# Roy Models of Policy Evaluation

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(Extracted from: Understanding Instrumental Variables in Models with Essential Heterogeneity)

Econ 312, Spring 2022

## 1. Policy adoption problem

- Suppose a policy is proposed for adoption in a country.
- What can we conclude about the likely effectiveness of the policy in countries?
- Build a model of counterfactuals.

$$
\begin{aligned}
Y_1 &= \mu_1(X) + U_1 \\
Y_0 &= \mu_0(X) + U_{0.}
\end{aligned}
\tag{1}
$$

## Consider the Basic Generalized Roy Model

- Two potential outcomes $(Y_0, Y_1)$.
- A choice equation

$$D = \mathbf{1}[\underbrace{\mu_D(Z, V)}_{\text{net utility}} > 0].$$

- Observed outcomes:

$$Y = DY_1 + (1 - D)Y_0$$

- Assume $\mu_D(Z, V) = \mu_D(Z) - V$.
- This separability plays a key role in the IV (LATE) and discrete choice.
- Can be relaxed, but things look much less traditional.

## Switching Regression Notation

$$Y = Y_0 + (Y_1 - Y_0)D \tag{2}$$
$$= \mu_0 + (\mu_1 - \mu_0 + U_1 - U_0)D + U_0.$$

(Quandt, 1958, 1972).

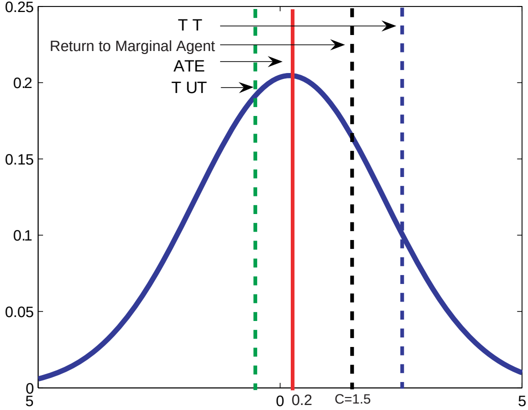
## In Conventional Regression Notation

$$Y = \alpha + \beta D + \varepsilon \tag{3}$$

$\alpha = \mu_0$, $\beta = (Y_1 - Y_0) = \mu_1 - \mu_0 + U_1 - U_0$, $\varepsilon = U_0$.

- $\beta$ is the "treatment effect."

## Figure 1: Distribution of gains, a Roy economy




$$\beta = Y_1 - Y_0$$

TT$= 2.666 = E(Y_1 - Y_0 | D = 1)$, TUT$= -0.632 = E(Y_1 - Y_0 | D = 0)$

Return to Marginal Agent $= C = 1.5$, ATE $= \mu_1 - \mu_0 = \bar{\beta} = 0.2$

# The model

| Outcomes | Choice Model |
|---|---|
| $Y_1 = \mu_1 + U_1 = \alpha + \bar{\beta} + U_1$ <br> $Y_0 = \mu_0 + U_0 = \alpha + U_0$ | $D = \begin{cases} 1 \text{ if } D^* > 0 \\ 0 \text{ if } D^* \leq 0 \end{cases}$ |

General Case

$$(U_1 - U_0) \not\perp\!\!\!\perp D$$
$$\text{ATE} \neq \text{TT} \neq \text{TUT}$$

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## Parameterizing the model

The Researcher Observes $(Y, D, C)$

$$Y = \alpha + \beta D + U_0 \text{ where } \beta = Y_1 - Y_0$$

Parameterization

$$\alpha = 0.67 \qquad (U_1, U_0) \sim N(\mathbf{0},\ ) \qquad D^* = Y_1 - Y_0 - C$$

$$\bar{\beta} = 0.2 \qquad = \begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix} \qquad C = 1.5$$

- In the case when $U_1 = U_0 = \varepsilon_0$, simple least squares regression of $Y$ on $D$ subject to a **selection bias** if $\varepsilon_0$ determines $D$.
- Notice that in a Roy model where $D = 1(Y_1 - Y_0 \geq 0)$ and $U_1 = U_0$, $D = 1(\mu_1(x) - \mu_0(x) \geq 0)$ where $\mu_1(\cdot)$ and $\mu_0(\cdot)$ depend on $X = x$.
- "Regression discontinuity" at set of points $x \in \{x | \mu_1(x) - \mu_0(x) = 0\}$.
- If

$$D = 1(Y_1 - Y_0 - C \geq 0)$$
$$C = \mu_C(Z) + U_C$$

there would be selection bias if $U_0 \not\perp\!\!\!\perp U_C$.

- Upward biased for $\beta$ if $\text{Cov}(D, \varepsilon_0) > 0$.
- In the example, if $\text{Cov}(\varepsilon_0, U_C) < 0$, you get upward bias for OLS. If $\text{Cov}(\varepsilon_0, U_C) > 0$, OLS is downward biased.
- **Prove.** How does this covariance relate to the question of whether a country is a meritocracy?

- Three main approaches have been adopted to solve this problem:
    1. Selection models
    2. Instrumental variable models (experiments; RDD is local IV)
    3. Matching: assumes that $\varepsilon \perp\!\!\!\perp D \mid X$.
- Matching is just nonparametric least squares and assumes access to rich data which happens to guarantee this condition.

## Instrumental Variables in Case I, the traditional case: $\beta$ is a constant

- If there is an instrument $Z$, with the property that

$$\text{Cov}(Z, D) \neq 0 \quad (4)$$
$$\text{Cov}(Z, \varepsilon) = 0, \quad (5)$$

then

$$\text{plim } \hat{\beta}_{\text{IV}} = \frac{\text{Cov}(Z, Y)}{\text{Cov}(Z, D)} = \beta.$$

- If other instruments exist, each identifies the same $\beta$.

**Case II, heterogeneous response case: $\beta$ is a random variable even conditioning on $X$**

**Sorting bias**

or sorting on the gain which is distinct from sorting on the level.

**Essential heterogeneity**

$$\text{Cov}(\beta, D) \neq 0.$$

Suppose (4), (5) and

$$\text{Cov}(Z, \beta) = 0. \tag{6}$$

- Can we identify the mean of $(Y_1 - Y_0)$ using IV?

- In general we cannot (Heckman and Robb, 1985).
- Let

$$\begin{aligned}\bar{\beta} &= (\mu_1 - \mu_0) \\ \beta &= \bar{\beta} + \eta \\ U_1 - U_0 &= \eta \\ Y = \alpha + \bar{\beta} D &+ [\varepsilon + \eta D] .\end{aligned}$$

- Need $Z$ to be uncorrelated with $[\varepsilon + \eta D]$ to use IV to identify $\bar{\beta}$.
- This condition will be satisfied if policy adoption is made without knowledge of $\eta \, (= U_1 - U_0)$.
- If decisions about $D$ are made with partial or full knowledge of $\eta$, IV does not identify $\bar{\beta}$.
- Crucial Question: What is the agent's information set?

- The IV condition is

$$E[\varepsilon + \eta D \mid Z] = 0.$$

- $E(\varepsilon \mid Z) = 0, \quad E(\eta \mid Z) = 0.$
- Even if $\eta \perp\!\!\!\perp Z$, $\eta \not\perp\!\!\!\perp Z \mid D = 1$.
- $E(\eta D \mid Z) = E(\eta \mid D = 1, Z) \Pr(D = 1 \mid Z)$.
- But $E(\eta \mid Z, D = 1) \neq 0$, in general, if agents have some information about the gains.

- Draft Lottery example (Heckman, 1997).
- Linear IV does not identify ATE or any standard treatment parameters.

## Examples

$$D = 1(\mu_D(z) > V)$$

(Notice: lower case $z$ is a number; $Z$ is a random variable.)

**Example:**

$$\mu_D(z) = \gamma z$$

$$(V \perp\!\!\!\perp Z) \mid X.$$

**The propensity score or probability of selection into $D = 1$:**

$$P(z) = \Pr(D = 1 \mid Z = z) = \Pr(\gamma z > V) = F_V(\gamma z)$$

$F_V$ is the distribution of $V$.

**Generalized Roy model**

$$U_1 \neq U_0$$

$$D = \mathbf{1}[Y_1 - Y_0 - C \geq 0]$$

$$\begin{aligned} \text{Costs } C &= \mu_C(W) + U_C \\ Z &= (X, W) \\ \mu_D(Z) &= \mu_1(X) - \mu_0(X) - \mu_C(W) \\ V &= -(U_1 - U_0 - U_C). \end{aligned}$$

## Heterogeneous response model

In a general model with heterogenous responses, specification of $P(Z)$ and relationship with the rest of the model plays an essential role.

$$
\begin{aligned}
E &= (\eta D | Z = z) \\
&= E(\eta | D = 1, Z = z) Pr(D = 1 | Z = z) \\
&= E(\eta | \gamma z \geq V, Z = z) Pr(D = 1 | Z = z)
\end{aligned}
$$

If $F_V$ is weakly monotonic,

$$
= E(\eta | F_V(\gamma z) \geq F_V(V), Z = z) Pr(D = 1 | Z = z).
$$

Because $Z \perp\!\!\!\perp \eta | X$

$$
\begin{aligned}
& E(\eta | F_V(\gamma z) \geq F_V(V), Z = z) \\
= & E(\eta | F_V(\gamma z) \geq F_V(V)) \\
P(z) = & F_V(\gamma z) \text{ ``Propensity Score''} \\
U_D = & F_V(V) \text{ ``Uniform Random Variable''} \\
& E(\eta D | Z = z, D = 1) \\
= & E(\eta | P(z) \geq U_D) P(z).
\end{aligned}
$$

- Probability of selection enters this term, even though we use only one component of $Z$ as an instrument.

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- Selection models control for this dependence induced by choice.

## Selection models

Assume

$$(U_1, U_0, V) \perp\!\!\!\perp Z \tag{7}$$

$$\text{[Alternatively } (\varepsilon, \eta, V) \perp\!\!\!\perp Z\text{].}$$

$$\eta = (U_1 - U_0),\ \varepsilon = U_0 \tag{8}$$

$$\begin{aligned} E(Y \mid D = 0, Z = z) &= E(Y_0 \mid D = 0, Z = z) \\ &= \alpha + E(U_0 \mid \gamma z < V) \end{aligned}$$

$$E(Y \mid D = 0, Z = z) = \quad \alpha + \underbrace{K_0(P(z))}_{\text{control function}}$$

$$
\begin{aligned}
E(Y \mid D=1, Z=z) &= E(Y_1 \mid D=1, Z=z) \\
&= \alpha + \bar{\beta} + E(U_1 \mid \gamma z > V) \\
&= \alpha + \quad \bar{\beta} + \underbrace{K_1(P(z))}_{\text{control function}}
\end{aligned}
$$

- $K_0(P(z))$ and $K_1(P(z))$ are control functions in the sense of Heckman and Robb (1985, 1986).
- $P(z)$ is an essential ingredient in both matching and IV:
- Matching: $K_1(P(z)) = K_0(P(z))$. Why? $E(U_1|Z) = E(U_0|Z)$.
- Matching balances
- It may or may not be true that $E(U_1|Z) = 0$ or $E(U_2|Z) = 0$.
- Matching differences out the common term.