

# The Implications of Dynamic Complementarity for Investments across Children with Different Initial Endowments

Extract from  
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- The average family usually has more than one child, and society allocates public investments across multiple children.
- The problem of intra-child allocations is sometimes formulated as a problem in fairness.
- CES representation of parental utility  $V$  is often used:

$$V = \left( \sum_{k=1}^N \omega_k V_k^\sigma \right)^{\frac{1}{\sigma}} . \quad (1)$$

- $V_k$  is utility of child  $k$ .
- Parents can allocate resources (e.g., income) to each child.
- $V_k \uparrow$  with  $Y_k$ .
- $\sum_{k=1}^N Y_k = Y$  (family income for children).

- A Rawlsian version of maximal inequality aversion is obtained when  $\sigma \rightarrow -\infty$ , so utilities are perfect complements, and parents are concerned only with the maximization of the minimum outcome across all  $N$  children.

## Standard Intuition

- In a two-child version of the one-period-of-childhood model analyzed by Becker and Tomes (1979, 1986), under complementarity between initial endowment and investment, the optimal policy when  $\sigma = 1$  is to invest more in the advantaged child. (A force for promoting inequality: to those who have more should be given.)
- Under substitutability, it is optimal to invest more in the disadvantaged child.

## Dynamic Complementarity

- Story richer when we consider a multiperiod model with dynamic complementarity.
- *Investing relatively more in initially disadvantaged young children can be efficient even when the  $\omega_k$  are equal and  $\sigma = 1$ .*
- This can be true even if there is complementarity in each period of the life cycle.

- **Dynamic complementarity** is a force promoting compensating early stage investments.
- In a multiperiod model at stage  $t$

$$\theta_{t+1} = f^{(t)}(\theta_t, I_t) \quad (2)$$

- Even if there is complementarity at all stages, so  $f_{12}^{(t)}(\cdot) > 0$  (where  $(\cdot)$  denotes the argument of the function), output-maximizing investments can be compensating (i.e., invest most in the disadvantaged child).

- If  $f_{12}^{(1)}(\cdot) < 0$ , but  $f_{12}^{(2)}(\cdot) > 0$ , it is *always* efficient to invest relatively more in the initially disadvantaged child in the first period.

- It can also be productively efficient to invest in the disadvantaged child if  $f_{12}^{(1)}(\cdot) > 0$ , when initial endowments and investments are complements.



## Intuition for Second Result

- With increasing complementarity the stock of skills in the second period has a greater effect on the productivity of investments than it does in the first period  $\left( f_{12}^{(2)}(\cdot) > f_{12}^{(1)}(\cdot) \right)$ .
- First-period investment bolsters the stock of second-period skills and prepare disadvantaged children to make productive use of them in the second period.
- This effect is stronger when  $f_{12}^{(2)}(\cdot)$  is larger.

- Another force promoting greater initial investment in the disadvantaged child is diminishing self-productivity of skills in the first period  $\left(f_{11}^{(1)}(\cdot) < 0\right)$
- The greater the diminishing returns to investment for the better-endowed child, the lower the benefits of early advantage.
- Diminishing productivity of the stock of second-period skills  $\left(f_{11}^{(2)}(\cdot) < 0\right)$  operates in the same fashion to limit the effects of any initial advantage.

- The smaller the effect of the initial stock of skills on the productivity of investment in the first period  $\left(f_{12}^{(1)}(\cdot)\right)$ , the weaker is the disequalizing force of complementarity toward promoting investment in the initially advantaged child.

- Summarizing:
  - ① The more concave are the sub-period technologies in terms of stocks of skills (the more they exhibit decreasing returns in the stocks of skills), the more favorable is the case for investing in more disadvantaged children.
  - ② The stronger second-period complementarity  $(f_{12}^{(2)}(\cdot))$ , the stronger is the case for investing more in the initially disadvantaged child to build skill stocks take advantage of this opportunity.

- In general, even when investment is greater in the first period for the disadvantaged child, it is optimal for second-period investment to be greater for the initially advantaged child.
- It is generally not efficient to make the disadvantaged child whole in the first period.
- Greater second-period complementarity then kicks in to promote disequalizing second-period investments.