

Cost Benefit Analysis Using the MTE

Handbook of Econometrics

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- **Roy model:** agent selects into treatment if the benefits exceed the cost.
- Outcomes (Y_{1i}, Y_{0i}) :

$$Y_{1i} = \mu_1(X) + U_1$$

$$Y_{0i} = \mu_0(X) + U_0$$

- Ψ is generated as: $\Psi_i = \varphi(W_i) + V_i$. This is **cost**.
- W_i is a (possibly vector-valued) observed random variable.
- V_i is an unobserved random variable.

Assume:

$$D_z(\omega) = \mathbf{1}[\Delta \geq \Psi] \text{ where } \Psi = \varphi(W) + V$$

- i $\varphi(W)$ is a nondegenerate random variable, and $\varphi(W)$ is integrable.
- ii V is absolutely continuous with respect to Lebesgue measure.
- iii $Z = (W, X)$ is independent of (V, Y_0, Y_1) ,

Let $U \equiv V - (U_1 - U_0)$.

Let F denote the distribution function of U .

Let $\tilde{U} \equiv F(U)$.

Let $P(z) \equiv \Pr(D = 1|Z = z)$.

F is strictly increasing so that F will be invertible.

- Define the following treatment parameters corresponding to averages of the benefit received:

$$\begin{aligned}
 \Delta^{ATE}(x) &\equiv E(Y_1 - Y_0 | X = x) \\
 \Delta^{TT}(x, P(z), D = 1) &\equiv E(\Delta | X = x, P(Z) = P(z), D = 1) \\
 \Delta^{TT}(x, D = 1) &\equiv E(\Delta | X = x, D = 1) \\
 &= E(\Delta | X = x, \tilde{U} \leq P(Z)) \\
 \Delta^{LATE}(x, P(z), P(z')) &\equiv \frac{E(Y | X = x, P(Z) = P(z)) - E(Y | X = x, P(Z) = P(z'))}{P(z) - P(z')} \\
 \Delta^{MTE}(x, u) &\equiv E(\Delta | X = x, \tilde{U} = u)
 \end{aligned} \tag{1}$$

$$\begin{aligned}
\Delta^{ATE}(x) &= \int E(\Delta|X=x, \tilde{U}=u) du \\
\Delta^{TT}(x, P(z), D=1) &= \frac{1}{P(z)} \int_0^{P(z)} E(\Delta|X=x, \tilde{U}=u) du \\
\Delta^{TT}(x, D=1) &= \int \Delta^{TT}(x, P(z), D=1) dF_{P(Z)|X=x, D=1} \\
&= \int E(\Delta|X=x, \tilde{U}=u) g_x(u) du \\
\Delta^{LATE}(x, P(z), P(z')) &= \frac{1}{P(z) - P(z')} \int_{P(z')}^{P(z)} E(\Delta|X=x, \tilde{U}=u) du
\end{aligned} \tag{2}$$

where

$$g_x(u) = \frac{1 - F_{P(Z)|X=x}(u)}{\int (1 - F_{P(Z)|X=x}(t)) dt}$$

Mirror Image Cost Parameters

- Roy model produces mirror-image set of treatment parameters, averages of the cost of treatment.
- Define

$$\Psi^{LIV}(w, u) \equiv E(\Psi | W = w, \tilde{U} = u).$$

$$\begin{aligned}
\Psi^{ATE}(w) &\equiv E(\Psi|W = w) \\
&= \int E(\Psi|W = w, \tilde{U} = u)du \\
\Psi^{TT}(w, P(z), D = 1) &\equiv E(\Psi|X = x, P(Z) = P(z), D = 1) \\
&= \frac{1}{P(z)} \int_0^{P(z)} E(\Psi|X = x, \tilde{U} = u)du \\
\Psi^{TT}(w, D = 1) &\equiv E(\Psi|W = w, D = 1) \\
&= \int E(\Psi|X = x, \tilde{U} = u)g_w(u)du \\
\Psi^{LATE}(w, P(z), P(z')) &\equiv \frac{1}{P(z) - P(z')} \int_{P(z')}^{P(z)} E(\Psi|W = w, \tilde{U} = u)du
\end{aligned} \tag{3}$$

$$g_w(u) = \frac{1 - F_{P(Z)|W=w}(u)}{\int (1 - F_{P(Z)|W=w}(t)) dt}$$

$F_{P(Z)|W=w}$ denotes the distribution of $P(Z)$, $W = w$.

$$\Delta^{LIV}(x, P(z))$$

$$\Delta^{LIV}(x, P(z)) \equiv \frac{\partial E(Y|X = x, P(Z) = P(z))}{\partial P(z)}. \quad (4)$$

$$\Delta^{LIV}(x, P(z)) = \Delta^{MTE}(x, P(z))$$

$$\begin{aligned}\Delta^{MTE}(x, P(z)) &= E\left(\Delta|X = x, \tilde{U} = P(z)\right) \\ &= E\left(\Delta|X = x, \Delta(x) = \Psi(w)\right) \\ &= E\left(\Delta(x)|\Delta(x) = \Psi(w)\right)\end{aligned}$$

$$\begin{aligned}\Delta(x) &= \mu_1(x) - \mu_0(x) + U_1 - U_0 \\ \Psi(w) &= \varphi(w) + V\end{aligned}$$

$$\begin{aligned}
E\left(\Delta(x) \mid \Delta(x) = \Psi(w)\right) &= E\left(\Psi(w) \mid \Delta(x) = \Psi(w)\right) \\
&= E\left(\Psi(w) \mid W = w, \tilde{U} = P(z)\right) \\
&= \Psi^{MTE}(w, P(z))
\end{aligned}$$

$$\Delta^{LIV}(x, P(z)) = \Delta^{MTE}(x, P(z)) = \Delta^{LIV}(w, P(z)). \quad (5)$$

Benefit and Cost Parameters

- $\Delta^{LIV}(x, P(z)) = \frac{\partial E(Y|X = x, P(Z) = P(z))}{\partial P(z)}$.
- $\Delta^{LIV}(x, P(z))$ can be identified for any $(x, P(z))$.
- Follow the same arguments to use Ψ^{LIV} to identify or bound Ψ^{ATE} and Ψ^{TT} .

- Ψ^{LIV} identifies $\Psi^{ATE}(w)$ if the support of $P(Z)$ conditional on $W = w$ is the full unit interval.
- What information is available on the underlying benefit functions μ_0 and μ_1 .

$$\begin{aligned}\Delta^{MTE}(x, P(z)) &= E\left(\Delta \mid X = x, \tilde{U} = P(z)\right) \\ &= \mu_1(x) - \mu_0(x) + \Upsilon(P(z))\end{aligned}\quad (6)$$

$$\Upsilon(P(z)) = E(U_1 - U_0 \mid \tilde{U} = P(z))$$

$$\begin{aligned}\psi^{MTE}(w, P(z)) &= E\left(\Psi \mid W = w, U = P(z)\right) \\ &= \varphi(w) + \Gamma(P(z)).\end{aligned}\quad (7)$$

$$\Gamma(P(z)) = E(V \mid \tilde{U} = P(z)).$$

Let $\Delta^{LIV}(z) = \Delta^{LIV}(x, P(z))$.

$$\Delta^{LIV}(z) = \Delta^{MTE}(x, P(z)) = \Psi^{MTE}(w, P(z))$$

$$\begin{aligned}\Delta^{LIV}(z) - \Delta^{LIV}(z') &= (\mu_1(x) - \mu_0(x)) - (\mu_1(x') - \mu_0(x')) \\ &= \varphi(w) - \varphi(w')\end{aligned}$$

Can identify $\varphi(w)$ up to support of W conditional on $P(Z) = P(z)$.

Shifting z while conditioning on $P(z)$ shifts $(\mu_1(x) - \mu_0(x))$ and $\varphi(w)$.

- If V is degenerate,

$$D = 1 \text{ if } Y_1 - Y_0 > \varphi(W).$$

$$\Delta^{LIV}(z) = \varphi(w).$$

- Deterministic cost function.

Summary

- Roy model structure can be exploited to identify cost parameters without direct information on the cost of treatment.