)	381 (19.8%)
Panel B: Multiple Hypothesis Tests		
$\{\alpha_n = 0 \mid n \in \mathcal{N}\}$	**1,173 (61.0%)**	861 (44.8%)
$\{\beta_n = 0 \mid n \in \mathcal{N}\}$	**380 (19.8%)**	151 (7.9%)
$\{\alpha_n = 0 \mid n \in \mathcal{N}\} \bigcup \{\beta_n = 0 \mid n \in \mathcal{N}\}$	α_n: 1,097 (57.1%)	α_n: 793 (41.2%)
$\{\alpha_n = 0 \mid n \in \mathcal{N}\} \bigcup \{\beta_n = 0 \mid n \in \mathcal{N}\}$	β_n: 510 (26.5%)	β_n: 237 (12.3%)

This table summarizes results of hypothesis tests on individual neighborhood F.E. components α_n, β_n of the micro-level mobility coefficients $\alpha_n(\boldsymbol{\mathcal{X}}_{in}, \alpha_n)$, $\beta(\boldsymbol{\mathcal{X}}_{in}, \beta_n)$.

Heterogeneity after incl. All Family Chars.

- Conducted various F-tests on parameters such as...
  - $\alpha_n = \alpha$, $\beta_n = \beta$
  - $\boldsymbol{\alpha}_0 = 0$, $\boldsymbol{\beta}_0 = 0$
  - Combinations of the above
- Reject all nulls $\longrightarrow$ **neighborhood-level heterogeneity exists**
  - **Strong interpretation:** (α_n, β_n) represent pure "locational" effects
  - But this ignores neighborhood selection as well as other nonlinear dependence on family characteristics
  - Are there different "types" of neighborhoods?

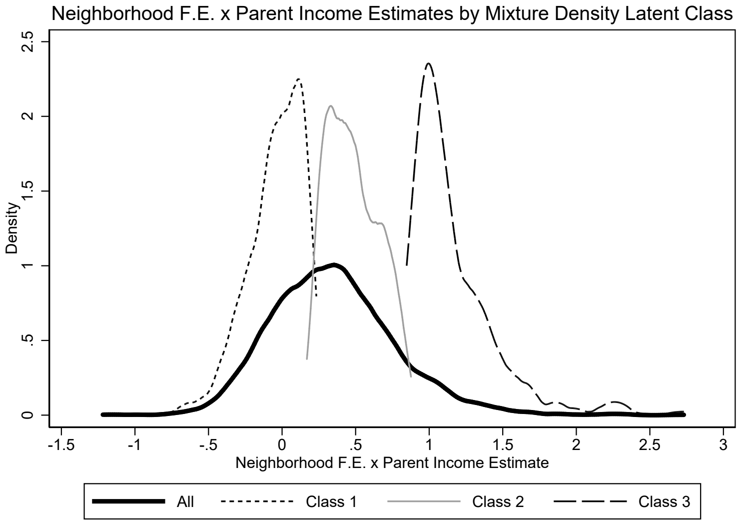
Outline

Classifying Neighborhood Types

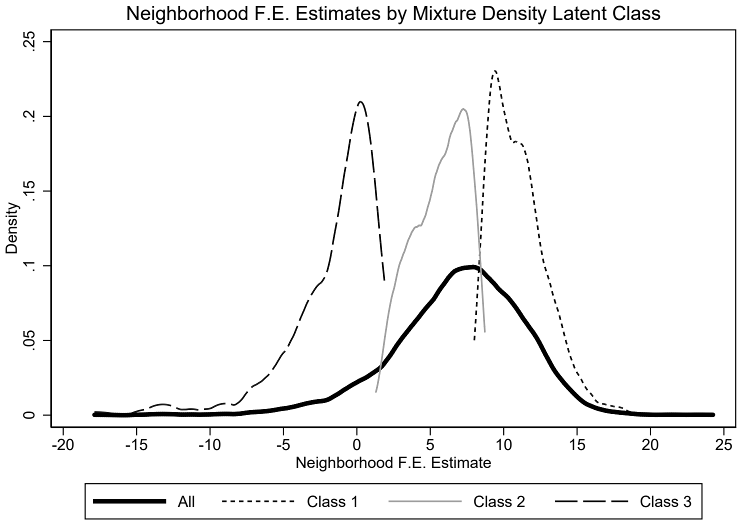
- **Is there irreducible heterogeneity in neighborhoods?**
- Use Gaussian finite mixture model on $F(\alpha_n, \beta_n)$ to classify neighborhoods into K latent classes.
- Check how these classes may differ from one another in estimates and across family characteristics

We find that three latent classes emerge...

Distribution of $\hat{\beta}_n$ by Neighborhood Class



Distribution of $\hat{\alpha}_n$ by Neighborhood Class

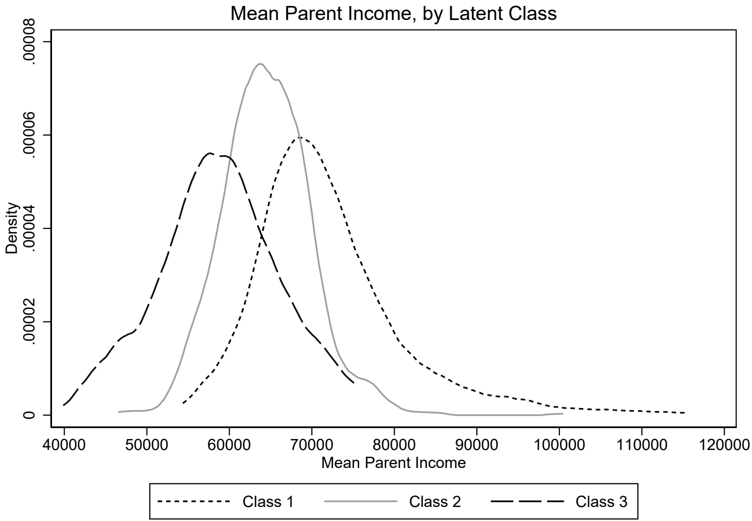


Summary Statistics of Neighborhood Classes

Class	N	β_n				α_n			
		Mean	S.D.	Min.	Max.	Mean	S.D.	Min.	Max.
1	746	−0.048	0.195	−1.218	0.233	11.054	2.041	8.002	24.259
2	974	0.482	0.175	0.169	0.876	5.633	1.839	1.297	8.719
3	203	1.159	0.294	0.847	2.731	−1.555	3.145	−17.872	1.877

- 5 of 746 statistically significant β_n are in Class 1
- 219 of 974 statistically significant β_n are in Class 2
- 156 of 203 statistically significant β_n are in Class 3

Local Population Characteristics by Type



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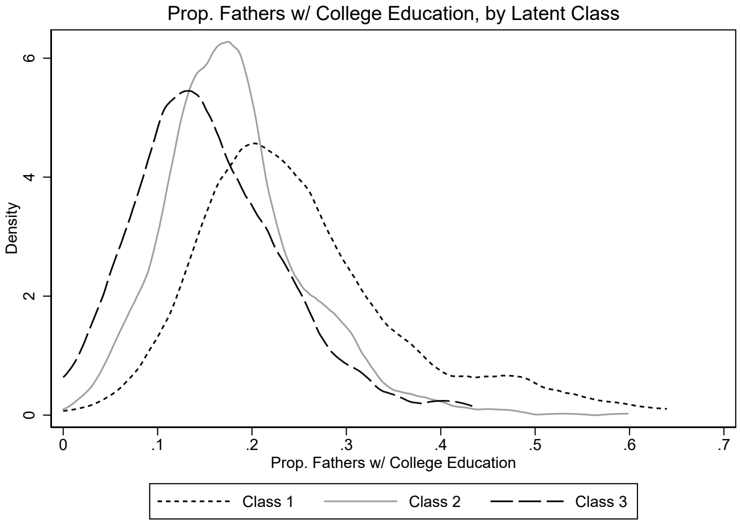

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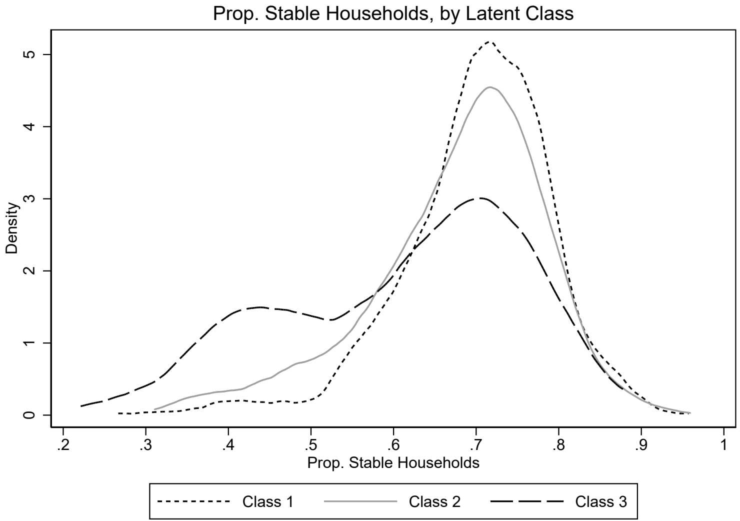

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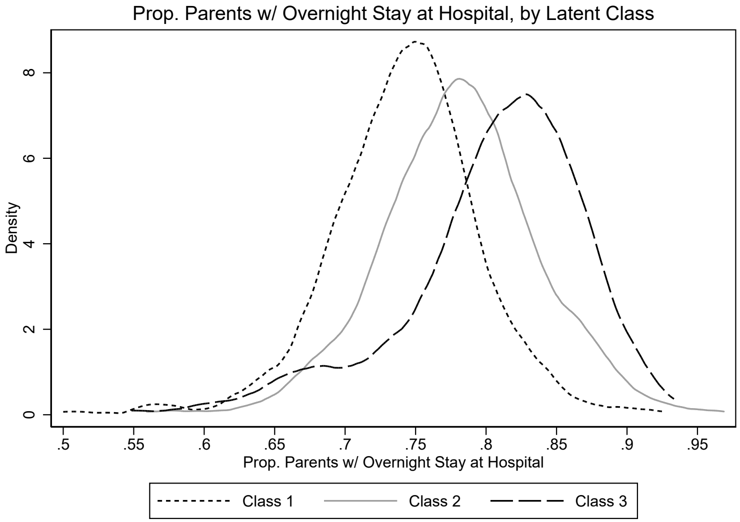

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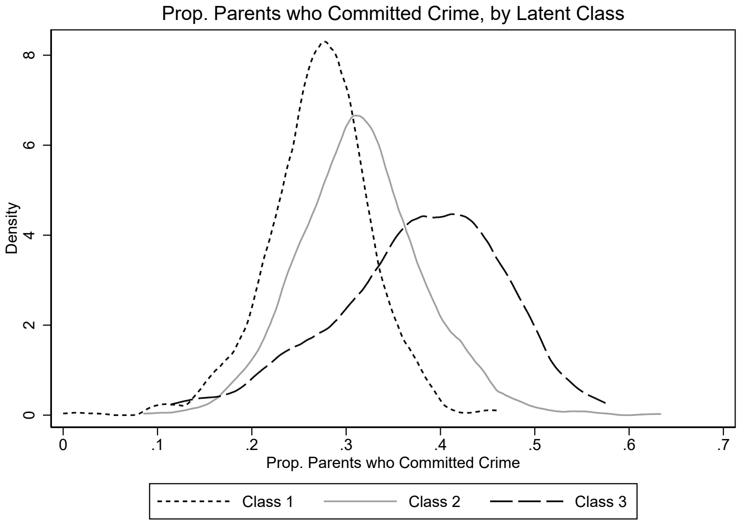

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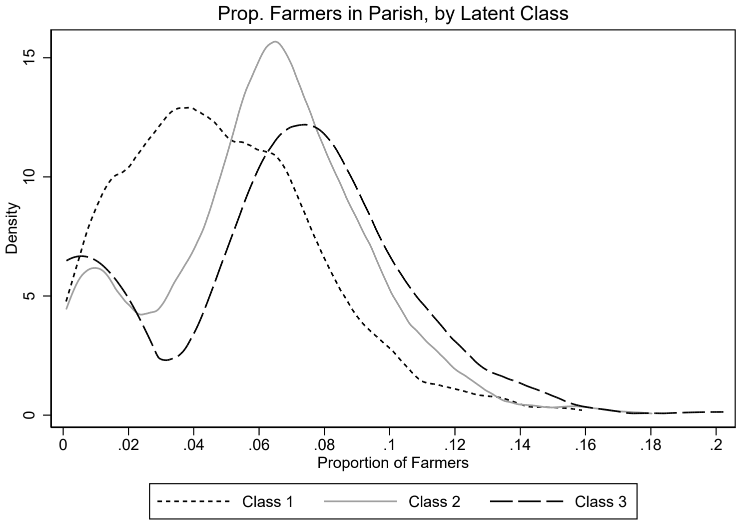
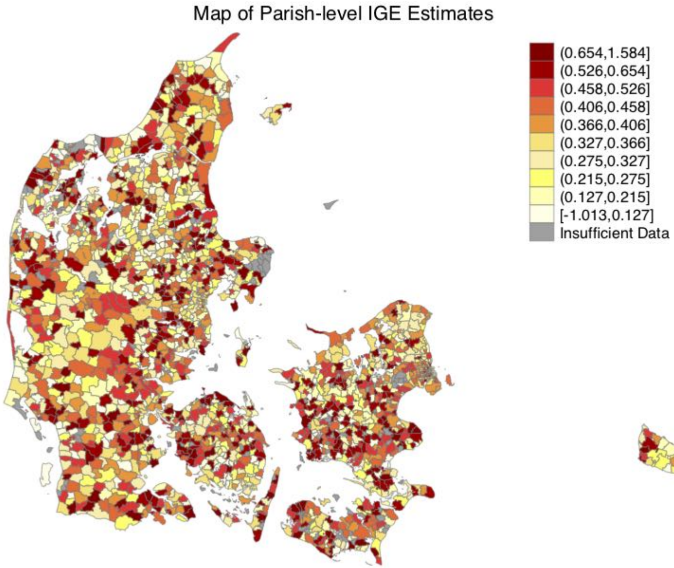


Summary of Differences across Types of Neighborhoods

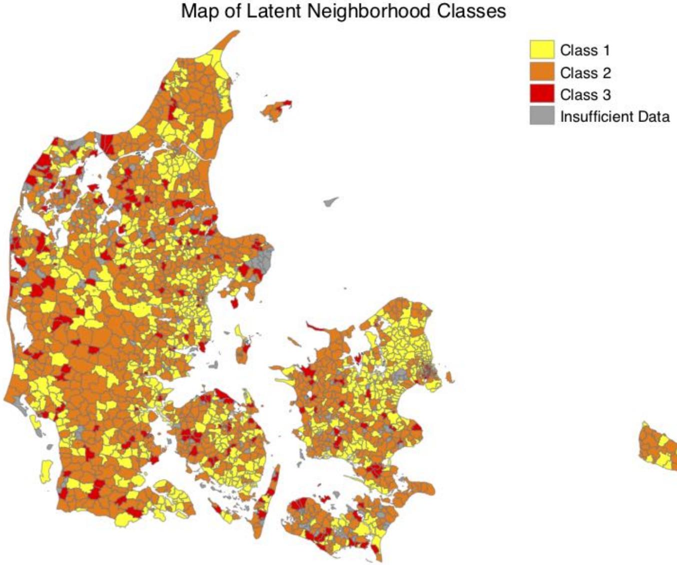
	Class 1	Class 2	Class 3
N	746	974	203
α_n	High	Medium	Low
β_n	Low	Medium	High
Parent Income	High	Medium	Low
Assets	High	Medium	Low
Parent Education	High	Medium	Low
Immigrants	Low	Medium	High
Hospitalizations	Low	Medium	High
Crime	Low	Medium	High
Rural	Low	Medium	High

This table summarizes characteristics of the three latent classes uncovered from the mixture of coefficient estimates of (α_n, β_n) from the micro-level mobility coefficients.

The Geography of Mobility in Denmark

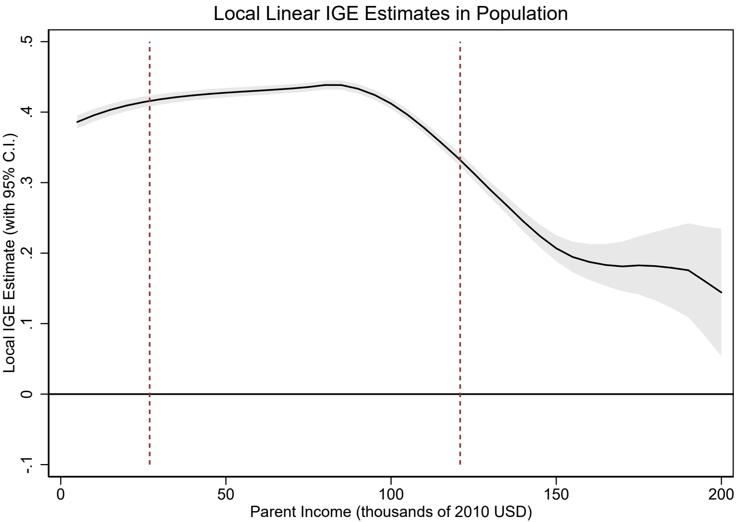


The Geography of Mobility in Denmark

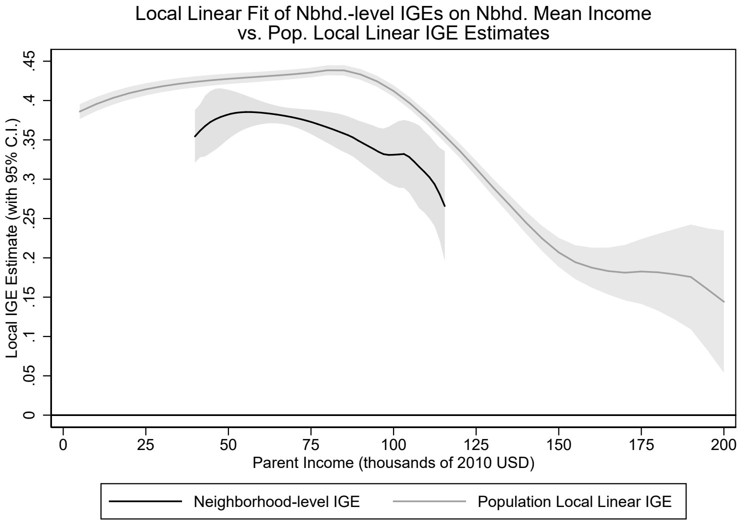


Outline

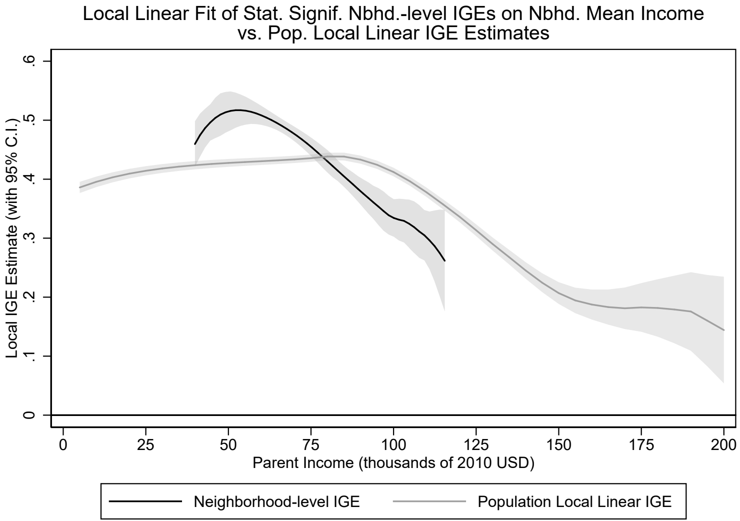
Population Nonlinear Income IGE



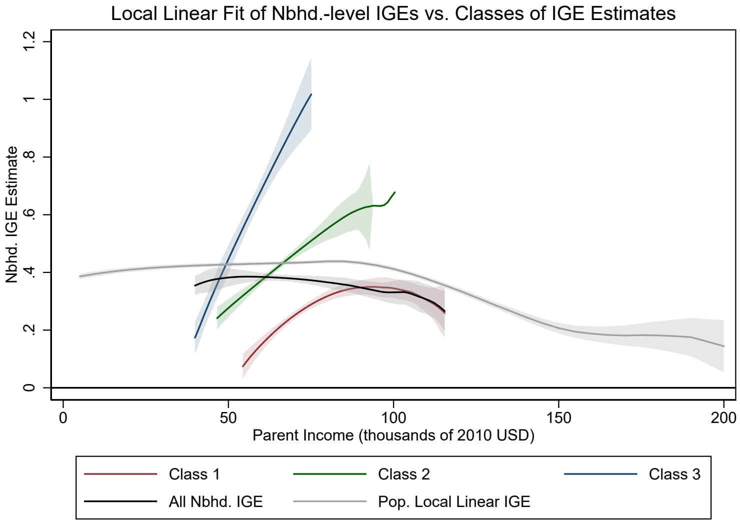
L.L. Pop. IGE vs. L.L. Fit of β_n^{IGE}



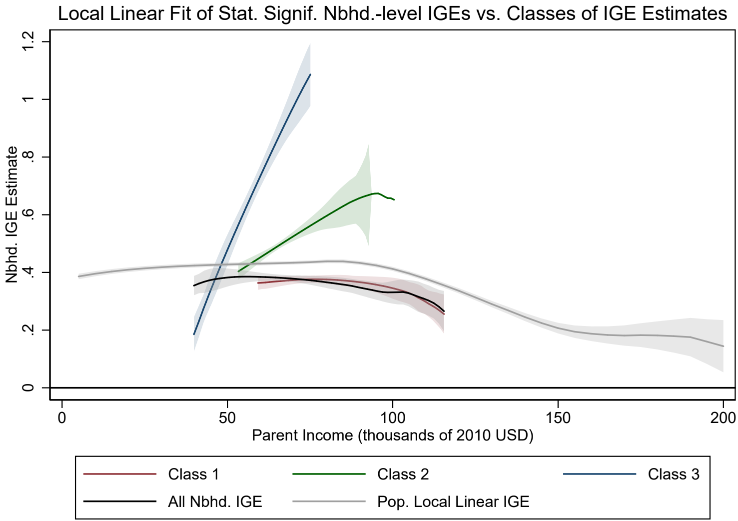
L.L. Pop. IGE vs. L.L. Fit of β_n^{IGE}



L.L. Pop. IGE vs. L.L. Fit of β_n^{IGE}

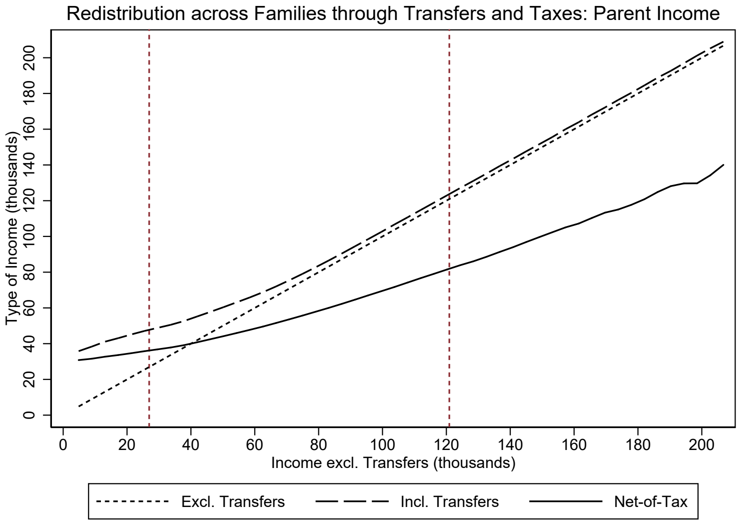


L.L. Pop. IGE vs. L.L. Fit of β_n^{IGE}

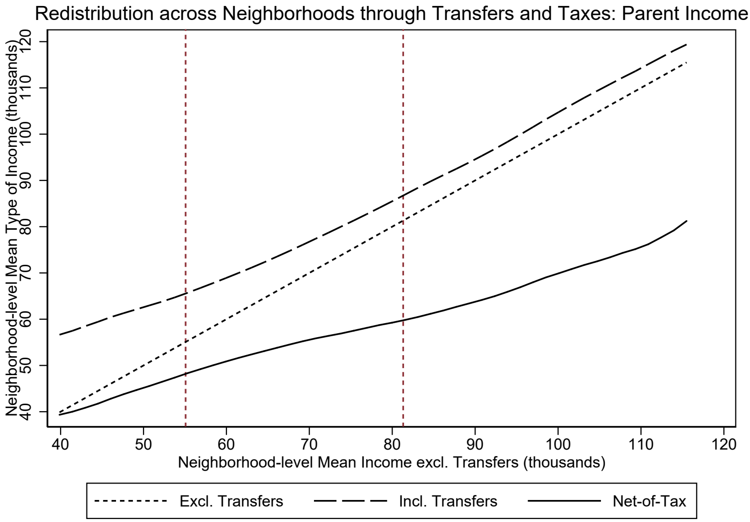


Outline

Redistribution across Families



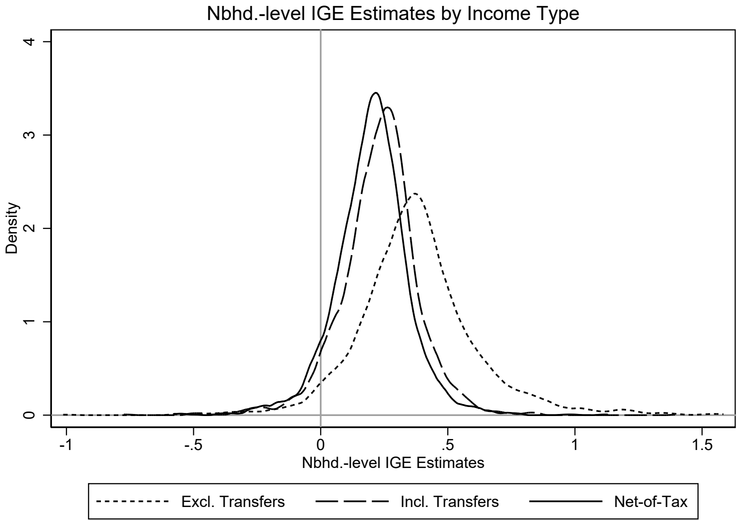
Average Redistribution across Neighborhoods



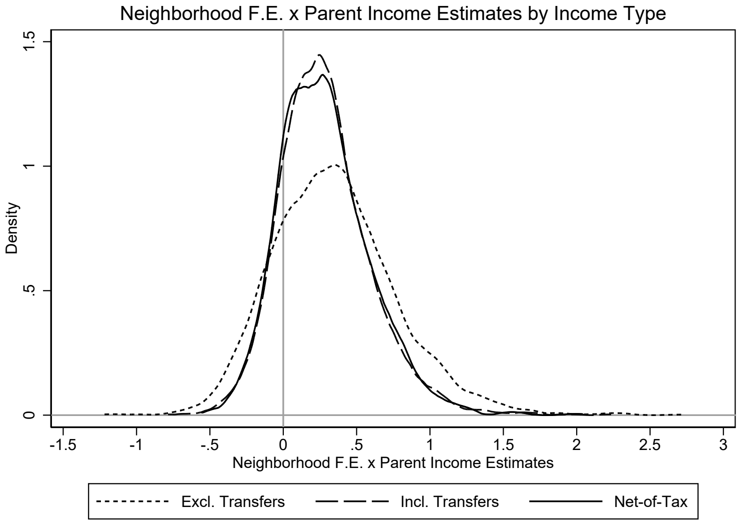
Implications of Redistribution on Neighborhood Mobility

- Large net redistribution from middle and top income earners to bottom income earners
  $\Longrightarrow$ large changes in average income in neighborhoods
- But there are strong incentives to sort on family characteristics

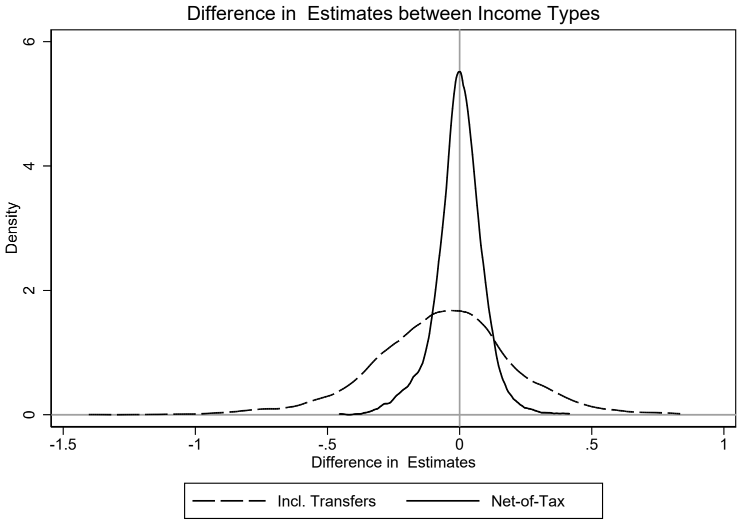
Neighborhood-level IGEs β_n^{IGE} by Income Type



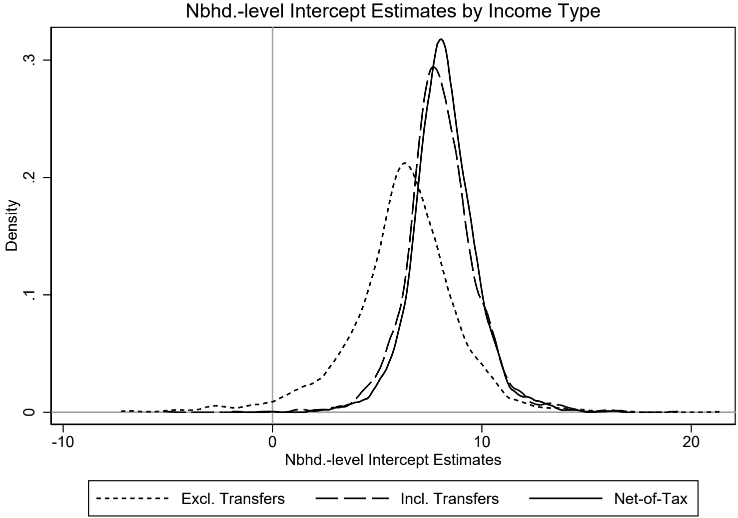
Neighborhood-level F.E. Component β_n by Income Type



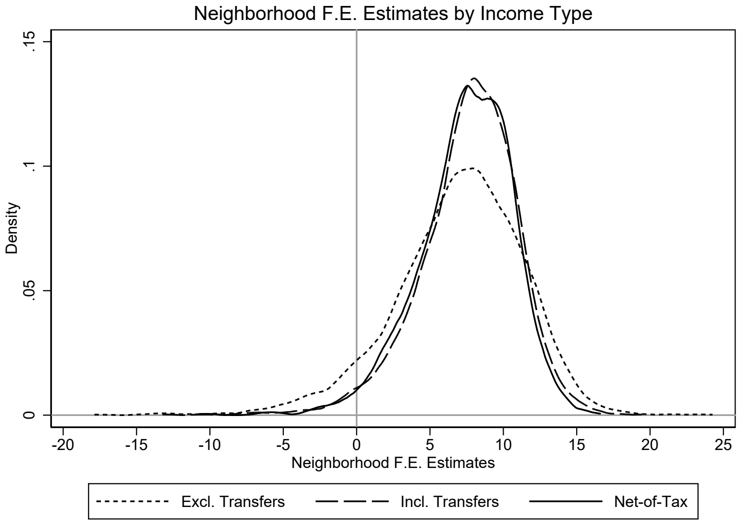
Neighborhood-level F.E. Component β_n by Income Type



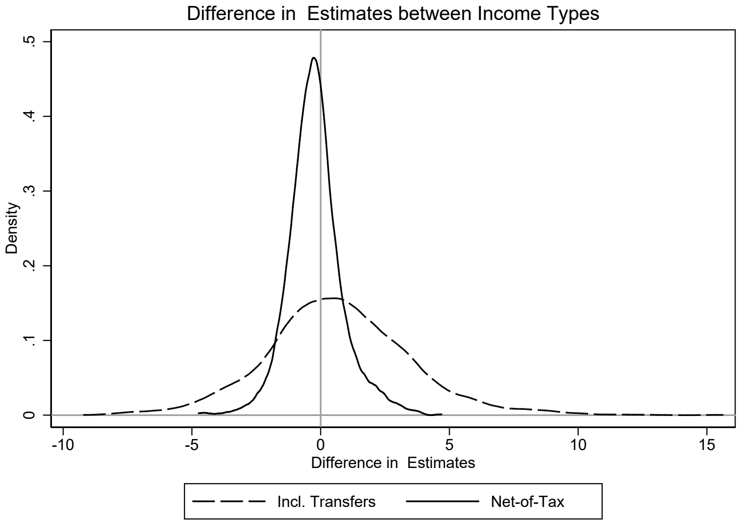
Neighborhood-level Intercepts α_n^{IGE} by Income Type



Neighborhood-level F.E. Component α_n by Income Type



Neighborhood-level F.E. Component α_n by Income Type



Implications of Redistribution on Neighborhood Mobility

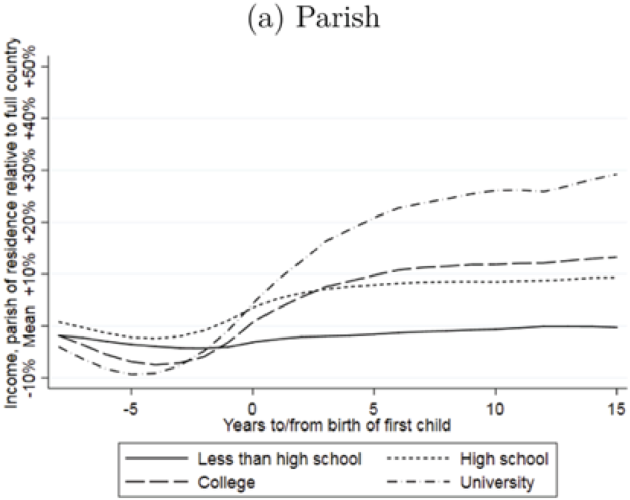
- After controlling for family characteristics, neighborhood F.E. estimates barely change!
- Redistribution to disadvantaged families does not ameliorate neighborhood differences in mobility
- Sorting generates persistent inequality across space

Outline

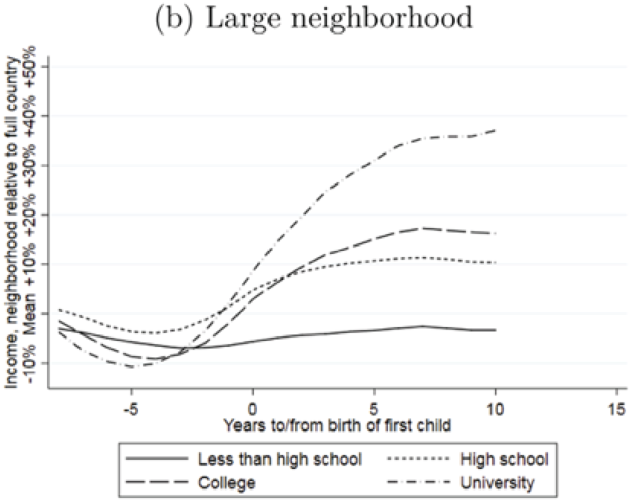
Summary

1. Much of the heterogeneity in neighborhood-level IGEs are driven by sampling error
2. Most neighborhoods have the same predicted child incomes
3. Much of the remaining heterogeneity in mobility can be explained by individual family characteristics and sorting
4. Neighborhoods can be categorized into three classes that vary in family characteristics
5. After accounting for sorting, redistribution does not appear effective

Consideration for Sorting Dynamics?



Consideration for Sorting Dynamics?



Consideration for Sorting Dynamics?