Wage and Employment Discrimination by Gender in Labor Market Equilibrium

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Abstract

This paper develops an equilibrium search model to study mechanisms underlying the life-cycle gender wage gap: workers' skill accumulation, amenity preferences, and employers' statistical discrimination in wage offers and hiring. Estimating the model on administrative employer-employee data from Finland, I find that statistical discrimination accounts for 44% of the gender wage gap in early career, whereas gender differences in labor force attachment explain most of the gap in late career. Policy counterfactuals highlight the importance of employers' decisions on both wage and employment margins; requiring equality on one margin might have unintended consequences on the other margin.

JEL-codes: J16, J24, J32, J64
Keywords: Gender wage gap, statistical discrimination, human capital, job search, child penalty, non-wage amenities

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1 Introduction

The gender wage gap expands substantially over the life-cycle, especially for highly educated workers.[1] An extensive literature emphasizes the role of child-related career interruptions and gender differences in labor force attachment in driving the divergence in wages.[2] However, less is known about the extent to which employers respond to the different labor market behaviors of men and women, and the mechanisms through which employers' choices affect gender disparities in career trajectories. Since match formation and wages are influenced by both workers and firms in a labor market with search frictions, it is important to consider both labor supply and demand sides when designing policies aimed at reducing gender inequality. On one hand, we have policies to foster labor market opportunities for women, their stable employment after childbirth and access to top-level jobs; but on the other hand, the same policies could have unintended consequences when employers' counteractions are taken into account.

This paper studies both the worker- and employer-side mechanisms underlying the gender wage gap: worker's human capital accumulation, preference for job amenities, and employer's statistical discrimination in wages and employment decisions. First, women might spend more time in non-employment (potentially due to family reasons), and thus accumulate less human capital than men on the job. Second, women might sort into jobs that pay lower wages but offer more flexibility and other non-wage amenities that allow them to balance work and family. Third, employers might anticipate women to have more fertility-related separations and absence than men, so they might offer men and women different jobs and different wages for the same job. Since the seminal paper of Becker (1962), economists have been aware that labor market frictions make turnover costly to both workers and firms. Given that finding a replacement is time-consuming and costly in a frictional environment, employers might transfer the expected future costs of turnover into lower wages for women, or avoid hiring women altogether and/or sort them into less productive jobs.[3]

[1] Barth, Kerr and Olivetti (2021) and Goldin, Kerr, Olivetti, Barth et al. (2017) document the life-cycle wage patterns of college and high school-educated men and women in the US.

[2] See Blau and Kahn (2017) and Altonji and Blank (1999) for comprehensive reviews on the explanations of the gender wage gap.

[3] Although employers' expectations might be correct on average and their decisions are rational, such differential wage-setting and hiring practices towards men and women would constitute the notion of statistical discrimination as in Arrow (1972) and Phelps (1972) – employers cannot observe the individual's labor force attachment, so they make decisions for each individual worker based on his/her group average characteristics. I use the terms "differential wage-setting" or "differential hiring" interchangeably with "statistical discrimination" throughout the paper.

In order to study both worker and firm behaviors in the presence of frictions, I develop an equilibrium search model to quantify the above mechanisms and their interactions. The model allows male and female workers to have different turnover rates, childcare behaviors, and preferences for job amenities in each of the three stages in life – before having children, after having children, and in non-fecund ages. Hiring a woman can be associated with a lower match value for several reasons. First, she is more likely to separate into unemployment and the employer might need to pay vacancy costs for some periods before hiring another worker. Second, she is more likely to take a longer parental leave, during which the employer suffers from a lower output and a lack of growth in her human capital. Third, a job with a low level of family-friendly amenity risks losing the woman to high-amenity jobs, whereas the employer is less likely to face such risks if matched with a man. All these considerations might serve as a basis for employers to statistically discriminate against women (some employers more than others).

A novel feature of the model is that workers and employers make decisions on both the wage margin and the employment margin. Employers have capacity constraints, and they can fill each job slot with only one worker.[4] To add to a close line of work that uses search models to analyze the gender pay gap (Bowlus, 1997; Flabbi, 2010; Bartolucci, 2013; Bagger, Lesner and Vejlin, 2019; Morchio and Moser, 2020; Gray, 2021; Amano, Baron and Xiao, 2021), the capacity constraint in this model puts men and women in direct competition with each other as they search for the same jobs. Such competition between the genders would be absent in any job ladder model where firms have unlimited capacity and operate under constant returns to scale. With a scarcity of jobs to allocate, employers have to carefully consider the trade-offs between hiring a woman versus a man. More productive jobs (such as managerial positions) might be especially concerned about employing women since these jobs forgo more production per period when the worker leaves.

The human capital and sorting channels offer further insights. First, women might sort into low productivity jobs if family-friendly positions are less productive, or if highly productive firms and positions offer fewer opportunities for women (or both). Therefore, the occupational gender segregation we see in the data might not be determined by workers' preferences alone. Second, if workers gain skills more quickly in a highly productive environment, then part of the gender productivity difference could be driven by women being stuck in low-end jobs where human capital grows slowly. These insights highlight the difficulties in interpreting worker and firm fixed effects in reduced-form decomposi-

[4]Even though a capacity constraint on the firm side is not necessary to generate sorting in the multi-dimensional settings (Lindenlaub and Postel-Vinay, 2017), the question I analyze naturally calls for a capacity constraint, so that men and women compete in the same market for the same jobs.

tions of the gender wage gap; women might have low worker or firm fixed effects because they face persistent consequences of differential job opportunities.

Using administrative matched employer-employee data combined with occupational level data on job amenities from Finland, I first document gender differences in labor market behaviors around childbirth for university-educated workers. I find that women are more than twice as likely as men to transition from employment to unemployment after having children. Compared to men, women are also more likely to reduce hours, switch to part-time jobs, and move to jobs with better amenities (in terms of a family-friendly index) after childbirth. Women in Finland spend on average 18 months in parental leave for each child, whereas men spend only 2 months. Over the life-cycle, the unconditional wage gap between highly educated men and women increases from 12 log points at labor market entry to 20 log points after 10 years, and then decreases to 15 log points in late career.

I estimate the model by the method of simulated moments, and find that 44% of the gender wage gap in early career is attributed to employers' statistical discrimination based on fertility concerns. As workers move beyond child-rearing ages, statistical discrimination fades away and a vast majority (75%) of the wage gap in late career is due to gender differences in labor force attachment and an accumulated shortage in women's human capital. I find that women value family-friendly amenities as much as men do before having children, but value them twice as much after children. This affects sorting patterns even before childbirth and is responsible for about 9% of the overall wage gap after having children. The residual wage gap, which could be due to employers' taste-based discrimination or initial productivity differences between men and women, accounts for approximately 18% of the total gap.

I compute three policy counterfactuals aimed at reducing gender inequality. First, a "daddy months" expansion that shifts two months of parental leave from women to men closes the wage gap by 13% throughout the life-cycle. The "daddy months" policy is not enough to correct hiring discrimination in early career, but it does change employer perceptions and reduces statistical discrimination in wages. It also reduces the human capital gap between men and women after childbirth. Second, an equal hiring policy in top jobs improves women's representation in managerial positions, but employers undo this policy by exerting more wage discrimination – the gender wage gap increases by 3% in the first 6 years. However, being employed in good jobs in early career allows young women to gain skills at a faster rate, and the human capital gains more than compensate for the initial wage loss. Third, the equal pay counterfactual shows that requiring firms

to pay the same wage to similar men and women closes the gender wage gap by 15% on average. However, the equal pay policy has unintended consequences as employers adjust on the hiring margin. Women are more likely to be unemployed, and the proportion of women in top job decreases slightly (by 3%) two decades after labor market entry.

Taken together, the results suggest that it would be difficult to achieve gender equality at the workplace without more equality in family responsibilities (e.g. sharing child-related leave more equally between men and women), given the sizable effect of employer statistical discrimination in both wages and employment in equilibrium.

This paper makes three contributions. First, it develops and estimates an equilibrium search model with employer capacity constraints, where men and women compete for the same jobs and employers may not match with both genders. While the capacity constraint is a natural feature in this context, it makes the problem considerably more complex, since it requires the solution of fixed point problems in not only the match surplus values, but also in the allocations of matched and unmatched agents.

Second, this paper is the first to bring together all three mechanisms – human capital, job preferences, and statistical discrimination in wages and employment – in one unified framework, opening an avenue to study the rich interactions between the channels. For example, statistical discrimination could be based on expected human capital stagnation during parental leave and/or anticipated job switches driven by amenity preferences. In turn, both hiring discrimination and amenity preferences push women into low-productivity jobs, affecting their human capital growth. The research question at hand requires many model features that are typically not present in standard search models in the literature, for example multi-dimensional firm and worker types, life-cycle dynamics, and human capital accumulation. These features make the model very rich but also post significant computational challenges.

Third, the paper combines administrative employer-employee data with survey data on job amenities, and documents workers' sorting patterns across jobs of different observable amenity levels. Exploiting the employer-employee linked nature of the data, I use the mobility patterns of men and women across jobs, gender ratios within jobs, wages and wage growths at various transitions over the life-cycle to separately identify human capital, preference and production parameters.

1.1 Related literature

There is an extensive literature examining the explanations for the gender wage gap (see Altonji and Blank (1999) and Blau and Kahn (2017) for comprehensive reviews). A growing recent literature highlights the importance of fertility-related career interruptions in explaining the gap. Angelov, Johansson and Lindahl (2016), Kleven, Landais and Søgaard (2019) and Andresen and Nix (2019) document a large and persistent income penalty experienced by women after having children, along with lower participation, fewer hours worked, and a higher tendency to work in the public sector after childbirth. Erosa, Fuster and Restuccia (2016) and Adda, Dustmann and Stevens (2017) develop dynamic models of human capital accumulation, fertility and labor supply choices of women to estimate the impact of children on the gender wage gap. While existing work focuses on the direct consequences of childbearing on female workers, this paper considers how women's behaviors around childbirth affect employers' wage and hiring decisions both before and after the fertility event.

Since Groshen (1991), several empirical studies have assessed the role of employers by distinguishing gender gaps *within* versus *across* firms (Card, Cardoso and Kline, 2016; Goldin, Kerr, Olivetti, Barth et al., 2017; Barth, Kerr and Olivetti, 2021). However, it is difficult to determine the mechanisms driving the within- and across-firm wage differentials between similar men and women. Bronson and Thoursie (2021) finds that the promotions gap between men and women is sizable within the firm especially in early career, consistent with a statistical discrimination model where it is costly to promote someone who might reduce labor supply in the future. The paper contributes to this literature by formalizing workers' mobility decisions across jobs as well as employers' pay-setting policies within each job in a unified equilibrium model, so that one is better-equipped to analyze the mechanisms driving within- and across-firm wage differentials.

On the labor demand side, existing literature has theorized the link between women's child-related career interruptions and firms' statistical discrimination (Barron, Black and Loewenstein, 1993; Albanesi and Olivetti, 2009; Gayle and Golan, 2012; Tô, 2018; Thomas, 2019). In these models, workers have private information about their labor force attachment, and employers' uncertainty about workers' types affects who gets assigned to high-paid jobs. This paper adds to the literature by including several sources of statistical discrimination, including workers' quit behaviors, human capital accumulation and amenity preferences. Moreover, the frictional search framework allows one to quantify the welfare loss due to worker-job mismatches that can be attributed to statistical discrimination.

Occupational segregation by gender can be driven by both labor demand and supply factors, and the literature has mainly focused on the supply side and highlighted gender differences in the willingness to pay for certain job amenities. Felfe (2012), Goldin (2014) and Wiswall and Zafar (2017) show that women sort into occupations with temporal flexibility and fewer working hours, which account for some of the gender wage gap. Adda, Dustmann and Stevens (2017) and Hotz, Johansson and Karimi (2017) point out long-term career consequences of working in family-friendly occupations, as skill accumulation is lower in these jobs. In light of this literature, my model incorporates non-wage amenities that help balance work and family, and investigates how these preferences interact with other channels in an equilibrium framework.[5]

The model in this paper is built on a body of search-matching literature with wage bargaining (Cahuc, Postel-Vinay and Robin, 2006), sorting (Lise, Meghir and Robin, 2016; Lindenlaub and Postel-Vinay, 2017), and with human capital accumulation (Herkenhoff, Lise, Menzio and Phillips, 2018; Lise and Postel-Vinay, 2020). The paper is closest to the equilibrium search models that analyze gender pay gaps (Bowlus, 1997; Flabbi, 2010; Morchio and Moser, 2020; Gray, 2021). In this line of work, Bagger, Lesner and Vejlin (2019) and Amano, Baron and Xiao (2021) are the only papers that consider human capital dynamics, fertility and parental leave. This study complements the above papers by allowing men and women to compete for the same jobs, and by modeling employers' decisions on the hiring margin so that one can analyze the consequences of unequal job opportunities.

2 Empirical Motivations

In this section, I will briefly describe the datasets and show a number of empirical patterns related to gender differences in the labor market.

2.1 Data

The Finnish Longitudinal Employer-Employee Data (FOLK) provides information on workers' demographics, monthly employment histories, children's birth dates and parental leave claims for the full population between years 1988 and 2016. Parental leave duration

[5] A strand of literature uses a revealed preference approach to study the importance of job amenities (Sorkin, 2018; Lamadon, Mogstad and Setzler, 2021; Taber and Vejlin, 2020). This paper uses a direct approach by focusing on observed amenities related to flexibility and hours which are particularly important for women's occupation choice.

is inferred from the annual parental leave allowance and home care allowance claims according to a schedule detailed in Appendix C. The Structure of Earnings Statistics (SES) 1995-2013 contains hourly wages, part-time status, (paid) contracted hours and 4-digit occupation codes that are typically not available from tax registers.[6]

Since educated people experience the largest increase in the gender wage gap over the life-cycle, in this paper I will focus on individuals who obtained master's degrees[7] in the years 1988 to 2005 so that we observe at least 10 years of labor market activities. Appendix B provides more details on sample restrictions.

2.2 Descriptive decomposition of the gender wage gap

Women have overtaken men in educational attainment in Finland. However, women's labor market outcomes do not seem to catch up with their male classmates.

To investigate the evolution of the gender wage gap over the life-cycle, I first decompose it descriptively by successively adding more controls. Figure 1a shows the difference between men and women's log hourly wages by years since graduation (potential experience): *(i)* unadjusted (only with year fixed effects); *(ii)* adjusted for a quadratic in actual experience in addition to *(i)*; and *(iii)* adjusted for a full set of interactions between 4-digit occupation and firm dummies in addition to *(i)* and *(ii)*.[8] Figure 1b does the same decomposition exercise around the first childbirth.

The unadjusted gender wage gap in Figure 1a increases from 12 log points at labor market entry to 20 log points in 10 years, and then declines slowly to 15 log points in 25 years (when workers are above age 50).

Actual experience is defined as the cumulative number of months a person has worked after college graduation.[9] Since women spend more time in non-employment especially after childbirth, actual experience explains more and more of the wage gap between men

[6]The SES covers 55 to 75 percent of private sector workers depending on the year, and under-samples small firms. Since I do not include small firms with 2 workers or less, data coverage is not a big issue. In the estimation, I use sample weights to account for potential missing data from small firms.

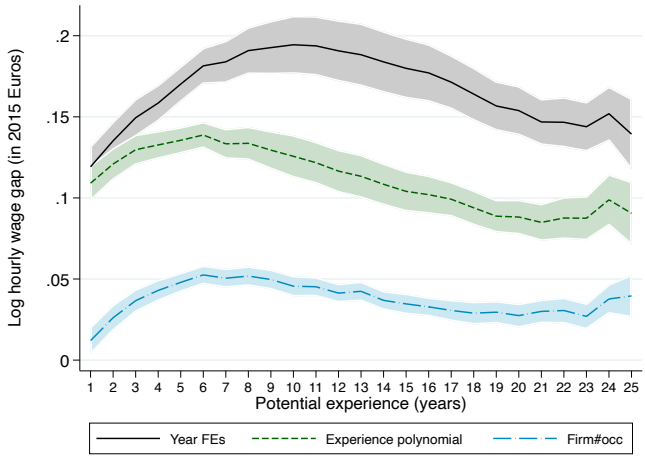
[7]Master's degree in Finland is roughly equivalent to US bachelor's degree, since Finnish students who get into academic-track bachelor's programs are automatically enrolled in the master's programs.

[8]Since university majors are highly correlated with occupations, they do not explain additional gender wage gap after controlling for 4-digit occupations. Results with university majors are available upon request.

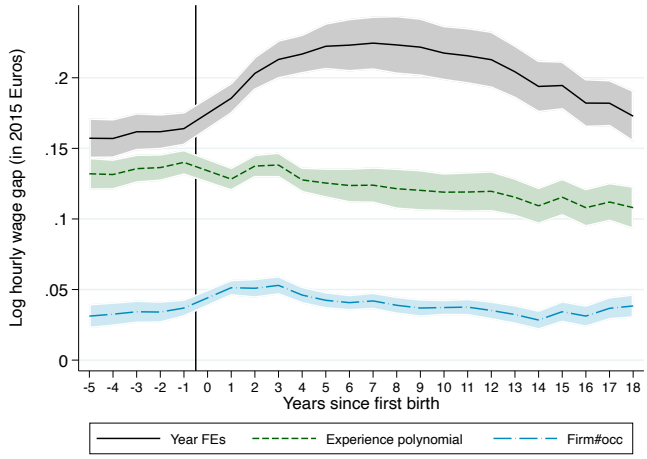
[9]Since both men and women might work before obtaining master's degrees, I calculate their formal labor market experience after bachelors' graduation, excluding short-term employment of 3 months or less and excluding summer internships. By the time they graduate with master's degree, men have 1.9 years of actual experience while women have 1.6 years. The difference is not statistically significant.

FIGURE 1. Descriptive decomposition of the gender wage gap

(A) Over the life-cycle



(B) Around childbirth



NOTES: The lines represent the coefficients on the male dummy interacted with potential experience or years since childbirth. Shaded areas represent 95% confidence intervals. Standard errors are clustered at the firm level. The coefficients are obtained from regressions of real log hourly wages on: (i) year dummies; (ii) a quadratic in actual experience in addition to (i); (iii) a full set of interactions of firm and occupation dummies in addition to (i) and (ii).

and women over the course of their careers.

There is still an "unexplained" gap (the bottom blue line in Figure 1a) of about 4 log points after adding occupation and firm interactions – men and women are offered different wages even when they have the same actual experience and work in the same detailed occupation within the same firm. This might suggest unequal pay for similarly qualified workers, potentially due to statistical or taste-based discrimination.

To investigate the impact of children, a similar descriptive decomposition is conducted for the years around the birth of the first child. Figure 1b shows that a substantial gender wage gap of 14 log points already exists before the birth of the first child, and increases to 21 log points 7 years afterwards. Notably, the "unexplained" gap also exists before childbirth, potentially suggesting that firms might anticipate their workers' fertility and treat men and women differently even before they have children.

Importantly, the descriptive patterns shown in both Figure 1a and Figure 1b must be interpreted with caution, since the "unexplained" gap might not represent the entire size of discrimination because actual experience, occupations and firms may themselves be a result of discrimination.

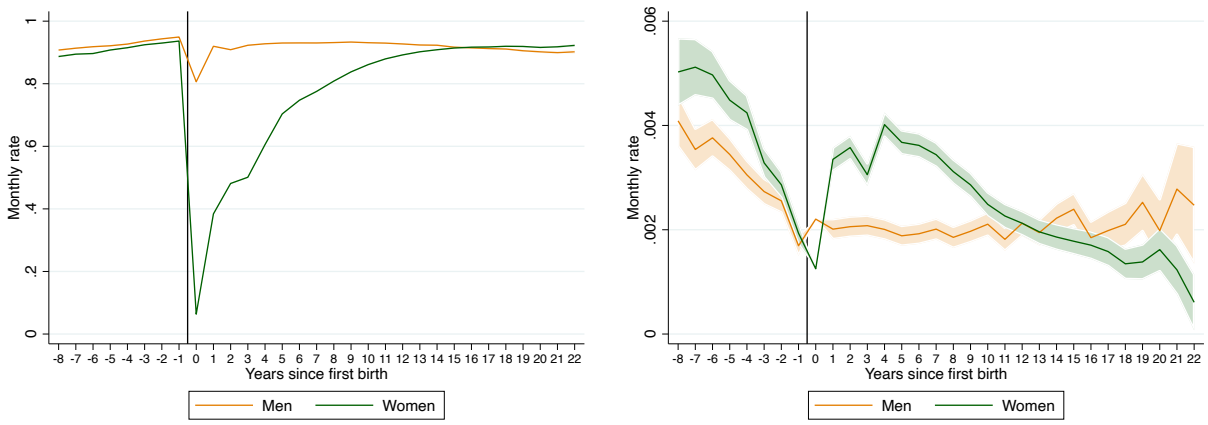
2.3 Gender differences in labor market behaviors

The Finnish parental leave system is very generous (see Appendix C for a detailed description). Master's graduated women take on average 18 months of paid leave for each child compared to only 2 months taken by men with master's degree.

Figure 2a shows that men and women have similar employment rates before having children (at about 90 percent), but their labor supply diverges drastically after the birth of their first child. Virtually all women take some months off in the year of childbirth; the female employment rate increases from 6 to 38 percent the year after birth, but takes time to recover to its pre-birth levels since many women have a second or third child. Eventually, women's labor supply does go back to 90 percent, but only some 14 years after the birth of the first child. In contrast, men only experience a small dip in labor supply in the year of childbirth, and do not seem to be affected afterwards.

FIGURE 2. Labor force attachment around childbirth

(A) Employment rate (PL=not working) (B) Separation rates



NOTES: The lines represent the coefficients obtained from regressions of outcome variables on the number of years since first birth, with calendar year fixed effects. Shaded areas represent 95% confidence intervals.

The large gender gap in employment rates is also driven by women's employment-to-unemployment (E-to-U) transitions after childbirth.[10] As shown in Figure 2b, women's monthly separation rate is slightly higher than men's prior to birth, but it spikes and remain well above men's level for nine years after childbirth.[11] This could be driven by

[10]Unemployment is defined as all months where the worker is not associated with any employer, whether or not he/she is actively looking for a job. Those who are on parental leave are associated with employers, so they are considered as employed for the purpose of computing E-to-U or E-to-E transition rates. See Appendix C for more details.

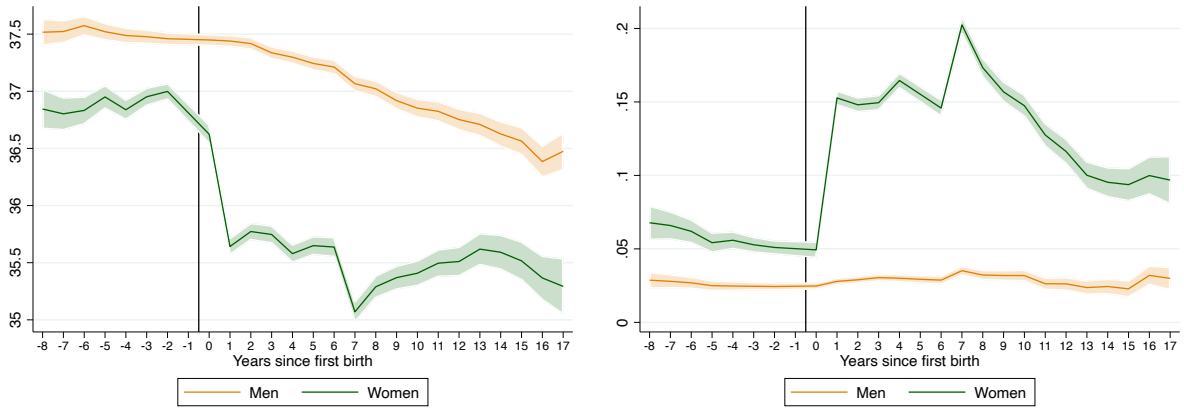
[11]Unemployment-to-employment transitions and job-to-job transitions are quite comparable between men and women, as shown in Figure A1.

voluntary or involuntary quits, although they cannot be distinguished in the data.

When women return to work after having children, they tend to work in different types of jobs. Figure 3a shows that women reduce weekly contracted hours from an average of 37 before childbirth to 35.5 immediately afterwards. Only a small proportion of educated workers have part-time jobs in Finland, but the proportion of women working part-time increases rapidly from 5 percent prior to birth to 15 percent the year after birth and remains at that level for 10 years, as shown in Figure 3b.

FIGURE 3. Demand for job amenities around birth

(A) Contracted weekly hours (B) Proportion with part-time status



NOTES: The lines represent the coefficients obtained from regressions of outcome variables on the number of years since first birth, separately for men and women, with individual fixed effects and calendar year fixed effects. Shaded areas represent 95% confidence intervals.

Even though Finnish workers are allowed to ask for reduced hours after having children, in practice the ability to do so might depend on specific employers. Out of those women who have always worked full-time before childbirth but have switched to part-time for at least one year afterwards, about 58 percent of them have to either change firms or change occupations within a firm in order to switch to part-time status. This is consistent with what Altonji and Paxson (1992) found for the US. I will use the availability of part-time work in a firm-occupation cell as part of the measure for job-specific amenities in section 2.4.

2.4 Family-friendly amenities

Workers' job choices can be driven by both wages and non-wage amenities e.g. job hazards, working conditions, stress and well-being and so on. In this paper, I focus only on the job amenities that are documented to be valued differentially by men and women,

such as reduced hours, part-time work, and flexible work schedules (Goldin and Katz, 2011; Flabbi and Moro, 2012; Felfe, 2012; Goldin, 2014; Edwards, 2014; Wiswall and Zafar, 2017).

I use several data sources to construct the amenity measure. The Finnish Quality of Work Life (QWL) Surveys[12] ask questions related to flexibility (positive amenities) and over-working (negative amenities), listed below:

Flexibility:

- Have you agreed with the employer to work occasionally at home?
- Can you influence starting and finishing times for your work by at least 30 minutes?
- Can you use flexible working hours sufficiently for your own needs?
- Do have the possibility for brief absences from work in the middle of the working day to run personal errands?

Overwork:

- Do you sometimes work overtime without compensation?
- Have you been contacted about work outside of working hours during the last two months?
- Do you have to do more overtime work than you would like to?

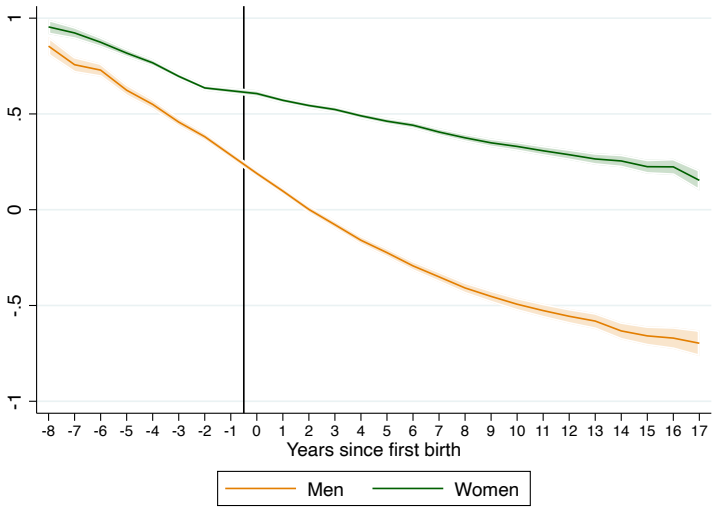
The "family friendly" amenity index of each job is the first principal component of the above 7 QWL questions aggregated at 2-digit occupation level,[13] actual hours worked from the labor force survey, and the opportunity to do part-time work in a firm-occupation cell from SES. Table A1 shows the factor loadings of these variables and the proportion of variation not explained by the first component. The amenity index is largely driven by the variables related to hours, as the first principal component loads more heavily on the QWL measures on overwork than on the measures of flexibility (in absolute value).[14]

[12]The QWL surveys are extensive studies of a representative sample of 4000 to 6000 wage or salary earners in Finland in each wave 1977, 1984, 1990, 1997, 2003, 2008 and 2013. It documents how people feel about their working conditions related to physical or social environment, job satisfaction, work orientation and so on.

[13]2-digit occupations may not be detailed enough to give a fully comprehensive picture. However, the 2013 QWL is the only wave that provides information on the 2-digit level, and one cannot go into more detailed occupations due to the sample size of the surveys.

[14]The loadings of the flexibility measures are negative, possibly because highly flexible occupations (e.g. managers, science and engineering professionals etc.) also have high overtime requirements, and the index aligns more with overtime. It would be interesting to analyze how workers trade off the two dimensions of family-friendly amenities (high flexibility and low overtime hours) in future work.

FIGURE 4. Amenity index of workers' jobs around childbirth



NOTES: The lines represent the coefficients obtained from a regression of the amenity index on the number of years since first birth, separately for men and women, with worker fixed effects. Shaded areas represent 95% confidence intervals.

Using this amenity index, several interesting patterns emerge. First, jobs with high amenity index values are more abundant in the middle and lower end of the wage distribution (see Figure 6). Second, workers move from high- to low-amenity jobs over the life-cycle, because jobs become more demanding and require more hours and overtime as people climb the career ladder (e.g. to managerial positions). Third, Figure 4 shows that women are in jobs with slightly higher amenity index than men before childbirth, but there is a clear divergence after childbirth when women switch into high-amenity jobs and are more likely to stay there.

3 Model

Motivated by the data patterns of men and women's labor market behaviors, the model incorporates important gender differences to study both workers' and employers' decisions. I first describe the characteristics of workers and firms, and their life-cycle stages. I then explain the matching process between workers and firms and the wage determination mechanisms. Lastly, the steady-state equilibrium of the labor market is characterized.

3.1 The environment

Workers Time is discrete and infinite. The labor market is populated by a continuum of female and male workers of measures μ and $(1-\mu)$, as well as a continuum of jobs of measure 1. Workers are risk-neutral, and maximize the present value of their utilities, discounted at factor $\beta \in (0,1)$. Workers are heterogeneous in the level of human capital x and their value for amenity ϵ. Human capital determines the worker's contribution to output when employed and the worker's home productivity $b(x)$ when unemployed.

Upon entering the labor market, workers of gender $g \in \{m, f\}$ draw their initial skills and value for amenities from an exogenous discrete distribution with probability mass function $\xi^g(x, \epsilon)$. The model focuses only on workers' lives after graduation, and takes as given their pre-labor market decisions in human capital investment (including choices in the level of education and field of study).[15]

Human capital evolves stochastically according to a law of motion $p_e(x, y)$ in employment and $p_u(x)$ in unemployment. The skill accumulation rate in employment is allowed to depend on job productivity y, capturing the idea that workers might learn faster on the job when matched with more productive employers (Gregory, 2021), either from knowledge spillovers by more productive coworkers (Nix, 2019), or from doing more complex tasks.

Employers A job is an occupation within a firm. Each job maximizes the present value of its profit, also discounted at factor β. Jobs are heterogeneous in productivity y and amenity provision α drawn from an exogenous distribution with joint density $\varphi(y, \alpha)$. If the job is vacant, it does not produce any output and has to pay a flow vacancy cost c. Importantly, each job can only match with one worker, and employers are not allowed to search for new hires when the job is filled. The distribution of jobs is fixed at $\varphi(y, \alpha)$ and there is no free entry of jobs. When an employer of type (y, α) matches with a worker of type (x, ϵ), they produce $f(x, y)$ units of output.

Life stages Workers go through four age segments in life. All workers start their careers in a stage with no child (the NC stage). At an exogenous fertility rate χ, the worker has a child and enters a stage with young child (the YC stage). Every time the worker has a child, he/she will enter a parental leave (PL) stage and stay out of the labor force. Men

[15] To the extent that the field of study is highly correlated with occupation choice, the preference for different university majors is partially incorporated in the model as men and women are allowed to have different tastes for occupations.

and women might stay in the PL stage for different durations governed by exit rates η_m and η_f, upon which they can go back to their previous employers. Workers can have children repeatedly until they become non-fecund (at rate γ), at which point they will be "done" with children (D stage). Workers retire at rate ϕ in stage D, and new workers enter the labor market at the same rate. Within each age segment $a \in \{NC, PL, YC, D\}$ of life, the search and matching process is analogous.

Fertility is taken to be exogenous in the model for the following two reasons. First, there is not much room for women to manipulate the timing of fertility given the late graduation age. In Finland, master's men and women graduate at the age of 26 on average, and both have the first child roughly 5 years after finishing school. Those who have children have on average 2 children, so there is not much time in the fecund period to postpone fertility.[16] Second, contrary to the idea that higher wages might encourage women to have fewer children and have them later, the data actually shows a small and positive correlation between wages and the number of children for both men and women. The model abstracts from the fact that it is expensive to have more children.

Exogenous gender differences include parental leave duration (governed by η_g), exogenous separation rates before and after having children (δ^g_{NC} and δ^g_{YC}), as well as preference for job amenities that are drawn from different distributions for men and women. Women's value for amenities is also allowed to change after children.

3.2 Search and matching

At each point in time, workers can be matched to a firm or be unemployed. The aggregate number of meetings between vacancies and searching workers is determined by a standard aggregate matching function $m(\hat{U}, V)$. This takes as inputs the total number of vacancies V and the total amount of effective job seekers $\hat{U} = U + s(1 - U)$, where U is the total number of unemployed workers and s is the search intensity in employment relative to unemployment. The matching function is assumed to be increasing in both arguments and exhibit constant returns to scale.

For ease of exposition, let $\kappa = \frac{m(\hat{U},V)}{\hat{U}\,V}$ summarize the effect of market tightness. κ is constant in a stationary equilibrium, but it is not invariant to policy, and it is important to allow it to change when evaluating interventions or counterfactual regulations.

Let $u^g_a(x, \epsilon)$ denote the measure of unemployed workers of gender g, age a and type

[16] Women in the highest decile of initial wage (average wage in the first 3 years) postpone having their first child by only 6 months compared to women in the lowest decile.

(x, ϵ), and let $v(y, \alpha)$ denote the measure of vacancies of type (y, α). The joint distribution of matches between workers of type (x, ϵ) and jobs of type (y, α) is denoted as $h_a^g(x, \epsilon, y, \alpha)$. While unemployed, workers randomly sample offers from the vacancies distribution, and the instantaneous rate at which an unemployed worker meets a vacancy of type (y, α) is $\kappa\, v(y, \alpha)$. Similarly, employed workers meet vacancies at rate $s\kappa\, v(y, \alpha)$, and vacancies meet employed workers at rate $s\kappa\, h_a^g(x, \epsilon, y, \alpha)$.

Upon a meeting between a worker and a job, a match will be formed if it generates positive surplus. In other words, match formation is assumed to be efficient.

Let $U_a^g(x, \epsilon)$ denote the lifetime value of an unemployed worker of type (x, ϵ), $\Pi_0(y, \alpha)$ denote the vacancy value of a job of type (y, α). Let $P_a^g(x, \epsilon, y, \alpha)$ denote the value of **joint** production of a match between worker (x, ϵ) and job (y, α). The surplus of a match is defined as $S_a^g(x, \epsilon, y, \alpha) = P_a^g(x, \epsilon, y, \alpha) - U_a^g(x, \epsilon) - \Pi_0(y, \alpha)$. A match is feasible and sustainable if the match surplus is positive, $S_a^g(x, \epsilon, y, \alpha) > 0$.

Workers have bargaining power denoted by σ and obtain a share of the match rent, following the formulation in Cahuc, Postel-Vinay and Robin (2006). Let $W_a^g(w, x, \epsilon, y, \alpha)$ (and respectively $\Pi_a^g(w, x, \epsilon, y, \alpha)$) denote the value of a wage contract w for a worker (x, ϵ) employed at a job (y, α) (respectively the firm's profit). The surplus can then be written as:

$$S_a^g(x, \epsilon, y, \alpha) = \underbrace{W_a^g(w, x, \epsilon, y, \alpha) - U_a^g(x, \epsilon)}_{\text{Worker's share}} + \underbrace{\Pi_a^g(w, x, \epsilon, y, \alpha) - \Pi_0(y, \alpha)}_{\text{Employer's share}}.$$

The way in which wage w splits the surplus between the worker and the employer will be discussed in the following section.

3.3 Wage determination

To define wages and renegotiations, I follow the setup in Cahuc, Postel-Vinay and Robin (2006). Workers' wages are determined by sequential auctions. Different wages are negotiated when a worker leaves unemployment, and when counteroffers are made for an employed worker upon poaching.

Wage bargaining with unemployed workers The starting wage $\phi_{0,a}^g(x, \epsilon, y, \alpha)$ obtained by a type-(x, ϵ) unemployed worker when matched with a type-(y, α) job is such that the worker receives the reservation utility $U(x, \epsilon)$ plus a share σ of the surplus:

$$W_a^g\left(\phi_{0,a}^g(x,\epsilon,y,\alpha),x,\epsilon,y,\alpha\right) = U_a^g(x,\epsilon) + \sigma\, S_a^g(x,\epsilon,y,\alpha) \tag{1}$$

for jobs where surplus $S_a^g(x,\epsilon,y,\alpha)$ is positive.

Wage at job-to-job transitions When a worker of type (x,ϵ) encounters an alternative job package (y',α') that produces more surplus than her current job, she will switch jobs with a wage $\phi_{1,a}(x,\epsilon,y,\alpha,y',\alpha')$ such that the value she receives at the new job (y',α') is $W_{NC}^g(\phi_1^g,x,\epsilon,y',\alpha')$. In this scenario, the worker extracts the maximum value from the incumbent match $P_a^g(x,\epsilon,y,\alpha) - \Pi_0(y,\alpha)$ plus a σ share of the surplus difference:

$$W_a^g\left(\phi_{1,a}^g(x,\epsilon,y,\alpha,y',\alpha'),x,\epsilon,y',\alpha'\right) = P_a^g(x,\epsilon,y,\alpha) - \Pi_0(y,\alpha) + \sigma\left[S_a^g(x,\epsilon,y',\alpha') - S_a^g(x,\epsilon,y,\alpha)\right] \tag{2}$$

Wage renegotiation upon poaching If the poaching job (y',α') generates a match surplus below that of the incumbent job, i.e. when $S_a^g(x,\epsilon,y',\alpha') < S_a^g(x,\epsilon,y,\alpha)$, the worker will stay in the same job. Incumbent employers will respond to outside offers and update wages only when there is a credible threat – when either the worker or the employer will credibly separate if they do not obtain an improved offer. In other words, wages will be re-negotiated when the poaching firm offers a value greater than what the worker currently receives, when $P_a^g(x,\epsilon,y',\alpha') - \Pi_0(y',\alpha') > W(w,x,\epsilon,y,\alpha)$. In this case, wages will be updated from w to $\phi_{2,a}(x,\epsilon,y',\alpha',y,\alpha)$ such that the worker receives an updated value $W_a^g\left(\phi_{2,a}^g,x,\epsilon,y,\alpha\right)$ at the incumbent job (y,α) that equals the maximum value the poaching employer is willing to offer:

$$W_a^g\left(\phi_{2,a}^g(x,\epsilon,y',\alpha',y,\alpha),x,\epsilon,y,\alpha\right) = P_a^g(x,\epsilon,y',\alpha') - \Pi_0(y',\alpha') + \sigma\left[S_a^g(x,\epsilon,y,\alpha) - S_a^g(x,\epsilon,y',\alpha')\right] \tag{3}$$

Note that when a worker's human capital appreciates from x to x_+ in the next period, her wage does not update until there is a credible outside option. Please refer to Appendix D for details of the workers' values.

3.4 Value functions

In order to define an equilibrium, I will describe the value functions and the distributions of workers and jobs across employment states and life stages. These define the decision rules for each agent.

3.4.1 Value in unemployment

In the "No Child" stage of life, the utility of an unemployed worker is:

$$U^g_{NC}(x,\epsilon) = b(x) + \beta\,\mathbb{E}\Big[\sum_{y,\alpha}\kappa\, v(y,\alpha)\,\Big(U^g_{NC}(x_+,\epsilon) + \sigma\,\max\{S^g_{NC}(x_+,\epsilon,y,\alpha),0\}\Big) \tag{4}$$
$$+\,\chi\, U^g_{PL}(x_+,\epsilon) + \gamma\, U^g_D(x_+,\epsilon) + (1-\chi-\gamma-\kappa V)\, U^g_{NC}(x_+,\epsilon)\Big].$$

The worker receives a flow value of $b(x)$ in the current period. In the next period, the worker's human capital level updates from x to x_+, where the transition matrix is given by the law of motion $p_u(x)$ in unemployment. The present discounted value takes the expected future payoff over the probability distribution $p_u(x)$.

The worker is subject to life-cycle shocks in the next period. When an unemployed worker has a child at rate χ, he/she exits the labor market and enters a period of parental leave and do not conduct job search in the PL stage. When parental leave terminates at rate η^g, the worker enters the labor market and resumes job search in unemployment in the "Young Child" stage. At any point in life, the worker ages at rate γ, upon which he/she enters a non-fecund period with unemployment value $U^g_D(x_+,\epsilon)$. The unemployment values in PL, YC and D stages are described as follows:

$$U^g_{PL}(x,\epsilon) = b(x) + \beta\,\mathbb{E}\Big[\eta^g\, U^g_{YC}(x_+,\epsilon) + \gamma\, U^g_D(x_+,\epsilon) + (1-\eta^g-\gamma)\, U^g_{PL}(x_+,\epsilon)\Big] \tag{5}$$

$$U^g_{YC}(x,\epsilon) = b(x) + \beta\,\mathbb{E}\Big[\sum_{y,\alpha}\kappa\, v(y,\alpha)\;\sigma\,\max\{S^g_{YC}(x_+,\epsilon,y,\alpha),0\} \tag{6}$$
$$+\,\chi\, U^g_{PL}(x_+,\epsilon) + \gamma\, U^g_D(x_+,\epsilon) + (1-\chi-\gamma)\, U^g_{YC}(x_+,\epsilon)\Big]$$

$$U^g_D(x,\epsilon) = b(x) + \beta\,\mathbb{E}\Big[\sum_{y,\alpha}\kappa\, v(y,\alpha)\;\sigma\,\max\{S^g_D(x_+,\epsilon,y,\alpha),0\} + (1-\phi)\, U^g_D(x_+,\epsilon)\Big] \tag{7}$$

In stage D, individuals no longer have any child, and retire at rate ϕ.

3.4.2 Value of vacancy

A vacant job could potentially hire a male or female worker of any age $a \in \{NC, YC, D\}$. The value of a vacancy of type (y,α) is:

$$\Pi_0(y,\alpha) = -c + \beta\Big[\sum_{a,g,x,\epsilon} \kappa\, u_a^g(x,\epsilon)\,\Big(\Pi_0(y,\alpha) + (1-\sigma)\max\{S_a^g(x,\epsilon,y,\alpha),0\}\Big)$$
$$+ \sum_{a,g,x,\epsilon,y',\alpha'} s\kappa\, h_a^g(x,\epsilon,y',\alpha')\Big(\Pi_0(y,\alpha) + (1-\sigma)\max\{S_a^g(x,\epsilon,y,\alpha) - S_a^g(x,\epsilon,y',\alpha'),0\}\Big)$$
$$+ \Big(1 - \kappa\, U - s\kappa\,(1-U)\Big)\Pi_0(y,\alpha)\Big] \tag{8}$$

c is a per-period cost of keeping a vacancy open, and U denotes the aggregate unemployment. Job vacancies have the opportunities to meet unemployed and employed workers of any age, gender, productivity and preference types. Since employers have capacity constraints, the option value of waiting Π_0 is typically positive.

3.4.3 Joint value of a match

In the "No Child" stage, the joint value of a match between worker (x,ϵ) and job (y,α) is:

$$P_{NC}^g(x,\epsilon,y,\alpha) = \underbrace{(1-\tau)\,f(x,y)}_{\text{after-tax flow output}} + \underbrace{q(\epsilon,\alpha)}_{\text{value for amenities}} + \tag{9}$$
$$\beta\,\mathbb{E}\Big[\underbrace{\delta_{NC}^g\Big(\Pi_0(y,\alpha) + U_{NC}^g(x_+,\epsilon)\Big)}_{\text{exogenous separation}}$$
$$+ \sum_{y',\alpha'} s\kappa\, v(y',\alpha')\,\Big(\widetilde{P}_{NC}^g(x_+,\epsilon,y,\alpha) + \sigma\,\max\{\underbrace{S_{NC}^g(x_+,\epsilon,y',\alpha')}_{\text{poaching job surplus}} - \underbrace{S_{NC}^g(x_+,\epsilon,y,\alpha)}_{\text{current job surplus}},\,0\}\Big)$$
$$+ \underbrace{\chi\,\widetilde{P}_{PL}^g(x_+,\epsilon,y,\alpha)}_{\text{fertility}} + \underbrace{\gamma\,\widetilde{P}_{D}^g(x_+,\epsilon,y,\alpha)}_{\text{ageing}} + (1 - \delta_{NC}^g - \chi - \gamma - sk\,V)\,\widetilde{P}_{NC}^g(x_+,\epsilon,y,\alpha)\Big]$$

In the current period, the match between worker of human capital x and job of productivity y produces $f(x,y)$ units of flow output, regardless of gender. There is a proportional tax τ on the flow output to finance parental leave benefits. The worker enjoys a flow utility that is a function of his/her value for amenities ϵ and the level of amenity provision at the job α.

In the next period, the worker's human capital level is x_+, where the transition matrix from x to x_+ is given by the law of motion $p_e(x,y)$ in employment. Upon exogenous separation δ_{NC}^g, the match dissolves and the worker and the employer both receive their outside options. The worker searches on-the-job, and employers Bertrand-compete for the worker.

All matches are efficient, and existing match are allowed to endogenously dissolve if

the joint value of the match falls below the sum of the agents' outside options in separation. There could be endogenous quits when human capital level x changes and at any age segment a in life:

$$\widetilde{P}_a^g(x,\epsilon,y,\alpha) = \max\{P_a^g(x,\epsilon,y,\alpha),\ \Pi_0(y,\alpha) + U_a^g(x,\epsilon)\}, \qquad a = \{NC, PL, YC, D\}$$

3.4.4 Parental leave

When a worker has a child, several changes take place. The woman's utility from amenities changes from $q(\epsilon,\alpha)$ to $q_{YC}^f(\epsilon,\alpha)$, whereas the men's value stays the same. Exogenous separation rates also change from δ_{NC}^g to δ_{YC}^g. The joint value in parental leave is:

$$\begin{aligned} P_{PL}^g(x,\epsilon,y,\alpha) = & \underbrace{R\,f(x,y)}_{\text{reduced flow output}} + \underbrace{q_{YC}^g(\epsilon,\alpha)}_{\text{value for amenities}} \\ & +\beta\,\mathbb{E}\Big[\underbrace{\delta_{YC}^g\Big(\Pi_0(y,\alpha) + U_{PL}^g(x_+,\epsilon)\Big)}_{\text{exogenous separations}} + \underbrace{\eta^g\,\widetilde{P}_{YC}^g(x_+,\epsilon,y,\alpha)}_{\text{PL ends}} + \underbrace{\gamma\,\widetilde{P}_D^g(x_+,\epsilon,y,\alpha)}_{\text{ageing}} \\ & + (1-\delta_{YC}-\eta^g-\gamma)\,\widetilde{P}_{PL}^g(x_+,\epsilon,y,\alpha)\Big] \end{aligned} \tag{10}$$

Mimicking the institutional settings in Finland as closely as possible, the model assumes the following. First, the worker goes into parental leave immediately after having a child, and gets paid a wage that is fully funded by the government for the whole duration of leave. Second, the worker on leave enjoys job protection and the employer has to keep the job available for when he/she returns. Third, the job still produces a flow output when the worker is absent, but production is slashed to a ratio R proportion of its previous amount.

One could think of parameter R as a reduced-form way of capturing various challenges and adjustment costs faced by firms whenever a worker goes on parental leave. Even though Finnish employers do not face direct costs of financing employees' wages while on leave, they may still encounter difficulties and costs in finding a replacement worker and/or coordinating schedules of existing workers to keep production going, potentially at a lower productivity. Ginja, Karimi and Xiao (2021) quantifies these adjustment costs experienced by firms in Sweden.[17]

[17] Ginja, Karimi and Xiao (2021) finds that firms hired temporary workers and increased incumbents' hours when parental leave was extended by 3 months in Sweden. Even though firms did not have to pay wages to the person on leave, the total wage bill cost of the re-organization was on average equivalent to 10 full-time months for each additional worker on extended leave.

From a modeling perspective, the employer continues production in this period but does not hire new workers; conceptually one could think of the job as being covered by co-workers working more hours. Abstracting from hiring temporary workers would simplify the model in parental leave stage as one does not need to keep track of any new workers being hired (and then laid off) in steady state balance flows.[18]

During the parental leave period, the worker is out of the labor force, so his/her human capital does not grow and there is no on-the-job search. Workers on leave are by default associated with their previous employers (in both the data and model), but can separate from their employers exogenously or endogenously in the parental leave period. Women and men exit parental leave at rate η^f and η^m respectively, upon which unemployed workers start searching for jobs and employed workers go back to pre-birth employers. The worker can have another child any time during fertile ages (including during parental leave). Upon having another child while employed, the worker will go into parental leave again.

The government runs a balanced budget. The tax rate τ is set such that total government transfers to job matches where workers are on parental leave are equal to the total tax revenues collected in stationary equilibrium:

$$\sum_{g}\sum_{x,\epsilon}\sum_{y,\alpha} \phi^{g}_{0,PL}(x,\epsilon,y,\alpha)\, h^{g}_{PL}(x,\epsilon,y,\alpha) = \sum_{g}\sum_{x,\epsilon}\sum_{y,\alpha}\sum_{a=NC,YC,D} \tau\, f(x,y)\, h^{g}_{a}(x,\epsilon,y,\alpha)$$

where $\phi^{g}_{0,PL}(x,\epsilon,y,\alpha)$ denotes the flow wage in PL stage.

The joint values of matches in "Young Child" and "Done with children" stages are analogous, and are listed below:

$$\begin{aligned} P^{g}_{YC}(x,\epsilon,y,\alpha) = (1-\tau)\, f(x,y) + q^{g}_{YC}(\epsilon,\alpha) + \beta\, \mathbb{E}\Big[& \delta^{g}_{YC}\Big(\Pi_0(y,\alpha) + U^{g}_{YC}(x_+,\epsilon)\Big) \\ & + \sum_{y',\alpha'} s\kappa\, v(y',\alpha')\ \sigma\ \max\{S^{g}_{YC}(x_+,\epsilon,y',\alpha') - S^{g}_{YC}(x_+,\epsilon,y,\alpha),\, 0\} \\ & + \gamma\, \widetilde{P}^{g}_{D}(x_+,\epsilon,y,\alpha) + \chi\, \widetilde{P}^{g}_{PL}(x_+,\epsilon,y,\alpha) + (1-\delta^{g}_{YC}-\gamma-\chi)\, \widetilde{P}^{g}_{YC}(x_+,\epsilon,y,\alpha)\Big] \end{aligned} \tag{11}$$

[18]This means that the employer would split the match surplus with the co-workers, not with workers on leave. I do not solve for equilibrium wages of the over-working co-workers. Workers on leave receive benefits outside the system of worker-job pairs – they get benefits directly from the government.

$$P_D^g(x,\epsilon,y,\alpha) = (1-\tau)\, f(x,y) + q^g(\epsilon,\alpha) + \beta\, \mathbb{E}\Big[\delta\Big(\Pi_0(y,\alpha) + U_D^g(x_+,\epsilon)\Big) + \phi\, \Pi_0(y,\alpha) \tag{12}$$
$$+ \sum_{y',\alpha'} s\kappa\, v(y',\alpha')\ \sigma\ \max\{S_D^g(x_+,\epsilon,y',\alpha') - S_D^g(x_+,\epsilon,y,\alpha),\, 0\}$$
$$+ (1-\phi-\delta)\, \widetilde{P}_D^g(x_+,\epsilon,y,\alpha)\Big]$$

The transition parameters and preference parameters in "Young Child" stage are the same as in "Parental Leave" stage, and one should think of these two stages as the period where workers have young children at home. The only difference is that individuals in "Parental Leave" stage are matched with some employers but are not working, whereas those in "Young Child" stage are actively participating in the labor force.

In stage D, individuals are non-fecund and will not have any additional child. Men and women have the same separation rate δ, and retire at rate ϕ, upon which the joint value of the match is just the vacancy value.

3.5 Steady-state balance flow conditions

In equilibrium all agents follow their optimal strategy. Denote the measure of workers of gender g in age segment $a \in \{NC, PL, YC, D\}$ as m_a^g. The total measure of women of all ages should add up to $\mu^f = \mu$, and men to $\mu^m = 1-\mu$.

$$m_{NC}^g + m_{YC}^g + m_{PL}^g + m_D^g = \mu^g \tag{13}$$

Also, the flows into and out of each age segment should balance.

$$\chi\,(m_{NC}^g + m_{YC}^g) = (\gamma + \eta^g)\, m_{PL}^g \tag{14}$$
$$\eta^g\, m_{PL}^g = (\chi + \gamma)\, m_{YC}^g \tag{15}$$
$$\gamma\,(m_{NC}^g + m_{YC}^g + m_{PL}^g) = \phi\, m_D^g \tag{16}$$

The equilibrium distribution of vacancies and matches will satisfy the following balance equation:

$$v(y,\alpha) + \sum_a \sum_{g=m,f} \sum_{x,\epsilon} h_a^g(x,\epsilon,y,\alpha) = \varphi(y,\alpha), \quad a \in \{NC, YC, PL, D\} \tag{17}$$

The equilibrium distribution of workers must be such that flows into and out of any worker stock must balance for each worker type (g, a, x, ϵ), in employed or unemployed

state, across all job types (if employed). Appendix E provides more details.

3.6 Definition of equilibrium

A stationary equilibrium is a tuple of value functions $\{U^m, U^f, P^m, P^f, \Pi_0\}$ together with a distribution of male and female workers across employment states and across job types $\{u^m, u^f, h^m, h^f\}$ as well as a distribution of job vacancies v such that:

(i) The value functions satisfy Bellman Equations (4) to (12).

(ii) The distributions $\{u^m, u^f, h^m, h^f, v\}$ are stationary given the transitions implied by the value functions, and satisfy balanced flow conditions (13) to (17) and flow equations in Appendix E.

(iii) Equilibrium wages are determined by surplus sharing rules defined in (1) to (3).

Note that the equilibrium values and allocations (points (i) and (ii) above) can be solved without making any reference to wages, just like in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). This is because utility is transferable between the worker and employer, so joint values and surpluses do not depend on wages. Moreover, match formation and worker mobility decisions are determined only by the sign of surpluses or difference in surpluses between two jobs, so the equilibrium worker and job allocations also do not depend on wages. The advantage of this transferable utility framework is that it makes the model very tractable, and the computation of the equilibrium fairly straightforward.

4 Estimation

In this section, I estimate the model using Simulated Method of Moments (SMM).[19] To this aim, I obtain a vector of moments from N individuals in the data, $\hat{m}^D = \frac{1}{N}\sum_{i=1}^{N} m_i$, for example mean wages out of unemployment in the first five years after graduation, etc. Model counterparts to these moments, $\hat{m}^S(\theta) = \frac{1}{M}\sum_{j=1}^{M} m_j^D$, are obtained from M simulated lives from the model based on a parameter vector θ. The estimation involves finding the vector θ that brings the simulated moments as close as possible to the data moments, i.e. minimizing the criterion function

$$L(\theta) = (\hat{m}^D - \hat{m}^S(\theta))^T \ \hat{W}^{-1} \ (\hat{m}^D - \hat{m}^S(\theta))$$

where $\hat{W}$ is a weighting matrix.

[19] See for example McFadden (1989) and Pakes and Pollard (1989). Constructing the likelihood function for this model is intractable.

Key parameters of interest are outlined below.

4.1 Model specification

The length of a model period is one month. Human capital of the worker takes discrete values $x \in H = \{x_1, x_2, ..., x_N\}$ and $0 < x_1 < x_2 < ... < x_N$. Human capital accumulation is assumed to take the form

$$p(x_i, y) = Prob(x_{i+1}|x_i, y) = d_1 + d_2\, y.$$

where $d_1, d_2 \in (0, 1)$. That is, every period an employed worker moves up by one category of human capital with a probability that is linear in his/her job productivity y. This captures the idea that workers might learn faster on the job when matched with more productive employers.

Central to the model is the sorting of men and women across jobs, which is intimately related to the production function. I specify the production of a match to be a CES function in the worker's human capital and the employer's productivity

$$f(x, y) = K\left[a\, x^{\rho} + (1 - a)\, y^{\rho}\right]^{\frac{1}{\rho}}.$$

This allows for various degrees of complementarity governed by the estimated value of ρ. Home production is assumed to take the form $b(x) = b\, x$.

Men and women draw their values for amenities ϵ^m and ϵ^f from normal distributions $N(\mu_m, sd_m)$ and $N(\mu_f, sd_f)$ respectively. In the "No Child" stage, value for amenities takes the simple form $q(\epsilon^g, \alpha) = \epsilon^g\, \alpha$. Women's value increases by M in motherhood, so that $q^f_{YC} = (\epsilon^f + M)\, \alpha$ in YC and PL stages, whereas men's values stay the same $q^m_{YC} = \epsilon^m\, \alpha$.

Finally, I assume the matching function has an elasticity of 0.5 and takes the functional form (see Petrongolo and Pissarides (2001)):

$$m(\hat{U}, V) = \vartheta \sqrt{\hat{U}\, V}$$

where effective job seekers $\hat{U} = U_{NC} + s_U(U_{YC} + U_D) + s_E(1 - U_{NC} - U_{YC} - U_D)$. I allow search in unemployment to be different in early and late stages in life. The search intensity for the unemployed in NC stage is normalized to one, and that of the unemployed in YC and D stages will be s_U. The relative search intensity of the employed is s_E and does not vary over the life-cycle.

In the next section I offer a heuristic argument on how the parameters are identified.

4.2 Estimation method and identification

Given the above specification, I estimate two sets of parameters in an iterative procedure. The first set of parameters involves exogenous separation rates δ_a^g and parameters from the matching function, denoted by $\boldsymbol{\lambda} = (\boldsymbol{\delta_a^g}, \vartheta, s_U, s_E)$. The second group includes model "core" parameters characterizing human capital processes, production functions, bargaining and preferences, denoted by $\boldsymbol{\theta} = (d_1, d_2, K, a, \rho, \sigma, b, \mu_m, \mu_f, M)$.

Note that separation rates, job-finding rates and job-to-job transition probabilities in the model depend on equilibrium surplus values and the equilibrium distribution of vacancies, and consequently cannot be obtained independently outside of the model. However, parameters in $\boldsymbol{\lambda}$ are directly related to workers' transitions in and out of work and between jobs given the equilibrium surpluses. Therefore, $\boldsymbol{\lambda}$ can be identified given $\boldsymbol{\theta}$. Estimating the two groups of parameters iteratively significantly reduces estimation time. For details of the estimation procedure and computation of standard errors, please refer to Appendix F.

Statistical discrimination depends on all the above parameters, but it can be distinguished from taste-based discrimination with the following intuition. After people become non-fecund (at age 40), men and women will no longer have children and face the same model primitives. Therefore, any gender wage gap after age 40, after conditioning on human capital histories up to that point, cannot be attributed to statistical discrimination based on child-related concerns.

The parameters in $\boldsymbol{\theta}$ are related to data moments in intuitive ways. Human capital growth rates d_1 and d_2 do not have a direct data counterpart since the assignment of workers to jobs is not random. However, with the aid of the full equilibrium structure of the model, these parameters can be related to the following aspects of the data. When a worker goes through an unemployment spell in the model, she falls off the job ladder and loses any "search capital" accumulated through job-to-job transitions. However, human capital is general and she will carry her accumulated experience to the next job. Comparing the wages immediately following a transition from unemployment to employment (UE wages) at different points of the life-cycle can inform us of the average human capital growth rate d_1 in the economy (Dustmann and Meghir (2005)).

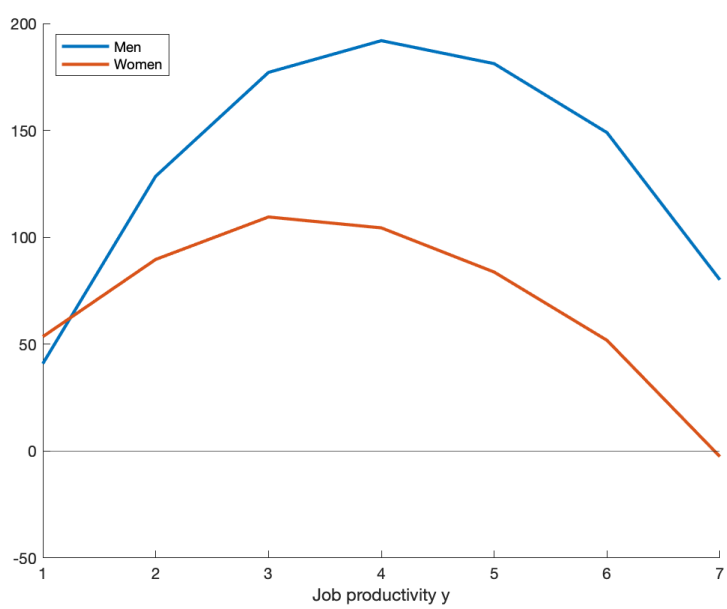
Moreover, human capital growth in each productivity category y is related to within-job wage growth in jobs of high- versus low-productivity types. Although wage gains within a job also depend on renegotiations triggered by poaching firms, the amount of contact with poachers is disciplined by s_E and ϑ that are pinned down in the previous

step. Therefore, the remaining within-job wage growth could be attributed to human capital growth.

Key to identification of production function parameters is the sorting of men and women across jobs. When production is very complementary (ρ very small or negative), the marginal return of employing a high-type worker is considerably higher for high-productivity jobs. In the presence of a capacity constraint of a firm, this implies that the match surplus might not be monotonically increasing in job productivity (Eeckhout and Kircher (2011)), since high-productivity jobs have a much higher option value of waiting for a better match.

Indeed, the values of match surplus might be an inverted-U shape (as shown in Figure 5), or even decreasing in job productivity for a low-type worker. The example in Figure 5 shows that with production complementarity, the medium-skilled workers are best matched with middle-level jobs where the surpluses peak. Top jobs (category 7, mainly managers) generate relatively low surpluses with mediocre workers, and this is more severe for women as they have higher turnovers and generate less surplus in general. High vacancy values of the top jobs imply that these employers might turn off matches with women even though they might still match with equally skilled men.

FIGURE 5. An example of surplus values of medium-skilled workers in NC stage



NOTES: The solid lines plot the surplus values of a male and female worker in "No Child" stage across jobs of different productivity levels. The man and woman have the same amenity preference and same productivity (both of skill type x_3). The production function in this example assumes high complementarity between worker and job productivities, with $\rho = -0.9$.

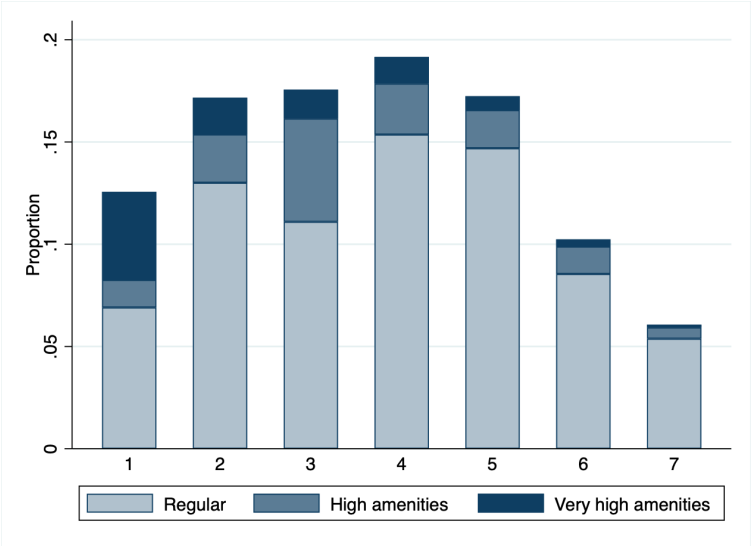
Consider the contrary case where production is perfectly substitutable ($\rho = 1$), then there are no productivity gains from sorting compared to random matching. Surpluses will be monotonically increasing in job productivity for a given worker type. Since match values are typically lower for women than men, it would imply that the low-productivity jobs are the first ones to stop matching with women, and we would see different sorting patterns of men and women vis à vis the case where production is complementary.

Relative productivity of labor (parameter a) is closely related to human capital parameters and wage growth over the life-cycle. When human capital appreciates, production grows more when a is high. Although both d_1, d_2 and a are positively related to wage growth moments, they could have opposite implications for UE wage levels. The intuition is that when a increases, all jobs are much better off matching with high-HC workers when production is complementary, and top jobs are actually worse off matching with low-type workers given the increased option value of hiring high-types. In contrast, an increase in d_1 or d_2 invariably raises surpluses and UE wages of all matches. As a result, in early career stages when most workers do not have much human capital, we will see lower UE wages when a increases but higher UE wages when d_1, d_2 increase. The extent of these effects depends on the strength of complementarity.

Amenity preference parameters μ_m and μ_f govern workers' mobility patterns across jobs of high- and low- amenity types, and do not affect the production of output. One caveat is that workers in high-amenity jobs might be positively selected in productivity (both in the data and in the model). High-HC workers might not be willing to accept low-productivity jobs in general, but if the low-type job provides enough amenities it might be enough to push the match surplus above zero. The extent to which female workers are drawn to high-amenity jobs helps to identify the magnitude of μ_f relative to μ_m. The increase in value for amenities during motherhood M is closely linked to the proportion of women who switch into high-amenity jobs immediately after childbirth.

The following parameters are fixed or calibrated without explicitly using the model. Firstly, the exogenous distribution of jobs $\Gamma(y, \alpha)$ in the model is fixed to the data distribution. The distribution of jobs along the productivity dimension is obtained through k-means clustering. The productivity of each firm-occupation cell is proxied by its long-term average wage – the average log wage of all workers who have worked in the job in all years from 1995 to 2013. The jobs are then grouped into seven productivity categories by clustering on these long-term average wages using k-means. The support of the distribution is normalized so that the bottom group takes a productivity value of 1. Summary statistics on job productivity categories are provided in Table A2.

FIGURE 6. Distribution of jobs by productivity and amenities



The distribution of jobs along the amenities dimension is obtained by ranking their amenity index constructed in subsection 2.4 and grouping them into 3 categories: very high amenity (above 90th percentile), high amenity (between 75 to 90th percentile) and regular jobs (below 75th percentile). The final distribution of jobs across both productivity and amenity dimensions are shown in Figure 6.

Secondly, I calibrate the life-cycle Poisson parameters. Fertility rate χ is calibrated to match the total number of children workers have, ageing rate γ is set to match the number of years between graduation and age 40, and retirement rate ϕ is set so that individuals retire at age 60. The rates at which parental leave ends for men and women, η_m and η_f, are calibrated to match the average length of parental leave taken for each child by men and women respectively.

Other calibrated parameters include R, c and the initial human capital distributions of men and women. The reduction in flow production R during parental leave is calibrated to the adjustment costs of extended parental leave estimated in Ginja, Karimi and Xiao (2021). The vacancy cost c is calibrated to that in Lise, Meghir and Robin (2016). The initial productivity distributions of male and female workers are calibrated to match the initial wage distributions at labor market entry. The monthly discount rate β is set to 0.988.

4.3 Results

The model fits the life-cycle wage profiles of men and women very well, and is able to replicate key moments of the data. Figure A2 summarizes the fit of the model moments compared to targeted data moments. Men have higher wages than women throughout the life-cycle, enjoy higher within-job wage growths, are less represented in low productivity jobs and more represented in high-end jobs (type 1 is lowest productivity and type 7 is highest). The proportion of women in high-amenity jobs increases after childbirth, and the gender wage gap increases in the first years after birth before coming down 10 years afterwards. All these important qualitative features of the data are captured by the model.

The allocation of women and men across jobs of different productivities is related to both human capital accumulation and the amount of statistical discrimination in the economy. While the model generally fits women's progression across jobs over time, it does not seem to push men into high-end jobs fast enough. This could be due to three reasons: (1) men and women might have different rates of human capital accumulation in the data, whereas they are assumed to accumulate at the same speed governed by d_1 and d_2 in the model; (2) there might be some element of directed search in the data whereas the model is random search; and (3) the model does not generate enough hiring discrimination at top jobs because of the transferable utility framework.

The complete set of parameter estimates is presented in Table A3. The estimate of ρ shows that production is strongly complementary between worker and firm productivity. Match surplus is declining by job productivity for low-HC workers, leading to some matches in the extreme off-diagonals not to form.

The human capital accumulation rate is positively related to job productivity – worker skills upgrade much faster when they work at highly productive firms. The estimates imply that in the job category with the lowest productivity, human capital appreciates at the rate of 0.011, whereas at the high end the rate is 0.034. There will be a divergence in human capital levels of men and women over time, not only because men spend more time working and accumulating skills, but also because men are more represented at top jobs that offer better learning opportunities.

Men and women have similar valuations for amenities before having children, but women's value increases to almost twice as much after childbirth. However, women's switch into high-amenity jobs are not as pronounced and sudden in the model as in the data. This is because in a frictional environment in the model, opportunities to move to

high-amenity jobs may not arise immediately after childbirth. Anticipating the rate of encountering high-amenity jobs, some women already sort into these jobs before childbirth and others gradually move into them after having children.

The estimates imply an equilibrium allocation where the most productive jobs (category 7) do not match with low-HC women in the "No Child" stage, whereas the same jobs do match with equally low-HC men. Such hiring discrimination against women in early career could have long-term consequences given the different rates of skill accumulation across high- and low-productivity jobs. In the "Young Child" stage, men of the highest HC type do not match with low productivity jobs, whereas high-HC women are willing to take the low-end jobs in YC stage. This is because high-HC men would rather wait for a great offer in unemployment than take a low-end job. In contrast, high-HC women have a lower reservation value than their male counterparts because women are subject to high separation rates in YC, so there is not as much value in waiting for better jobs. In the "Done with children" stage where workers have moved beyond child-rearing ages, match formation decisions are the same for men and women.

5 Gender gap decomposition and policy counterfactuals

Given model estimates, I first decompose the life-cycle gender gaps by sequentially shutting off the channels. Then I compare three policies aimed at reducing gender inequality: (1) more parental leave months earmarked for fathers; (2) equal hiring at top jobs; and (3) equal pay for men and women of the same type in the same job.

5.1 Decomposition of the life-cycle gender wage gap

I decompose the gender gaps in wages and employment into components due to: (a) labor force attachment, (b) statistical discrimination, and (c) amenity preferences. There is no straightforward way of decomposing the gender wage gap, since all three channels mentioned above interact with each other. In the following decomposition exercise, I focus on the impact of child-related career interruptions on human capital accumulation and its interactions with statistical discrimination, while considering preference for amenities separately. Figure 7 shows how much of the total wage gap is explained by each of the channels, and Table A4 shows the responding proportions.

The gender wage gap is decomposed in three steps. First, I allow men and women to have the same child-related interruptions, while keeping equilibrium wages and em-

ployment decisions fixed. That is, men and women will have the same parental leave duration and face the same exogenous separation rates.[20] Since equilibrium effects are not considered at this point, any wage change after equalizing parental leave duration can be attributed to human capital gains (losses) of women (men). On the other hand, when separation rates decrease, women's wages could increase because of two reasons. First, women now stay longer on the job and gain more human capital and second, they fall off the job "ladder" less often and can extract more match surplus by re-negotiations and by climbing the career ladder.

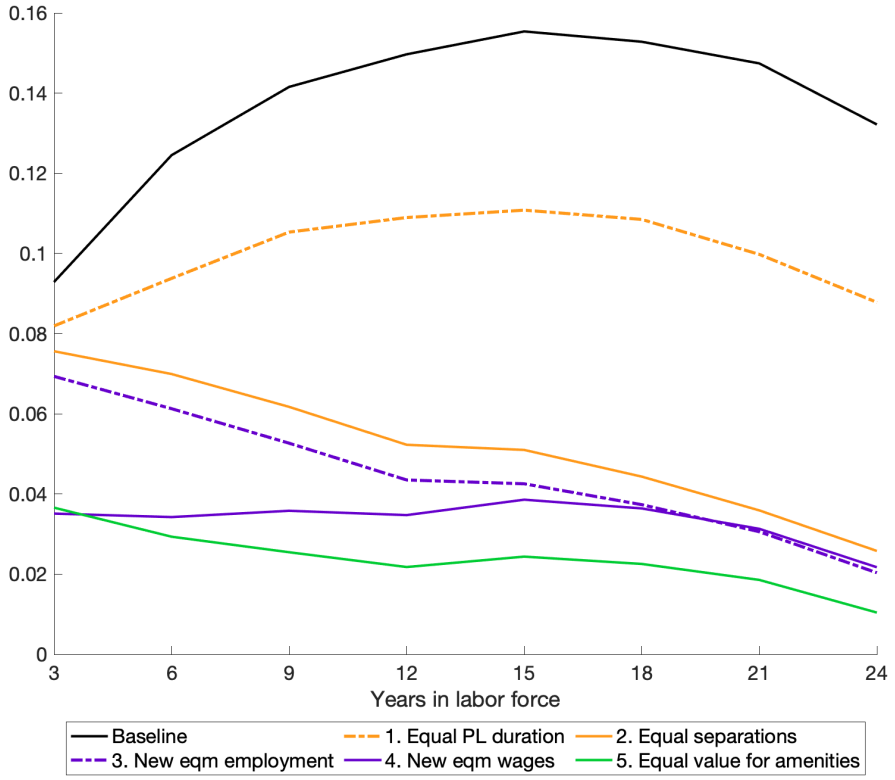
The top black solid line in Figure 7 is the gender wage gap implied by model estimates (which fits the data closely). The orange dotted line and orange solid line show the decreased gaps as a result of equalizing parental leave and equalizing separation rates, respectively. These labor force attachment aspects do not explain much of the gender wage gap in early career since educated men and women behave similarly before having children. However, the effects of parental leave duration and separation rates compound over time as women have an accumulated shortage of human capital compared to men. The compounding effects are substantial because of workers' sorting across jobs – having a low human capital means the worker would sort towards low-end jobs where learning content is low, thus creating a vicious cycle. Taken together, labor force attachment explains over half of the gender wage gap 10 years into the labor force, and is responsible for 3/4 of the gap in 20 years.

The second step is to measure the effects of child-related interruptions on (i) equilibrium employment and (ii) equilibrium wages. When parental leave durations and separation rates are equalized between men and women, employers will anticipate similar behaviors of male and female workers around childbirth and reduce statistical discrimination in both hiring and wage decisions.

In order to measure changes in equilibrium employment, I allow match formation and mobility to change to the new equilibrium while keeping wage policies the same as before. In the new equilibrium, jobs in the highest productivity category that did not hire low-HC women now start matching with both men and women in NC stage. High-HC men who did not accept low-end jobs in YC stage now start taking them. Even though match formation decisions only change for a handful of types of workers and firms, changes in job allocations would propagate to the rest of the distribution. More

[20] Instead of women taking 18 months and men 2 months, they will each take 10 months in the counterfactual so the total number of PL months remains the same as before. The counterfactual separation rate is chosen such that the total measure of employed workers is fixed to the estimated level.

FIGURE 7. Gender wage gap decomposition



NOTES: The lines represent the log wage gap between men and women over the life-cycle. The top black solid line is the wage gap based on model estimates. The colored lines are the counterfactual wage gaps under: 1. Equal PL duration by gender without changing equilibrium wages and employment. 2. Equal separation rates in addition to 1., without equilibrium effects. 3. Implement the new equilibrium **job allocations** implied by equal PL and separations. 4. Implement new equilibrium **wages**. 5. Equal value for amenities by gender before and after childbirth.

women at top jobs implies some men would be "pushed" to lower jobs. Vice versa, more men being drawn to bottom jobs means women will contact these vacancies with lower probability and encounter vacancies elsewhere with a relatively higher probability. These changes in allocations, however, have only a small impact on the overall gender wage gap as shown in Figure 7. They explains about 6 percent of the gap on top of what was explained by fertility-related interruptions in the first step. The small effect might be driven by the fact that allocation changes only occur for a small group of people, who do not influence average wages considerably. Another reason might be that wages are kept to the previous equilibrium where there is still substantial wage discrimination especially at top firms.

Next, I implement new equilibrium wages under equal PL and separations on top of the new equilibrium employment changes. Employer's statistical discrimination in

wages plays an important role in early career, accounting for 37 percent of the gender wage gap in the first 3 years since labor market entry. As the employer anticipates men and women to spend the same amount of time in parental leave and separate at the same rate, the expected future costs associated with leave-taking and turnover also become equal whether the job is given to a man or a woman. As a result, employers in the new equilibrium revise wage offers downwards for men and upwards for women in early career stages (in both NC and YC life stages) when workers are prone to fertility events. Wage discrimination fades over time as more and more workers move beyond child-rearing ages, although the human capital effects from earlier job allocations are carried over to infertile ages.

In the third step, I compute a new equilibrium based on equal valuations of amenities between men and women and no change in preferences after childbirth, in addition to equal parental leave and separation rates. There are both wage and mobility changes in the new equilibrium, and altogether these changes explain an additional 9 percent of the gender wage gap in late career. Since men and women have very similar values for amenities in the "No Child" stage, preference for job amenities explains little of the gap in early career.

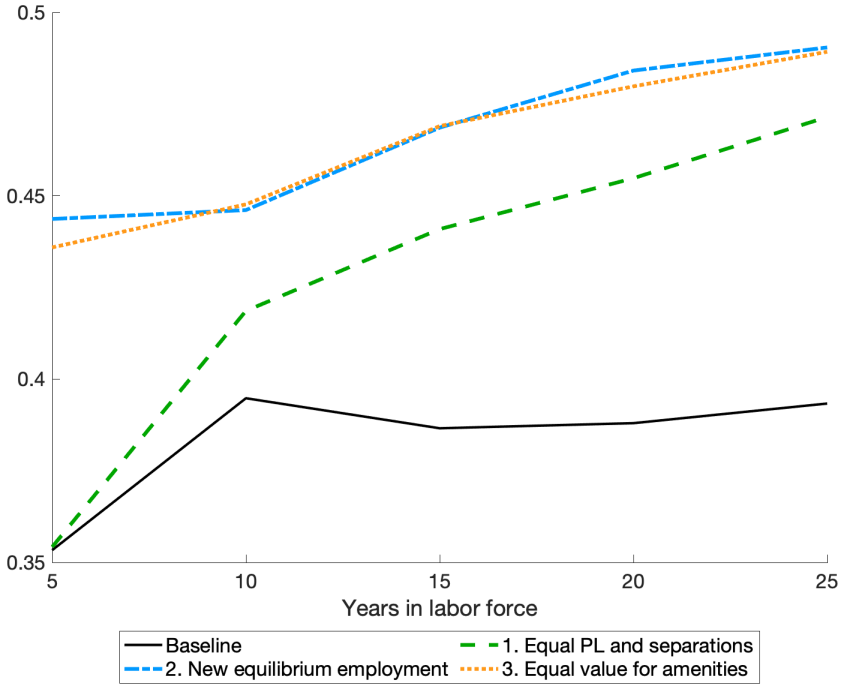
5.2 Under-representation of women at top jobs

Although wages are the most common statistic to investigate in issues revolving gender inequality, another relevant and related question is: why do so few women make it into top-level positions compared to men? How much of the gender wage gap come from the top versus bottom of the productivity distribution?

I answer the first question by investigating the share of women at the most productive jobs. These are jobs in categories 6 and 7 which are mostly management and professional positions in high-productivity firms. In the estimated model, 35% to 39% of the workforce in top jobs are women, as shown by the bottom black solid line in Figure 8.

Similar to the decomposition in subsection 5.1, I proceed in 3 steps. First, men and women are given equal parental leave durations and equal separation rates without changing the equilibrium. The gap between the black solid line and the green dotted line in Figure 8 shows that over half of the gender imbalance in top jobs during late career (year 20 and onward) could be eliminated by the labor force attachment channel alone. The optimal decision rule at top jobs is to hire a high- or medium-HC worker whenever they encounter one, regardless of gender. Therefore it is unsurprising that most of the problem

FIGURE 8. Counterfactual proportion female in top jobs



NOTES: The lines represent the proportion of workers in the most productive two job categories who are women. The bottom black solid line is the female share implied by model estimates. The colored lines are counterfactual female shares under: 1. Equal PL duration and separation rates by gender, without equilibrium effects. 2. Implement the new equilibrium allocations implied by equal PL and separations. 3. Equal value for amenities by gender before and after childbirth, in addition to 1. and 2.

could be attributed to the human capital factor – there are simply not as many encounters between these top jobs and high-HC women as compared to high-HC men.

Second, I implement the new equilibrium implied by equal parental leave and equal separations. The blue dotted line in Figure 8 shows the resulting female share at top jobs with the new equilibrium allocations – without hiring discrimination. Statistical discrimination in hiring starts years before childbirth and accounts for almost half of the gender disparity in top jobs during early career (before year 10). Women who do not have access to good job opportunities earlier on also do not accumulate as much human capital as their male counterparts, and the impact of hiring discrimination persists over time.

Third, I compute the new equilibrium implied by equal amenity preferences by gender before and after childbirth. Preference for amenities does not seem to play a big role in women's under-representation at top jobs.

5.3 Counterfactual policy experiments

The section below considers three counterfactual policies that aim to reduce gender gaps in the labor market – a "daddy month" parental leave expansion, equal hiring in top jobs, and an equal pay policy. I compute the new equilibrium and quantify the effects of each policy on the gender wage gap and on gender disparities in top positions over the life-cycle.

5.3.1 Daddy months

In Finland and many other Nordic countries, there is generous wage-replaced parental leave of durations from 6 months to over a year that could be shared between the parents, but it is almost always the mother who takes up all of the shared leave. Many of these countries have then introduced 1 to 3 months of "daddy months" to encourage fathers to spend more time with the baby (Dahl, Løken and Mogstad, 2014).

I consider a policy that expands daddy's leave by 2 months per child and reduce mother's parental leave by 2 months. To do this, I calibrate the parental leave exit shocks η_m and η_f so that men's leave duration per child increases from 2 to 4 months, while that of women's decreases from 18 to 16 months.

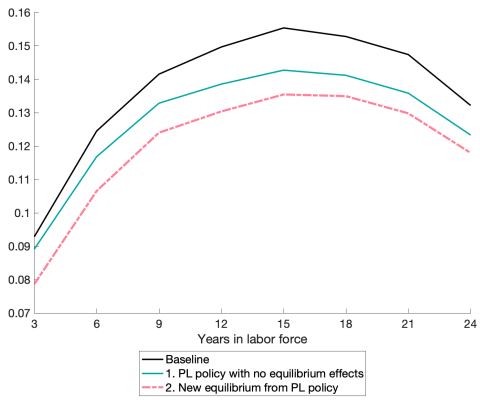
The daddy month policy is quite effective in reducing the gender wage gap throughout the life-cycle. As shown in Figure 9a, the wage gap closes by 15% during the first 3 years of working, and over 10% afterwards. About half of the impact on wages comes from a reduction in statistical discrimination in pre-child years. Even though hiring discrimination still persists in years prior to childbirth, women's wages are now closer to men's when they are hired. Women also gain more human capital during mid-career because they return to work sooner after having children, while men accumulate less. This slightly balances the gender ratio in top jobs as the proportion of women increases from 39 to 41 percent by year 25 (see Figure 9b).

One caveat of this policy is that it might not result in a pareto improvement – the progress in women's careers might come at the expense of men's. In order to assess the overall social value of the policy, define social welfare (SW) as the sum of the production of the employed matches and the home production of the unemployed net of the total cost of vacancies:

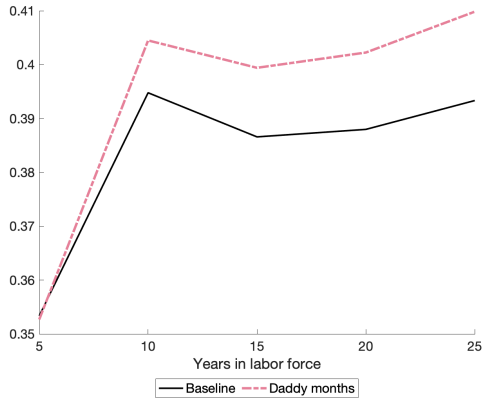
$$SW = \sum_{g,a,x,\epsilon} b(x)\, u_a^g(x,\epsilon) + \sum_{g,a,x,\epsilon,y,\alpha} f(x,y)\, h_a^g(x,\epsilon,y,\alpha) - \sum_{y,\alpha} c\, v(y,\alpha).$$

FIGURE 9. Counterfactuals under daddy months policy

(A) Gender wage gap



(B) Female proportion at top jobs



By the time men become fathers, they are already in slightly more advanced positions than women and are producing more output, so the output loss of having men spend 2 months at home cannot be fully compensated by output gains of women working 2 months more. However, the net loss in social welfare is very small (only 0.02% of total welfare). Similarly, paying men on parental leave is more costly since the benefits are proportional to wages and men typically earn more than women. In order to fund the new policy, the tax rate on flow output has to increase modestly from 2.80% to 2.88%.

5.3.2 Equal hiring policy in top jobs

To address the under-representation of women in top-earning jobs, many countries have passed legislature to require a certain percentage of female board members in public companies. Finland requires state-owned enterprises to reserve 40% of board seats to female directors. However, the evidence on the effectiveness of these policies in reducing gender gaps is mixed at best (Bertrand, Black, Jensen and Lleras-Muney, 2018).

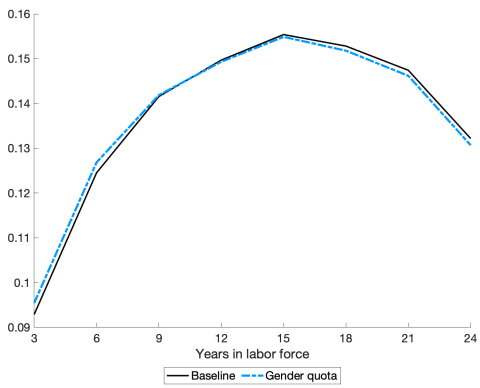
There is no direct way of implementing a gender quota in the model since the proportion of women in a particular job category depends not only on the optimal hiring rule of the job, but also on the transition rates and workers' mobility to all other jobs in equilibrium. In practice, I implement an "equal hiring" policy that requires the top jobs (those in the highest productivity category) to have the same hiring rule towards a woman and a man of the same (x, ϵ) type.

The policy essentially changes hiring rules of productive employers towards low-HC women in the "No Child" stage. Since these matches would not have been formed in the absence of the equal hiring policy, there is no standard wage protocol about how to split

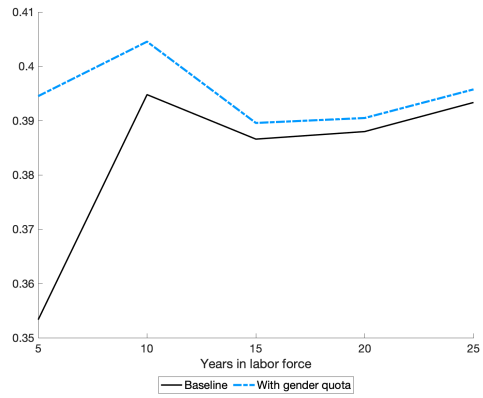
the (negative) match surplus. In this exercise, I assume that the employer sets the wage to cover the vacancy value of the job, and the worker gets the rest of the match value.

FIGURE 10. Counterfactuals under equal hiring policy

(A) Gender wage gap



(B) Female proportion at top jobs



Unsurprisingly, banning hiring discrimination at top jobs improves women's representation in those jobs during the early years of workers' professional lives. Figure 10b shows that the female share increases from 35.5 to 39.5 percent in top jobs during the first 5 years of work. However, this effect is very short-lived. Since the equal hiring policy does not address child-related interruptions, women start falling behind men in human capital levels soon after childbirth, and are thus less likely to stay in highly productive jobs later on due to forces of sorting. The proportion female in top jobs almost falls back to baseline levels during child-rearing years. The overall effect of the policy on the share of women in top jobs is only slightly positive by the end of the life-cycle.

Even though the equal hiring policy improves women's representation at top jobs, employers undo this policy by exerting more wage discrimination. Women hired under the new policy receive lower wages than men in the same job during the early years of the life-cycle. This is because employers are now required to form matches with all women even though some matches generate negative surpluses; as a result, the new female hires have to "compensate" the employers by accepting sub-par wages. Since the new hires are a small proportion of the working population, the overall wage gap only increases by a small amount (by 3% in 6 years). However, being employed in high-productivity jobs in early career allows young women to gain skills at a faster rate, and the human capital gains more than compensate for the initial wage loss. Figure 10a shows that the negative impact of the policy on women's wages disappears after year 9.

5.3.3 Equal pay policy

Many OECD countries have passed some form of Equal Pay Act that requires men and women in the same workplace be given equal pay for equal work. The Finnish Equality Act requires companies with 30 or more full-time employees to draft a gender equality plan, which should include an assessment of pay differences between men and women who perform work of equal value. [21]

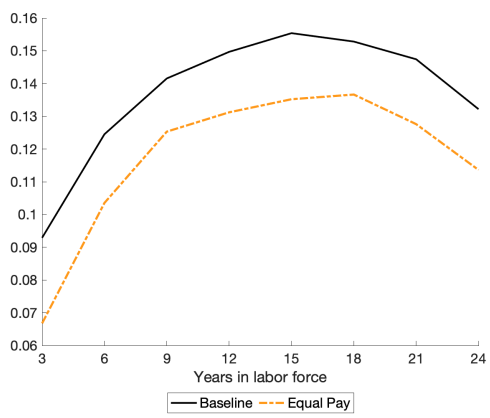
In the equal pay counterfactual, I require men and women of the same (x, ϵ) type working in the same (y, α) job to receive the same flow wage. I compute the equivalent lifetime value of the female worker $W_a^f(\phi_{0,a}^m, x, \epsilon, y, \alpha)$ implied by having men's wages $\phi_{0,a}^m$ in each age segment a, and re-calculate employer's share in the surplus:

$$\underbrace{\Pi_a^f(\phi_{0,a}^m, x, \epsilon, y, \alpha) - \Pi_0(y, \alpha)}_{\text{employer's share}} = S_a^f(x, \epsilon, y, \alpha) - \underbrace{\left(W_a^f(\phi_{0,a}^m, x, \epsilon, y, \alpha) - U_a^f(x, \epsilon)\right)}_{\text{worker's share}}.$$

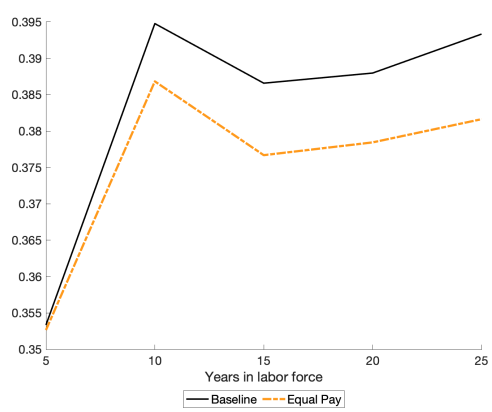
When the worker's value W_a^f is required to increase, the employer's portion might become negative, and the match would dissolve.

FIGURE 11. Counterfactuals under equal pay policy

(A) Gender wage gap



(B) Female proportion at top jobs



I simulate the workers' careers with the equal wage policy, allowing matches where the employer's value $\Pi_a^g(\phi_{0,a}^m, x, \epsilon, y, \alpha)$ fall below the vacancy value $\Pi_0(y, \alpha)$ to no longer form. Figure 11 shows that the equal pay policy unsurprisingly reduces the gender wage gap by 28% in the first 3 years since labor market entry, and over 10% thereafter. However, some matches are no longer sustained in the stages after having children. As a result, women are more likely to fall off the "career ladder" and more likely to be unemployed,

[21] Details of the Equality Act and related reforms can be found at: https://www.finlex.fi/en/laki/kaannokset/1986/en19860609

although the effect size is very small. Figure 11b shows that the proportion of women in top jobs decreases from 39 to 38 percent by year 25.

6 Conclusion

This paper studies the mechanisms underlying the gender wage gap over the life-cycle — workers' human capital accumulation, preference for amenities, and employers' statistical discrimination in wages and hiring. I propose an equilibrium search model with capacity constraints, production complementarities, fertility and parental leave, and taste for job amenities. The model is estimated using matched employer-employee data from Finland combined with occupation-level data on amenities from the Finnish Quality of Work Life Survey.

Men and women behave very differently in the labor market especially after having children. Employers take into account of these gender differences and statistically discriminate women even before they have children. The model estimates imply that statistical discrimination based on fertility concerns explains a large portion of the gender wage gap in early career, while labor force attachment accounts for the majority of the gap in late career.

The most effective policies in reducing gender gaps are those that alleviate women's childcare responsibilities, for example childcare expansions that help to reduce women's separation rates, and more parental leave for fathers. These policies would not only help women gain more human capital on the job, but also shift firms' expectations and reduce statistical discrimination in both wages and employment. However, eliminating hiring discrimination at top jobs through an equal hiring policy reduces women's average wage in early career, and eliminating wage discrimination through an equal pay policy reduces the proportion of women in top positions as employers adjust on the hiring margin. Taken together, the policy counterfactuals show that it would be difficult to achieve gender equality at the workplace without more equality in family responsibilities, given the sizable effect of employer statistical discrimination in equilibrium. Requiring equality in one margin (either wages or employment) induces firms to counteract the policy on the other margin, and does not address the main source of statistical discrimination – career interruptions of women around childbirth.

An extension of the model might involve formalizing intra-household decisions where spouses jointly choose their parental leave lengths and separation rates, taking into ac-

count their labor market prospects. Employers' priors that women are more prone to higher separations might become a self-fulfilling prophecy if the resulting discrimination in wages and job opportunities induce women to specialize in household production. Greater gender equality in the labor market might reinforce gender equality in family responsibilities and vice versa. I leave it for future research to quantify the long-run consequences of such propagating effects.

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Appendix

Appendix A Tables and figures

TABLE A1. Principal component analysis for the amenity index

Variables	Factor loading	Unexplained proportion
1. Work from home	-0.315	.522
2. Flexible start/end	-0.378	.312
3. Flexible hours	-0.256	.683
4. Run errands during work	-0.338	.450
5. Overtime without pay	0.369	.344
6. Contacted after work	0.403	.217
7. Too much overtime	0.375	.322
8. Actual hours worked (LFS)	0.347	.419
9. Proportion part-time	0.140	.905

NOTES: The table shows the factor loading of each variable for the first principal component. Negative amenities (variables 5 to 8) are multiplied by -1 before entering the principal component analysis, so all the variables can be interpreted as good amenities.

TABLE A2. Summary statistics by job productivity types

Job productivity types	1	2	3	4	5	6	7
Number of workers	27,192	37,155	38,003	41,466	37,309	22,161	13,136
Number of workers per job	2.00	4.20	4.03	4.05	4.04	2.91	2.24
Mean log-wages	2.64	2.96	3.10	3.24	3.39	3.55	3.83
SD of log-wages	0.212	0.043	0.041	0.041	0.044	0.056	0.133
% **Clerical jobs**	33.51%	7.37%	4.49%	2.91%	1.47%	1.01%	0.70%
% **Associates**	23.03%	18.19%	28.42%	19.54%	13.02%	9.50%	3.46%
% **Professionals**	42.01%	72.26%	63.6%	70.03%	70.89%	59.97%	35.27%
% **Managers**	1.45%	2.17%	3.49%	7.52%	14.62%	29.52%	60.56%

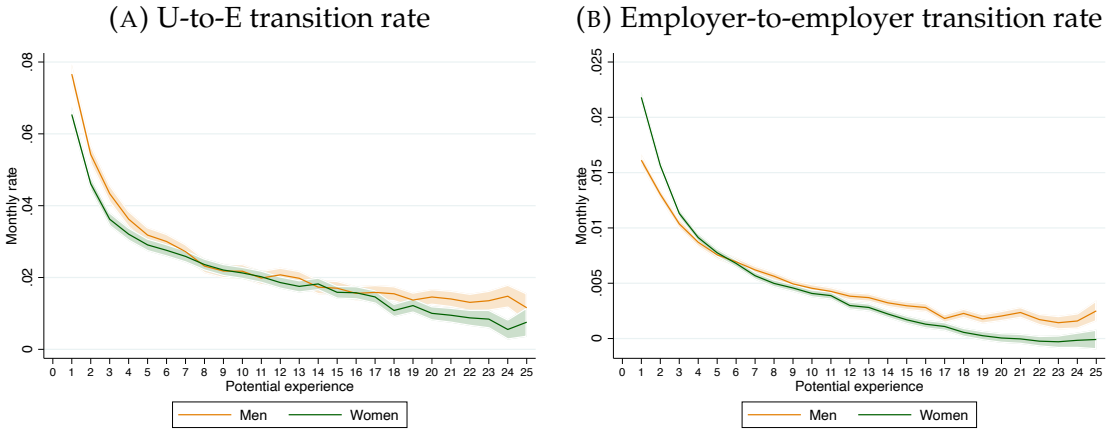
TABLE A3. Parameter Estimates

Parameters		Estimates	SEs
Complementarity	ρ	-15.531	1.136
Relative productivity	a	0.856	0.019
TFP	K	29.230	1.044
Baseline HC rate	d_1	0.001	0.002
Proportional HC rate	d_2	0.010	0.003
Men's value for amenities	μ_m	0.783	0.020
Women's value for amenities	μ_f	0.867	1.042
Preference increase in motherhood	M	1.744	2.045
Worker's bargaining	σ	0.522	0.015
Home productivity	b	5.164	0.791
Women's separation rate in NC	δ_{NC}	0.012	0.001
Women's separation rate in YC	δ_{YC}	0.016	0.015
Men's separation rate	δ	0.008	0.002
Matching efficiency	ϑ	0.107	0.006
Relative search intensity in unemployment	s_U	0.719	0.431
Relative search intensity in employment	s_E	0.531	0.164

TABLE A4. Proportion of gender wage gap explained by each channel (%)

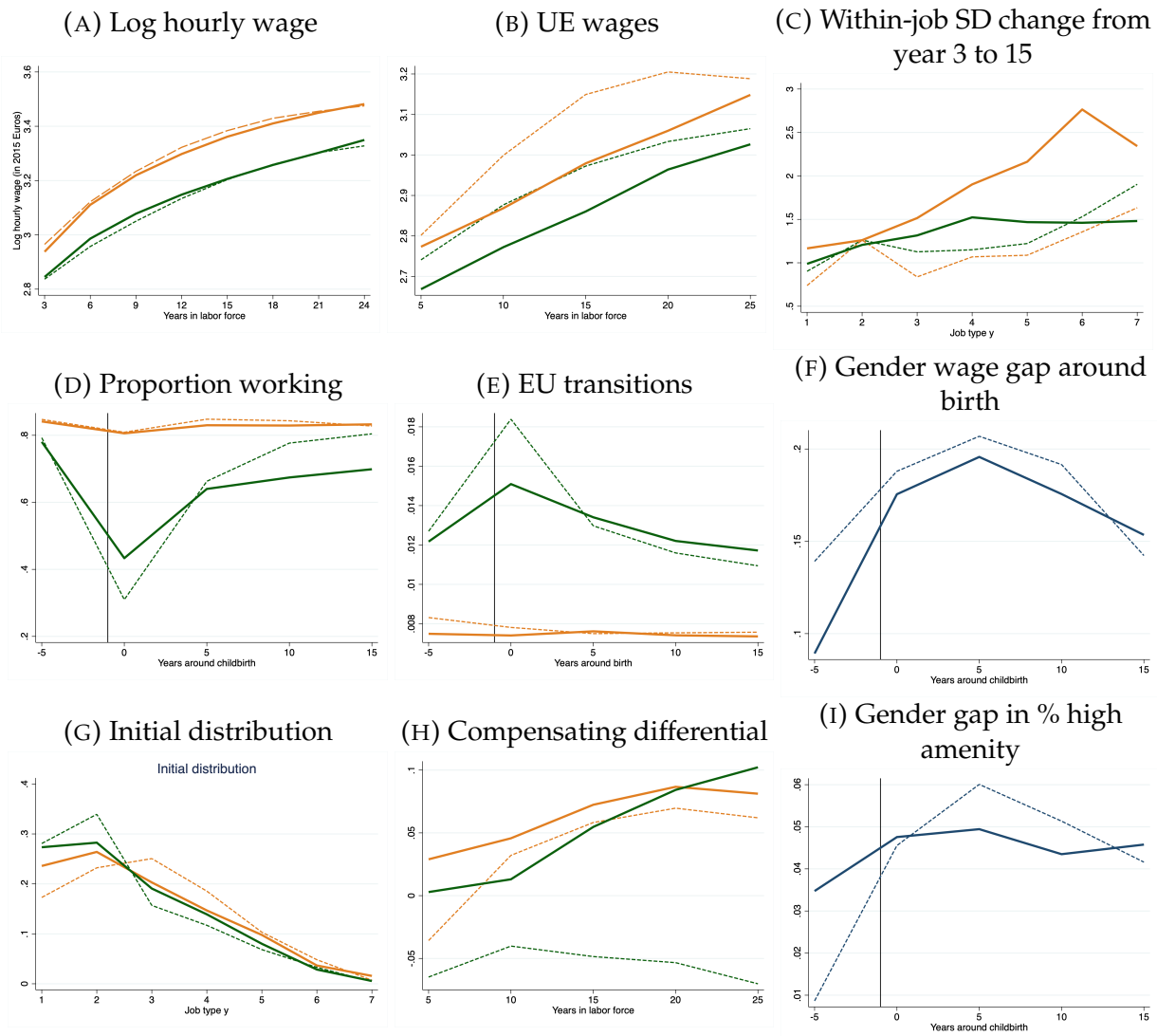
Years in labor force	(1) Equal PL duration	(2) Equal E-to-U rate	(3) Equilibrium employment	(4) Equilibrium wages	(5) Preference for amenities	(6) Residual
3	11.80	6.79	6.75	36.85	-1.59	39.39
6	24.68	19.16	6.94	21.72	3.93	23.57
9	25.59	30.79	6.41	11.91	7.30	17.99
12	27.20	37.87	5.87	5.85	8.65	14.55
15	28.69	38.49	5.44	2.55	9.14	15.69
18	29.00	41.98	4.58	0.61	9.06	14.76
21	32.32	43.32	3.61	-0.49	8.65	12.60
24	33.56	46.92	4.12	-1.04	8.55	7.89

FIGURE A1. Transitions over the life-cycle



NOTES: The lines represent the coefficients obtained from regressions of outcome variables on potential experience, separately for men and women. Shaded areas represent 95% confidence intervals.

FIGURE A2. Model fit



NOTES: The solid lines represent model-predicted moments, the dashed lines are data moments, and green denotes women while orange denotes men. Shaded areas correspond to 95% confidence intervals.

Appendix B Data description and sample selection

The Finnish Longitudinal Employer-Employee Data (FOLK) is assembled by Statistics Finland from numerous administrative registers, and covers the entire resident population aged 15 to 70 between years 1988 and 2016. FOLK provides detailed employment histories for each worker. Using the start and end dates of each employment relationship, I create a monthly employment status for each worker – employed, unemployed, or on parental leave. Since FOLK can be linked to the official population register, I also observe the birth date of each child of the worker and use it to infer the worker's parental leave status when he/she starts collecting benefits around that date.

The hourly wage data comes from the Structure of Earnings Statistics (SES). The SES consists of large-scale surveys collected by the Employers' Association in the last quarter of each year from 1995 to 2013. It covers all public sector workers and 55 to 75 percent of private sector workers depending on the year. The following groups in the private sector are either entirely excluded or at least severely under-represented: 1. small (less than 5 persons) enterprises; 2. the vast majority of non-organized (mainly small) enterprises; 3. agriculture, forestry and fisheries; 4. international organizations; 5. company management and owners and their family members; 6. the employment relationships beginning or ending during the reference month.

SES observations are on the yearly level (as opposed to daily in FOLK), and some firms might not be surveyed by SES in certain years. In the estimation, I use sample weights in the simulations to account for potential missing data from small firms.

I drop workers whose age is in the bottom or top 5 percentiles of the age distribution at graduation, so that workers in my sample are aged between 24 and 31 when they graduated master's. I drop small firms that have never had more than 2 workers during the sample period.

I only include periods after the individuals have completed their master's education. Unemployment of 2 months or less is counted as the final tenure of the previous spell. Similarly, employment of 2 months or less is counted as non-employment.

Wages and occupations are observed in the SES once a year from 1995 to 2013 in the last quarter of the calendar year. If the worker has wages from more than one employer in a quarter, I keep only the wage from the "main" job – the full-time job if there is one, or the job with the most earnings if all jobs are part-time. I trim the top 0.5% of the wage distributions in each year, which tend to be very thin and cover wide ranges. After

sample selection, I have an unbalanced panel of 116,781 workers, and 25,951 distinct firm-occupations over the course of 18 years.

I remove macroeconomic fluctuations in wages and transition rates by taking out year fixed effects in all moments calculations.

Appendix C Parental leave system in Finland

The Finnish maternity allowance system was first introduced in 1964. Currently, parents are entitled to wage-replaced leave for a total of 12 months, in which 4 months are reserved for mothers, 2 months for fathers, and 6 months can be shared between the spouses. In addition, parents are entitled to Child Home Care Allowances until the child turns 3 years old. Both biological and adoptive parents are entitled to parental leave on the basis of permanent residence in Finland.

The amount of parental leave benefits is a piece-wise linear function of annual earnings in the previous employment, or social benefits collected in the case of unemployment. The rate of wage replacement depends on income tiers as shown in the following table:

TABLE A5. Maternity, paternity and parental allowances pay schedule

Annual earnings (€)	Calculation formula (annual amount in €)
up to 11,942	8,358
11,943 - 37,861	0.7 x annual earnings
37,862 - 58,252	26,503 + 0.40 x (annual earnings - 37,861)
over 58,252	34,659 + 0.25 x (annual earnings - 58,252)

After the parental leave is over, parents can continue to care for the child at home and receive the Home Care Allowances (HCA). The HCA may be paid to either parent, although it is predominantly the mother who takes up the allowance. The HCA benefit amount consists of two parts – there is a fixed amount of 338.34 euros per month for one child under 3, and a means-tested amount targeted at low-income families up to 180 euros per month. In addition, there is sibling extra and municipality-based supplements. For

details of HCA, please refer to Kosonen (2014).

The benefit amount of the parental leave allowance and the HCA claimed are separately reported in the FOLK data for each individual in each calendar year. This paper uses the pay schedule in Table A5 and the fixed HCA amount adjusted by inflation to infer the total number of months of parental leave taken for each worker.

Since I observe the exact amount of parental leave benefits collected around the time of childbirth, I can pinpoint the month at which the worker stops collecting benefits. If a worker is not associated with an employer and is not collecting parental leave benefits in a particular month, he/she is considered to be unemployed.[22] According to this measure, female separation rate is already a little higher than male's prior to birth, but the big difference appears right after childbirth, where women's separation spikes and remain well above men's for many years after childbirth.

Appendix D Wage determination and workers' values

To facilitate notation, define function $A(\cdot,\cdot)$:

$$A(\nu_P,\nu_I) = \begin{cases} \nu_I + \sigma(\nu_P - \nu_I) & \text{if } \nu_P > \nu_I \\ \nu_P + \sigma(\nu_I - \nu_P) & \text{otherwise} \end{cases}$$

where $v^g_{a,P}(x,\epsilon,y',\alpha') = P^g_a(x,\epsilon,y',\alpha') - \Pi_0(y',\alpha')$ is the maximum value the poaching job offers, and $v^g_{a,I}(x,\epsilon,y,\alpha) = P^g_a(x,\epsilon,y,\alpha) - \Pi_0(y,\alpha)$ is the maximum the incumbent job offers.

The equation below illustrates an example of the worker's value when he/she gets a wage ϕ_0 out of unemployment in the "No Child" stage:

$$\begin{aligned} W^g_{NC}\big(\phi^g_{0,NC}(x,\epsilon,y,\alpha),x,\epsilon,y,\alpha\big) &= U^g_a(x,\epsilon) + \sigma\, S^g_a(x,\epsilon,y,\alpha) \\ &= \phi^g_{0,NC}(x,\epsilon,y,\alpha) + q^g(\epsilon,\alpha) + \beta\,\mathbb{E}\Big[\delta\, U^g_{NC}(x_+,\epsilon) + \gamma\, \widetilde{W}^g_D(w_+,x_+,\epsilon,y,\alpha) + \chi\, \widetilde{W}^g_{PL}(w_+,x_+,\epsilon,y,\alpha) \\ &\quad + \sum_{y',\alpha'} s\kappa\, v(y',\alpha') \max\Big\{A\big(v^g_{NC,P}(x_+,\epsilon,y',\alpha'), v^g_{NC,I}(x_+,\epsilon,y,\alpha)\big) - \widetilde{W}^g_{NC}(w_+,x_+,\epsilon,y,\alpha), 0\Big\} \\ &\quad + (1-\delta-\gamma-\chi)\, \widetilde{W}^g_{NC}(w_+,x_+,\epsilon,y,\alpha)\Big] \end{aligned}$$

[22] If someone is unemployed for only two months or less after she stops collecting parental leave benefits, I consider it as measurement error in leave duration calculations and do not count the months as unemployment. A separation is only indicated for unemployment of 3 months or more.

where w_+ denotes the wage in the next period, and x_+ denotes the worker's human capital type in the next period. When a worker's human capital changes from x to x_+ in the next period, the wage does not update until there is a credible outside option. At any point in time, the match can dissolve endogenously if surplus falls below zero.

Appendix E Steady-state balance equations

In a stationary equilibrium, flows into and out of any worker stock must balance. Every period is divided into 3 stages. Let $u_a^-(x,\epsilon)$ and $h_a^-(x,\epsilon,y,\alpha)$ denote the distributions of workers in unemployment and employment at the beginning of the current search period at age $a \in \{NC, YC, PL, D\}$. In the human capital evolution stage (Stage I), the worker's skill type changes from x to x_+ according to stochastic processes $p_e(x_+|x,y)$ during employment (except in PL stage) and $p_u(x_+|x)$ during unemployment.

$$u_a^I(x,\epsilon) = u_a^-(x,\epsilon) + \sum_{x' \neq x} u_a^-(x',\epsilon) p_u(x|x') - \sum_{x' \neq x} u_a^-(x,\epsilon) p_u(x'|x) \tag{18}$$

$$h_a^I(x,\epsilon,y,\alpha) = h_a^-(x,\epsilon,y,\alpha) + \sum_{x' \neq x} h_a^-(x',\epsilon,y,\alpha) p_e(x|x',y) - \sum_{x' \neq x} h_a^-(x,\epsilon,y,\alpha) p_e(x'|x,y)$$

for stages $a \in \{NC, YC, D\}$. Workers in PL stage do not accumulate human capital, so $h_{PL}^I(x,\epsilon,y,\alpha) = h_{PL}^-(x,\epsilon,y,\alpha)$.

In the search stage (Stage II):

$$\begin{aligned} u_{NC}^{II}(x,\epsilon) = {} & u_{NC}^I(x,\epsilon)\Big(1 - \gamma - \chi - \kappa \sum_{y,\alpha} v(y,\alpha)\, \mathbb{1}[S_{NC}^f(x,\epsilon,y,\alpha) > 0]\Big) \\ & + (0.5\phi\, D)\, \xi_0(x,\epsilon) + \delta_{NC} \sum_{y,\alpha} h_{NC}^I(x,\epsilon,y,\alpha) \end{aligned} \tag{19}$$

$$\begin{aligned} h_{NC}^{II}(x,\epsilon,y,\alpha) = {} & h_{NC}^I(x,\epsilon,y,\alpha)(1 - \gamma - \chi - \delta_{NC}) \\ & + \kappa u_{NC}^I(x,\epsilon)\, v(y,\alpha)\, \mathbb{1}[S_{NC}^f(x,\epsilon,y,\alpha) > 0] \\ & + s\kappa\, v(y,\alpha) \sum_{y',\alpha'} h_{NC}^I(x,\epsilon,y',\alpha')\, \mathbb{1}[S_{NC}^f(x,\epsilon,y,\alpha) > S_{NC}^f(x,\epsilon,y',\alpha')] \\ & - s\kappa\, h_{NC}^I(x,\epsilon,y,\alpha) \sum_{y',\alpha'} v(y',\alpha') \mathbb{1}[S_{NC}^f(x,\epsilon,y',\alpha') > S_{NC}^f(x,\epsilon,y,\alpha)] \end{aligned}$$

$$u_{PL}^{II}(x,\epsilon) = u_{PL}^{I}(x,\epsilon)(1-\gamma-\eta) + \chi\Big(u_{NC}^{I}(x,\epsilon) + u_{YC}^{I}(x,\epsilon)\Big) + \delta_{YC}\sum_{y,\alpha} h_{PL}^{I}(x,\epsilon,y,\alpha)$$

$$h_{PL}^{II}(x,\epsilon,y,\alpha) = h_{PL}^{I}(x,\epsilon,y,\alpha)(1-\gamma-\delta_{YC}-\eta) + \chi\left(h_{NC}^{I}(x,\epsilon,y,\alpha) + h_{YC}^{I}(x,\epsilon,y,\alpha)\right)$$

$$\begin{aligned}
u_{YC}^{II}(x,\epsilon) &= u_{YC}^{I}(x,\epsilon)\Big(1-\gamma-\chi-\kappa\sum_{y,\alpha} v(y,\alpha)\,\mathbb{1}[S_{YC}^{f}(x,\epsilon,y,\alpha)>0]\Big) \\
&\quad + \eta\, u_{PL}^{I}(x,\epsilon) + \delta_{YC}\sum_{y,\alpha} h_{YC}^{I}(x,\epsilon,y,\alpha) \\
h_{YC}^{II}(x,\epsilon,y,\alpha) &= h_{YC}^{I}(x,\epsilon,y,\alpha)(1-\gamma-\delta_{YC}-\chi) + \eta\, h_{PL}^{I}(x,\epsilon,y,\alpha) \\
&\quad + \kappa u_{YC}^{I}(x,\epsilon)\, v(y,\alpha)\,\mathbb{1}[S_{YC}^{f}(x,\epsilon,y,\alpha)>0] \\
&\quad + s\kappa\, v(y,\alpha)\sum_{y',\alpha'} h_{YC}^{I}(x,\epsilon,y',\alpha')\,\mathbb{1}[S_{YC}^{f}(x,\epsilon,y,\alpha) > S_{YC}^{f}(x,\epsilon,y',\alpha')] \\
&\quad - s\kappa\, h_{YC}^{I}(x,\epsilon,y,\alpha)\sum_{y',\alpha'} v(y',\alpha')\mathbb{1}[S_{YC}^{f}(x,\epsilon,y',\alpha') > S_{YC}^{f}(x,\epsilon,y,\alpha)]
\end{aligned}$$

$$\begin{aligned}
u_{D}^{II}(x,\epsilon) &= u_{D}^{I}(x,\epsilon)\Big(1-\phi-\kappa\sum_{y,\alpha} v(y,\alpha)\,\mathbb{1}[S_{D}^{f}(x,\epsilon,y,\alpha)>0]\Big) \\
&\quad + \gamma\left(u_{NC}^{I}(x,\epsilon) + u_{YC}^{I}(x,\epsilon)\right) + \delta\sum_{y,\alpha} h_{D}^{I}(x,\epsilon,y,\alpha) \\
h_{D}^{II}(x,\epsilon,y,\alpha) &= h_{D}^{I}(x,\epsilon,y,\alpha)(1-\phi-\delta) + \gamma\left(h_{NC}^{I}(x,\epsilon,y,\alpha) + h_{YC}^{I}(x,\epsilon,y,\alpha) + h_{PL}^{I}(x,\epsilon,y,\alpha)\right) \\
&\quad + \kappa u_{D}^{I}(x,\epsilon)\, v(y,\alpha)\,\mathbb{1}[S_{D}^{f}(x,\epsilon,y,\alpha)>0] \\
&\quad + s\kappa\, v(y,\alpha)\sum_{y',\alpha'} h_{D}^{I}(x,\epsilon,y',\alpha')\,\mathbb{1}[S_{D}^{f}(x,\epsilon,y,\alpha) > S_{YC}^{f}(x,\epsilon,y',\alpha')] \\
&\quad - s\kappa\, h_{D}^{I}(x,\epsilon,y,\alpha)\sum_{y',\alpha'} v(y',\alpha')\mathbb{1}[S_{D}^{f}(x,\epsilon,y',\alpha') > S_{D}^{f}(x,\epsilon,y,\alpha)]
\end{aligned}$$

In the endogenous quits stage:

$$u_a^{+}(x,\epsilon) = u_a^{II}(x,\epsilon) + \sum_{y,\alpha} h_a^{II}(x,\epsilon,y,\alpha)\mathbb{1}[S_a^{f}(x,\epsilon,y,\alpha) < 0] \tag{20}$$

$$h_a^{+}(x,\epsilon,y,\alpha) = h_a^{II}(x,\epsilon,y,\alpha)(1-\mathbb{1}[S_a^{f}(x,\epsilon,y,\alpha) < 0]), \quad \forall \quad a \in \{NC, PL, YC, D\}$$

After the dismissals (or endogenous quits) occur, u_a^+ and h_a^+ become the initial distributions for the next period. In stationary equilibrium, $u_a^- = u_a^+$ and $h_a^- = h_a^+$.

Appendix F Estimation procedures and standard errors

I use the following iterative procedure to estimate two sets of parameters, the transition parameters $\boldsymbol{\lambda} = (\delta_{NC}^f, \delta_{YC}^f, \delta, \vartheta, s_U, s_E)$ and the core parameters $\boldsymbol{\theta} = (d_1, d_2, K, a, \rho, \sigma, b, \mu_m, \mu_f, M)$.

Step 1: Core moments given transition parameters Given a value for the transition parameters $\boldsymbol{\lambda}$ obtained from the previous iteration (or an initial guess at the start), I estimate $\boldsymbol{\theta}$ by minimizing the following quadratic distance

$$L_1(\boldsymbol{\theta}|\boldsymbol{\lambda}) = (\hat{m}_1^D - \hat{m}_1^S(\boldsymbol{\theta}|\boldsymbol{\lambda}))^T \ \hat{W}_1^{-1} \ (\hat{m}_1^D - \hat{m}_1^S(\boldsymbol{\theta}|\boldsymbol{\lambda}))$$

where $\hat{m}_1^D$ is a vector of data moments related to wage profiles of men and women, U-to-E wages and wage growths, proportion of men and women in high- and low-amenity jobs etc. that are described in section 4.2. The vector $\hat{m}_1^S$ are the corresponding model moments from simulations, taking $\boldsymbol{\lambda}$ as given.

Step 2: Transition moments given core parameters Given the estimate of $\boldsymbol{\theta}$ obtained from the previous step, I update the estimate of $\boldsymbol{\lambda}$ by matching appropriate moments related to transitions:

$$L_2(\boldsymbol{\lambda}|\boldsymbol{\theta}) = (\hat{m}_2^D - \hat{m}_2^S(\boldsymbol{\lambda}|\boldsymbol{\theta}))^T \ \hat{W}_2^{-1} \ (\hat{m}_2^D - \hat{m}_2^S(\boldsymbol{\lambda}|\boldsymbol{\theta}))$$

I iterate over these two steps using MCMC until the functions L_1 and L_2 are minimized and the estimates of $\boldsymbol{\lambda}$ and $\boldsymbol{\theta}$ converge. The estimation strategy is a good fit for my problem because MCMC is derivative-free, so it is able to handle the non-linearities in the criterion functions due to the discreteness in the model. MCMC can also deal with large parameter spaces and multiple local minima quite well.[23]

I use the sandwich formula to estimate standard errors. Normally, the variance of the converged MCMC chain would provide a direct way to construct valid confidence intervals for the parameter estimates if the optimal weighting matrix is used. But I use a diagonally weighted approach. I will illustrate the computation for the core parameters

[23]See the discussion in Chernozhukov and Hong (2003) for more details.

$\boldsymbol{\theta}$ below (the calculation is analogous for the transition parameters $\boldsymbol{\lambda}$). The estimated covariance matrix has the form

$$\hat{V}(\hat{\boldsymbol{\theta}}) = \left(G'(\hat{\boldsymbol{\theta}})\Omega G(\hat{\boldsymbol{\theta}})\right)^{-1} G'(\hat{\boldsymbol{\theta}})\Omega\, \hat{E}\left[(m_1^S(\hat{\boldsymbol{\theta}}) - \hat{m}_1^D)(m_1^S(\hat{\boldsymbol{\theta}}) - \hat{m}_1^D)'\right]\Omega G(\hat{\boldsymbol{\theta}}) \left(G'(\hat{\boldsymbol{\theta}})\Omega G(\hat{\boldsymbol{\theta}})\right)^{-1}$$

where Ω is the weight matrix used in the estimation, $G(\hat{\boldsymbol{\theta}})$ is the gradient matrix evaluated at the estimated parameters $\hat{\boldsymbol{\theta}}$.

Estimates for the gradient G are obtained through simulation. Suppose m_1 consists of K moments and $\boldsymbol{\theta}$ consists of J parameters. Then the numerical derivatives $\hat{G}(\hat{\boldsymbol{\theta}})$ is a $K \times J$ matrix where the j-th column is computed as:

$$\hat{G}_j = \frac{m_1^S(\hat{\boldsymbol{\theta}} + h\,\hat{\theta}_j) - m_1^S(\hat{\boldsymbol{\theta}} - h\,\hat{\theta}_j)}{2\,h\,\hat{\theta}_j}$$

where m_1^S is the vector of simulated moments evaluated at $\hat{\boldsymbol{\theta}} + h\,\hat{\theta}_j$ and $\hat{\boldsymbol{\theta}} - h\,\hat{\theta}_j$ respectively. The step size of deviation h is a vector of zeros except for one positive element at the j-th position equal to 1%. $\hat{\theta}_j$ is the j-th element of $\hat{\boldsymbol{\theta}}$.