Modeling the Income Process

Partially Extracted from "Earnings, Consumption and Lifecycle Choices" by Costas Meghir and Luigi Pistaferri (2011)

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- How much risk do households face, to what extent does risk affect basic household choices such as consumption, labor supply, and human capital accumulation, and what types of risks matter for explaining behavior?
- A fruitful distinction is between ex-ante and ex-post responses to risk.

- Ex-ante – Behavioural responses in expectation of shocks
    1. precautionary saving or precautionary labor supply
    2. delay in the adjustment in the optimal stock of durable goods in models with fixed adjustment costs
    3. a shift in the optimal asset allocation towards safer assets
    4. responding to “background risk”, i.e., increase the amount of insurance against formally insurable events (i.e., a fire in the home) when the risk of facing an independent, uninsurable event (i.e., lay-off) increases.

- Ex-post responses are the answer to the question: What do people do when they're hit by shocks?
  1. run down assets or even borrow at high(er) cost
  2. change (family) labor supply (at the intensive and extensive margin)
  3. sell durables (eBay)
  4. use family networks, loans from friends, etc.
  5. relocate or migrate (presumably for lack of local job opportunities) or change job (presumably because of increased firm risk)
  6. apply for government provided insurance, use charities, etc.

Micro risk

- The structure of the income process, including the persistence and the volatility of shocks as well as the sources of risk underlies both the ex-ante and the ex-post responses.
- Browning, Hansen and Heckman (1998): ". . . calibrating model economies under imperfect insurance requires a measure of the magnitude of microeconomic uncertainty."
- There has been a large increase in the cross sectional dispersion of wages/earnings.
- This has happened despite the "great moderation" taking place at the aggregate level.

We need some theory to make the point why knowing the income process is important to understand how consumption responds to income changes: Classical PIH

Classical PIH

- Quadratic period utility
- $\beta(1+r)=1$
- Finite horizon T
- $A_T \geq 0$

Recall:

- Hall (1978) and Bewley (1976) before him shows that if people have a single asset, the within period utility is quadratic, and the discount rate is equal to the interest rate Then optimal inter-temporal choice implies the PIH and consumption follows a martingale (random walk)

$$c_{it} = c_{i,t-1} + u_{it} \qquad \text{or} \qquad \Delta c_{it} = u_{it}$$

where u_{it} is a consumption innovation that is related to income shocks

Specifically

$$\Delta c_{it} = \frac{r}{1+r}\left[1 - \frac{1}{(1+r)^{T-t+1}}\right]^{-1} \sum_{j=0}^{T} \frac{\mathbb{E}\left(y_{i,t+j}|\Omega_{it}\right) - \mathbb{E}\left(y_{i,t+j}|\Omega_{it-1}\right)}{(1+r)^{j}}$$

- Consumption changes only if new information arrives ($\Omega_{it} \neq \Omega_{i,t-1}$)
- The extent of consumption adjustment to news (the shock) depends on the the persistence of the shock and the remaining time horizon
- Anticipated income changes have no effect on consumption growth (recall $\beta(1+r) = 1$)

- To get a clearer characterization, suppose income follows an ARMA(1,1) process

$$y_{it} = \rho y_{i,t-1} + \varepsilon_{it} + \Theta \varepsilon_{i,,t-1}$$

$$\Delta c_{it} = \frac{r}{1+r}\left[1 - \frac{1}{(1+r)^{T-t+1}}\right]^{-1} \times \left[1 + \frac{\rho + \Theta}{1 + r - \rho}\left(1 - \left(\frac{\rho}{1+r}\right)^{T-t}\right)\right]\varepsilon_{it}$$

or

$$\Delta c_{it} = \kappa\left(r, \rho, \Theta, T - t\right)\varepsilon_{it}$$

The Response of Consumption to Income Shocks with Quadratic Preferences

ρ	θ	$T-t$	κ
1	-0.2	40	0.81
1	0	10	1
0.99	-0.2	40	0.68
0.95	-0.2	40	0.39
0.8	-0.2	40	0.13
0.95	-0.2	30	0.45
0.95	-0.2	20	0.53
0.95	-0.2	10	0.65
0.95	-0.1	40	0.44
0.95	-0.01	40	0.48
1	0	∞	1
0	-0.2	40	0.03

The Response of Consumption to Income Shocks with Quadratic Preferences

1. If income is random walk process ($\rho = 1$, $\theta = 0$), consumption responds one-to-one regardless of time horizon
2. A decrease in the persistence of the shock lowers the value of κ. When $\rho = 0.8$ (and $\theta =$ -0.2) for example, the value of κ is a modest 0.13.
3. A decrease in the persistence of the MA component acts in the same direction (but the magnitude of the response is much attenuated).
4. Finally, a shortening of the planning horizon increases the value of κ.

Permanent vs Transitory Shocks

- The previous ARMA(1,1) process has a single shock ε_{it}
- A very popular generalization (still parsimonious) is to model income as the sum of a random walk and a transitory iid component

Identifying Permanent and Transitory Shocks

- Use data on choices to help learn about the stochastic process for income

- A Stochastic Process for Income

$$y_{it} = \underbrace{y_{it}^{P}}_{\text{permanent component of income}} + \underbrace{u_{it}}_{\text{transitory component of income}}$$

Let

$$y_{it}^{P} = \underbrace{y_{i,t-1}^{P}}_{\text{last period permanent income}} + \underbrace{v_{it}}_{\text{shock}}$$

Identifying assumptions: $cov(u_{it}, v_{it}) = 0$, $cov(v_{it}, y_{i,t-1}) = cov(u_{it}, y_{i,t-1}) = 0$.

- Lagging y^P_{it} we have

$$
\begin{aligned}
y^P_{i,t-1} &= y^P_{i,t-2} + v_{i,t-1} \\
y^P_{i,t-2} &= y^P_{i,t-3} + v_{i,t-2} \\
y^P_{i,t-3} &= y^P_{i,t-4} + v_{i,t-3}
\end{aligned}
$$

Subbing into above expressions we have

$$
y_{it} = \underbrace{v_{it} + v_{i,t-1} + v_{i,t-2} + \ldots + v_{i,0}}_{\text{Here we see permanent shocks accumulating}} + u_{it}
$$

- So, take differences

$$y_{it} - y_{i,t-1} = v_{it} + u_{it} - u_{i,t-1}$$

or

$$\underbrace{y_{it}}_{\text{income today}} = \underbrace{y_{i,t-1}}_{\text{income last period}} + \underbrace{v_{it}}_{\text{permanent shock}} + \underbrace{u_{it}}_{\text{transitory shock}} - u_{i,t-1}$$

we can see that u is transitory because it gets subtracted off again next period.

Aside

Suppose we knew the variances of the shocks didn't change over time

$$u_{it} \sim iid\left(0, \sigma_u^2\right)$$
$$v_{it} \sim iid\left(0, \sigma_v^2\right)$$

then if we have panel data on income, we can write

$$var\left(\Delta y_{it}\right) = \sigma_v^2 + 2\sigma_u^2$$
$$cov\left(\Delta y_{it}, \Delta y_{i,t-1}\right) = \sigma_u^2$$

and use the data moments on the left to identify the variances on the right

- If, however, we are not willing to assume the variances are constant, we don't have enough information to separate the permanent and transitory components.

Bring in the PIH and consumption data

- With quadratic preferences

$$\Delta c_{it} = v_{it} + \frac{r}{1+r}\frac{1}{\rho_t}u_{it}$$

  where $\rho_t = 1 - (1+r)^{-(T-t+1)}$

- Consumption responds one-for-one to permanent shocks
- Response to the transitory shock depends on the time horizon

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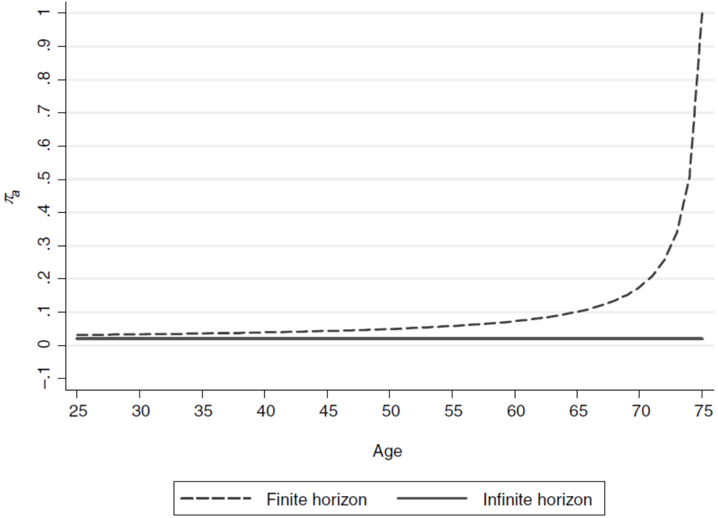
Response to the transitory shock depends on the time horizon



Basic Specification

- Notation changes for permanent and transitory shocks. Now $p_{i,a,t}$ is permanent. v_{it} transitory
- Specification that encompasses many of the ideas in the literature:

$$u_{i,a,t} = a \times f_i + v_{i,a,t} + p_{i,a,t} + m_{i,a,t}$$

$$v_{i,a,t} = \Theta_q(L)\varepsilon_{i,a,t} \qquad \text{Transitory process} \tag{1}$$

$$P_p(L)p_{i,a,t} = \zeta_{i,a,t} \qquad \text{Permanent process}$$

- L is a lag operator such that $Lz_{i,a,t} = z_{i,a-1,t-1}$.

- Individual specific lifecycle trend: $(a \times f_i)$
- A transitory shock $v_{i,a,t}$: MA process, lag polynomial of order q : $\Theta_q(L)$
- A permanent shock $P_p(L) p_{i,a,t} = \zeta_{i,a,t}$, $P_p(L)$
- Measurement error $m_{i,a,t}$ which may be taken as classical *iid* or not.

A Simple Model of Earnings Dynamics

- Ignore $a \times f_i$ is excluded.
- Lag polynomials $\Theta(L)$ and $P(L)$.
- Not generally possible to identify $\Theta(L)$ and $P(L)$ without any further restrictions.

- Typical specification used in MaCurdy (1982) :

$$u_{i,a,t} = v_{i,a,t} + p_{i,a,t} + m_{i,a,t}$$

$$\begin{aligned} v_{i,a,t} &= \varepsilon_{i,a,t} - \Theta\varepsilon_{i,a-1,t-1} \quad \text{Transitory process} \\ p_{i,a,t} &= p_{i,a-1,t-1} + \zeta_{i,a,t} \quad \text{Permanent process} \\ p_{i,0,t-a} &= h_i \end{aligned} \tag{2}$$

$m_{i,a,t}$ measurement error at age a and time t

- $m_{i,a,t}$, $\zeta_{i,a,t}$ and $\varepsilon_{i,a,t}$ independently and identically distributed
- h_i reflects initial heterogeneity.

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- Review the identification of the variance of the permanent shock.
- Define unexplained earnings growth as:

$$g_{i,a,t} \equiv \Delta y_{i,a,t} = \Delta m_{i,a,t} + (1 + \Theta L)\Delta \varepsilon_{i,a,t} + \zeta_{i,a,t}. \tag{3}$$

- From now on ignore measurement error

- Key moment condition for identifying the variance of the permanent shock is

$$E\left(\zeta_{i,a,t}^{2}\right) = E\left[g_{i,a,t}\left(\sum_{j=-(1+q)}^{(1+q)} g_{i,a+j,t+j}\right)\right] \qquad (4)$$

- q is the order of the moving average process in the original levels equation; in our example $q = 1$.
- If we know the order of serial correlation of the log income we can identify the variance of the permanent shock without any need to identify the variance of the measurement error or the parameters of the MA process.

- If the MA process is order one, need at least six individual-level observations to construct this moment.
- The moment is averaged over individuals and the relevant asymptotic theory for inference is one that relies on a large number of individuals N.

- The order of the MA process for $v_{i,a,t}$ can be found by estimating the autocovariance structure of $g_{i,a,t}$ and deciding *a priori* on the suitable criterion for judging whether they should be taken as zero.

How to identify σ_ε^2 and Θ ?

$$
\begin{aligned}
g_{it} =& (1+\Theta L)\Delta\varepsilon_{it} + \zeta_{i,a,t} \\
\text{Cov}(g_{it}, g_{i,t-1}) =& (\varepsilon_{it} - \Theta\varepsilon_{i,t-1} - (\varepsilon_{i,t-1} - \Theta\varepsilon_{i,t-2})) \\
\text{Cov}(g_{it}, g_{i,t-1}) =& E\{[\varepsilon_{it} - (\Theta+1)\varepsilon_{i,t-1} + \Theta\varepsilon_{i,t-2}]\cdot \\
& [\varepsilon_{i,t-1} - (\Theta+1)\varepsilon_{i,t-2} + \Theta\varepsilon_{i,t-3}]\} \\
& \text{Assume stationarity of } \varepsilon_{it} \\
=& [-(\Theta+1)\Theta^2 - \Theta(\Theta+1)\Theta^2]\sigma_\varepsilon^2 \\
=& -\Theta^2[(\Theta+1)(\Theta+1)]\sigma_\varepsilon^2
\end{aligned}
$$

$$
\begin{aligned}
& \text{Cov}(g_{i,t}, g_{i,t-2}) \\
=& E[(\varepsilon_{it} - (\Theta + 1)\varepsilon_{i,t-1} + \Theta\varepsilon_{i,t-2}) \cdot \\
& (\varepsilon_{i,t-2} - (\Theta + 1)\varepsilon_{i,t-3} + \Theta\varepsilon_{i,t-4})] \\
=& \Theta\sigma_{\varepsilon}^2
\end{aligned}
$$

- We know left hand side of the following equation (K):

$$K = \frac{\text{Cov}(g_{i,t}, g_{i,t-1})}{\text{Cov}(g_{i,t}, g_{i,t-2})} = \frac{(\Theta + 1)^2}{\Theta}$$

$$\therefore \Theta^2 + (2 - K)\Theta + 1 = 0$$

$$\Theta = \frac{-(2 - K) \pm \left[(2 - K)^2 - 4\right]^{1/2}}{2}$$

Estimating Alternative Income Processes

Time varying impacts

- An alternative specification with very different implications is one where

$$\ln Y_{i,a,t} = \rho \ln Y_{i,a-1,t-1} + d_t(X'_{i,a,t}\beta + h_i + v_{i,a,t}) \qquad (5)$$

  where h_i is a fixed effect while $v_{i,a,t}$ follows some MA process (see Holtz-Eakin, Newey and Rosen, 1988).
- This process can be estimated by method of moments following a suitable transformation of the model.

- Define $\Theta_t = d_t / d_{t-1}$and quasi-difference to obtain:

$$\ln Y_{i,a,t} = (\rho + \Theta_t) \ln Y_{i,a-1,t-1} - \Theta_t \rho \ln Y_{i,a-2,t-2} + d_t (\Delta X'_{i,a,t} \beta + \Delta v_{i,a,t}) \quad (6)$$

- In this model the persistence of the shocks is captured by the autoregressive component of $\ln Y$ which means that the effects of time varying characteristics are persistent to an extent.
- Given estimates of the levels equation in (6) the autocovariance structure of the residuals can be used to identify the properties of the error term $d_t \Delta v_{i,a,t}$

- Alternatively, the fixed effect with the autoregressive component can be replaced by a random walk in a similar type of model.
- This could take the form

$$\ln Y_{i,a,t} = d_t(X'_{i,a,t}\beta + p_{i,a,t} + v_{i,a,t}) \quad (7)$$

- In this model $p_{i,a,t} = p_{i,a-1,t-1} + \zeta_{i,a,t}$ as before, but the shocks have a different effect depending on aggregate conditions.

- Given fixed T a linear regression in levels can provide estimates for d_t, which can now be treated as known.
- Now define $\Theta_t = d_t/d_{t-1}$ and consider the following transformation

$$\ln Y_{i,a,t} - \Theta_t \ln Y_{i,a-1,t-1} = d_t(\zeta_{i,a,t} + \Delta v_{i,a,t}) \quad (8)$$

- The autocovariance structure of $\ln Y_{i,a,t} - \Theta_t \ln Y_{i,a-1,t-1}$ can be used to estimate the variances of the shocks, very much like in the previous examples.

Stochastic growth in Earnings

- Now consider generalizing in a different way the income process and allow the residual income growth (3) to become

$$g_{i,a,t} = f_i + (1 + \Theta L)\Delta\varepsilon_{i,a,t} + \zeta_{i,a,t} \tag{9}$$

  where the f_i is a fixed effect.
- The fundamental difference of this specification from the one presented before is that income growth of a particular individual will be correlated over time.
- In the particular specification above, all theoretical autocovariances of order three or above will be equal to the variance of the fixed effect f_i.
- Consider starting with the null hypothesis that the model is of the form presented in (2) but with an unknown order for the MA process governing the transitory shock $v_{i,a,t} = \Theta_q(L)\varepsilon_{i,a,t}$.

Meghir and Pistaferri (2004), *Econometrica*

- Returning to the model previously discussed, we can extend this by allowing the variances of the shocks to follow a dynamic structure with heterogeneity.
- A relatively simple possibility is to use ARCH(1) structures of the form

$$\begin{aligned} E_{t-1}\left(\varepsilon_{i,a,t}^2\right) &= \gamma_t + \gamma\varepsilon_{i,a-1,t-1}^2 + \nu_i \quad \textit{Transitory} \\ E_{t-1}\left(\zeta_{i,a,t}^2\right) &= \varphi_t + \varphi\zeta_{i,a-1,t-1}^2 + \xi_i \quad \textit{Permanent} \end{aligned} \tag{10}$$

where $E_{t-1}(.)$ denotes an expectation conditional on information available at time $t-1$.