Tasks, Automation, and the Rise in U.S. Wage Inequality

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NBER Working Paper Series No. 28920, June 2021

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Econ 350, Winter 2023

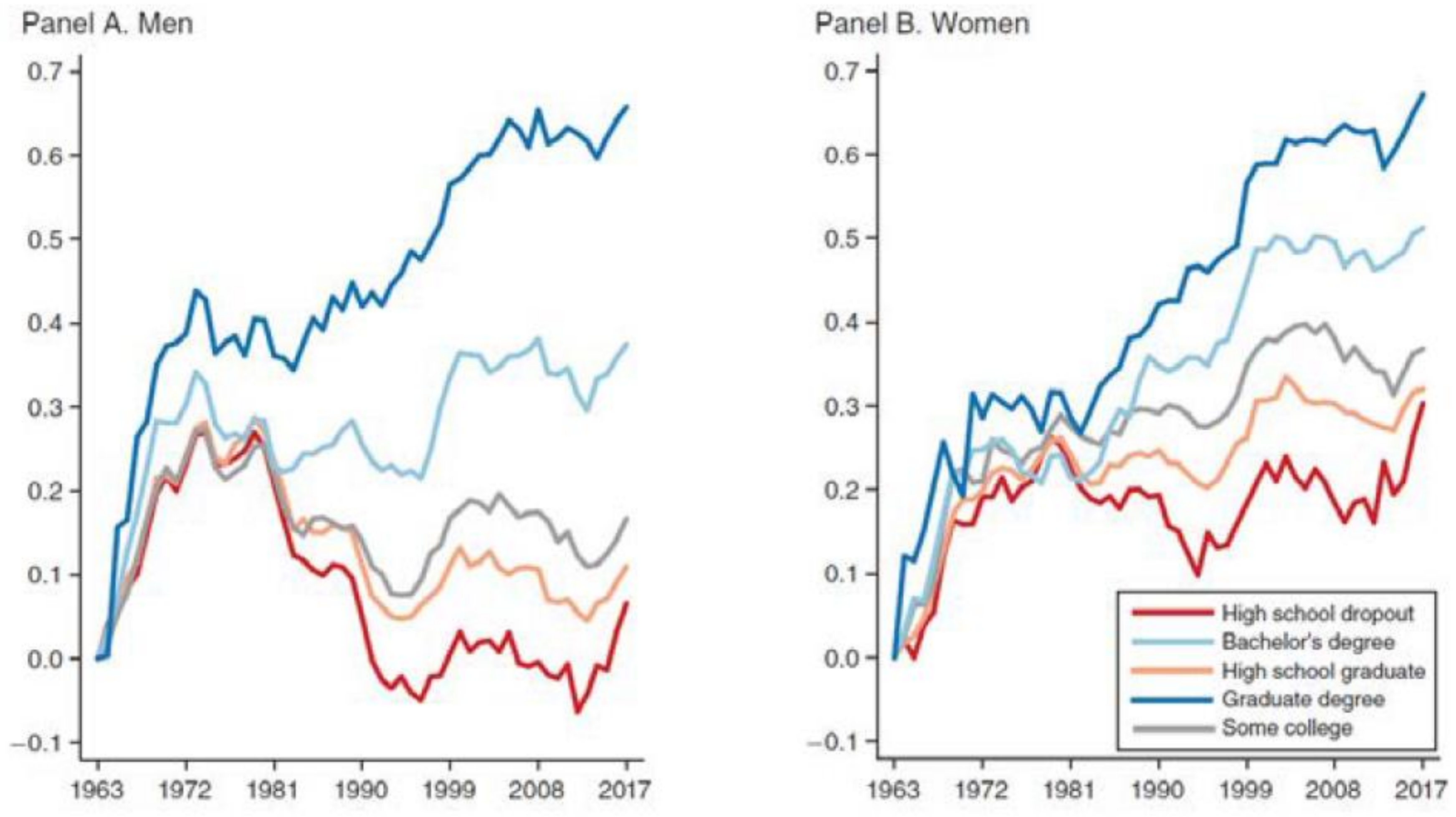
1. Introduction

- Wage and earnings inequality have risen sharply in the US and other industrialized economies over the last four decades.

- Figure 1 depicts some salient aspects of US developments: while the real wages of workers with a post-graduate degree rose, the real wages of low-education workers declined significantly.

- The real earnings of men without a high-school degree are now 15% lower than they were in 1980.

Figure 1: Cumulative Growth of Real Wages by Gender and Education



- The most popular explanation for these changes is based on skill-biased technological change (SBTC).

- According to this framework, the demand for different types of workers comes from an aggregate production function of the form $F(A_H H, A_L L)$, where H and L are employment levels of high-skill and low-skill workers, and A_H and A_L represent technologies (or equipment) augmenting these two types of workers.

- SBTC corresponds to technology becoming more favorable to high-skill workers (e.g., a greater increase in A_H than in A_L, provided that F has an elasticity of substitution greater than one).

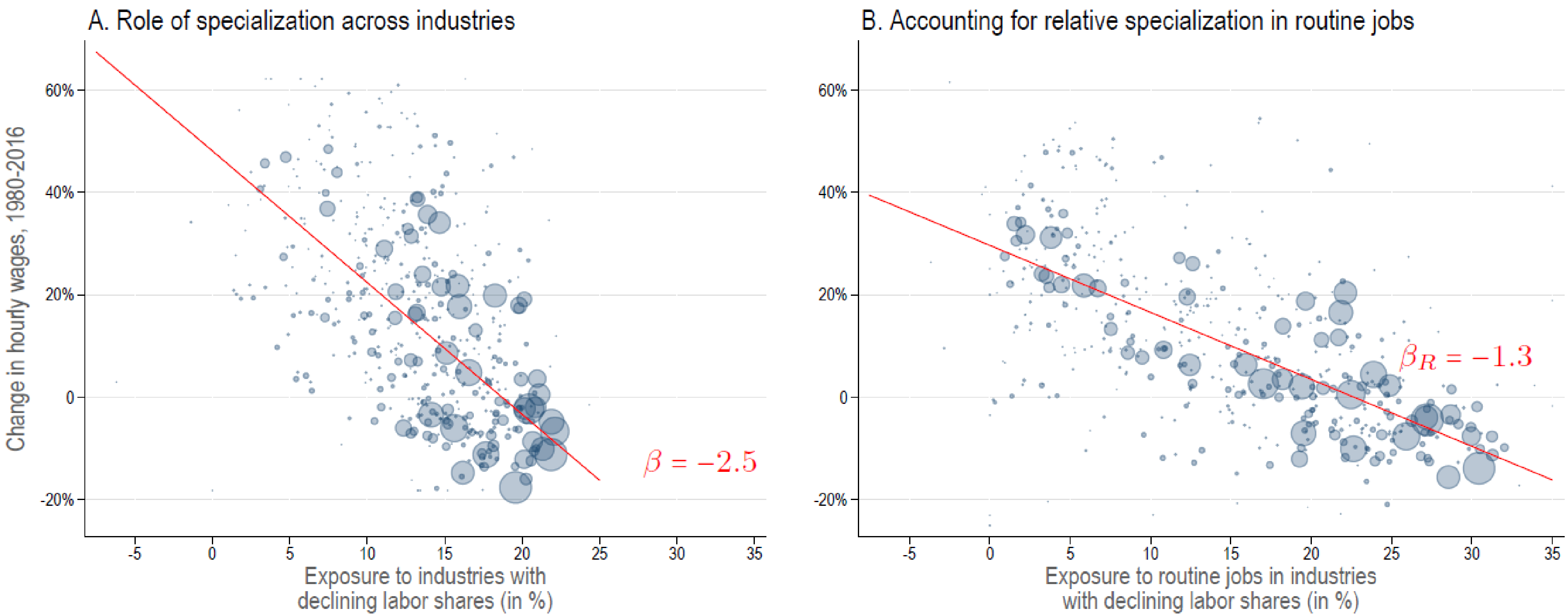
- This paper proposes an alternative approach for thinking about wage inequality.

- We argue that much of the changes in US wage structure are driven by the automation of tasks previously performed by certain types of workers in some industries (e.g., numerically-controlled machinery or industrial robots replacing blue-collar workers in manufacturing or specialized software replacing clerical workers).

- Workers who are not displaced from the tasks in which they have a comparative advantage, such as those with a postgraduate degree or women with a college degree, enjoyed real wage gains, while those, including low-education men, who used to specialize in tasks and industries undergoing rapid automation, experienced stagnant or declining real wages.

- Figure 2 provides motivating evidence for our explanation by revisiting the role of industry and declining labor shares—a telltale sign of automation—in US wage inequality.

Figure 2: Relationships between change in real wages and a demographic group's exposure to industries with declining labor share (left panel) and exposure to routine jobs in industries with declining labor share (right panel).



- Our framework delivers three key results.
- First, and in contrast to models of SBTC with factor-augmenting technologies, in our framework automation can have a negative effect on workers who are displaced from tasks they used to perform, and such changes can take place with limited increases in total factor productivity (TFP).
- Hence, real wage declines and slow productivity growth despite rapid automation are not puzzles within this framework.

- Second, we derive a simple equation linking wage changes of a demographic group to the task displacement it experiences, which forms the basis of our reduced-form analysis.

- Third, our framework implies that the task displacement experienced by a group can be measured by its employment share in routine tasks in industries undergoing automation.

- Moreover, the extent of automation in an industry can be inferred from declines in its labor share, thus providing an explanation for the relationship reported in the right panel of Figure 2.

- Our work contributes to various literatures.
- The first is the literature on SBTC, with papers such as Bound and Johnson (1992), Katz and Murphy (1992) and Card and Lemieux (2001) that explored the evolution of between-group wage inequality in response to changes in factor supplies and technologies augmenting the productivity of educated workers.
- We differ from this literature because of our distinct conceptual framework and focus on task displacement as the main driving force of changes in wage structure.

- The second is the literature exploring the effects of lower equipment and computer prices on wage inequality through capital-skill complementarity.
- Our framework complements this work by underscoring the role of task displacement as a separate mechanism contributing to wage inequality.
- We also clarify the distinction between automation and the capital-skill complementarity studied in this literature.
- Notably, we show that automation has a powerful impact on inequality even if there are no direct capital-skill complementarities.

- Third, and most closely related to our paper is Autor, Levy and Murnane (2003), who explore the effects of technologies automating routine tasks and complementing non-routine ones on the occupational and task structure of the economy.

- Our paper can be seen as a generalization of their conceptual framework, enabling us to clarify the role of task displacement and quantify its effects on changes in US inequality.

- Finally, our conceptual framework builds on previous task models, in particular, Zeira (1998), Acemoglu and Zilibotti (2001), Acemoglu and Autor (2011), and Acemoglu and Restrepo (2018), as well as Grossman and Rossi-Hansberg's (2008) model of offshoring.

- Our two main innovations relative to these papers are:

    i. The general structure of comparative advantage and the flexible manner in which technologies affect the allocation of tasks to workers.

    ii. Our derivation of explicit formulas linking a group's wage change to its task displacement.

- These formulas underpin all of our empirical work.

2. Conceptual Framework: Tasks, Wages, and Inequality

2.1 Single Sector

- **Environment and equilibrium**: Output is produced by combining a mass M of tasks in a set $\mathcal{T}$ using a CES aggregator with elasticity of substitution $\lambda \geq 0$,

$$y = \left(\frac{1}{M} \int_{\mathcal{T}} \left(M \cdot y(x) \right)^{\frac{\lambda-1}{\lambda}} \cdot dx \right)^{\frac{\lambda}{\lambda-1}},$$

where x indexes tasks.

- The key economic decision in this model is how to perform these different tasks.

- Each task can be produced using capital or different types of labor indexed by g (where $g \in \mathcal{G} = \{1,2, \dots, G\}$):

$$y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_{g \in \mathcal{G}} A_g \cdot \psi_g(x) \cdot \ell_g(x).$$

- Here, $\ell_g(x)$ denotes the amount of labor of type g allocated to task x, while $k(x)$ is the amount of capital allocated to task x.

- In addition, A_k and the A_g's represent standard factor-augmenting technologies, which make factors uniformly more productive at all tasks.

- More novel and important for our purposes, productivity also has a task-specific component, represented by the functions $\psi_k(x)$ and $\{\psi_g(x)\}_{g \in \mathcal{G}}$, which determine comparative advantage and specialization patterns.

- Task-specific productivity is zero for factors that cannot perform the relevant task.

- Capital is supplied elastically and can be produced using the final good at a constant marginal cost $1/q(x)$.

- Net output, which is equal to consumption, is therefore obtained by subtracting the production cost of capital goods from output:

$$c = y - \int_{\mathcal{T}} (k(x)/q(x)) \cdot dx.$$

- We assume that all types of labor are supplied inelastically, and we denote the total supply of labor of type g by ℓ_g.

- A *market equilibrium* in this economy is defined as an allocation of tasks to factors and a production plan for capital goods that maximize consumption.

- To formalize this notion, we define a critical object for the rest of our analysis: $\mathcal{T}_g$, which represents the set of tasks allocated to labor of type g; and $\mathcal{T}_k$, which is analogously the set of tasks allocated to capital.

- Given a supply of labor $\ell = (\ell_1, \ell_2, \ldots, \ell_G)$, a market equilibrium is given by wages $w = (w_1, w_2, \ldots, w_G)$, capital production decisions $k(x)$, and an allocation of tasks to factors $\{\mathcal{T}_k, \mathcal{T}_1, \ldots, \mathcal{T}_G\}$, such that:

  i. The allocation of tasks to factors minimizes costs.

  ii. The choice of capital maximizes net output.

  iii. The markets for capital and different types of labor clear.

- Throughout, we set the final good as the numeraire, so that the w_g's correspond to real wages and the real user cost of capital is $R(x) = 1/q(x)$.

- **Task shares**: Cost minimization implies that the sets of tasks allocated to factors satisfy:

$$\mathcal{T}_g = \left\{ \begin{array}{c} x: \dfrac{w_g}{\psi_g(x) \cdot A_g} \le \dfrac{w_j}{\psi_j(x) \cdot A_j} \text{ for } j < g; \\ \dfrac{w_g}{\psi_g(x) \cdot A_g} < \dfrac{w_j}{\psi_j(x) \cdot A_j}, \dfrac{1}{\psi_k(x) \cdot q(x) \cdot A_k} \text{ for } j > g \end{array} \right\}$$

$$\mathcal{T}_k = \left\{ x: \frac{1}{\psi_k(x) \cdot q(x) \cdot A_k} \le \frac{w_j}{\psi_j(x) \cdot A_j} \text{ for all } j \right\}$$

- Given an allocation of tasks to factors, we define:

$$\Gamma_g(w,\Psi) = \frac{1}{M}\int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1} \cdot dx \quad \text{and}$$

$$\Gamma_k(w,\Psi) = \frac{1}{M}\int_{\mathcal{T}_k} \big(\psi_k(x) \cdot q(x)\big)^{\lambda-1} \cdot dx.$$

- The quantities Γ_g and Γ_k, which we refer to as the *task shares* of workers of type g and capital, respectively, give the measure of the set of tasks allocated to a factor weighted by the "importance" of the tasks.

- Task shares depend on the sets $\mathcal{T}_g$ and $\mathcal{T}_k$, and thus on wages, factor-augmenting technologies and task productivities.

- Hence we write them as functions of the vectors of wages w and technology $\Psi = \left(\{\psi_k(x), \psi_g(x)\}_{x\in\mathcal{T}}, A_k, \{A_g\}_{g\in\mathcal{G}}\right)$, but will omit this dependence when it causes no confusion.

- The next proposition characterizes the equilibrium (all proofs are in Appendix A-1), and expresses factor prices, shares, and output as functions of task shares.

- Because production in this economy is "roundabout" (capital is produced linearly from the final good), output can be infinite.

- In Appendix B-1, we derive an Inada condition that ensures finite output (in the one-sector case, this condition implies $A_k^{\lambda-1} \cdot \Gamma_k < 1$) and assume throughout that it is satisfied.

- ***Proposition 1 (Equilibrium)*** *There exists a unique equilibrium. In this equilibrium, output, wages, and factor shares can be expressed as functions of task shares:*

$$(1) \qquad y = \left(1 - A_k^{\lambda-1} \cdot \Gamma_k\right)^{\frac{\lambda}{1-\lambda}} \cdot \left(\sum_{g \in \mathcal{G}} \Gamma_g^{\frac{1}{\lambda}} \cdot \left(A_g \cdot \ell_g\right)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}},$$

$$(2) \qquad w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \Gamma_g^{\frac{1}{\lambda}} \; for \; all \; g \in \mathcal{G},$$

$$(3) \qquad s^K = A_k^{\lambda-1} \cdot \Gamma_k.$$

- Equation (2) shows that real wages are given by the marginal product of each type of labor, which is a function of output per worker (raised to the power $1/\lambda$ for standard reasons) and the factor-augmenting technology, A_g, raised to the power $(\lambda - 1)/\lambda$.

- This exponent captures the fact that improvements in the productivity of workers from group g reduce the price of tasks they produce, and if $\lambda < 1$ this price effect dominates.

- More novel is that real wages also depend directly on task shares, the Γ_g's, highlighting a key aspect of our model: the real wage of a factor is linked to the set of tasks allocated to that factor.

- **The effects of technology**: Our conceptual framework clarifies that different types of technologies have distinct impacts on wages, productivity, and output.

- We now discuss the effects of three types of technologies.

1. **Factor augmenting**: higher A_g or A_k resulting in uniform increases in productivity in all tasks.

    - Factor-augmenting technologies have been the focus of much of the macro and labor literatures, and as we will see, they are qualitatively different from task displacement (and arguably a significant abstraction, since there are no examples of technologies that increase factor productivity in all tasks).

2. **Productivity deepening**: increases in $\psi_g(x)$ for $x \in \mathcal{T}_g$ or in $\psi_k(x)$ for $x \in T_k$—which result in an increase in the productivity of a factor at the tasks it is currently performing.

   - For example, we may have improvements in the tools used by workers to perform one of their tasks (think of GPS making drivers better at navigation, or upgrades in the capital equipment used to produce the same task).

   - The defining feature of this type of technological progress is that it does not directly displace factors from the tasks they were performing.

3. **Task displacement**: increases in $\psi_k(x)$ for $x \in \mathcal{T}_g$—which therefore lead to automation and a reallocation of tasks away from workers toward capital.

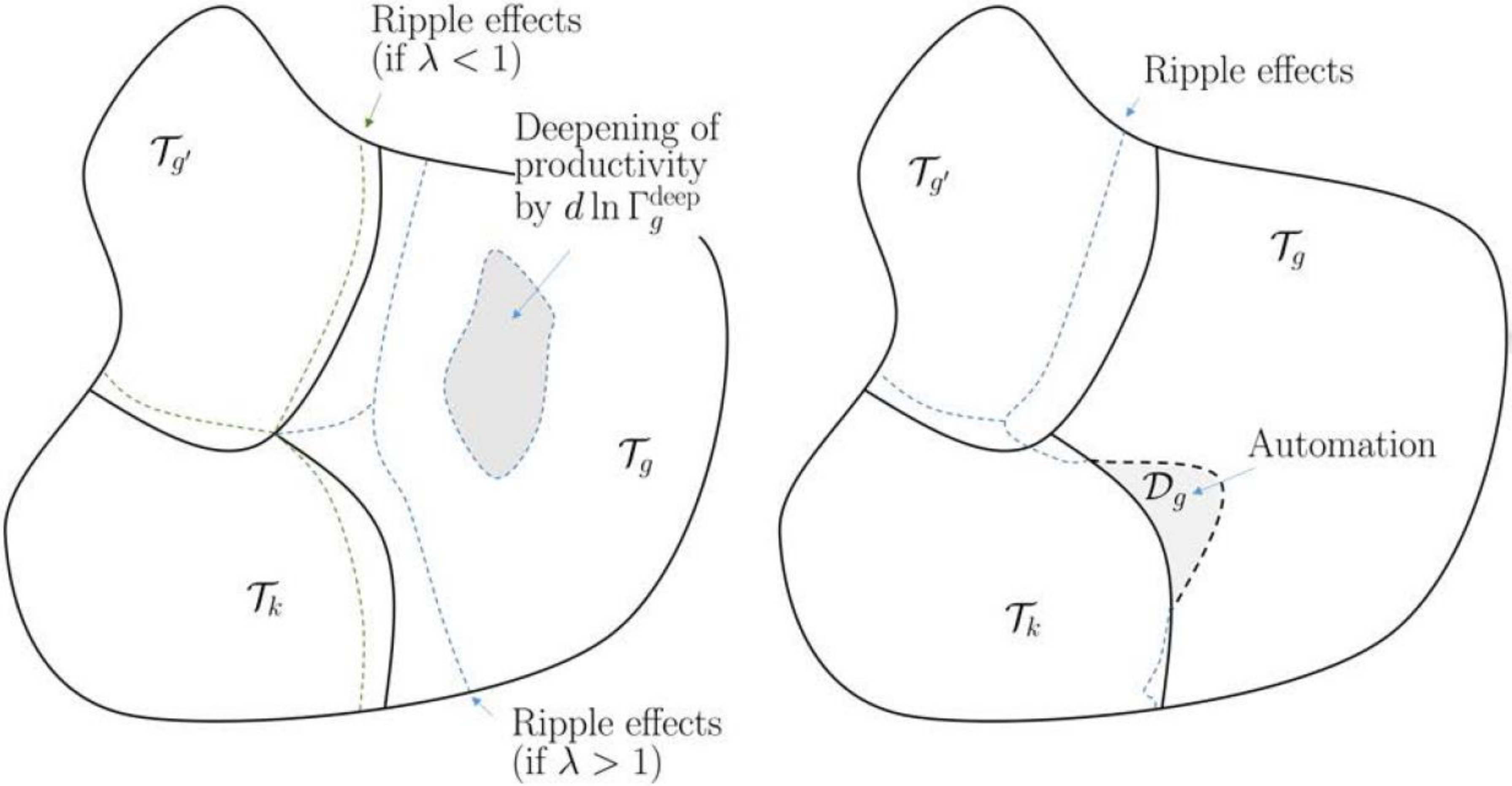
- Well-known examples of technologies causing task displacement include the introduction of numerical control or industrial robots for blue-collar tasks previously performed by manual workers or the introduction of specialized software automating various back-office and clerical tasks.

- Offshoring also leads to task displacement, and one can think about it in this framework by assuming that tasks can be performed abroad and imported in exchange of the final good.

- Figure 3 depicts the effects of productivity deepening and task displacement on the allocation of tasks to factors.

- The figure highlights that the total impact of a change in technology on task shares is comprised of a *direct effect*, given by the changes in the Γ_g's and Γ_k driven by productivity deepening and displacement *holding all prices* constant; and indirect or *ripple* effects, driven by the reallocation of tasks across factors in response to changes in factor prices.

Figure 3: The direct effects of technology and ripple effects



- The following assumption rules out ripple effects and is maintained until Section 5:

- ***Assumption 1***

  1. *Workers can only produce non-overlapping sets of tasks (i.e.,* $\psi_g(x) > 0$ *only if* $\psi_{g'}(x) = 0$ *for all* $g' \neq g$*).*

  2. *There exist* $\underline{\psi} > 0$ *and* $\overline{q} > 0$ *such that* $\psi_k(x) > \underline{\psi}$ *and* $q(x) > \overline{q}$ *for all* $x \in \mathcal{S} = \{x: \psi_k(x) > 0\}$*.*

- In the next proposition, we characterize the effects of different types of technologies on factor prices, TFP, and output under Assumption 1.

- We present a characterization in terms of the infinitesimal changes in the direct effects of these technologies.

- In particular, we let $d \ln \Gamma_g^{\text{deep}} \geq 0$ denote the direct effect of productivity deepening (for capital or some types of labor) on the task share of group g; and $d \ln \Gamma_g^{\text{disp}}$ denote the direct displacement effect experienced by group g due to automation (i.e., because capital productivity $\psi_k(x)$ increases at tasks previously performed by this group).

- These direct effects can be expressed as follows:

$$d\ln\Gamma_g^{\text{deep}} = \frac{1}{M}\int_{\mathcal{T}_g} \frac{\psi_g(x)^{\lambda-1}}{\Gamma_g} \cdot d\ln\psi_g(x)dx$$

$$d\ln\Gamma_g^{\text{disp}} = \frac{1}{M}\int_{\mathcal{D}_g} \psi_g(x)^{\lambda-1}dx \Big/ \frac{1}{M}\int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1}dx,$$

where $\mathcal{D}_g \subseteq \mathcal{T}_g$ is the subset of tasks in which, after the technological change, capital outperforms workers from group g (as shown in the right panel of Figure 3).

- Finally, we define

$$\pi_g = \frac{1}{M}\int_{\mathcal{D}_g} \psi_g(x)^{\lambda-1} \cdot \pi_g(x)dx \Big/ \frac{1}{M}\int_{\mathcal{D}_g} \psi_g(x)^{\lambda-1}dx$$

as the average cost savings from producing the tasks in $\mathcal{D}_g$ with the now more cost effective capital.

- In this expression, $\pi_g(x)$ is the cost saving of automating task x previously performed by workers group g.

- ***Proposition 2 (Technology Effects)*** *Suppose Assumption 1 holds, so that there are no ripple effects. Consider a change in technology (including factor-augmenting, productivity deepening, and task-displacement). The impact on real wages, TFP, output, and the capital share are*

$$(4) \qquad d\ln w_g = \frac{1}{\lambda} d\ln y + \frac{\lambda - 1}{\lambda} d\ln A_g + \frac{\lambda - 1}{\lambda} d\ln \Gamma_g^{deep} - \frac{1}{\lambda} d\ln \Gamma_g^{disp},$$

$$(5) \qquad d\ln TFP = \sum_{g \in \mathcal{G}} s_g^L \cdot \left(d\ln A_g + d\ln \Gamma_g^{deep}\right) + s^K \cdot \left(d\ln A_k + d\ln \Gamma_k^{deep}\right) + \sum_{g \in \mathcal{G}} s_g^L \cdot d\ln \Gamma_g^{disp} \cdot \pi_g,$$

$$(6) \qquad d\ln s^K = (\lambda - 1) \cdot \left(d\ln A_k + d\ln \Gamma_k^{deep}\right) + \frac{1}{s^K} \cdot \sum_{g \in \mathcal{G}} s_g^L \cdot d\ln \Gamma_g^{disp} \cdot \left(1 + (\lambda - 1) \cdot \pi_g\right),$$

$$(7) \qquad d\ln y = \frac{1}{1 - s^K} \cdot \left(d\ln TFP + s^K \cdot d\ln s^K\right).$$

- The impact of factor-augmenting technologies on TFP can be computed from (5) as $\sum_{g \in \mathcal{G}} s_g^L \cdot d \ln A_g + s^K \cdot d \ln A_k$.

- This formula, which follows from Hulten's theorem, has a simple envelope logic: a 1% increase in the productivity of all workers in group g leads to an increase in TFP of s_g^L%, where s_g^L is the share of skilled labor in GDP.

- Likewise, a 1% increase in the productivity of capital at all tasks leads to an increase in TFP of s^K%.

- Thus, relative to their modest effects on the wage structure (especially for values of λ close to 1), factor-augmenting technologies have large productivity effects.

- These results contrast with the effects of automation, which displaces some workers from the tasks they are performing, and whose effects are captured by the term $d \ln \Gamma_g^{\text{disp}}$ in the proposition.

- The impact of this type of technology on wages in (4) becomes $\frac{1}{\lambda} d \ln y - \frac{1}{\lambda} d \ln \Gamma_g^{\text{disp}}$.

- Equation (6) also shows that task displacement always results in an increase in the capital share and a reduction in the labor share of value added—an observation that will motivate our measurement approach in Section 2.3.

- This is also in stark contrast to what one would get from factor-augmenting technologies, whose impact on factor shares depends on whether $\lambda \lessgtr 1$(with no ripple effects, is also the elasticity of substitution between capital and labor).

2.2 Full Model: Multiple Sectors

- Our full model generalizes the one-sector setup in the previous subsection.
- There are multiple industries indexed by $i \in \mathcal{I} = \{1,2, \dots, I\}$.
- Output in industry i is produced by combining the tasks in some set $\mathcal{T}_i$, with measure M_i, using a CES aggregator with elasticity $\lambda \geq 0$:

$$y_i = A_i \cdot \left(\frac{1}{M_i} \int_{\mathcal{T}_i} \left(M_i \cdot y(x) \right)^{\frac{\lambda - 1}{\lambda}} \cdot dx \right)^{\frac{\lambda}{\lambda - 1}},$$

- where x again indexes tasks and A_i is a Hicks-neutral industry productivity term.
- As before, tasks, $\mathcal{T}_{gi}$ denotes the set of tasks in industry i allocated to workers of type g and $\mathcal{T}_{ki}$ denotes those allocated to capital.

- Likewise, we define industry-level task shares, Γ_{gi} and Γ_{ki}, as:

$$\Gamma_{gi}(w, \Psi) = \frac{1}{M_i} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} \cdot dx; \Gamma_{ki}(w, \Psi) = \frac{1}{M_i} \int_{\mathcal{T}_{ki}} \left(\psi_k(x) \cdot q(x)\right)^{\lambda-1} \cdot dx.$$

- We assume that industry outputs are combined into a single final good using a constant returns to scale aggregator.

- Rather than specifying this aggregator, we work with the implied expenditure shares, $s_i^Y(\boldsymbol{p})$, where $\boldsymbol{p} = (p_1, p_2, \dots, p_I)$ is the vector of industry prices.

- The next proposition generalizes Proposition 1 to this environment and characterizes the equilibrium in terms of task shares.

- ***Proposition 3 (Equilibrium in multi-sector economy)*** *There is a unique equilibrium. In this equilibrium, output, wages, and industry prices can be expressed as functions of task shares defined implicitly by the solution to the system of equations:*

$$w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p}) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{gi}\right)^{\frac{1}{\lambda}} \tag{8}$$

$$p_i = \frac{1}{A_i}\left(A_k^{\lambda-1} \cdot \Gamma_{ki} + \sum_{g\in\mathcal{G}} w_g^{1-\lambda} \cdot A_g^{\lambda-1} \cdot \Gamma_{gi}\right)^{\frac{1}{1-\lambda}} \tag{9}$$

$$1 = \sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p}). \tag{10}$$

2.3 Mapping the Model to Data

- In this subsection, we use Proposition 3 to derive an equation that links the change in wages to the direct effects of task displacement (and other technologies), extending (4) to this environment.

- This equation will form the basis of our reduced-form analysis.

- We will then use our model to derive a measure of task displacement that captures its direct effects across groups of workers.

- **Task displacement and wage structure**: As before, denote the effects productivity deepening and automation on task shares in industry i by $d \ln \Gamma_{gi}^{\text{deep}}$ and $d \ln \Gamma_{gi}^{\text{disp}}$, respectively.

- Differentiating Equation (8) and using Assumption 1, we obtain a generalization of (4):

$$(11) \qquad d\ln w_g = \frac{1}{\lambda} d\ln y + \frac{\lambda - 1}{\lambda}\left(d\ln A_g + \sum_{i\in\mathcal{I}} \omega_g^i \cdot d\ln \Gamma_{gi}^{\text{deep}}\right) + \frac{1}{\lambda}\sum_{i\in\mathcal{I}} \omega_g^i \cdot \left(d\ln s_i^Y + (1-\lambda)(d\ln p_i + d\ln A_i)\right) - \frac{1}{\lambda}\sum_{i\in\mathcal{I}} \omega_g^i \cdot d\ln \Gamma_{gi}^{\text{disp}},$$

  where ω_g^i denotes the share of group g's wage income earned in industry i, so that $\sum_{i\in\mathcal{I}} \omega_g^i = 1$.

- Equation (11) shows that wages depend on four terms.

1. ***The common expansion of output***: $d \ln y$, which captures the productivity effect.

   - In our reduced-form regressions, this effect will be absorbed by the constant term.

2. ***Group-specific shifters***: $\frac{\lambda-1}{\lambda}\left(d\ln A_g + \sum_{i\in\mathcal{J}} \omega_g^i \cdot d\ln \Gamma_{gi}^{\text{deep}}\right)$, which represent the contribution of factor-augmenting technologies and productivity deepening.

- Following the SBTC literature, in our reduced-form regressions we will assume that these technologies augment certain well-defined skills associated with education and also allow them to be gender-biased.

- In particular, we parameterize these as:

$$\frac{\lambda-1}{\lambda}\left(d\ln A_g + \sum_{i\in\mathcal{J}} \omega_g^i \cdot d\ln \Gamma_{gi}^{\text{deep}}\right) = \alpha_{\text{edu}(g)} + \gamma_{\text{gender}(g)} + v_g,$$

  where v_g is an additional unobserved component, and $\alpha_{\text{edu}(g)}$ and $\gamma_{\text{gender}(g)}$ will be absorbed by dummies for education levels and gender.

- As a further refinement, we allow group-specific shifters to also depend on baseline group wages, which may proxy for skills as well.

3. Industry shifters:

$$\text{Industry shifter}_g = \frac{1}{\lambda} \sum_{i \in \mathcal{I}} \omega_g^i \cdot (d \ln s_i^Y + (1-\lambda)(d \ln p_i + d \ln A_i)),$$

which capture the effects coming from the expansion or contraction of industries in which a demographic group specializes (for example, due to trade in final goods, structural transformation, or the uneven effects of automation in some sectors).

- In our reduced-form regressions, we control for this term by including the exposure of a group to the change in value added of the sectors in which it specializes.

4. *Task displacement*:

$$\text{Task displacement}_g = \sum_{i \in \mathcal{J}} \omega_g^i \cdot d \ln \Gamma_{gi}^{\text{disp}}.$$

- This term represents the direct effect of task displacement on a demographic group's wages and will be the focus of our empirical work.
- As equation (11) shows, the key prediction of our model is that groups exposed to task displacement should experience a decline in their relative wages.
- Unlike other technologies, this effect is always negative—independently of whether the elasticity of substitution λ is above or below 1.
- Task displacement could come from automation or offshoring, and we will later study their contribution to this process.

- **Measuring task displacement**: We now turn to measuring task displacement.
- Our measure summarizes the direct effects of task-displacing technologies on different groups of workers, and will form the basis of our reduced-form regression analysis and quantitative evaluation.
- We use two complementary strategies to measure task displacement, both of which rely on an initial observation: task displacement takes place mainly in tasks that can be automated, which we initially proxy with routine tasks.
- Formally, we impose the following assumption:
- ***Assumption 2*** *Only routine tasks are automated and, within an industry, different groups of workers are displaced from their routine tasks at a common rate.*

- The next component of our measurement requires a proxy for the extent of task displacement taking place in each industry.

- Our two strategies take different approaches to this problem.

- Our first strategy develops a more comprehensive measure based on the idea that task displacement is tightly linked to declines in industry labor shares, and uses the “unexplained” portion of the change in labor share to infer task displacement at the industry level.

- Specifically, and as we show in Appendix B-3, when $\lambda = 1$ (so that the task aggregator is Cobb-Douglas) and there are no changes in industry markups, we have:

$$\text{(12)} \qquad \text{Task displacement}_g = \sum_{i \in \mathcal{I}} \omega_g^i \cdot (\omega_{gi}^R / \omega_i^R) \cdot (-d \ln s_i^L).$$

- This measure comprises three terms: (1) a group's exposure to different industries, ω_g^i, which is given by the share of wages earned by workers of group g in industry i; (2) the percent decline in the labor share, $-d \ln s_i^L$, which in our framework is tightly linked to automation in industry i; (3) $\omega_{gi}^R / \omega_i^R$, which captures the relative specialization of group g in industry i's routine jobs, where displacement takes place.

- The measure of task displacement in Equation (12) is precisely the one used in the right panel of Figure 2, while the left panel focuses on exposure to industries with declining labor shares and ignores the relative specialization of workers in routine jobs.

- Our second strategy uses direct measures of automation technologies (and offshoring):

(13) $$\text{Task displacement due to automation}_g = \sum_{i \in \mathcal{I}} \omega_g^i \cdot (\omega_{gi}^R / \omega_i^R) \cdot \text{automation in industry}_i.$$

- Although these measures can be included directly on the right-hand side of our wage regressions, we focus on specifications where they are used as instruments for the measure of task displacement based on labor share declines, which enables us to compare coefficient estimates across specifications.

3. Data, Measurement, and Descriptive Patterns

3.1 Main Data Sources

- We use data from the BEA Integrated Industry-Level Production Accounts on industry labor shares, factor prices, and value added for 49 industries that can be tracked consistently from 1987 to 2016.

- We complement these industry data with proxies of the adoption of automation technologies, including BLS data on the change in the share of specialized machinery and software in value added from 1987 to 2016, and measures of robot adoption by industry from the International Federation of Robotics (IFR).

- On the worker side, we use US Census and American Community Survey (ACS) data to trace the labor market outcomes of 500 demographic groups defined by gender, education (less than high school, high school graduate, some college, college degree, and post-graduate degree), age (proxied by 10-year age bins, from 16-25 years to 56-65), race/ethnicity (White, Black, Asian, Hispanic, Other), and native vs. foreign-born.

3.2 Task Displacement and Changes in the Labor Share across Industries

- Figure 4 reveals considerable variation in task displacement across industries, with the largest levels of task displacement seen in mining, chemical products, petroleum, car manufacturing, and computers and electronics.

- The figure also plots our index of automation, which points to an important role of technology in generating task displacement and declining labor shares across industries.

- This is further confirmed in Figure 5, where we see a significant positive association between three measures of automation and industry task displacement.

Figure 4: Task displacement 1987-2016 and index of automation

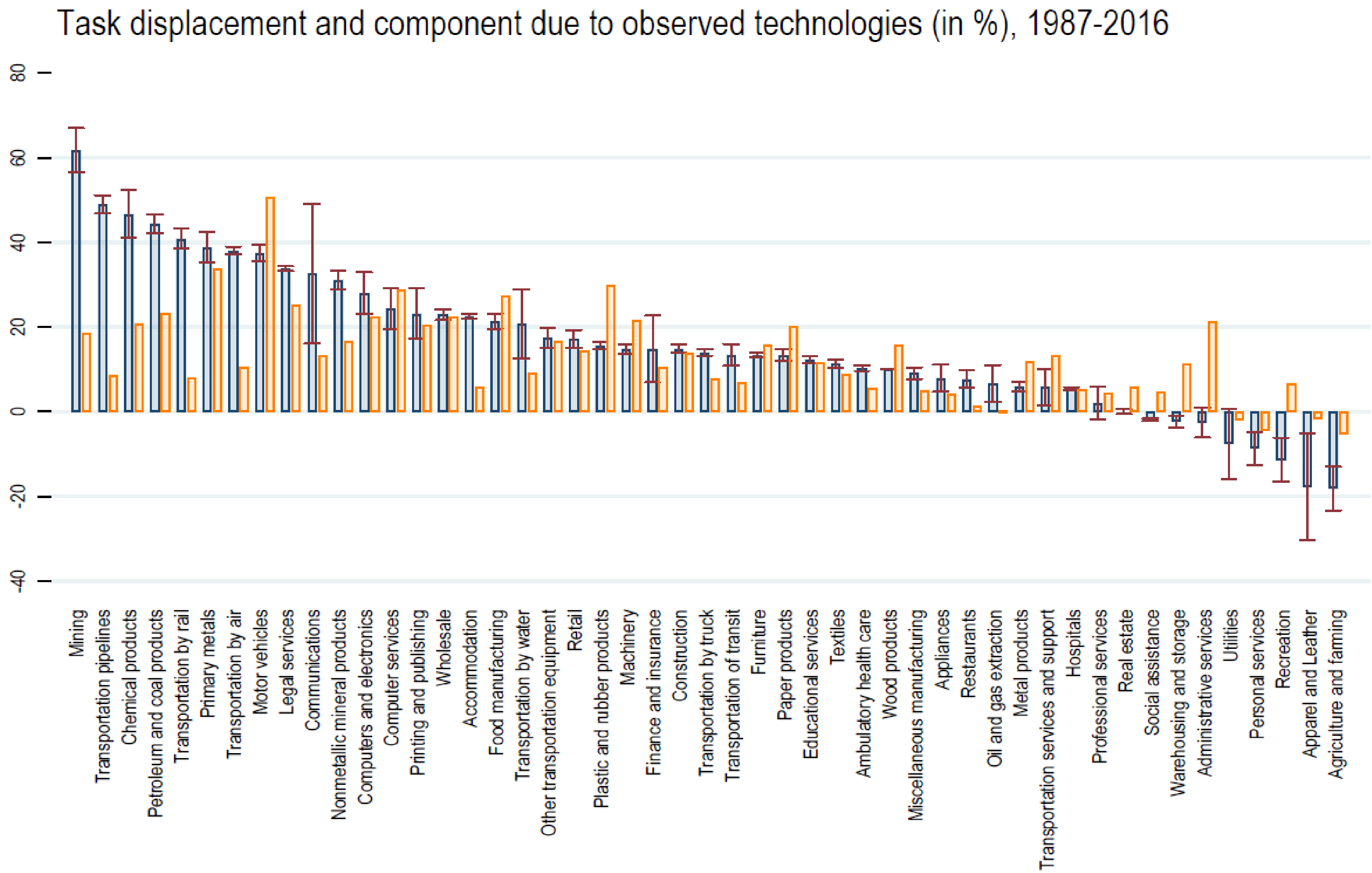
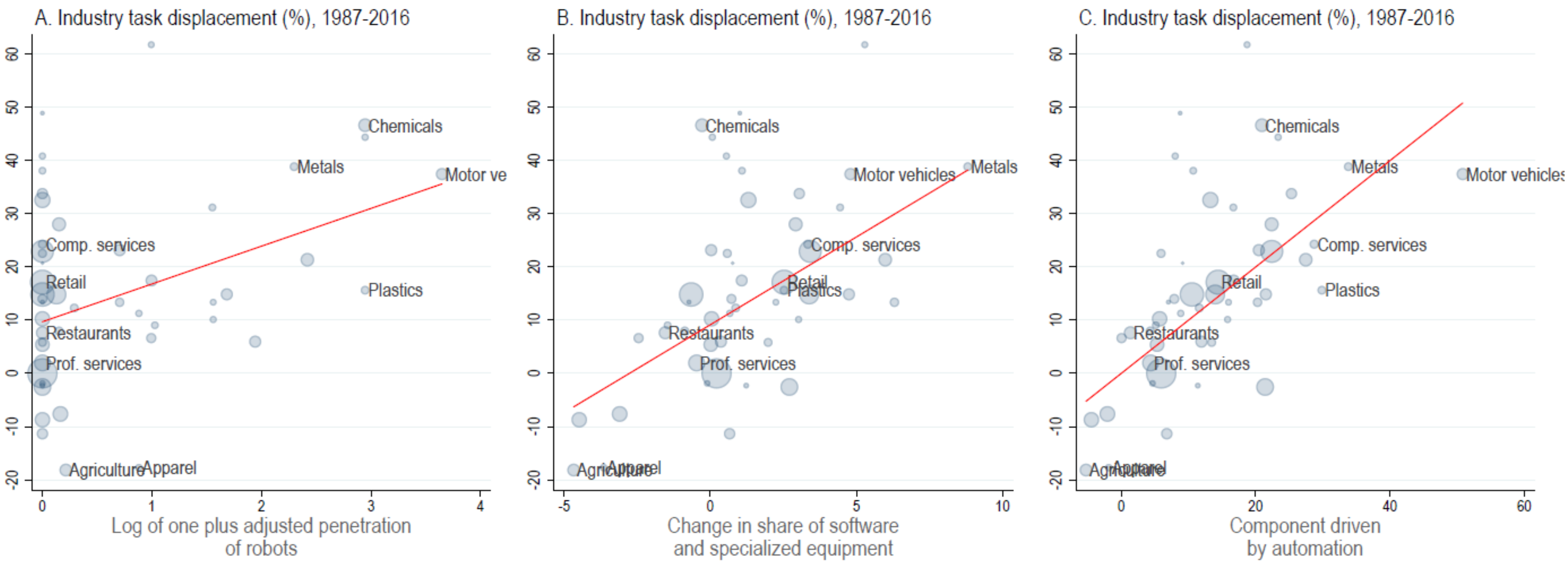
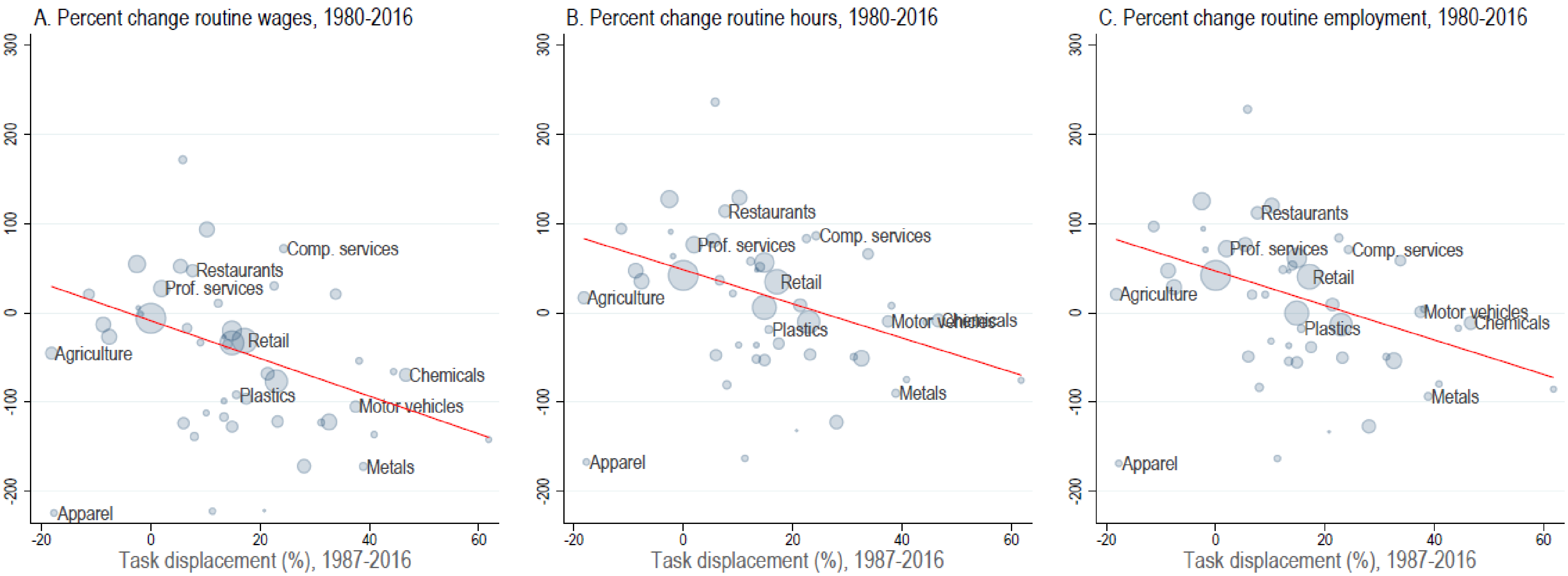


Figure 5: Relationship between automation technologies and task displacement across industries



- The industry-level variation in addition provides support for Assumption 2.
- In particular, Figure 6 depicts a strong negative association between task displacement and reductions in the demand for routine tasks across industries (measured in one of three ways: total wages in routine jobs, total hours in routine jobs, or total number of workers in routine jobs).
- With all three measures, there is a significant decline in routine jobs in industries experiencing task displacement.

Figure 6: Relationship between task displacement and the decline of routine jobs across industries



3.3 Task Displacement and Wages across Demographic Groups

- We now present descriptive statistics for our measure of task displacement at the level of demographic groups and take a preliminary look at its association with real wage changes.

- Figure 7 shows large differences across demographic groups in terms of task displacement between 1980 and 2016, with some experiencing a 25% reduction in their task shares, while others saw no change in theirs.

- Importantly, 95% of the variation in task displacement across groups is driven by our index of automation technologies, as can be seen from the left panel of Figure 7, which depicts task displacement by demographic group (computed using Equation (12)) against the component driven by automation technologies (computed using Equation (13)).

Figure 7: Task displacement across 500 demographic groups

- Figure 8 provides a first glimpse at the relationship between task displacement and real wage changes across demographic groups.

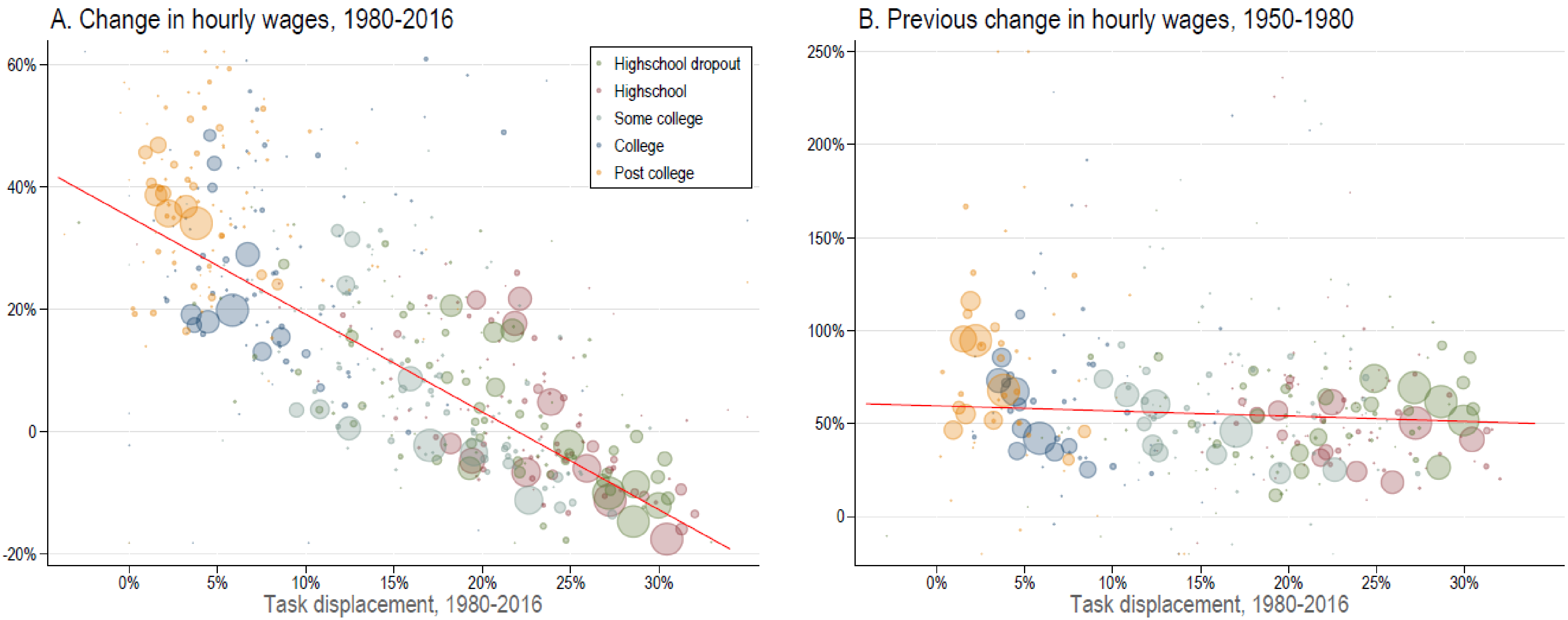
- The left panel plots the bivariate correlation between our task displacement measure and real wage changes between 1980 and 2016 (as in the right panel of Figure 2).

- The figure reveals a powerful negative relationship between task displacement and changes in real wages, with groups experiencing the highest levels of task displacement seeing their real wages fall or stagnate.

- The right panel displays a falsification exercise demonstrating that the relationship depicted in the left panel is not driven by secular declines in labor market fortunes of some demographic groups.

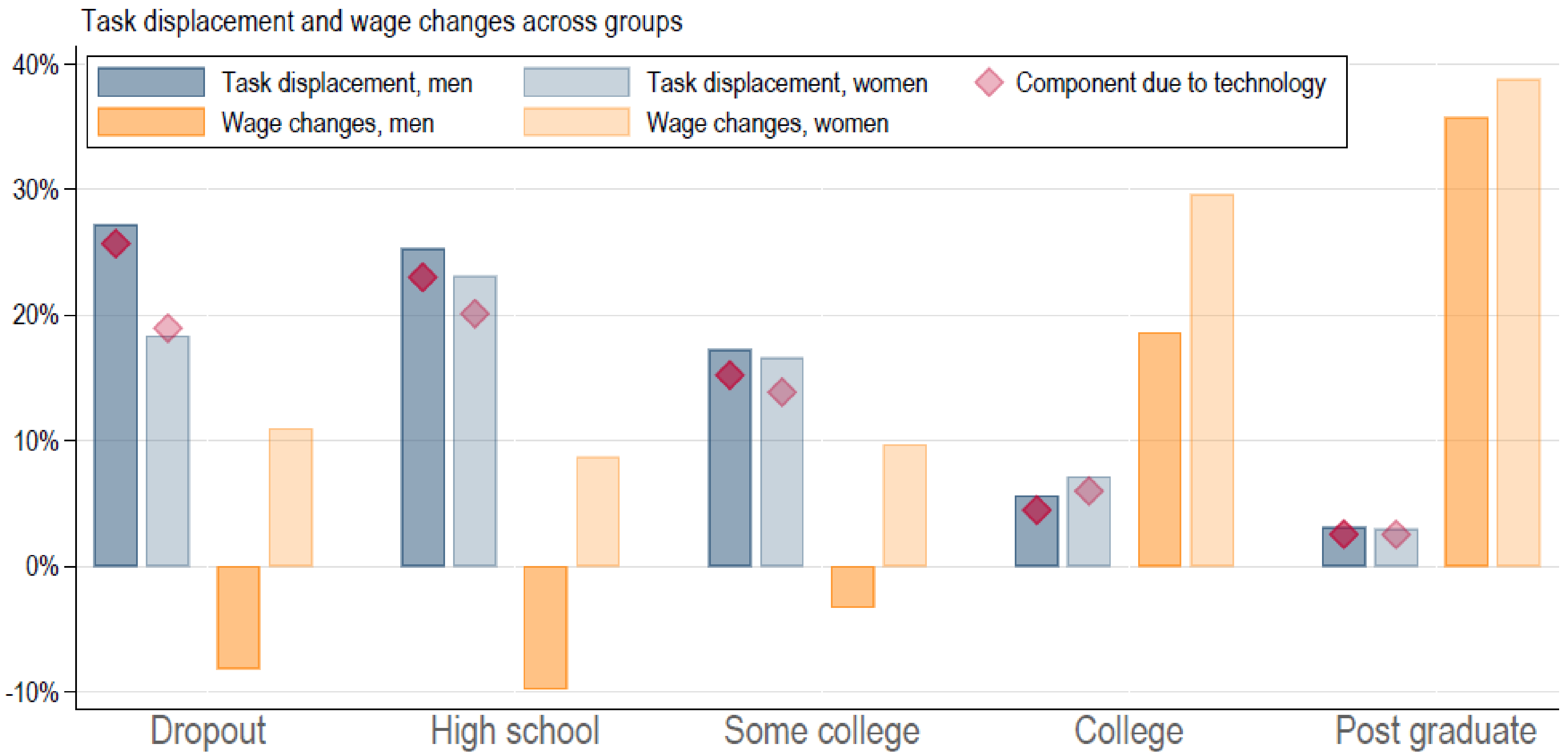
- All demographic groups, including those who later on experienced adverse task displacement after 1980, enjoyed robust real wage growth, of about 50%, between 1950 and 1980.

Figure 8: Reduced-form relationship between task displacement and real wage changes



- Figure 8—like Figure 7—identifies different education levels, highlighting that task displacement has been much higher for workers without a college degree.
- Still, the negative association between change in wages and task displacement is visible within education groups.
- Relatedly, Figure 9 displays average task displacement and average real wage change by gender and education.
- It reveals that men without college degree have experienced the highest levels of task displacement as well as substantial real wage declines, while men and women with a post-graduate degree and women with a college degree have been subject to negligible task displacement and have enjoyed robust real wage growth.
- Once again, these patterns are explained by the component of task displacement driven by automation technologies.

Figure 9: Task displacement, component of task displacement, driven by automation, and real wage changes by education level and gender



4. Reduced-Form Evidence of the Effects of Task Displacement

4.1 Baseline OLS Results

- Table 1 presents estimates from an empirical analogue of Equation (11):

(14) $$\Delta \ln w_g = \beta^d \cdot \text{Task displacement}_g + \beta^s \cdot \text{Industry shifter}_g + \alpha_{\text{edu}(g)} + \gamma_{\text{gender}(g)} + v_g.$$

- Here g indexes our 500 demographic groups, and $\Delta \ln w_g$ denotes the log change in real hourly wages for workers in group g between 1980 and 2016.

- The error term v_g represents residual group-specific changes in supply or demand, which are assumed to be orthogonal to task displacement.

- As in all of our other results, regressions are weighted by the share of hours worked by each group and standard errors are robust to heteroskedasticity.

- Column 1 presents a bivariate regression identical to the one shown in Figure 8.

Table 1: Task Displacement and Changes in Real Hourly Wages, 1980-2016

TABLE 1: TASK DISPLACEMENT AND CHANGES IN REAL HOURLY WAGES, 1980-2016.

	DEPENDENT VARIABLES: CHANGE IN WAGES AND WAGE DECLINES, 1980–2016			
	(1)	(2)	(3)	(4)
Task displacement	-1.598	-1.323	-1.307	-1.659
	(0.094)	(0.158)	(0.188)	(0.444)
Industry shifters		0.210	0.310	0.347
		(0.091)	(0.120)	(0.158)
Exposure to industry labor share decline				0.178
				(0.664)
Relative specialization in routine jobs				0.072
				(0.073)
Share variance explained by task displacement	0.67	0.55	0.55	0.70
R-squared	0.67	0.70	0.84	0.84
Observations	500	500	500	500
Other covariates:				
Manufacturing share, and education and gender dummies			✓	✓

Notes: This table presents estimates of the relationship between task displacement and the change in real wages across 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is the change in real wages for each group from 1980 to 2016. Besides the covariates reported in the table, columns 3 and 4 control for baseline wage shares in manufacturing and dummies for education (for no high school degree, some college, college degree and postgraduate degree) and gender. All regressions are weighted by the share of hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

4.2 Baseline IV Results

- We now exploit information on measures of automation and offshoring to instrument for task displacement (constructed from industry labor share declines).

- This strategy thus focuses on the component of task displacement driven by automation technologies.

- Table 2 presents our findings.

- Panel A shows the reduced-form relationship between real wage changes and the automation measure in Equation (13), focusing on our baseline specification from column 3 in Table 1.

Table 2: Estimates Instrumenting Task Displacement with Automation and Offshoring Measures

Instrument:	Robot APR (1)	Specialized machinery (2)	Software (3)	Robot APR and software (4)	Machinery and software (5)	Combined measure (6)	Offshoring (7)
	Dependent variable: Change in wages 1980–2016						
	Panel A. Reduced-form estimates						
Routine at industries adopting robots	-2.014			-2.029			
	(0.451)			(0.439)			
Routine at industries adopting specialized machinery		-0.969			-1.352		
		(0.430)			(0.315)		
Routine at industries adopting software			-4.076	-4.097	-4.645		
			(0.823)	(0.880)	(0.918)		
Routine at automating industries						-1.334	
						(0.210)	
Routine at offshoring industries							-2.243
							(1.009)
Share variance explained by task displacement	0.30	0.16	0.17	0.47	0.42	0.52	0.09
R-squared	0.79	0.77	0.80	0.83	0.82	0.83	0.76
Observations	500	500	500	500	500	500	500
	Panel B. IV estimates						
Task displacement	-1.216	-0.894	-1.480	-1.345	-1.216	-1.279	-0.813
	(0.246)	(0.317)	(0.357)	(0.214)	(0.184)	(0.193)	(0.299)
Share variance explained by task displacement	0.51	0.38	0.62	0.56	0.51	0.54	0.34
R-squared	0.84	0.83	0.83	0.84	0.84	0.84	0.82
First-stage F	98.00	44.98	67.40	439.92	831.72	785.80	30.62
Overid p-value				0.56	0.31		
Observations	500	500	500	500	500	500	500
	Panel C. Role of industry and occupation in driving results						
Task displacement	-1.265	-0.186	-2.951	-2.075	-1.425	-1.677	-2.494
	(0.830)	(0.934)	(1.003)	(0.534)	(0.462)	(0.466)	(0.714)
Exposure to industry labor share decline	0.302	-1.656	-0.857	0.103	0.362	0.310	0.063
	(0.913)	(1.558)	(1.346)	(0.792)	(0.743)	(0.686)	(0.959)
Relative specialization in routine jobs	0.011	-0.165	0.269	0.137	0.037	0.075	0.201
	(0.143)	(0.161)	(0.147)	(0.088)	(0.080)	(0.081)	(0.110)
R-squared	0.83	0.82	0.79	0.83	0.84	0.84	0.83
First-stage F	6.32	33.72	5.46	32.80	229.90	156.33	23.71
Observations	500	500	500	500	500	500	500

4.3 Task Displacement versus SBTC

- How important is task displacement relative to other forms of SBTC?
- Table 3 explores this question by considering different specifications of SBTC.
- In summary, these results suggest that task displacement has been at the root of the changes in the wage structure from 1980 to today, while other forms of SBTC have limited explanatory power.

Table 3: Task Displacement vs. SBTC, 1980-2016

	Dependent variable: Change in real wages 1980–2016					
	SBTC by education level			Allowing for SBTC by wage level		
	OLS (1)	OLS (2)	IV (3)	OLS (4)	OLS (5)	IV (6)
Gender: women	0.173	0.104	0.105	0.245	0.154	0.165
	(0.019)	(0.020)	(0.020)	(0.024)	(0.026)	(0.027)
Education: no high school	0.016	0.023	0.023	0.051	0.039	0.040
	(0.024)	(0.020)	(0.019)	(0.023)	(0.018)	(0.018)
Education: some college	0.053	-0.070	-0.068	0.027	-0.057	-0.046
	(0.031)	(0.032)	(0.033)	(0.024)	(0.031)	(0.031)
Education: full college	0.245	-0.019	-0.013	0.180	0.005	0.027
	(0.039)	(0.050)	(0.052)	(0.036)	(0.049)	(0.050)
Education: more than college	0.416	0.083	0.090	0.292	0.093	0.118
	(0.046)	(0.062)	(0.064)	(0.048)	(0.061)	(0.061)
Log of hourly wage in 1980				0.235	0.115	0.130
				(0.046)	(0.043)	(0.046)
Task displacement		-1.307	-1.279		-1.028	-0.902
		(0.188)	(0.193)		(0.185)	(0.194)
Share variance explained by:						
- educational dummies	0.55	0.08	0.09	0.37	0.09	0.12
- baseline wage				0.15	0.07	0.08
- task displacement		0.55	0.54		0.43	0.38
R-squared	0.76	0.84	0.84	0.81	0.85	0.84
First-stage F			785.80			562.20
Observations	500	500	500	500	500	500
Other covariates:						
Industry shifters and manufacturing share	✓	✓	✓	✓	✓	✓

4.4 Employment Outcomes

- If task displacement leads to lower labor demand for a demographic group, we could see an impact not just on its wage but on its employment as well.
- Table 4 provides results for the 1980-2016 period, focusing on the employment to population ratios in the top panel and non-participation rate in the bottom panel.
- We find that task displacement is associated with lower employment to population ratios, both in OLS (columns 1-3) and IV specifications using our index of automation as instrument (columns 4-6).
- Overall, our task displacement measure explains between 15% and 36% of the variation in employment and participation changes over this time period.

Table 4: Task Displacement and Employment Outcomes, 1980-2016

	Dependent variable: Labor market outcomes 1980–2016					
	OLS Estimates			IV Estimates		
	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A. Employment to population ratio					
Task displacement	-0.676	-0.465	-0.785	-0.720	-0.422	-0.729
	(0.112)	(0.141)	(0.317)	(0.112)	(0.149)	(0.366)
Share variance explained by:						
- task displacement	0.31	0.21	0.36	0.33	0.19	0.34
- educational dummies		0.10	0.12		0.12	0.12
R-squared	0.31	0.77	0.78	0.31	0.77	0.78
First-stage F				3246.45	785.80	156.33
Observations	500	500	500	500	500	500
	Panel B. Non-participation rate					
Task displacement	0.668	0.374	0.772	0.718	0.337	0.747
	(0.120)	(0.138)	(0.312)	(0.120)	(0.149)	(0.361)
Share variance explained by:						
- task displacement	0.30	0.17	0.34	0.32	0.15	0.33
- educational dummies		0.16	0.19		0.18	0.19
R-squared	0.30	0.80	0.81	0.30	0.80	0.81
First-stage F				3246.45	785.80	156.33
Observations	500	500	500	500	500	500
Covariates:						
Industry shifters, manufacturing share, education and gender dummies		✓	✓		✓	✓
Exposure to labor share declines and relative specialization in routine jobs			✓			✓

4.5 Confounding Trends: Imports, Deunionization and Other Capital

- In this and the next subsection, we control for other changes affecting the US labor market and contrast their effects with those of task displacement.
- See Table 5.
- In all of these specifications, there is no evidence of a sizable role for these other forces, and task displacement's effects continue to be precisely-estimated and similar to our baseline results.

Table 5: Task Displacement and Changes in Real Hourly Wages—Controlling for Other Trends, 1980-2016

	Dependent variable: Change in wages 1980–2016							
	OLS estimates				IV estimates			
	Chinese imports' competition	Decline in unionization rates	Rising K/L ratio by industry	Rising TFP by industry	Chinese imports' competition	Decline in unionization rates	Rising K/L ratio by industry	Rising TFP by industry
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Controlling for main effect of other shocks								
Task displacement	-1.259	-1.308	-1.306	-1.314	-1.235	-1.277	-1.274	-1.285
	(0.203)	(0.219)	(0.189)	(0.187)	(0.203)	(0.210)	(0.193)	(0.189)
Effect of other shocks by industry	0.012	0.017	0.014	-0.042	0.012	-0.032	0.015	-0.031
	(0.013)	(0.841)	(0.078)	(0.371)	(0.012)	(0.821)	(0.078)	(0.367)
Share variance explained by task displacement	0.53	0.55	0.55	0.55	0.52	0.54	0.53	0.54
R-squared	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84
First-stage F					851.32	1126.35	883.48	952.66
Observations	500	500	500	500	500	500	500	500
Panel B. Controlling for effects on workers in routine jobs								
Task displacement	-1.312	-1.629	-1.128	-1.299	-1.285	-1.580	-1.055	-1.267
	(0.184)	(0.451)	(0.221)	(0.199)	(0.185)	(0.523)	(0.264)	(0.208)
Effect of other shocks on routine jobs	0.001	0.678	-0.049	-0.035	0.001	0.601	-0.059	-0.044
	(0.006)	(0.806)	(0.054)	(0.196)	(0.006)	(0.899)	(0.059)	(0.197)
Share variance explained by task displacement	0.55	0.68	0.47	0.54	0.54	0.66	0.44	0.53
R-squared	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84
First-stage F					929.19	224.85	362.42	686.25
Observations	500	500	500	500	500	500	500	500

4.6 Confounding Trends: Concentration and Markups

- We next explore the role of rising sales concentration and markups, which impact industry labor shares and might directly affect the wage structure.
- Table 6 provides our findings.
- The findings in this table suggest that our task displacement variable is not picking up confounding effects of rising markups or concentration.
- Overall, it appears that it is task displacement rather than rising market power that has played a defining role in the surge in US wage inequality over the last four decades.

Table 6: Task Displacement and Changes in Real Hourly Wages—Controlling for Changes in Markups and Industry Concentration, 1980-2016

	Dependent variable: Change in wages 1980–2016							
	OLS estimates				IV estimates			
	Rising sales concentration	Markups from accounting approach	Markups from materials share	Markups from DLEU (2020)	Rising sales concentration	Markups from accounting approach	Markups from materials share	Markups from DLEU (2020)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Controlling for main effect of markups and concentration								
Task displacement	-1.368	-1.315	-1.417	-1.314	-1.339	-1.283	-1.365	-1.286
	(0.178)	(0.204)	(0.204)	(0.183)	(0.186)	(0.204)	(0.205)	(0.187)
Exposure to rising markups or concentration	1.874	0.261	-0.767	-0.670	1.835	0.211	-0.721	-0.663
	(1.429)	(1.442)	(0.425)	(1.005)	(1.433)	(1.419)	(0.417)	(1.000)
Share variance explained by:								
- task displacement	0.57	0.55	0.59	0.55	0.56	0.54	0.57	0.54
- markups/concetration	0.04	-0.00	-0.07	0.01	0.04	-0.00	-0.07	0.01
R-squared	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84
First-stage F					747	798	704	784
Observations	500	500	500	500	500	500	500	500
Panel B. Net out markups from construction of task displacement								
Task displacement	-1.738	-1.712	-1.122	-1.323	-1.809	-1.759	-1.090	-1.177
	(0.223)	(0.238)	(0.149)	(0.161)	(0.257)	(0.277)	(0.164)	(0.151)
Exposure to rising markups or concentration	0.694	-0.684	-2.089	-2.127	0.721	-0.654	-2.016	-1.930
	(1.503)	(1.397)	(0.528)	(0.748)	(1.482)	(1.377)	(0.535)	(0.794)
Share variance explained by:								
- task displacement	0.57	0.56	0.54	0.50	0.60	0.58	0.53	0.44
- markups/concetration	0.02	0.01	-0.19	0.03	0.02	0.01	-0.19	0.03
R-squared	0.83	0.83	0.85	0.86	0.83	0.83	0.85	0.86
First-stage F					471	405	301	197
Observations	500	500	500	500	500	500	500	500

4.7 Regional Variation

- Task and industry composition vary greatly across regions and commuting zones in the US.
- To further test the association between task displacement and wages, we now investigate whether regional variation in task displacement also predicts changes in sub-national wage structures.
- Table 7 provides estimates that exploit regional differences in specialization patterns.
- The main difference is that now the unit of observation is given by group-region cells, and we exploit differences in specialization across these cells to construct our task displacement measures.

Table 7: Task Displacement and Changes in Real Hourly Wages, 1980-2016: Regional Variation

	Dependent variable: Change in real wages 1980–2016					
	OLS estimates			IV estimates		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Variation across US regions						
Task displacement	-1.601	-1.070	-1.307	-1.650	-1.134	-1.497
	(0.111)	(0.118)	(0.252)	(0.107)	(0.121)	(0.285)
R-squared	0.62	0.81	0.82	0.62	0.81	0.81
First-stage F				1548.83	893.34	146.35
Observations	2633	2633	2633	2633	2633	2633
Panel B. Variation across US regions absorbing national trends by group						
Task displacement	-1.296	-0.263	-0.373	-1.714	-0.412	-0.601
	(0.100)	(0.082)	(0.119)	(0.097)	(0.112)	(0.171)
R-squared	0.88	0.95	0.95	0.26	0.70	0.71
First-stage F				293.03	546.96	150.16
Observations	2633	2633	2633	2633	2633	2633
Panel C. Variation across commuting zones						
Task displacement	-1.234	-0.943	-1.119	-1.385	-1.225	-1.472
	(0.146)	(0.140)	(0.221)	(0.189)	(0.180)	(0.286)
R-squared	0.36	0.56	0.56	0.35	0.55	0.54
First-stage F				558.65	487.63	92.12
Observations	20768	20768	20768	20768	20768	20768
Panel D. Variation across commuting zones absorbing national trends by group						
Task displacement	-0.767	-0.418	-0.414	-1.169	-0.522	-0.567
	(0.070)	(0.065)	(0.147)	(0.097)	(0.061)	(0.188)
R-squared	0.71	0.78	0.79	0.10	0.36	0.39
First-stage F				694.53	137.33	69.31
Observations	20768	20768	20768	20768	20768	20768
Covariates:						
Industry shifters, manufacturing share, education and gender dummies		✓	✓		✓	✓
Exposure to labor share declines and relative specialization in routine jobs			✓			✓

4.8 Further Robustness Checks

- The Appendix provides a number of additional checks, all of which support our main conclusions:

- First, in Table A-8, we provide estimates of the effects of task displacement excluding immigrants, as well as separate estimates for men and women.

- Second, in Table B-3, we checked our results for 1980-2007, thus avoiding any persistent effects of the Great Recession, with similar results.

- Third, in Table A-9 we present stacked-differences models with two periods, 1980-2000 and 2000-2016, which explore the differential patterns of task displacement between these sub-periods.

- Fourth, Table B-4 verifies that our results are similar when we compute the task displacement measure using different values of the elasticity of substitution to account for changes in factor prices using the formulas in footnote 15.

- Table B-5 confirms that the results are robust to using labor share data from the BLS, excluding extractive industries, winsorizing the labor share changes, or focusing only on industries with a declining labor share to construct our measure of task displacement (rather than our baseline measure which exploited both declines and increases in industry labor shares).

- Table A-10 reports similar results when we utilize several alternative measures of which jobs can be automated.

- Most importantly, the results are similar when we rely on the measure of automatable jobs from Webb (2020).

5. General Equilibrium Effects and Quantitative Analysis

- Our reduced-form evidence documented a strong negative relationship between task displacement and relative wage changes.
- This evidence misses three general equilibrium effects, however.
- First, in our regressions the common effect of productivity on real wages is included in the intercept, making our estimates uninformative about wage *level* changes.
- Second, our regression estimates focus on the direct effects of task displacement and do not account for the resulting ripple effects, which also impact the wage structure.
- Third, although our regressions control for *observed* industry changes, they do not separate out industry shifts *induced* by task displacement, thus missing one component of the total effect of automation and offshoring.
- In this section, we develop our full general equilibrium model, which enables us to quantify these mechanisms.

5.1 General Equilibrium Effects and the Propagation Matrix

- We first generalize Proposition 2 to the case in which Assumption 1 is relaxed and there are ripple effects.

- For this purpose, let us define *aggregate task shares* as

$$\Gamma_g(w, \zeta, \Psi) = \sum_{i \in \mathcal{I}} \underbrace{s_i^Y(p,c) \cdot (A_i \cdot p_i)^{\lambda-1}}_{=\zeta_i} \cdot \underbrace{\frac{1}{M_i} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} \cdot dx,}_{=\Gamma_{gi}}$$

$$\Gamma_k(w, \zeta, \Psi) = \sum_{i \in \mathcal{I}} \underbrace{s_i^Y(p,c) \cdot (A_i \cdot p_i)^{\lambda-1}}_{=\zeta_i} \cdot \underbrace{\frac{1}{M_i} \int_{\mathcal{T}_{ki}} \psi_k(x)^{\lambda-1} \cdot dx,}_{=\Gamma_{ki}}$$

which are given by a weighted sum of industry-specific task shares, Γ_{gi} (or Γ_{ki}), and also depend on industry shifters, $\zeta = (\zeta_1, \dots, \zeta_I)$.

- To characterize ripple effects, consider any technological change with a direct effect of z_g on the real wage of group g, and denote by z the column vector of z_g's.

- Differentiating (2), we obtain:

$$d \ln w = z + \frac{1}{\lambda}\frac{\partial \ln \Gamma(w, \zeta, \Psi)}{\partial \ln w} \cdot d \ln w \Rightarrow d \ln w = \underbrace{\left(1 - \frac{1}{\lambda}\frac{\partial \ln \Gamma(w, \zeta, \Psi)}{\partial \ln w}\right)^{-1}}_{\Theta} \cdot z,$$

where $\partial \ln \Gamma(w, \zeta, \Psi)/\partial \ln w$ is the $G \times G$ Jacobian of the function $\ln \Gamma(w, \zeta, \Psi) = (\ln \Gamma_1(w, \zeta, \Psi), \ln \Gamma_2(w, \zeta, \Psi), \dots, \ln \Gamma_G(w, \zeta, \Psi))$ with respect to the vector of wages w.

- We refer to the $G \times G$ matrix Θ as the *propagation matrix*.

- In the Appendix, we prove that Θ is well defined and has positive entries.
- In particular $\theta_{gg'} \geq 0$ captures the extent to which workers of type g' compete for marginal tasks against workers of type g.
- Second, we show that the row sum of Θ, which we label by ε_g, is always between 0 and 1.
- Third, the propagation matrix satisfies the following symmetry property: $\varepsilon_g - \frac{\theta_{gg'}}{s_{g'}^L} = \varepsilon_{g'} - \theta_{g'g}/s_g^L$ for any two groups g and g' (where s_g^L is the labor share of group g in output).

- Finally, the propagation matrix tells us whether different workers are q−complements or q−substitutes: an increase in the supply of workers of type g' reduces the real wage of type g if and only if $\theta_{gg'} > s^L_{g'} \cdot \varepsilon_g$.

- In what follows, we denote row g of the propagation matrix by $\Theta_g = \left(\theta_{g1}, \dots, \theta_{gG}\right)$.

- The next proposition characterizes the general equilibrium effects of task displacement on wages, industry prices, TFP, and output.

- We use $d \ln x$ to designate the column vector of $(d \ln x_1, \dots, d \ln x_G)$ across groups of workers, and with some abuse of notation, we denote the vector of industry prices by $d \ln p = (d \ln p_1, d \ln p_2, \dots, d \ln p_I)$.

- ***Proposition 4 (Counterfactuals)*** *The effect of task displacement on wages, industry prices, and aggregates is given by the solution to the system of equations:*

$$d\ln w_g = \Theta_g \cdot \left(\frac{1}{\lambda} \cdot d\ln y + \frac{1}{\lambda} d\ln \boldsymbol{\zeta} - \frac{1}{\lambda} d\ln \boldsymbol{\Gamma}^{disp}\right) \; for\ all\ g \in \mathcal{G},$$

$$d\ln \zeta_g = \sum_{i\in\mathcal{I}} \omega_{gi} \cdot \left(\frac{\partial \ln s_i^Y(\boldsymbol{p})}{\partial \ln \boldsymbol{p}} \cdot d\ln \boldsymbol{p} + (\lambda - 1) \cdot d\ln p_i\right) \; for\ all\ g \in \mathcal{G},$$

$$d\ln p_i = \sum_{g\in\mathcal{G}} s_{gi}^L \cdot \left(d\ln w_g - d\ln \Gamma_{gi}^{disp} \cdot \pi_{gi}\right) \; for\ all\ i \in \mathcal{I},$$

$$d\ln TFP = \sum_{i\in\mathcal{I}} s_i^Y(\boldsymbol{p}) \sum_{g\in\mathcal{G}} s_{gi}^L \cdot d\ln \Gamma_{gi}^{disp} \cdot \pi_{gi},$$

$$d\ln y = \frac{1}{1 - s^K} \cdot \left(d\ln TFP + s^K \cdot d\ln s^K\right),$$

$$d\ln s^K = -\frac{1}{s^K} \sum_{g\in\mathcal{G}} s_g^L \cdot (d\ln w_g - d\ln y).$$

5.2 Parametrization, Calibration, and Estimation

- **Measuring task displacement and the cost savings from automation**: Recall that λ is the elasticity of substitution between capital and labor in an industry holding the task allocation constant, while the elasticity of substitution incorporating task reallocation, σ_i, exceeds λ. As a result, when we incorporate ripple effects and task reallocation, the task displacement experienced by group g in industry i can be expressed as

(15) $$d \ln \Gamma_{gi}^{\text{disp}} = (\omega_{gi}^R / \omega_i^R) \cdot \frac{-d \ln s_i^L - s_i^K \cdot (1 - \sigma_i) \cdot (d \ln w_i - d \ln R_i)}{1 + (\lambda - 1) \cdot s_i^L \cdot \pi_i},$$

and the task displacement measure in Equation (12) becomes

(16) $$\text{Task displacement}_g = \sum_{g \in \mathcal{G}} \omega_g^i \cdot (\omega_{gi}^R / \omega_i^R) \cdot \frac{-d \ln s_i^L - s_i^K \cdot (1 - \sigma_i) \cdot (d \ln w_i - d \ln R_i)}{1 + (\lambda - 1) \cdot s_i^L \cdot \pi_i}$$

- **Industry demand**: We use a simple CES demand system across industries: $s_i^Y(p) = \alpha_i \cdot p_i^{1-\eta}$.

- Following Buera, Kaboski and Rogerson (2015), we set the elasticity of substitution between industries to $\eta = 0.2$.

- **Propagation matrix**: Motivated by the symmetry property of the propagation matrix described above, we parameterize the extent of competition for tasks between two demographic groups g and g' as a function of their distance (dissimilarity) across $n \in \mathcal{N}$ dimensions.

- In particular, we assume that

$$\theta_{gg'} = \frac{1}{2}\left(\varepsilon_g - \varepsilon_{g'}\right) \cdot s^L_{g'} + \sum_{n \in \mathcal{N}} \beta_n \cdot f\left(d^n_{gg'}\right) \cdot s^L_{g'} \text{ for all } g' \neq g \text{ and}$$

$$\theta_{gg} = \theta \text{ for all } g,$$

where f is a decreasing function of the distance along a given dimension n between groups g' and g, denoted here by $d^n_{gg'}$.

- The parameter $\beta_n \geq 0$ gives the importance of dimension n in mediating ripple effects.

- Using this parameterization, the wage effects from Proposition 4 can be written as:

$$(17) \qquad d\ln w_g = \frac{\varepsilon_g}{\lambda} \cdot d\ln y - \frac{\theta}{\lambda} \cdot \text{Task displacement}_g$$
$$- \sum_{g' \neq g} \left(\frac{1}{2} \left(\frac{\varepsilon_g}{\lambda} - \frac{\varepsilon_{g'}}{\lambda} \right) + \sum_{n \in \mathcal{N}} \frac{\beta_n}{\lambda} \cdot f(d^n_{g,g'}) \right) \cdot s^L_{g'} \cdot \text{Task displacement}_{g'} + v_g,$$
$$\text{subject to: } \varepsilon_g = \theta + \sum_{g' \neq g} \left(\frac{1}{2}(\varepsilon_g - \varepsilon_{g'}) + \sum_x \beta_n \cdot f(d^n_{g,g'}) \right) \cdot s^L_{g'}, \text{ and } \beta_n \geq 0,$$

where f is chosen as an inverted sigmoid function of the distance between two groups.

- The parameters of this system can be estimated by GMM exploiting the moment conditions

$$\mathbb{E}\left[v_g \cdot \left(1, z, \sum_{g' \neq g} f\left(d^1_{g,g'}\right) \cdot s^L_{g'} \cdot z_{g'}, \dots, \sum_{g' \neq g} f\left(d^N_{g,g'}\right) \cdot s^L_{g'} \cdot z_{g'}\right)\right] = 0,$$

- where z_g is either our measure of task displacement, or alternatively, our index of automation (so that we only exploit automation-induced changes in demand).

- Table 8 provides our estimates θ/λ and β_n/λ in Equation (17).
- Columns 1-3 use our task displacement measure in Equation (16) to form moment conditions, while columns 4-6 use the index of automation.
- The estimates in Panel A of Table 8 provide evidence of significant ripple effects by occupation, industry, and within age $\times$ education cells, and suggest that these are all of comparable importance.
- These estimates also imply that demographic groups suffering displacement will compete for tasks performed by other groups that have similar age and education and specialize in similar occupations and industries.

Table 8: Estimates of the Propagation Matrix

	Dependent variable: Change in wages 1980–2016					
	GMM estimates			GMM using automation index		
	(1)	(2)	(3)	(4)	(5)	(6)
Own effect, θ/λ	0.885	0.881	0.818	0.878	0.872	0.800
	(0.047)	(0.050)	(0.053)	(0.048)	(0.050)	(0.054)
Contribution of ripple effects via occupational similarity	0.362	0.357	0.310	0.366	0.360	0.321
	(0.087)	(0.090)	(0.091)	(0.087)	(0.091)	(0.091)
Contribution of ripple effects via industry similarity	0.221	0.222	0.363	0.225	0.225	0.366
	(0.105)	(0.105)	(0.113)	(0.105)	(0.105)	(0.113)
Contribution of ripple effects via education–age groups	0.179	0.179	0.170	0.178	0.179	0.167
	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)
Observations	500	500	500	500	500	500
Covariates:						
Industry shifters		✓	✓		✓	✓
Manufacturing share			✓			✓

Notes: This table presents estimates of the propagation matrix using the parametrization in equation (17). Here, ripple effects are parametrized as functions of the similarity of groups in terms of their 1980 occupational distribution, industry distribution, and education×age groups. The table reports our estimates of the common diagonal term θ and a summary measure of the strength of ripple effects operating through each of these dimensions, defined by

$$\text{Contribution of ripple effects}_n = \frac{\beta_n}{\lambda} \cdot \left(\frac{1}{s^L} \sum_g \sum_{g' \neq g} f(d^n_{gg'}) \cdot s^L_g \cdot s^L_{g'} \right),$$

which equals the average sum of the off diagonal terms of the propagation matrix explained by each dimension of similarity. Estimates and standard errors are obtained via GMM. Columns 1–3 provide GMM estimates using our measure of task displacement to construct the instruments used in the moment conditions. Columns 4–6 provide GMM estimates using our index of automation to construct the instruments used in the moment conditions. All models are weighted by the share of hours worked by each group in 1980.

5.3 Quantitative Results

- This subsection presents our quantitative results using the estimates of the propagation matrix from column 1 of Table 8.
- We use Proposition 4 to compute the effects of task displacement across workers and industries.
- We treat task displacement, as measured in Equations (15) and (16), as the driving force affecting the wage structure.
- Thus, this exercise leaves out other forms of technological progress (including factor-augmenting technologies, productivity deepening, new tasks, and sectoral TFPs) and changes in factor supplies driven by education and demographics.
- Table 9 summarizes our findings.

Table 9: Results from Quantitative Exercise

	Data for 1980–2016 (1)	Model prediction computed using Proposition 4 (2)	Variation due to automation index (3)
Wage structure:			
Share wage changes explained:			
-due to industry shifts		6.78%	5.72%
-adding direct displacement effects		100.54%	84.21%
-accounting for ripple effects		48.35%	41.10%
Rise in college premium	25.51%	21.82%	18.29%
-part due to direct displacement effect		40.92%	33.91%
Rise in post-college premium	40.42%	24.06%	19.88%
-part due to direct displacement effect		48.04%	39.11%
Change in gender gap	15.37%	1.83%	1.75%
-part due to direct displacement effect		6.31%	5.38%
Share with declining wages	53.10%	41.71%	44.89%
-part due to direct displacement effects		49.61%	49.62%
Wages for men with no high school	-8.21%	-7.18%	-7.09%
-part due to direct displacement effects		-13.97%	-13.52%
Wages for women with no high school	10.94%	1.24%	-1.47%
-part due to direct displacement effects		6.21%	-0.06%
Aggregates:			
Change in average wages, $d\ln w$	29.15%	5.71%	4.61%
Change in GDP per capita, $d\ln y$	70.00%	23.42%	18.93%
Change in TFP, $d\ln \text{tfp}$	35%	3.77%	3.04%
Change in labor share, ds^L	-8 p.p.	-11.69 p.p	-9.45 p.p
Change in K/Y ratio	30.00%	41.93%	35.15%
Sectoral patterns:			
Share manufacturing in GDP	-8.80 p.p	-0.41 p.p	-0.43 p.p
Change in manufacturing wage bill	-35.00%	-8.23%	-9.98%

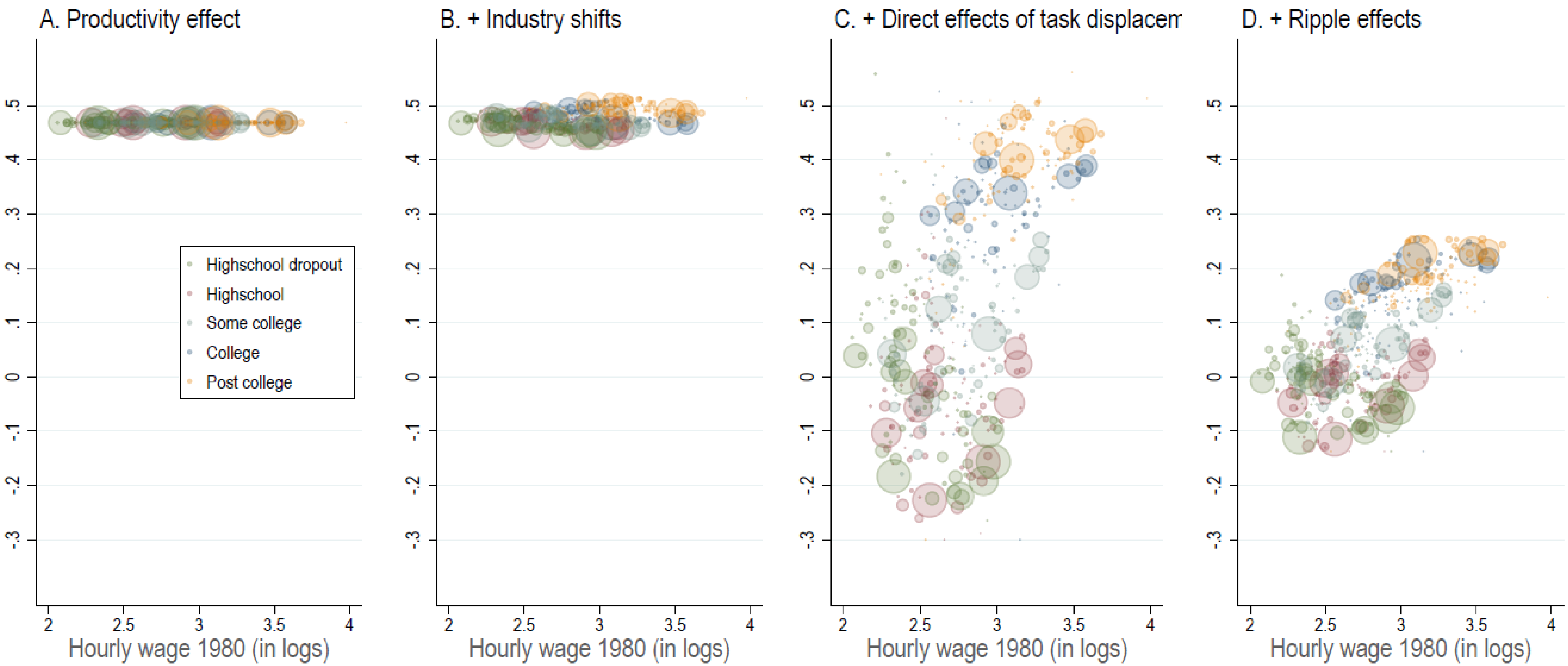
- This information is also displayed in Figure 10, which decomposes the effects of the various mechanisms via which task displacement affects the wage structure.

- There are, nonetheless, some notable differences from the reduced-form evidence.

- While in the reduced-form regressions, task displacement accounted for 50%-70% of the changes in the US wage structure, the second row of Table 9 shows that it accounts for as much as 100% of the variation here.

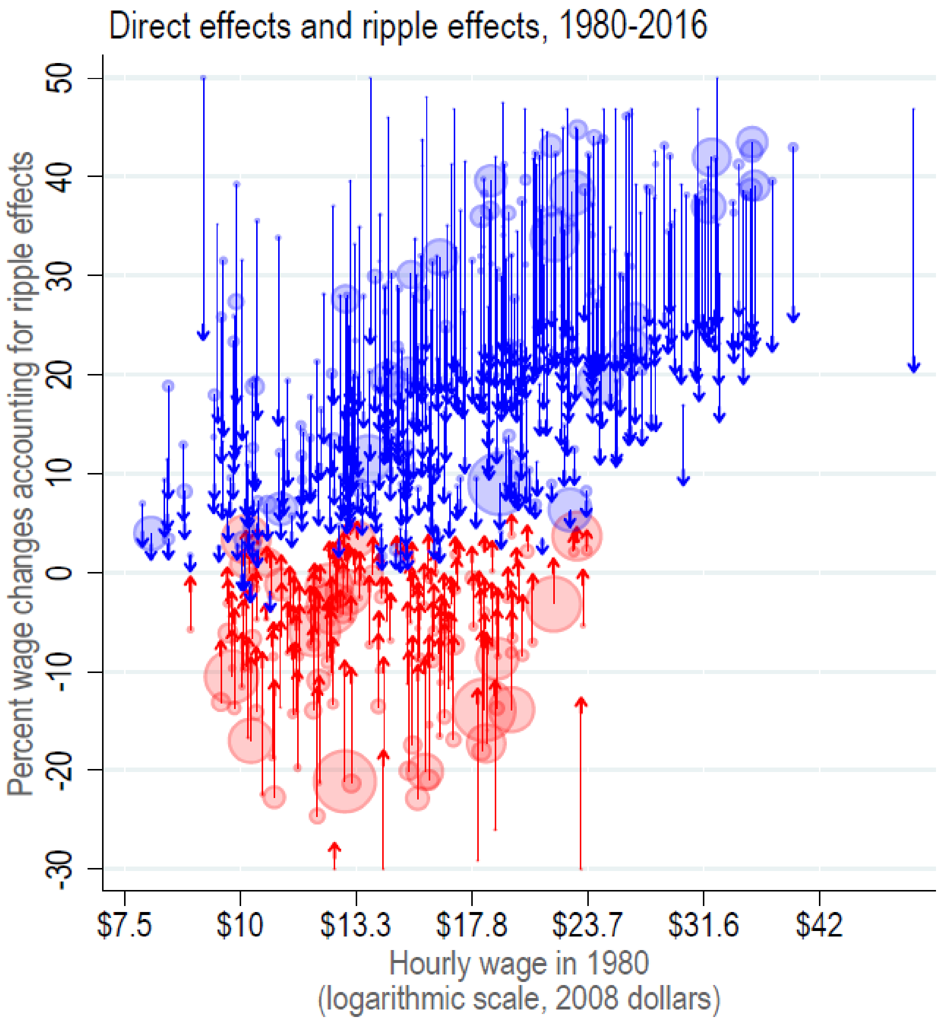
- This, however, overstates the full impact of task displacement, because of the ripple effects shown in the next row of the table and in Panel D of Figure 10.

Figure 10: Contribution of productivity effects, industry shifts, direct displacement effects, and ripple effects to the predicted change in wages for 1980-2016



- The consequences of ripple effects are further illustrated in Figure 11.
- The figure plots the direct effects of task displacement against the baseline wage of demographic groups (marker sizes are proportional to hours worked by each group).

Figure 11: Direct effects of task displacement compared to general equilibrium effects after accounting for ripple effects



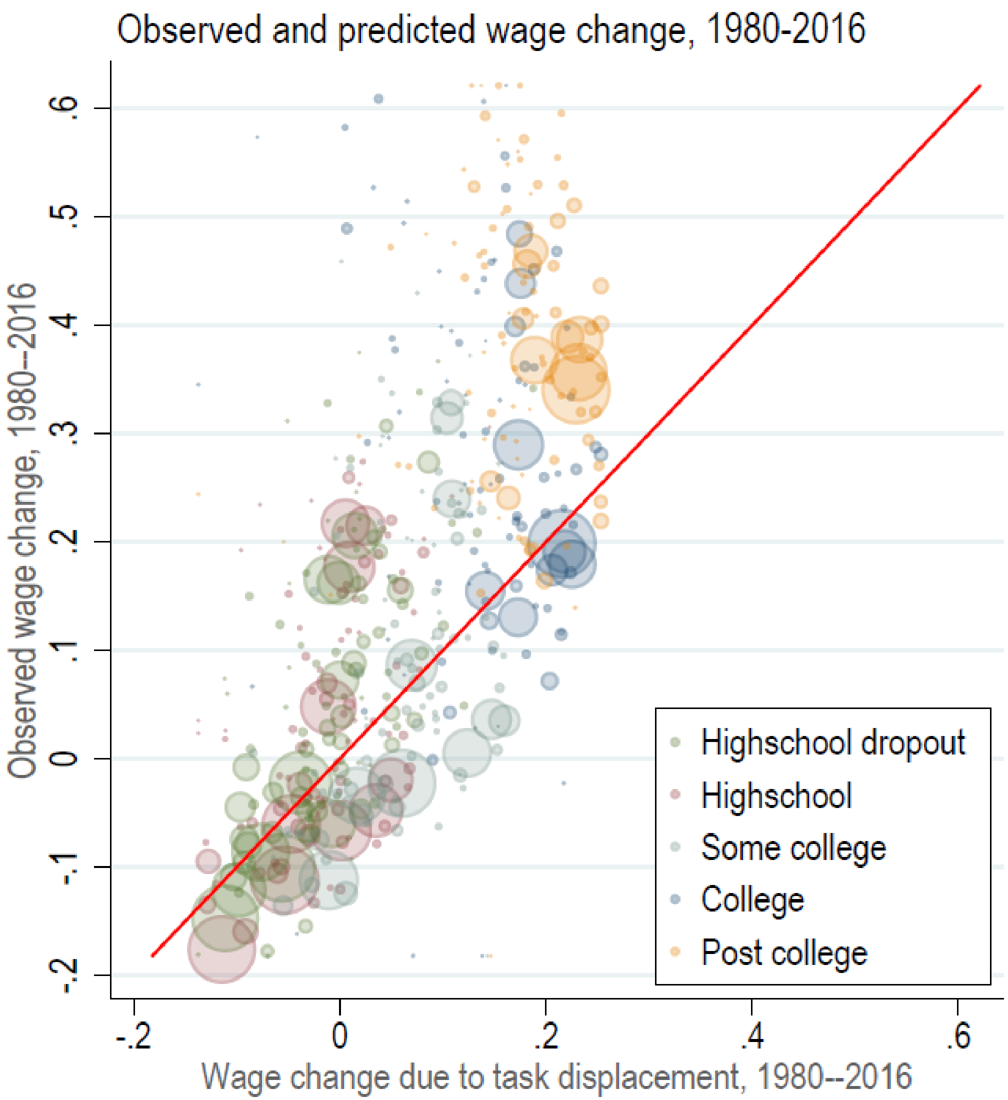
- As a summary, Figure 12 plots the predicted wage changes in the model and the observed real wage changes between 1980 and 2016.

- In addition to accounting for a large fraction of the variation in US wage structure, task displacement can explain several other salient aspects of the labor market this period.

- It is also worth noting, however, that our model misses a significant portion of wage growth coming from highly-educated workers at the top of the wage distribution.

- This may reflect the complementarity between some of the new technologies and post-graduate skills or other forces, such as winner-take-all dynamics in some high-skill professions, which are both absent from our model.

Figure 12: Predicted (horizontal axis) vs. observed (vertical axis) wage changes



6. Concluding Remarks

- This paper argued that a significant portion of the rise in US wage inequality over the last four decades has been driven by automation (and to a lesser extent offshoring) displacing certain workgroups from employment opportunities for which they had comparative advantage.

- To develop this point, we proposed a conceptual framework where tasks are allocated to different types of labor and capital, and automation technologies expand the set of tasks performed by capital and displace workers previously employed in these tasks.

- We derived a simple equation linking wage changes of a demographic group to the task displacement it experiences.

- Our reduced-form evidence is based on estimating this equation and reveals a number of striking new facts.

- Most notably, we documented that between 50% and 70% of the changes in US wage structure between 1980 and 2016 are accounted for by the relative wage declines of worker groups specialized in routine tasks in industries experiencing rapid automation.

- In our first set of regression models, industry level task displacement is approximated by (the unexplained component of) labor share declines.

- We also estimate very similar results using explicit measures of industry-level automation and offshoring, confirming that our task displacement variable captures the effects of automation technologies (and to a lesser degree offshoring) rather than increasing markups, industry concentration, or import competition.

- These alternative economic trends themselves do not appear to play a major role in the evolution of the US wage structure.

- Our reduced-form regressions estimate the direct effects of task displacement on relative wages, but miss important general equilibrium forces.
- We developed a methodology to quantify the general equilibrium effects of task displacement, which can account for the implications of automation working through productivity gains, ripple effects and changes in industry composition.
- Our full quantitative evaluation shows that task displacement explains close to 50% of the observed changes in US wage structure.
- Most notably, task displacement leads to sizable increases in wage inequality, but only small productivity gains—thus providing a possible resolution to a puzzling feature of US data.

- There are several interesting areas for future research.
- First, our framework has been static, and thus any effects from capital accumulation, dynamic incentives for the development of new technologies and education and skill acquisition are absent. Incorporating those is an important direction for future research.
- Second, and relatedly, we did not attempt to model and estimate the effects of technologies introducing new labor-intensive tasks.
- Finally, our empirical work has been confined to the US and the 1980-2016 period, for which we have all the data components necessary for implementing our reduced-form and structural estimation.
- Expanding these data sources and the empirical exploration of the role of task displacement to earlier periods and other economies is an important direction for research that may help us understand the technological and institutional reasons why the US wage structure was quite stable for the three decades leading up to the mid-1970s.

Appendix

A-1 Proofs of the Results in the Main Text

- **Proof of Proposition 1**. We first show that an equilibrium exists and is unique.

- The equilibrium of this economy solves the following optimization problem

$$\max_{\{k(x),\ell_1(x),\ldots,\ell_G(x)\}_{x\in\mathcal{T}}} y - \int_{\mathcal{T}} (k(x)/q(x)) \cdot dx$$

$$\text{subject to: } y = \left(\frac{1}{M}\int_{\mathcal{T}} (M \cdot y(x))^{\frac{\lambda-1}{\lambda}} \cdot dx\right)^{\frac{\lambda}{\lambda-1}},$$

$$y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_{g\in\mathcal{G}} A_g \cdot \psi_g(x) \cdot \ell_g(x) \quad \forall x \in \mathcal{T},$$

$$\ell_g = \int_{\mathcal{T}} \ell_g(x) \cdot dx \quad \forall g \in \mathcal{G}.$$

- This involves the maximization of a concave objective function subject to a convex constraint set.
- As a result, this optimization problem is i. unbounded or ii. has a unique solution (up to a set of measure zero).

- Suppose the problem is not unbounded (Proposition B-2 in this appendix provides conditions under which the maximization problem is bounded).

- Let w_g be the Lagrange multiplier associated with the constraint for labor of type g. It follows that the solution to this optimization problem is given by an allocation of tasks to factors such that

$$\mathcal{T}_g \subseteq \left\{x : \frac{w_g}{A_g \cdot \psi_g(x)} \leq \frac{w_{g'}}{A_{g'} \cdot \psi_{g'}(x)}, \frac{1}{\psi_k(x) \cdot q(x) \cdot A_k} \text{ for all } g'\right\},$$

$$\mathcal{T}_k \subseteq \left\{x : \frac{1}{\psi_k(x) \cdot q(x) \cdot A_k} \leq \frac{w_g}{A_g \cdot \psi_g(x)}, \text{ for all } g\right\}.$$

- The tie-breaking rule described in footnote 8 then selects a unique equilibrium allocation.
- This argument shows that, when the maximization problem is bounded, there is a unique equilibrium, where the task allocation is as described in the main text.
- In what follows, we characterize the equilibrium as a function of this unique task allocation.

- The demand for task x is

$$y(x) = \frac{1}{M} \cdot y \cdot p(x)^{-\lambda}, \tag{A-1}$$

where $p(x)$ is this task's price.

- Given the allocation of tasks $\{T_k, T_1, \ldots, T_G\}$, this price is

$$p(x) = \begin{cases} \dfrac{1}{A_k \cdot q(x) \cdot \psi_k(x)} & \text{if } x \in \mathcal{T}_k \\[2ex] \dfrac{w_g}{A_g \cdot \psi_g(x)} & \text{if } x \in \mathcal{T}_g. \end{cases} \tag{A-2}$$

- This implies that the demand for capital and labor at the task level is given by:

$$k(x)/q(x) = \begin{cases} \frac{1}{M} \cdot y \cdot (A_k \cdot q(x) \cdot \psi_k(x))^{\lambda-1} & \text{if } x \in \mathcal{T}_k \\ 0 & \text{if } x \notin \mathcal{T}_k. \end{cases}$$

$$\ell_g(x) = \begin{cases} \frac{1}{M} \cdot y \cdot (A_g \cdot \psi_g(x))^{\lambda-1} \cdot w_g^{-\lambda} & \text{if } x \in \mathcal{T}_g \\ 0 & \text{if } x \notin \mathcal{T}_g. \end{cases}$$

- To derive Equation (2), we integrate over the demand for labor across tasks in the previous expression and rearrange to obtain:

$$\ell_g = \int_{\mathcal{T}_g} \frac{1}{M} \cdot y \cdot (A_g \cdot \psi_g(x))^{\lambda-1} \cdot w_g^{-\lambda} \cdot dx \Rightarrow w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\frac{1}{M} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1} dx\right)^{\frac{1}{\lambda}}.$$

- Equation (1) follows by noting that by definition gross output y is

$$y = \int_{\mathcal{T}} y(x) p(x) dx.$$

- Substituting for $y(x)$ from Equation (A-1), we obtain the ideal price condition:

$$1 = \frac{1}{M}\int_{\mathcal{T}} p(x)^{1-\lambda}dx. \tag{A-3}$$

- Substituting for the equilibrium task prices from equation (A-2) yields

$$1 = A_k^{\lambda-1}\cdot\left(\frac{1}{M}\int_{\mathcal{T}_k}(q(x)\cdot\psi_k(x))^{\lambda-1}dx\right) + \sum_{g\in\mathcal{G}}\left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\left(\frac{1}{M}\int_{\mathcal{T}_g}\psi_g(x)^{\lambda-1}dx\right).$$

- Next substituting for w_g from Equation (2), we can rewrite this equation in terms of task shares as

$$1 = A_k^{\lambda-1} \cdot \Gamma_k + \sum_{g \in \mathcal{G}} \Gamma_g^{\frac{1}{\lambda}} \cdot \left(\frac{y}{A_g \cdot \ell_g} \right)^{\frac{1-\lambda}{\lambda}} .$$

- Rearranging this equation and using the fact that $A_k^{\lambda-1}\Gamma_k < 1$ yields the expression for output in Equation (1).

- Finally, we can compute factor shares as:

$$s^K = \frac{1}{M}\int_{\mathcal{T}_k} y \cdot p(x)^{1-\lambda} dx \Big/ y = A_k^{\lambda-1} \cdot \Gamma_k.$$

- Because of constant-returns to scale, we must have $s^L = 1 - s^K$.

- To conclude, note that in any competitive equilibrium we have $s^L, s^K \in [0,1]$, and so

$$1 \geq A_k^{\lambda-1} \cdot \Gamma_k,$$

as claimed in the main text. ■

- **Proof of Proposition 2.** We now characterize the effects of a small change in technology.
- As in the text, we use $D_g \subset T_g$ to denote the set of tasks that used to be performed by group g and where, after the technological change, capital now outperforms labor.
- The definitions of $d\ln\Gamma_g^{\text{deep}}$ and $d\ln\Gamma_g^{\text{disp}}$ in the main text imply

$$d\ln\Gamma_g = (\lambda - 1)d\ln\Gamma_g^{\text{deep}} - d\ln\Gamma_g^{\text{disp}}. \tag{A-4}$$

- To characterize the effects of technology on wages, we first log-differentiate Equation (2):

$$d \ln w_g = \frac{1}{\lambda} d \ln y + \frac{\lambda - 1}{\lambda} d \ln A_g + \frac{1}{\lambda} d \ln \Gamma_g .$$

- Plugging the formula for $d \ln \Gamma_g$ in (A-4) yields the expression for wage changes in (4).

- Let us next define changes in TFP, which are:

$$d \ln \text{TFP} = d \ln y - s^K \cdot d \ln k,$$

where $k = \int_{T_k} k(x)/q(x)dx$.

- This definition corresponds to gross TFP, defined as the change in gross output that is not explain by the change in capital and intermediate inputs, k.

- This can also be written in its dual representation as:

$$\text{(A-5)} \qquad d \ln \text{TFP} = \sum_{g \in \mathcal{G}} s_g^L \cdot d \ln w_g - \int_{T_k} s^K(x) d \ln q(x) dx,$$

- where $s^K(x)$ denotes the share of capital $k(x)$ in gross output and s_g^L denote the share of labor of type g in gross output.

- To obtain this expression, note that because of constant returns to scale, Euler's theorem implies

$$y = \sum_{g \in \mathcal{G}} w_g \ell_g + \int_{\mathcal{T}_k} k(x)/q(x)dx.$$

- For any small change in technology, we have

$$d\ln y = \sum_{g \in \mathcal{G}} s_g^L \cdot d\ln w_g + \int_{\mathcal{T}_k} s^K(x) d\ln k(x) dx - \int_{\mathcal{T}_k} s^K(x) d\ln q(x) dx$$
$$+ \frac{1}{y} \sum_{g \in \mathcal{G}} \int_{\mathcal{D}_g} (k^{\text{new}}(x)/q^{\text{new}}(x)) dx,$$

where the $k^{\text{new}}(x)$ and $q^{\text{new}}(x)$ denote the new capital usage and prices in the newly-automated tasks.

- Rearranging, we have

$$d\ln y - \left(\int_{\mathcal{T}_k} s^K(x) d\ln k(x) dx + \frac{1}{y} \sum_{g \in \mathcal{G}} \int_{\mathcal{D}_g} (k^{\text{new}}(x)/q^{\text{new}}(x)) dx \right)$$
$$= \sum_{g \in \mathcal{G}} s_g^L \cdot d\ln w_g - \int_{\mathcal{T}_k} s^K(x) d\ln q(x) dx.$$

- Finally, using the fact that

$$s^K d\ln k = \frac{1}{y} dk = \int_{\mathcal{T}_k} s^K(x) d\ln k(x) dx + \frac{1}{y} \sum_{g \in \mathcal{G}} \int_{\mathcal{D}_g} (k^{\text{new}}(x)/q^{\text{new}}(x)) dx,$$

we obtain the dual representation of TFP.

- We now return to determining the contribution of different types of technologies to TFP.

- For this, we use the ideal price index condition in Equation (A-3), which we can rewrite as

$$1 = A_k^{\lambda-1} \cdot \left(\frac{1}{M} \int_{\mathcal{T}_k} (q(x) \cdot \psi_k(x))^{\lambda-1} dx \right) + \sum_{g \in \mathcal{G}} \left(\frac{w_g}{A_g} \right)^{1-\lambda} \cdot \left(\frac{1}{M} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1} dx \right).$$

- Log-differentiating this equation following an arbitrary change in technology and capital prices, we obtain:

$$\text{(A-6)} \qquad \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln w_g - \int_{\mathcal{T}_k} s^K(x) d\ln q(x) dx = s^K \cdot \left(d\ln A_k + d\ln \Gamma_k^{\text{deep}}\right)$$
$$+ \sum_{g\in\mathcal{G}} s_g^L \cdot \left(d\ln A_g + d\ln \Gamma_g^{\text{deep}}\right)$$
$$+ \frac{1}{\lambda - 1}\left[s^K \cdot d\ln \Gamma_k^{\text{disp}} - \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln \Gamma_g^{\text{disp}}\right].$$

- Let us define the last line as

$$\Delta = \frac{1}{\lambda - 1}\left[s^K \cdot d\ln \Gamma_k^{\text{disp}} - \sum_{g\in\mathcal{G}} s_g^L \cdot d\ln \Gamma_g^{\text{disp}}\right],$$

which represents the reallocation of tasks from labor to capital.

- To develop this expression further, let us recall the definition of the *cost-saving gains* from automating task x:

$$\pi_g(x) = \frac{1}{\lambda - 1}\left[\left(w_g \frac{A_k \cdot q(x) \cdot \psi_k(x)}{A_g \cdot \psi_g(x)}\right)^{\lambda-1} - 1\right] > 0.$$

- Averaging this across tasks, we obtain the average cost-saving gains from automating tasks in D_g (which was also defined in the text):

$$\pi_g = \frac{1}{M}\int_{\mathcal{D}_g} \psi_g(x)^{\lambda-1} \cdot \pi_g(x)dx \bigg/ \frac{1}{M}\int_{\mathcal{D}_g} \psi_g(x)^{\lambda-1} dx.$$

- Using these definitions, Δ can be rewritten as

$$
\begin{aligned}
\Delta &= \sum_{g\in\mathcal{G}} \frac{1}{\lambda-1}\left[A_k^{\lambda-1}\cdot\frac{1}{M}\int_{\mathcal{D}_g}(q(x)\cdot\psi_k(x))^{\lambda-1}dx - \left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\frac{1}{M}\int_{\mathcal{D}_g}\psi_g(x)^{\lambda-1}dx\right]\\
&= \sum_{g\in\mathcal{G}} \frac{1}{M}\int_{\mathcal{D}_g}\frac{1}{\lambda-1}\left[(A_k\cdot q(x)\cdot\psi_k(x))^{\lambda-1} - \left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\psi_g(x)^{\lambda-1}\right]dx\\
&= \sum_{g\in\mathcal{G}} \frac{1}{M}\int_{\mathcal{D}_g}\left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\psi_g(x)^{\lambda-1}\cdot\pi_g(x)dx\\
&= \sum_{g\in\mathcal{G}} \left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\left(\frac{1}{M}\int_{\mathcal{D}_g}\psi_g(x)^{\lambda-1}dx\right)\cdot\pi_g.
\end{aligned}
$$

- Next, using the fact that $s_g^L = \left(\frac{w_g}{A_g}\right)^{1-\lambda} \cdot \left(\frac{1}{M}\int_{T_g} \psi_g(x)^{\lambda-1} dx\right)$, we can rewrite Δ as:

$$\Delta = \sum_{g \in \mathcal{G}} s_g^L \cdot \frac{\frac{1}{M}\int_{\mathcal{D}_g} \psi_g(x)^{\lambda-1} dx}{\frac{1}{M}\int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1} dx} \cdot \pi_g = \sum_{g \in \mathcal{G}} s_g^L \cdot d \ln \Gamma_g^{\text{disp}} \cdot \pi_g.$$

- Substituting this expression for Δ into Equation (A-6) and using the dual representation of TFP in Equation (A-5), we obtain the TFP expressions in Equation (5) as desired.

- The output equation, (7), can be obtained from the TFP equation, (5).
- Note that by definition we have

$$d \ln y = d \ln \mathrm{TFP} + s^K \cdot d \ln k.$$

- Moreover, $k = s^K \cdot y$, which implies

$$d \ln k = d \ln s^K + d \ln y.$$

- Combining these two equations yields

$$d\ln y = \frac{1}{1-s^K}\left(d\ln \text{TFP} + s^K \cdot d\ln s^K\right)$$
$$d\ln k = \frac{1}{1-s^K}\left(d\ln \text{TFP} + d\ln s^K\right).$$

- To obtain the factor share changes, note that

$$d\ln s^K = (\lambda - 1)\cdot\left(d\ln A_k + d\ln \Gamma_k^{\text{deep}}\right) + d\ln \Gamma_k^{\text{disp}},$$

which follows from the fact that $s^K = A_k^{\lambda-1} \cdot \Gamma_k$.

- We can rewrite this expression as follows:

$$
\begin{aligned}
d\ln s^K =& (\lambda-1)\cdot\left(d\ln A_k + d\ln\Gamma_k^{\text{deep}}\right) + \frac{1}{s^K}\cdot\left((\lambda-1)\cdot\Delta + \sum_{g\in\mathcal{G}} s_g^L\cdot d\ln\Gamma_g^{\text{disp}}\right)\\
=& (\lambda-1)\cdot\left(d\ln A_k + d\ln\Gamma_k^{\text{deep}}\right) + \frac{1}{s^K}\cdot\sum_{g\in\mathcal{G}} s_g^L\cdot d\ln\Gamma_g^{\text{disp}}\cdot(1+(\lambda-1)\cdot\pi_g),
\end{aligned}
$$

- which yields Equation (6) in the proposition. ■

- **Proof of Proposition 3.** We first show that an equilibrium exists and is unique.
- Denote the aggregator of industry output by $H(y_1, \ldots, y_I)$. The equilibrium of this economy solves the following optimization problem

$$\max_{\{k(x),\ell_1(x),\ldots,\ell_G(x)\}_{x\in\mathcal{T}_i, i\in\mathcal{I}}} H(y_1,\ldots,y_I) - \int_{\mathcal{T}} (k(x)/q(x)) \cdot dx$$

$$\text{subject to: } y_i = \left(\frac{1}{M}\int_{\mathcal{T}} (M \cdot y(x))^{\frac{\lambda-1}{\lambda}} \cdot dx\right)^{\frac{\lambda}{\lambda-1}} \quad \forall i \in \mathcal{I},$$

$$y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_{g\in\mathcal{G}} A_g \cdot \psi_g(x) \cdot \ell_g(x) \quad \forall x \in \mathcal{T},$$

$$\ell_g = \int_{\mathcal{T}} \ell_g(x) \cdot dx \quad \forall g \in \mathcal{G}.$$

- This involves the maximization of a concave objective function subject to a convex constraint set.

- As a result, this optimization problem is i. unbounded or ii. has a unique solution (up to a set of measure zero).

- Suppose the problem is not unbounded (Proposition B-2 in this appendix
- provides conditions under which the maximization problem is bounded).

- Let w_g be the Lagrange multiplier associated with the constraint for labor of type g.

- It follows that the solution is given by an allocation of tasks to factors such that

$$\mathcal{T}_{gi} \subseteq \left\{ x : \frac{w_g}{A_{gi} \cdot \psi_g(x)} \leq \frac{w_{g'}}{A_{g'i} \cdot \psi_{g'}(x)}, \frac{1}{\psi_k(x) \cdot q(x) \cdot A_{ki}} \text{ for all } g' \right\},$$

$$\mathcal{T}_{ki} \subseteq \left\{ x : \frac{1}{\psi_k(x) \cdot q(x) \cdot A_{ki}} \leq \frac{w_g}{A_{gi} \cdot \psi_g(x)}, \text{ for all } g \right\}.$$

- The tie-breaking rule described in footnote 8 then selects a unique equilibrium allocation.

- This argument shows that, when the maximization problem is bounded, there is a unique equilibrium, where the task allocation is as described in the main text.

- In what follows, we characterize the equilibrium as a function of this unique task allocation (we provide a sufficient condition for finite output at the end of the proof).

- The demand for task x in sector i is

$$y(x) = \frac{1}{M_i} \cdot y \cdot s_i^Y(\boldsymbol{p}) \cdot p(x)^{-\lambda} \cdot (A_i p_i)^{\lambda-1}.$$

- Given $\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \dots, \mathcal{T}_{Gi}\}$, the price of task x is

$$p(x) = \begin{cases} \dfrac{1}{A_k \cdot q(x) \cdot \psi_k(x)} & \text{if } x \in \mathcal{T}_{ki} \\[2ex] \dfrac{w_g}{A_g \cdot \psi_k(x)} & \text{if } x \in \mathcal{T}_{gi}. \end{cases}$$

- The demand for capital and labor at task x can be written as

$$k(x)/q(x) = \begin{cases} \dfrac{1}{M_i} \cdot y \cdot s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot (A_k \cdot q(x) \cdot \psi_k(x))^{\lambda-1} & \text{if } x \in \mathcal{T}_{ki} \\ 0 & \text{if } x \notin \mathcal{T}_k. \end{cases}$$

$$\ell_g(x) = \begin{cases} \dfrac{1}{M_i} \cdot y \cdot s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot (A_g \cdot \psi_g(x))^{\lambda-1} \cdot w_g^{-\lambda} & \text{if } x \in \mathcal{T}_g \\ 0 & \text{if } x \notin \mathcal{T}_g. \end{cases}$$

- Integrating these demands, as in the proof of Proposition 1, and rearranging, we have

$$\ell_g = \sum_{i \in \mathcal{I}} \int_{\mathcal{T}_{gi}} \frac{1}{M_i} \cdot y \cdot s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot (A_g \cdot \psi_g(x))^{\lambda-1} \cdot w_g^{-\lambda} \cdot dx$$

$$\Rightarrow w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\sum_{i \in \mathcal{I}} s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \frac{1}{M_i} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} dx\right)^{\frac{1}{\lambda}},$$

which thus establishes Equation (8) as desired.

- To derive the industry price index in Equation (9), we observe that

$$p_i \cdot y_i = \int_{\mathcal{T}_i} p(x) \cdot y(x) dx \Rightarrow p_i = \frac{1}{A_i} \left(\frac{1}{M_i} \int_{\mathcal{T}_i} p(x)^{1-\lambda} dx \right)^{\frac{1}{1-\lambda}}.$$

- Using the allocation of tasks, $\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \ldots, \mathcal{T}_{Gi}\}$, this implies

$$\begin{aligned} p_i &= \frac{1}{A_i} \left(\frac{1}{M_i} \int_{\mathcal{T}_i} p(x)^{1-\lambda} dx \right)^{\frac{1}{1-\lambda}} \\ &= \frac{1}{A_i} \left(A_k \cdot \left(\frac{1}{M_i} \int_{\mathcal{T}_{ki}} (q(x) \cdot \psi_k(x))^{\lambda-1} dx \right) + \sum_{g \in \mathcal{G}} w_g^{1-\lambda} \cdot A_g^{\lambda-1} \cdot \left(\frac{1}{M_i} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} dx \right) \right)^{\frac{1}{1-\lambda}}, \end{aligned}$$

which yields Equation (10) in the proposition.

- Finally, because industry shares must add up to 1, Equation (10) holds, completing the proof.

- Although not reported, factor shares can be computed as

$$s^K = A_k^{\lambda-1} \cdot \sum_{i \in \mathcal{I}} s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{ki} \tag{A-7}$$

$$s^L = 1 - A_k^{\lambda-1} \cdot \sum_{i \in \mathcal{I}} s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{ki}. \tag{A-8}$$

■

- Proof of Proposition 4. We first provide a proof for the existence and some of the properties of the propagation matrix Θ.

- Define the matrix

$$\Sigma = \mathbb{1} - \frac{1}{\lambda} \frac{\partial \ln \Gamma(w, \zeta, \Psi)}{\partial \ln w}.$$

- This matrix satisfies several properties.

- First, because $\partial \Gamma_g / \partial w_{g'} \geq 0$, all of its off-diagonal entries are negative.

- This implies that $\sum$ is a Z−matrix.

- Second, $\sum$ has a positive dominant diagonal.

- This follows from the fact that

$$\Sigma_{gg} = 1 - \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln w_g} > 0,$$

and

$$\Sigma_{gg} - \sum_{g' \neq g} |\Sigma_{gg'}| = 1 - \sum_{g'} \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} > 1.$$

- This last inequality follows from the fact that $\sum_{g'} \frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} \leq 0$, which is true since when all wage rise by the same amount, workers lose tasks to capital but do not experience task reallocation among them.

- Third, all eigenvalues of $\sum$ have a real part that exceeds 1.

- This follows from an application of Gershgorin circle theorem, which states that for each eigenvalue ε pf $\sum$, we can find a dimension g such that

$$\|\varepsilon - \Sigma_{gg}\| < \sum_{g' \neq g} |\Sigma_{gg'}|.$$

- This inequality requires that

$$\Re(\varepsilon) \in \left[\Sigma_{gg} - \sum_{g' \neq g} |\Sigma_{gg'}|, \Sigma_{gg} + \sum_{g' \neq g} |\Sigma_{gg'}|\right].$$

- Because $\sum_{gg} - \sum_{g' \neq g} |\sum_{gg'}| > 1$ for all g, as shown above, all eigenvalues of $\sum$ have a real part that is greater than 1.

- Fourth, since $\sum$ has negative off diagonal elements and all of its eigenvalues have a positive real part, we can conclude that it is an $M-$matrix.
- Because $\sum$ is an $M-$matrix, its inverse Θ exists and has positive and real entries, $\theta_{gg'} \geq 0$, as desired.
- Moreover, each eigenvalue of Θ has a real part that is positive and less than 1.
- Finally, the row and column sums of Θ are also less than 1.
- In particular, denote by θ_g^r the sum of the elements of row g of Θ.
- Then:

$$\Theta \cdot (1, 1, \ldots, 1)'_1 = (\theta_1^r, \theta_2^r, \ldots, \theta_G^r)' \Rightarrow \Sigma \cdot (\theta_1^r, \theta_2^r, \ldots, \theta_G^r)' = (1, 1, \ldots, 1)'.$$

- This equality requires that

$$\Sigma_{gg} \cdot \theta_g^r + \sum_{g' \neq g} \Sigma_{gg'} \cdot \theta_{g'}^r = 1.$$

- Now, suppose without loss of generality, that $\theta_1^r > \theta_2^r > \cdots > \theta_G^r > 0$ (all rows must have strictly positive sums, since $\theta_{gg'} = 0$ for all g' would imply that Θ is singular, contradicting the fact that all its eigenvalues have real parts in $(0,1)$).

- We have

$$\Sigma_{11} \cdot \theta_1^r + \sum_{g' \neq 1} \Sigma_{1g'} \cdot \theta_{g'}^r = 1,$$

which implies that

$$\left(1 - \frac{1}{\lambda} \frac{\partial \ln \Gamma_1}{d \ln w_1}\right) \cdot \theta_1^r = 1 + \frac{1}{\lambda} \sum_{g' \neq 1} \frac{\partial \ln \Gamma_1}{d \ln w_{g'}} \cdot \theta_{g'}^r \leq 1 + \frac{1}{\lambda} \sum_{g' \neq 1} \frac{\partial \ln \Gamma_1}{d \ln w_{g'}} \cdot \theta_1^r.$$

- Because $\sum_{g'} \frac{\partial \ln \Gamma_1}{d \ln w_{g'}} \leq 0$, we can rewrite this inequality as

$$\theta_1^r < 1 + \frac{1}{\lambda} \sum_{g'} \frac{\partial \ln \Gamma_1}{d \ln w_{g'}} \cdot \theta_1^r \leq 1.$$

- An identical argument establishes that column sums of Θ lie in $(0,1)$.

- Having introduced the propagation matrix Θ, we are now in a position to derive the formulas characterizing the effects of technology on wages, sectoral prices, and TFP.

- First, define $w_g^e = w_g / A_g$ as the wage per efficiency unit of labor of g workers.

- Equation (8) then implies

$$w_g^e = \left(\frac{y}{A_g \cdot \ell_g} \right)^{\frac{1}{\lambda}} \cdot \Gamma_g(w, \zeta, \Psi)^{\frac{1}{\lambda}}.$$

- Log-differentiating this equation in response to an automation (task-displacing) technology, we obtain:

$$d \ln w_g^e = \frac{1}{\lambda} d \ln y - \frac{1}{\lambda} d \ln \Gamma_g^{\text{disp}} + \frac{1}{\lambda} \sum_{i \in \mathcal{I}} \omega_{gi} \cdot d \ln \zeta_i + \frac{1}{\lambda} \frac{\partial \ln \Gamma_g}{\partial \ln w} \cdot d \ln w.$$

- Stacking these equations for all groups, we can write:

$$\begin{pmatrix} d\ln w_1^e \\ d\ln w_1^e \\ \cdots \\ d\ln w_G^e \end{pmatrix} = \frac{1}{\lambda}\begin{pmatrix} d\ln y \\ d\ln y \\ \cdots \\ d\ln y \end{pmatrix} + \frac{1}{\lambda}\begin{pmatrix} \sum_{i\epsilon\mathcal{I}} \omega_{1i}\cdot d\ln\zeta_i \\ \sum_{i\epsilon\mathcal{I}} \omega_{2i}\cdot d\ln\zeta_i \\ \cdots \\ \sum_{i\epsilon\mathcal{I}} \omega_{Gi}\cdot d\ln\zeta_i \end{pmatrix} - \frac{1}{\lambda}\begin{pmatrix} d\ln\Gamma_1^{\text{disp}} \\ d\ln\Gamma_2^{\text{disp}} \\ \cdots \\ d\ln\Gamma_G^{\text{disp}} \end{pmatrix} + \frac{1}{\lambda}\frac{\partial\ln\Gamma}{\partial\ln\boldsymbol{w}}\cdot\begin{pmatrix} d\ln w_1^e \\ d\ln w_1^e \\ \cdots \\ d\ln w_G^e \end{pmatrix}.$$

- We can solve this system of equations as

$$\begin{pmatrix} d\ln w_1^e \\ d\ln w_1^e \\ \dots \\ d\ln w_G^e \end{pmatrix} = \frac{1}{\lambda}\Theta \cdot \begin{pmatrix} d\ln y \\ d\ln y \\ \dots \\ d\ln y \end{pmatrix} + \frac{1}{\lambda}\Theta \cdot \begin{pmatrix} \sum_{i\in\mathcal{I}} \omega_{1i} \cdot d\ln\zeta_i \\ \sum_{i\in\mathcal{I}} \omega_{2i} \cdot d\ln\zeta_i \\ \dots \\ \sum_{i\in\mathcal{I}} \omega_{Gi} \cdot d\ln\zeta_i \end{pmatrix} - \frac{1}{\lambda}\Theta \cdot \begin{pmatrix} d\ln\Gamma_1^{\text{disp}} \\ d\ln\Gamma_2^{\text{disp}} \\ \dots \\ d\ln\Gamma_G^{\text{disp}} \end{pmatrix},$$

which implies

$$d\ln w_g = \frac{\varepsilon_g}{\lambda} d\ln y + \frac{1}{\lambda}\Theta_g \cdot d\ln\zeta - \frac{1}{\lambda}\Theta_g \cdot d\ln\Gamma^{\text{disp}},$$

where

$$d\ln\zeta_g = \sum_{i\in\mathcal{I}} \omega_{gi} \cdot d\ln\zeta_i = \sum_{i\in\mathcal{I}} \omega_{gi} \cdot \left(\frac{\partial \ln s_i^Y(\boldsymbol{p})}{\partial \ln \boldsymbol{p}} \cdot d\ln\ \boldsymbol{p} + (\lambda - 1) \cdot d\ln p_i \right).$$

- Turning to industry prices, note that these are given by Equation (10).

- By definition, the equilibrium task allocation $\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \dots, \mathcal{T}_{Gi}\}$ solves the cost-minimization problem:

$$p_i = \min_{\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \dots, \mathcal{T}_{Gi}\}} \frac{1}{A_i} \left(A_k^{\lambda-1} \cdot \Gamma_{ki} + \sum_{g \in \mathcal{G}} w_g^{1-\lambda} \cdot A_g^{\lambda-1} \cdot \Gamma_{gi} \right)^{\frac{1}{1-\lambda}} .$$

- The envelope theorem then implies that

$$d \ln p_i = \sum_{g \in \mathcal{G}} s_{gi}^L \cdot d \ln w_g - (A_i p_i)^{\lambda-1} \frac{1}{\lambda - 1} \left[A_k^{\lambda-1} \cdot d\Gamma_{ki}^{\text{disp}} - \sum_{g \in \mathcal{G}} w_g^{1-\lambda} \cdot A_g^{\lambda-1} \cdot d\Gamma_{gi}^{\text{disp}} \right],$$

- since the reallocation of tasks across factors in response to changes in factor prices has a second-order effect on industry prices.

- Here, the term

$$\Delta_i = (A_i p_i)^{\lambda-1} \frac{1}{\lambda - 1} \left[A_k^{\lambda-1} \cdot d\Gamma_{ki}^{\text{disp}} - \sum_{g \in \mathcal{G}} w_g^{1-\lambda} \cdot A_g^{\lambda-1} \cdot d\Gamma_{gi}^{\text{disp}} \right]$$

- is a generalization of the term Δ in the proof of Proposition 2, and again corresponds to cost savings from the reallocation of tasks from labor to capital, but now in industry i.

- Similarly, we define the industry versions of cost savings at the task level (when tasks in the set $\mathcal{D}_{gi}$ in industry i previous to perform by factor g are automated):

$$\pi_{gi}(x) = \frac{1}{\lambda - 1}\left[\left(w_g \frac{A_k \cdot q(x) \cdot \psi_k(x)}{A_g \cdot \psi_g(x)}\right)^{\lambda-1} - 1\right] > 0,$$

and *average percent cost-saving gains* in industry i as

$$\pi_{gi} = \frac{1}{M_i}\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1} \cdot \pi_{gi}(x)dx \bigg/ \frac{1}{M_i}\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}dx.$$

- Using these definitions, we can write Δ_i as

$$\begin{aligned}
\Delta_i =& (A_i p_i)^{\lambda-1} \sum_{g\in\mathcal{G}} \frac{1}{\lambda-1}\left[A_k^{\lambda-1}\cdot\frac{1}{M_i}\int_{\mathcal{D}_{gi}}(q(x)\cdot\psi_k(x))^{\lambda-1}dx - \left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\frac{1}{M_i}\int_{\mathcal{D}_{gi}}\psi_g(x)^{\lambda-1}dx\right] \\
=& (A_i p_i)^{\lambda-1} \sum_{g\in\mathcal{G}} \frac{1}{M_i}\int_{\mathcal{D}_{gi}}\frac{1}{\lambda-1}\left[(A_k\cdot q(x)\cdot\psi_k(x))^{\lambda-1} - \left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\psi_g(x)^{\lambda-1}\right]dx \\
=& (A_i p_i)^{\lambda-1} \sum_{g\in\mathcal{G}} \frac{1}{M_i}\int_{\mathcal{D}_{gi}}\left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\psi_g(x)^{\lambda-1}\cdot\pi_{gi}(x)dx \\
=& (A_i p_i)^{\lambda-1} \sum_{g\in\mathcal{G}} \left(\frac{w_g}{A_g}\right)^{1-\lambda}\cdot\left(\frac{1}{M_i}\int_{\mathcal{D}_{gi}}\psi_g(x)^{\lambda-1}dx\right)\cdot\pi_{gi}.
\end{aligned}$$

- Again as in the proof of Proposition 2, using the fact that $s_{gi}^{L} = (A_i p_i)^{\lambda-1} \left(\frac{w_g}{A_g}\right)^{1-\lambda} \cdot \left(\frac{1}{M_i} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1} dx\right)$, we get

$$\Delta_i = \sum_{g \in \mathcal{G}} s_{gi}^{L} \cdot \frac{\frac{1}{M_i} \int_{\mathcal{A}_{gi}} \psi_g(x)^{\lambda-1} dx}{\frac{1}{M_i} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1} dx} \cdot \pi_{gi} = \sum_{g \in \mathcal{G}} s_{gi}^{L} \cdot d \ln \Gamma_{gi}^{\text{disp}} \cdot \pi_{gi},$$

which yields the desired formula for $d \ln p_i$ in the proposition.

- We now turn to TFP.
- As before, we use the dual definition of TFP, which now implies

$$d \ln \text{TFP} = \sum_{g \in \mathcal{G}} s_g^L \cdot d \ln w_g. \quad \text{(A-9)}$$

- To derive a formula for TFP, first note that given a price vector p, we can define the cost of producing the final good as $c^h(p)$.
- Moreover, Shephard's lemma implies that

$$s_i^Y(p) = \frac{\partial c^h(p)}{\partial p_i} \frac{p_i}{c^h}.$$

- Our choice of numeraire, which implies that the final good has a price of 1, then implies

$$1 = c^h(p).$$

- Log-differentiating this expression yields

$$\begin{aligned} 0 &= \sum_{i \in \mathcal{I}} s_i^Y(p) \cdot d \ln p_i \\ &= \sum_{i \in \mathcal{I}} s_i^Y(p) \cdot \left(\sum_{g \in \mathcal{G}} s_{gi}^L \cdot \left(d \ln w_g - d \ln \Gamma_{gi}^{\text{disp}} \cdot \pi_{gi} \right) \right) \\ &= \sum_{g \in \mathcal{G}} s_g^L \cdot d \ln w_g - \sum_{i \in \mathcal{I}} s_i^Y(p) \sum_{g \in \mathcal{G}} \cdot s_{gi}^L \cdot \pi_{gi} \end{aligned}$$

- Rearranging this expression, and using the dual definition of TFP in Equation (A-9), yields the formula for the contribution of automation to TFP in the proposition.

- Turning to output, the primal definition of TFP implies

$$d \ln y = d \ln \mathrm{TFP} + s^K \cdot d \ln k.$$

- Moreover, $k = s^K \cdot y$, which implies

$$d \ln k = d \ln s^K + d \ln y.$$

- Combining these two equations yields

$$d\ln y = \frac{1}{1-s^K}\left(d\ln \mathrm{TFP} + s^K \cdot d\ln s^K\right)$$
$$d\ln k = \frac{1}{1-s^K}\left(d\ln \mathrm{TFP} + d\ln s^K\right).$$

- Finally, we provide a derivation for the change in the capital share.

- Recall that the capital share is given by

$$d\ln s^K = -\frac{1-s^K}{s^K} d\ln s^L = -\frac{1}{s^K}\sum_{g\in\mathcal{G}} s_g^L \cdot (d\ln w_g - d\ln y).$$

■

A-2 Additional Figures and Tables

Figure A-1: Task displacement across 500 demographic groups sorted by their hourly wage in 1980

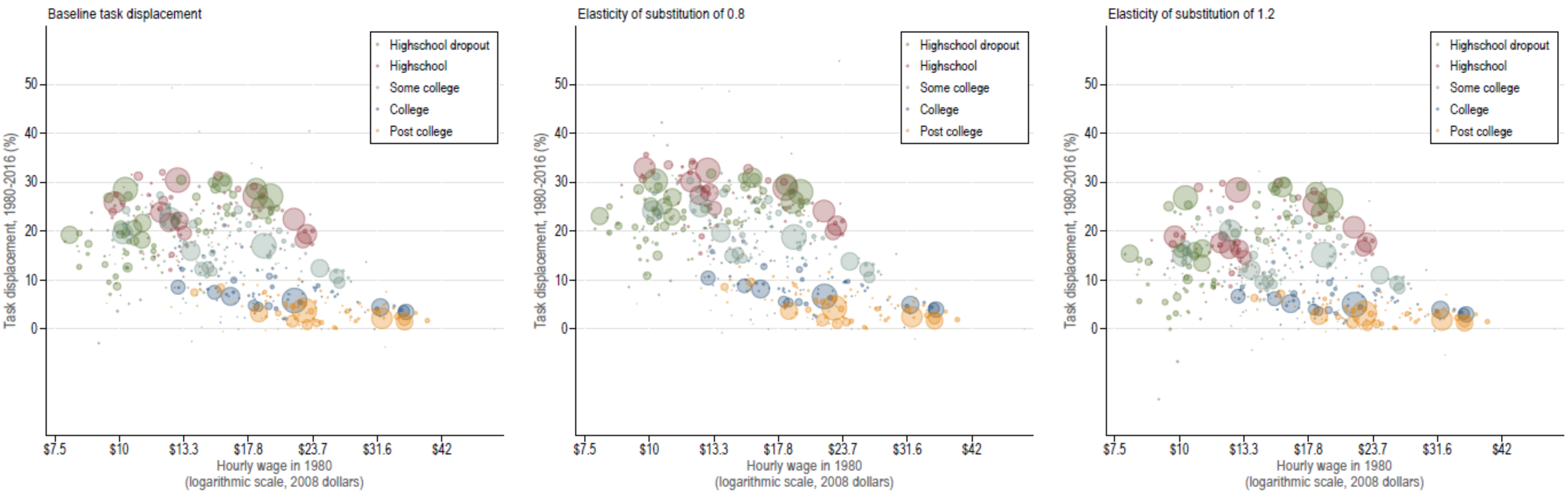


Figure A-2 (first panel): Relationship between task displacement 1987-2016 and sales concentration

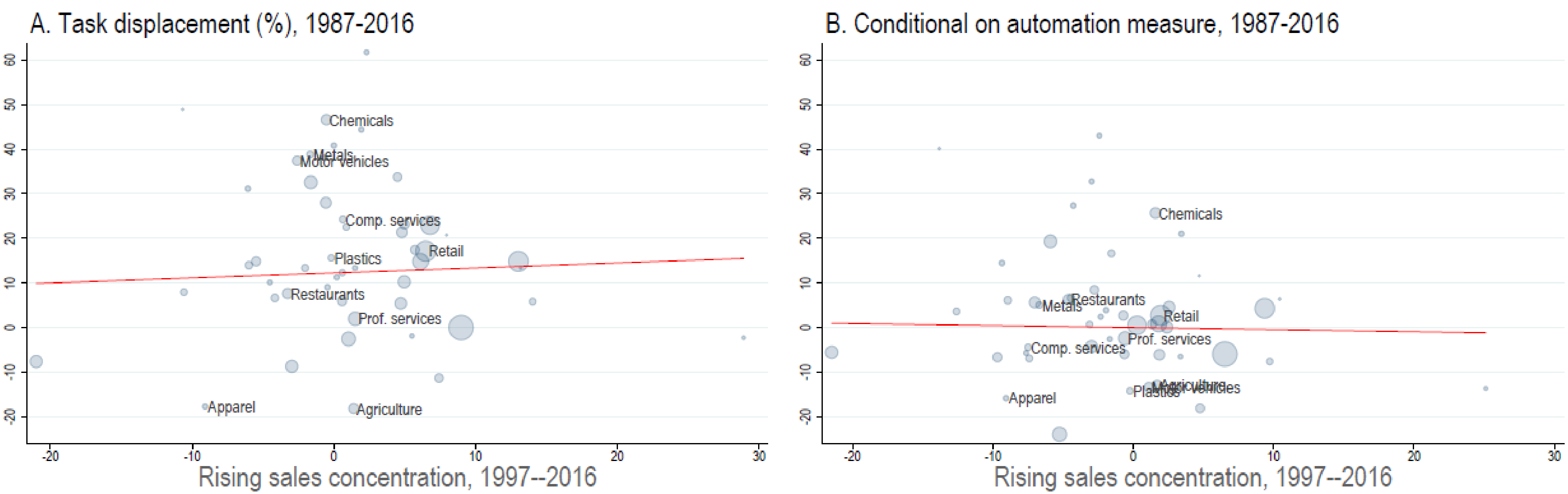


Figure A-2 (second panel): Relationship between task displacement 1987-2016 and rising

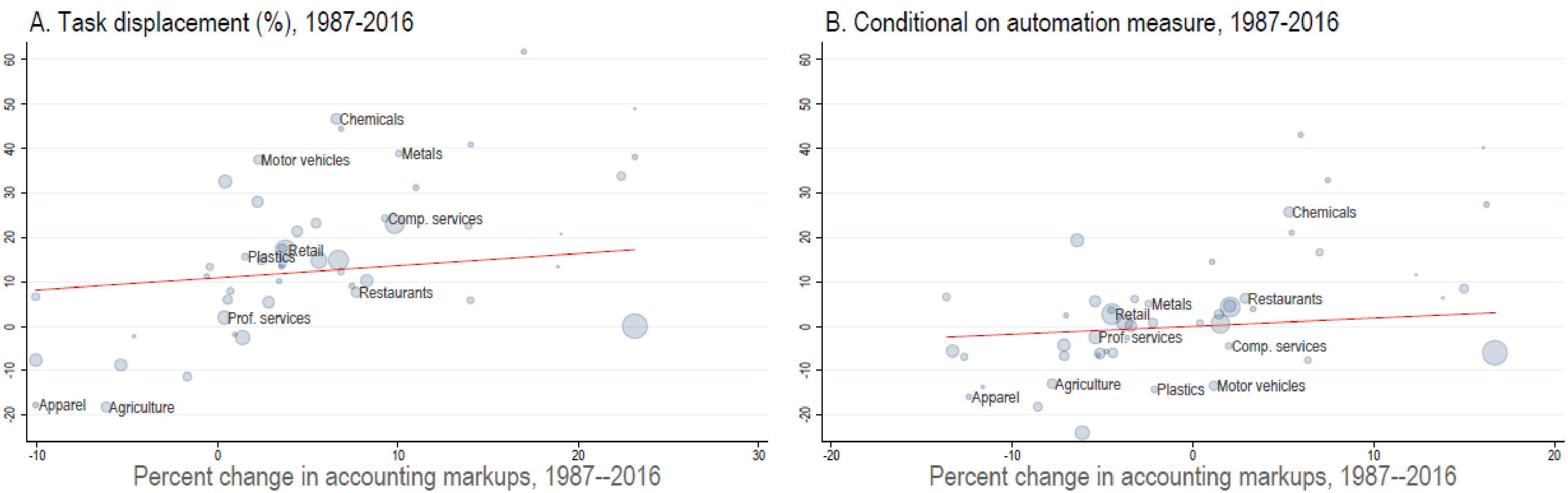


Figure A-2 (third panel): Relationship between task displacement 1987-2016 and Chinese import penetration

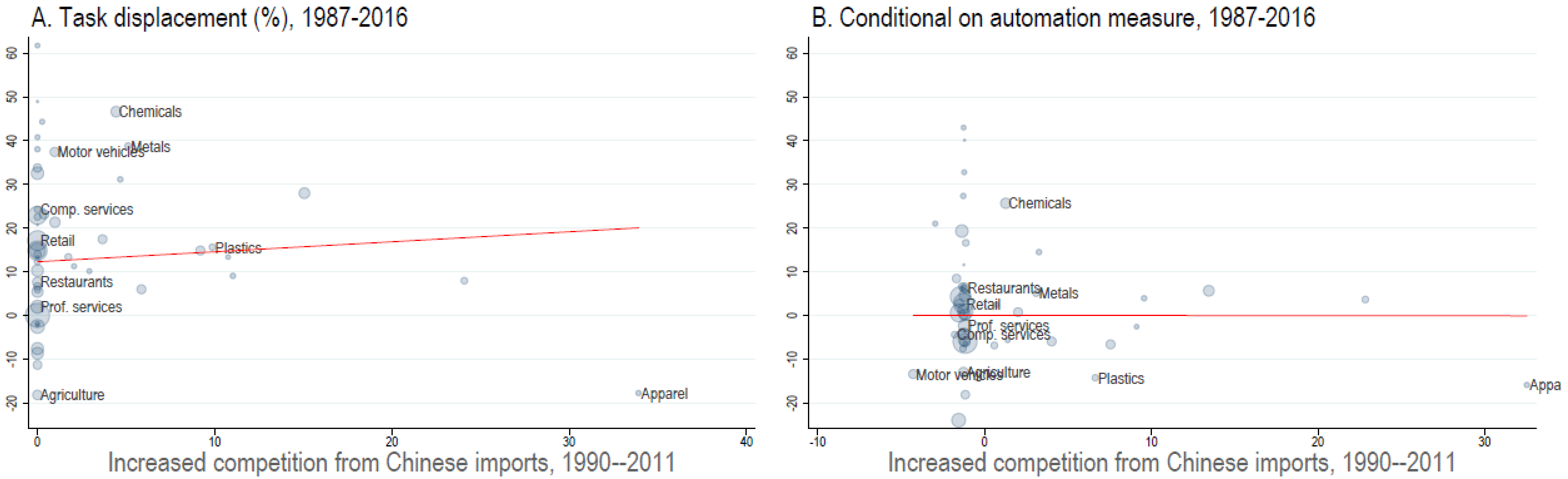
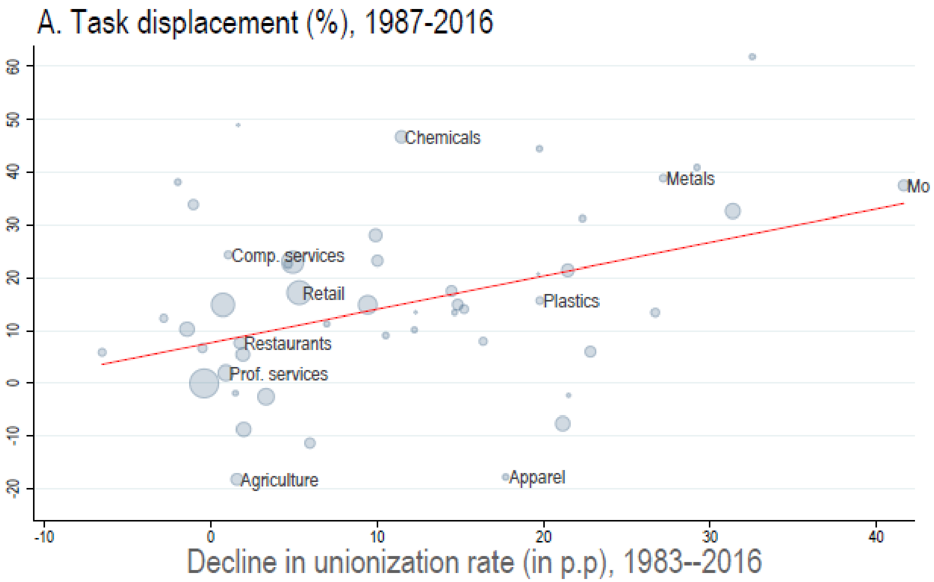


Figure A-2 (fourth panel): Relationship between task displacement 1987-2016 and declining unionization



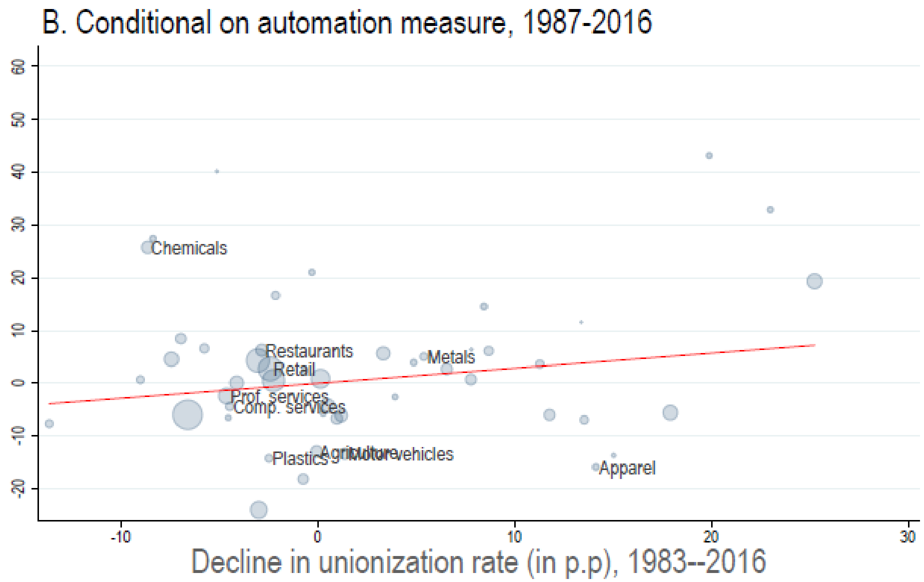


Table A-1: Determinants of task displacement and labor share declines across industries, 1987-2016

	Dependent variable: task displacement and labor share declines, 1987–2016							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Labor share decline, 1987–2016								
Adjusted penetration of robots	0.582		0.341	0.323	0.340	0.388	0.345	0.307
	(0.103)		(0.122)	(0.115)	(0.125)	(0.121)	(0.123)	(0.157)
Change in share of specialized machinery		1.940	1.655	1.629	1.659	1.484	1.654	1.611
		(0.301)	(0.330)	(0.340)	(0.354)	(0.365)	(0.333)	(0.340)
Change in share of software	3.157	3.072	3.489	3.323	3.501	3.419	3.493	3.489
	(1.206)	(1.247)	(1.206)	(1.274)	(1.345)	(1.262)	(1.231)	(1.210)
Change in share of imported intermediates				0.496				
				(0.295)				
Change tail index of revenue concentration					-0.008			
					(0.112)			
Percent change in accounting markups						0.174		
						(0.180)		
Chinese imports penetration							-0.027	
							(0.170)	
Decline in unionization rate								0.045
								(0.084)
F-stat technology variables	15.88	21.21	19.07	17.41	18.41	15.85	17.88	11.25
Share variance explained by technology	0.27	0.45	0.50	0.49	0.51	0.49	0.51	0.49
R-squared	0.27	0.45	0.50	0.51	0.51	0.54	0.51	0.51
Observations	49	49	49	49	49	49	49	49
Panel B: Task displacement, 1987–2016								
Adjusted penetration of robots	1.298		0.967	0.936	0.957	1.016	0.967	0.745
	(0.353)		(0.414)	(0.398)	(0.425)	(0.420)	(0.419)	(0.533)
Change in share of specialized machinery		3.089	2.282	2.238	2.307	2.104	2.282	2.001
		(0.502)	(0.579)	(0.599)	(0.638)	(0.659)	(0.585)	(0.657)
Change in share of software	5.943	5.222	6.401	6.124	6.474	6.328	6.402	6.403
	(1.956)	(2.176)	(1.925)	(2.038)	(2.195)	(2.073)	(1.957)	(1.881)
Change in share of imported intermediates				0.829				
				(0.559)				
Change tail index of revenue concentration					-0.046			
					(0.253)			
Pecrent change in accounting markups						0.182		
						(0.327)		
Chinese imports penetration							-0.003	
							(0.244)	
Decline in unionization rate								0.284
								(0.252)
F-stat technology variables	9.35	19.10	15.27	13.50	13.51	11.08	14.65	6.52
Share variance explained by technology	0.35	0.35	0.48	0.46	0.48	0.47	0.48	0.42
R-squared	0.35	0.35	0.48	0.49	0.48	0.49	0.48	0.50
Observations	49	49	49	49	49	49	49	49

Table A-2: Summary statistics for demographic groups by quintiles of task displacement

			Labor-market outcomes				Educational levels				
Quintile	N	Task displacement	Wage change 1980–2016	Wage change 1980–2007	Employment to population ratio change 1980–2016	Hourly wage 1980	Completed high-school	Some college	Completed college	Post-college	Share male
1—Lowest	191	4.8%	26.5%	24.2%	0.00 pp	$26.9	0.0%	12.2%	42.1%	44.8%	80.0%
2	141	15.5%	5.9%	7.1%	-0.80 pp	$18.3	17.5%	69.2%	1.8%	0.1%	61.8%
3	63	21.0%	3.1%	3.6%	-3.71 pp	$17.3	73.0%	13.2%	0.2%	0.0%	55.5%
4	69	24.9%	-5.1%	-3.4%	-8.72 pp	$15.1	36.9%	19.4%	0.0%	0.0%	66.3%
5—Highest	36	28.9%	-12.0%	-8.5%	-16.23 pp	$15.7	61.2%	1.2%	0.0%	0.0%	99.3%
All	500	16.8%	7.2%	7.6%	-4.80 pp	$19.9	32.8%	22.3%	13.4%	13.9%	73.0%

Notes: This table presents summary statistics for the 500 demographic groups used in our analysis. These groups are defined by gender, education, age, race, and native/immigrant status. The table breaks down these groups by quintiles of exposure to task displacement and provides summary statistics for groups in each quintile and for all groups pooled together. See the main text and Appendix B-4 for definitions and data sources.

Table A-3: Task displacement and real wage declines, 1980-2016

	Dependent variables: Change in wages and wage declines, 1980–2016			
	(1)	(2)	(3)	(4)
	Panel A. Dummy for declining real wages 1980–2016			
Task displacement	4.071 (0.265)	3.691 (0.639)	4.164 (1.062)	6.586 (1.614)
Industry shifters		-0.290 (0.383)	-0.495 (0.641)	-0.317 (0.789)
Exposure to industry labor share decline				-2.880 (2.499)
Relative specialization in routine jobs				-0.446 (0.266)
Share variance explained by task displacement	0.48	0.44	0.49	0.78
R-squared	0.48	0.49	0.65	0.66
Observations	500	500	500	500
	Panel B. Real wage declines, 1980–2016			
Task displacement	-0.445 (0.072)	-0.418 (0.080)	-0.647 (0.074)	-1.149 (0.166)
Industry shifters		0.021 (0.025)	0.272 (0.077)	0.185 (0.069)
Exposure to industry labor share decline				0.789 (0.208)
Relative specialization in routine jobs				0.087 (0.023)
Share variance explained by task displacement	0.50	0.47	0.73	1.30
R-squared	0.50	0.51	0.78	0.80
Observations	500	500	500	500
Other covariates: Manufacturing share, and education and gender dummies			✓	✓

Table A-4: Task displacement vs. SBTC—Controlling for changes in relative supply, 1980-2016

	Dependent variable: Change in real wages 1980–2016					
	SBTC by education level			Allowing for SBTC by wage level		
	OLS	OLS	IV	OLS	OLS	IV
	(1)	(2)	(3)	(4)	(5)	(6)
Gender: women	0.193	0.094	0.099	0.254	0.159	0.175
	(0.029)	(0.020)	(0.019)	(0.028)	(0.026)	(0.028)
Education: no high school	-0.098	-0.041	-0.044	0.039	0.014	0.018
	(0.076)	(0.050)	(0.051)	(0.034)	(0.033)	(0.032)
Education: some college	0.128	-0.066	-0.056	0.035	-0.052	-0.038
	(0.063)	(0.034)	(0.036)	(0.025)	(0.030)	(0.031)
Education: full college	0.375	-0.027	-0.008	0.192	0.006	0.037
	(0.084)	(0.054)	(0.055)	(0.036)	(0.049)	(0.049)
Education: more than college	0.499	0.026	0.049	0.292	0.070	0.107
	(0.067)	(0.079)	(0.076)	(0.047)	(0.067)	(0.063)
Log of hourly wage in 1980				0.254	0.137	0.156
				(0.055)	(0.049)	(0.053)
Change in supply	-0.104	-0.060	-0.062	-0.014	-0.026	-0.024
	(0.062)	(0.035)	(0.036)	(0.024)	(0.023)	(0.023)
Task displacement		-1.718	-1.634		-1.152	-0.962
		(0.312)	(0.297)		(0.208)	(0.201)
Share variance explained by:						
- educational dummies	0.75	0.04	0.08	0.39	0.08	0.13
- baseline wage				0.16	0.09	0.10
- supply changes	-0.28	-0.16	-0.17	-0.04	-0.07	-0.06
- task displacement		0.72	0.69		0.48	0.40
R-squared	0.43	0.75	0.74	0.80	0.83	0.83
First-stage F	9.98	18.73	2.96	34.42	34.96	5.82
Observations	493	493	493	493	493	493
Other covariates:						
Industry shifters and manufacturing share	✓	✓	✓	✓	✓	✓

Table A-5: Task displacement and changes in real hourly wages—Controlling for other trends and for exposure to industry labor share declines and relative specialization in routine jobs

	Dependent variable: Change in wages 1980–2016							
	OLS estimates				IV estimates			
	Chinese imports' competition (1)	Decline in unionization rates (2)	Rising K/L ratio by industry (3)	Rising TFP by industry (4)	Chinese imports' competition (5)	Decline in unionization rates (6)	Rising K/L ratio by industry (7)	Rising TFP by industry (8)
	Panel A. Controlling for main effect of other shocks							
Task displacement	-1.601	-1.816	-1.767	-1.813	-1.483	-1.753	-1.672	-1.763
	(0.524)	(0.467)	(0.574)	(0.476)	(0.669)	(0.520)	(0.640)	(0.586)
Effect of other shocks by industry	0.003	1.136	-0.028	-0.164	0.006	1.065	-0.017	-0.149
	(0.019)	(1.532)	(0.094)	(0.380)	(0.022)	(1.560)	(0.097)	(0.411)
Share variance explained by task displacement	0.67	0.76	0.74	0.76	0.62	0.73	0.70	0.74
R-squared	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84
First-stage F					206.62	277.15	210.06	187.47
Observations	500	500	500	500	500	500	500	500
	Panel B. Controlling for effects on workers in routine jobs							
Task displacement	-1.730	-2.388	-1.785	-1.726	-1.675	-2.447	-1.828	-1.695
	(0.459)	(0.715)	(0.444)	(0.438)	(0.512)	(0.905)	(0.478)	(0.496)
Effect of other shocks on routine jobs	-0.008	1.013	-0.127	-0.144	-0.007	1.060	-0.128	-0.141
	(0.012)	(0.854)	(0.064)	(0.247)	(0.012)	(0.983)	(0.063)	(0.252)
Share variance explained by task displacement	0.73	1.00	0.75	0.72	0.70	1.03	0.77	0.71
R-squared	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84
First-stage F					291.82	90.39	272.01	268.17
Observations	500	500	500	500	500	500	500	500

Table A-6: Task displacement and changes in real hourly wages—Controlling for differential effect of markups and concentration on routine jobs

	Dependent variable: Change in wages 1980–2016							
	OLS estimates				IV estimates			
	Rising sales concentration	Markups from accounting approach	Markups from materials share	Markups from DLEU (2020)	Rising sales concentration	Markups from accounting approach	Markups from materials share	Markups from DLEU (2020)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Controlling for effects of markups and concentration on workers in routine jobs								
Task displacement	-1.200	-1.363	-1.290	-1.106	-1.152	-1.238	-1.249	-1.082
	(0.196)	(0.354)	(0.237)	(0.171)	(0.218)	(0.440)	(0.240)	(0.174)
Effects of rising markups or	-0.526	0.207	0.041	1.870	-0.603	-0.146	0.074	1.891
concentration on routine jobs	(0.798)	(1.354)	(0.221)	(0.535)	(0.815)	(1.551)	(0.217)	(0.523)
Share variance explained by:								
- task displacement	0.50	0.57	0.54	0.46	0.48	0.52	0.52	0.45
- markups/concetration	0.01	-0.02	0.00	-0.08	0.01	0.01	0.00	-0.08
R-squared	0.84	0.84	0.84	0.85	0.84	0.84	0.84	0.85
First-stage F					515	178	723	721
Observations	500	500	500	500	500	500	500	500
Panel B. Net out markups from construction of task displacement								
Task displacement	-1.499	-1.363	-1.290	-1.106	-1.440	-1.238	-1.249	-1.082
	(0.238)	(0.354)	(0.237)	(0.171)	(0.270)	(0.440)	(0.240)	(0.174)
Effects of rising markups or	-1.101	-1.157	-1.249	0.764	-1.155	-1.384	-1.176	0.809
concentration on routine jobs	(0.745)	(1.064)	(0.419)	(0.609)	(0.757)	(1.169)	(0.419)	(0.593)
Share variance explained by:								
- task displacement	0.49	0.45	0.63	0.42	0.47	0.41	0.61	0.41
- markups/concetration	0.01	0.10	-0.08	-0.03	0.01	0.12	-0.08	-0.03
R-squared	0.84	0.84	0.84	0.85	0.84	0.84	0.84	0.85
First-stage F					348	178	723	721
Observations	500	500	500	500	500	500	500	500

Table A-7: Task displacement and changes in real hourly wages—Controlling for changes in markups and concentrations and for exposure to industry labor share declines and relative specialization in routine jobs

	Dependent variable: Change in wages 1980–2016							
	OLS estimates				IV estimates			
	Rising sales concentration	Markups from accounting approach	Markups from materials share	Markups from DLEU (2020)	Rising sales concentration	Markups from accounting approach	Markups from materials share	Markups from DLEU (2020)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Controlling for main effect of markups and concentration								
Task displacement	-1.389	-1.614	-2.074	-1.573	-1.374	-1.584	-2.131	-1.499
	(0.461)	(0.454)	(0.460)	(0.542)	(0.481)	(0.486)	(0.527)	(0.584)
Exposure to rising markups or concentration	1.969	0.721	-1.290	-0.404	1.984	0.749	-1.312	-0.468
	(1.556)	(1.754)	(0.552)	(1.159)	(1.485)	(1.766)	(0.576)	(1.163)
Share variance explained by:								
- task displacement	0.58	0.68	0.87	0.66	0.58	0.66	0.89	0.63
- markups/concetration	0.04	-0.01	-0.12	0.01	0.04	-0.01	-0.12	0.01
R-squared	0.84	0.84	0.85	0.84	0.84	0.84	0.85	0.84
First-stage F					326	385	252	214
Observations	500	500	500	500	500	500	500	500
Panel B. Net out markups from construction of task displacement								
Task displacement	-1.216	-1.651	-1.244	-1.970	-1.722	-1.881	-1.378	-1.228
	(0.554)	(0.560)	(0.229)	(0.401)	(0.605)	(0.574)	(0.345)	(0.436)
Exposure to rising markups or concentration	1.662	-0.640	-2.511	-2.045	0.951	-1.042	-2.731	-1.935
	(1.677)	(1.903)	(0.645)	(0.719)	(1.670)	(2.013)	(0.805)	(0.786)
Share variance explained by:								
- task displacement	0.40	0.54	0.60	0.74	0.57	0.62	0.67	0.46
- markups/concetration	0.04	0.01	-0.23	0.03	0.02	0.02	-0.25	0.03
R-squared	0.84	0.83	0.85	0.87	0.83	0.83	0.85	0.86
First-stage F					236	355	108	47
Observations	500	500	500	500	500	500	500	500

Table A-8: Task displacement and changes in real hourly wages for men, women, and native-born workers, 1980-2016

	Dependent variables: Change in wages, 1980–2016			
	(1)	(2)	(3)	(4)
	Panel A. Change in real wages for native-born workers 1980–2016			
Task displacement	-1.573	-1.288	-1.482	-1.660
	(0.099)	(0.191)	(0.231)	(0.526)
Industry shifters		0.212	0.113	0.213
		(0.115)	(0.176)	(0.292)
Share variance explained by task displacement	0.68	0.56	0.64	0.72
R-squared	0.68	0.71	0.85	0.85
Observations	250	250	250	250
	Panel B. Change in real wages for men 1980–2016			
Task displacement	-1.515	-1.083	-0.827	-1.570
	(0.107)	(0.193)	(0.085)	(0.302)
Industry shifters		0.374	0.604	0.520
		(0.158)	(0.124)	(0.123)
Share variance explained by task displacement	0.84	0.60	0.46	0.87
R-squared	0.84	0.86	0.96	0.96
Observations	250	250	250	250
	Panel C. Change in real wages for women 1980–2016			
Task displacement	-1.568	-1.676	-2.657	-2.805
	(0.182)	(0.234)	(0.367)	(0.790)
Industry shifters		-0.077	0.754	0.240
		(0.084)	(0.282)	(0.358)
Share variance explained by task displacement	0.53	0.57	0.90	0.95
R-squared	0.53	0.54	0.66	0.68
Observations	250	250	250	250
Other covariates:				
Manufacturing share, and education and gender dummies			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓

Table A-9: Task displacement and changes in real hourly wages, stacked-differences models, 1980-2000 and 2000-2016

	Dependent variable: Change in real wages 1980–2000, 2000–2016			
	(1)	(2)	(3)	(4)
	Panel A. Common coefficients across periods			
Task displacement	-1.310	-1.045	-0.938	-0.612
	(0.102)	(0.130)	(0.204)	(0.352)
Industry shifters		0.248	-0.438	-0.438
		(0.058)	(0.112)	(0.132)
Exposure to industry labor share decline				0.191
				(0.402)
Exposure to routine occupations				-0.056
				(0.041)
Share variance explained by				
- task displacement	0.46	0.36	0.33	0.21
- task displacement in 80s	0.26	0.21	0.19	0.12
- task displacement in 00s	0.59	0.47	0.42	0.27
R-squared	0.42	0.46	0.56	0.57
Observations	1000	1000	1000	1000
	Panel B. Allow covariates to have period-specific coefficients			
Task displacement	-1.310	-1.205	-1.273	-1.419
	(0.102)	(0.132)	(0.144)	(0.275)
Share variance explained by				
- task displacement	0.46	0.42	0.44	0.50
- task displacement in 80s	0.26	0.24	0.26	0.29
- task displacement in 00s	0.59	0.54	0.57	0.64
R-squared	0.42	0.58	0.74	0.74
Observations	1000	1000	1000	1000
	Panel C. Period specific estimates of task displacement			
Task displacement 80–00	-2.081	-1.333	-1.364	-2.109
	(0.277)	(0.248)	(0.252)	(0.728)
Task displacement 00–16	-1.100	-1.159	-1.220	-1.077
	(0.113)	(0.141)	(0.169)	(0.391)
Share variance explained by				
- task displacement	0.45	0.42	0.44	0.45
- task displacement in 80s	0.42	0.27	0.27	0.42
- task displacement in 00s	0.49	0.52	0.55	0.48
R-squared	0.46	0.58	0.74	0.74
Observations	1000	1000	1000	1000
Covariates:				
Industry shifters		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓

Table A-10: Task displacement and changes in real hourly wages—Alternative measures of jobs that can be automated

	Dependent variable: Change in real wages 1980–2016			
	(1)	(2)	(3)	(4)
	Panel A. Top 40			
Task displacement	-1.391	-1.016	-1.100	-2.496
	(0.147)	(0.164)	(0.193)	(0.534)
Share variance explained by task displacement	0.52	0.38	0.41	0.93
R-squared	0.52	0.64	0.82	0.84
Observations	500	500	500	500
	Panel B. Alternative definitions			
Task displacement	-1.875	-1.674	-1.673	-1.793
	(0.082)	(0.148)	(0.197)	(0.469)
Share variance explained by task displacement	0.76	0.67	0.67	0.72
R-squared	0.76	0.77	0.85	0.85
Observations	500	500	500	500
	Panel C. Occupations suitable to automation via robots			
Task displacement	-1.184	-1.163	-0.848	-0.658
	(0.080)	(0.111)	(0.157)	(0.291)
Share variance explained by task displacement	0.69	0.68	0.49	0.38
R-squared	0.69	0.69	0.81	0.82
Observations	500	500	500	500
	Panel D. Occupations suitable to automation via software			
Task displacement	-1.757	-1.709	-1.456	-1.546
	(0.132)	(0.150)	(0.222)	(0.513)
Share variance explained by task displacement	0.68	0.66	0.56	0.59
R-squared	0.68	0.68	0.81	0.82
Observations	500	500	500	500
	Panel E. Occupations suitable to automation via robots or software			
Task displacement	-1.459	-1.417	-1.027	-0.869
	(0.092)	(0.116)	(0.173)	(0.324)
Share variance explained by task displacement	0.71	0.69	0.50	0.42
R-squared	0.71	0.71	0.81	0.82
Observations	500	500	500	500
Covariates:				
Industry shifters		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓

Table A-11: Robustness checks for estimates of full general equilibrium effects

	Data for 1980–2016 (1)	Baseline calibration but setting $\lambda = 0.625$ (2)	Baseline calibration but setting $\sigma_i = 0.8$ (3)	Baseline calibration but setting $\sigma_i = 1.2$ (4)	Baseline calibration but setting $\pi = 0.5$ (5)	Estimates of propagation matrix for $\kappa = 1$ (6)	Estimates of propagation matrix for $\kappa = 5$ (7)
Wage structure:							
Share wage changes explained:							
-due to industry shifts		7.99%	4.85%	8.40%	8.08%	6.82%	6.88%
-adding direct displacement effects		83.00%	99.88%	100.90%	101.84%	100.58%	100.64%
-accounting for ripple effects		50.61%	34.75%	58.59%	46.40%	48.63%	49.17%
Rise in college premium	25.51%	22.40%	19.38%	22.31%	21.33%	21.63%	22.45%
-part due to direct displacement effect		32.74%	45.96%	35.89%	40.92%	40.92%	40.92%
Rise in post-college premium	40.42%	25.00%	20.54%	25.21%	23.19%	24.01%	24.87%
-part due to direct displacement effect		38.43%	54.06%	42.01%	48.04%	48.04%	48.04%
Change in gender gap	15.37%	2.53%	-4.45%	8.52%	1.17%	2.13%	1.73%
-part due to direct displacement effect		5.05%	-0.19%	12.81%	6.31%	6.31%	6.31%
Share with declining wages	53.10%	44.66%	48.97%	34.20%	26.49%	40.84%	46.24%
-part due to direct displacement effects		49.48%	55.52%	38.49%	34.58%	51.41%	48.83%
Wages for men with no high school	-8.21%	-7.67%	-2.74%	-11.32%	-3.16%	-7.27%	-7.13%
-part due to direct displacement effects		-11.02%	-10.02%	-17.50%	-9.32%	-15.32%	-13.25%
Wages for women with no high school	10.94%	1.47%	-2.18%	5.38%	4.66%	1.52%	1.07%
-part due to direct displacement effects		5.13%	2.48%	10.37%	10.86%	4.86%	6.93%
Aggregates:							
Change in average wages, $d\ln w$	29.15%	5.71%	6.41%	5.02%	9.52%	5.71%	5.71%
Change in GDP per capita, $d\ln y$	70.00%	23.52%	26.72%	20.34%	25.75%	22.75%	23.78%
Change in TFP, $d\ln \text{tfp}$	35%	3.77%	4.23%	3.31%	6.29%	3.77%	3.77%
Change in labor share, ds^L	-8 p.p.	-11.75 p.p	-13.40 p.p	-10.11 p.p.	-10.71 p.p	-11.24 p.p	-11.93 p.p.
Change in K/Y ratio	30.00%	42.12%	46.83%	37.21%	39.01%	40.62%	42.62%
Sectoral patterns:							
Share manufacturing in GDP	-8.80 p.p	-0.42 p.p	-0.36 p.p.	-0.46 p.p.	-0.60 p.p.	-0.42 p.p.	-0.42 p.p.
Change in manufacturing wage bill	-35.00%	-8.19%	-7.36%	-8.84%	-6.93%	-8.91%	-7.89%

B-1 Additional Theory Results

- **Existence of $\overline{q}$ and conditions for finite output**

- This section proves the existence of the threshold $\bar{q}$ introduced in Assumption 1 and provides primitive conditions under which the economy will produce finite output.

- ***Proposition B-1 (Existence of $\overline{q}$)*** *Suppose that workers can only produce non-overlapping sets of tasks (i.e., $\psi_g(x) > 0$ only if $\psi_{g'}(x) = 0$ for all $g \neq g'$). Consider the set of tasks where capital has positive productivity, $\mathcal{S} = \{x: \psi_k(x) > 0\}$. Suppose that there exists $\underline{\psi} > 0$, such that for all $x \in \mathcal{S}$ we have $\psi_k(x) > \underline{\psi}$. Then there exists a threshold $\bar{q}$ such that, if $q(x) > \bar{q}$ for all $x \in \mathcal{S}$, all the tasks in $\mathcal{S}$ are allocated to capital.*

- **Proof**. Consider an allocation with $\mathcal{T}_k = \mathcal{S}$ and where $\mathcal{T}_g = \{x: \psi_g(x) > 0, x \notin \mathcal{S}\}$.

- This allocation is the unique equilibrium of the economy if and only if

$$\frac{w_g}{A_g \cdot \psi_g(x)} \geq \frac{1}{q(x) \cdot A_k \cdot \psi_k(x)} \text{ for all } x \in \mathcal{S} \text{ and } g \in \mathcal{G}.$$

- Using the formula for wages in Equation (2) and the fact that $\psi_k(x) > \underline{\psi}$, it follows that a sufficient condition for this inequality is that

$$\text{(B-1)} \qquad \frac{\left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\frac{1}{M} \int_{x:\psi_g(x)>0, x \notin \mathcal{S}} \psi_g(x)^{\lambda-1} dx\right)^{\frac{1}{\lambda}}}{A_g \cdot \psi_g(x)} \geq \frac{1}{q_0 \cdot A_k \cdot \underline{\psi}} \text{ for all } x \in \mathcal{S} \text{ and } g \in \mathcal{G},$$

where $q_0 = \inf_{x \in \mathcal{S}} q(x)$.

- The left-hand side of (B-1) is increasing in q_0 (since output increases in $q(x)$ and the candidate task allocation remains unchanged); while the right-hand side is decreasing in q_0 and converges to zero as q_0 goes to infinity.

- Let $\bar{q}$ denote the point at which (B-1) holds with equality.

- It follows that if $q_o \geq \bar{q}$ (that is, $q(x) \geq \bar{q}$ for all $x \in \mathcal{S}$), inequality (B-1) holds and the task allocation described in Assumption 1 is the unique equilibrium. ■

- Finally, we provide conditions under which the economy produces final output.
- To do so, it is convenient to introduce the derived production function of the economy as

$$
\begin{aligned}
F(k,\ell) &= \max H(y_1,\dots,y_I) \\
\text{subject to: } y_i &= \left(\frac{1}{M}\int_{\mathcal{T}} (M\cdot y(x))^{\frac{\lambda-1}{\lambda}}\cdot dx\right)^{\frac{\lambda}{\lambda-1}} \quad \forall i\in\mathcal{I}, \\
y(x) &= A_k\cdot\psi_k(x)\cdot k(x) + \sum_{g\in\mathcal{G}} A_g\cdot\psi_g(x)\cdot\ell_g(x) \quad \forall x\in\mathcal{T}, \\
\ell_g &= \int_{\mathcal{T}} \ell_g(x)\cdot dx \quad \forall g\in\mathcal{G}, \\
k &= \int_{\mathcal{T}} (k(x)/q(x))\cdot dx.
\end{aligned}
$$

- This gives a standard constant-returns to scale production function that depends on the supply of labor and the total resources used to produce capital, k.

- ***Proposition B-2 (Finite output)*** *The economy produces finite output if and only if the following Inada condition holds:*

$$\lim_{k \to \infty} F_k(k, \ell) < 1. \qquad \text{(B-2)}$$

Moreover, in any equilibrium with positive and finite consumption, we have that $s^K \in [0,1)$*. Instead, in any equilibrium with infinite output,* $s^K = 1$*.*

- **Proof**. A competitive equilibrium maximizes $c(k) = F(k, \ell) - k$.
- When the Inada condition (B-2) holds, we have that $c(k)$ reaches a unique maximum at some $k^* \geq 0$.
- Moreover, $c(k^*) = (1 - s^K)F(k^*, \ell)$, which requires $s^K \in [0,1)$.
- When the Inada condition fails, $c(k)$ is an increasing function and the economy achieves infinite output.

- Moreover, because $\lim_{k \to \infty} F_k(k, \ell) > 1$, we have that $\lim_{k \to \infty} F_k(k, \ell) = m > 1$.
- Thus, the capital share is given by

$$s^K = \lim_{k \to \infty} \frac{F_k(k, \ell) \cdot k}{F(k, \ell)} = m \cdot \lim_{k \to \infty} \frac{k}{F(k, \ell)} = m \cdot \lim_{k \to \infty} \frac{1}{F_k(k, \ell)} = 1,$$

- where we used l'Hôpital's rule in the third step. ■

- **Extensions with Markups and Endogenous Labor supply**

- ***Proposition B-3 (Extension with markups)*** *Given labor-supply levels $\ell = (\ell_1, \ell_2, \dots, \ell_G)$ and industry markups $\mu = (\mu_1, \mu_2, \dots, \mu_I)$, and conditional on an allocation of tasks $\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \dots, \mathcal{T}_{Gi}\}$, equilibrium wages, industry prices, and output are a solution to the system of equations*

(B-3) $$w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\sum_{i \in \mathcal{I}} s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \mu_i^{-\lambda} \cdot \Gamma_{gi}\right)^{\frac{1}{\lambda}}$$

(B-4) $$p_i = \frac{\mu_i}{A_i} \cdot \left(A_k^{\lambda-1} \cdot \Gamma_{ki} + \sum_{g \in \mathcal{G}} w_g^{1-\lambda} \cdot A_g^{\lambda-1} \cdot \Gamma_{gi}\right)^{\frac{1}{1-\lambda}}$$

(B-5) $$1 = \sum_{i \in \mathcal{I}} s_i^Y(p).$$

- ***Proposition B-3 (Cont'd)*** *Moreover, following advances in automation or changes in markups, the change in the real wage of group g is given by*

$$d\ln w_g = \frac{\varepsilon_g}{\lambda} \cdot d\ln y + \frac{1}{\lambda}\Theta_g \cdot d\ln \zeta - \frac{1}{\lambda}\Theta_g \cdot d\ln \boldsymbol{\Gamma}^{disp} \ \textit{for all } g \in \mathcal{G},$$

where the industry shifters are now given by

$$d\ln \zeta_g = \sum_{i \in \mathcal{I}} \omega_{gi} \cdot \left(\frac{\partial \ln s_i^Y(\boldsymbol{p})}{\partial \ln \boldsymbol{p}} \cdot d\ln \boldsymbol{p} + (\lambda - 1) \cdot d\ln p_i - d\ln \mu_i \right) \ \textit{for all } g \in \mathcal{G}.$$

- **Proof**. Let

$$\mu_i : \frac{p}{\mathrm{mc}_i}$$

denote the markup charged in industry i, where p_i is the industry price and mc_i the marginal cost.

- Production optimality requires that

$$\mathrm{mc}_i = p(x) \Big/ \frac{\partial y}{\partial y(x)} \Rightarrow p(x) = \frac{p_i}{\mu_i} \cdot \frac{\partial y}{\partial y(x)}.$$

- Using this last equation, we can solve for the quantity of task x used in sector i as

$$y(x) = \frac{1}{M_i} \cdot y \cdot s_i^Y(p) \cdot (\mu \cdot p(x))^{-\lambda} \cdot (A_i p_i)^{\lambda-1} \cdot \mu_i^{-\lambda}, \tag{B-6}$$

where $p(x)$ is the price of task x.

- Following the same steps as in the proof of Proposition 3, we can therefore compute the demand for capital and labor at task x as

$$k(x)/q(x) = \begin{cases} \frac{1}{M_i} \cdot y \cdot \mu_i^{-\lambda} \cdot s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot (A_k \cdot q(x) \cdot \psi_k(x))^{\lambda-1} & \text{if } x \in \mathcal{T}_{ki} \\ 0 & \text{if } x \notin \mathcal{T}_k. \end{cases}$$

$$\ell_g(x) = \begin{cases} \frac{1}{M_i} \cdot y \cdot \mu_i^{-\lambda} \cdot s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot (A_g \cdot \psi_g(x))^{\lambda-1} \cdot w_g^{-\lambda} & \text{if } x \in \mathcal{T}_g \\ 0 & \text{if } x \notin \mathcal{T}_g. \end{cases}$$

- To derive Equation (B-3), we add-up the demand for labor across tasks, and rearrange the resulting expression:

$$\ell_g = \sum_{i \in \mathcal{I}} \int_{\mathcal{T}_{gi}} \frac{1}{M_i} \cdot y \cdot \mu_i^{-\lambda} \cdot s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot (A_g \cdot \psi_g(x))^{\lambda-1} \cdot w_g^{-\lambda} \cdot dx$$

$$\Rightarrow w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\sum_{i \in \mathcal{I}} \mu_i^{-\lambda} \cdot s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \frac{1}{M_i} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} dx\right)^{\frac{1}{\lambda}}.$$

- To derive the industry price index in Equation (B-5), note that due to constant returns to scale and the presence of markups, we must have

$$\frac{1}{\mu_i} \cdot p_i \cdot y_i = \int_{\mathcal{T}_i} p(x) \cdot y(x) dx \Rightarrow p_i = \frac{\mu_i}{A_i} \left(\frac{1}{M_i} \int_{\mathcal{T}_i} p(x)^{1-\lambda} dx\right)^{\frac{1}{1-\lambda}}.$$

- Using the allocation of tasks $\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \dots, \mathcal{T}_{Gi}\}$, this implies

$$p_i = \frac{\mu_i}{A_i}\left(\frac{1}{M_i}\int_{\mathcal{T}_i} p(x)^{1-\lambda}dx\right)^{\frac{1}{1-\lambda}}$$

$$= \frac{1}{A_i}\left(A_k \cdot \left(\frac{1}{M_i}\int_{\mathcal{T}_{ki}} (q(x)\cdot\psi_k(x))^{\lambda-1}dx\right) + \sum_{g\in\mathcal{G}} w_g^{1-\lambda}\cdot A_g^{\lambda-1}\cdot\left(\frac{1}{M_i}\int_{\mathcal{T}_{gi}}\psi_g(x)^{\lambda-1}dx\right)\right)^{\frac{1}{1-\lambda}},$$

which yields the expression for industry prices in the proposition.

- Finally, because industry shares must add up to 1, we have Equation (B-5), which is equivalent to a price-index condition for industries.

- The expressions for wage changes and industry shifters are derived using the same steps as in the proof of Proposition 4, but now accounting for the markup term in Equation (B-4). ■

- ***Proposition B-4 (Extension with labor supply)*** *Suppose that households choose their labor supply and consumption to maximize*

$$\max_{\ell_g, c_g} \frac{c_g^{1-\varsigma_c}}{1-\varsigma_c} - \frac{\ell_g^{1+\varsigma_\ell}}{1+\varsigma_\ell} \quad \textit{subject to: } c_g \leq w_g \cdot \ell_g,$$

and let $\varsigma = (1-\varsigma_c)/(\varsigma_c + \varsigma_\ell)$. *Conditional on an allocation of tasks* $\{\mathcal{T}_{ki}, \mathcal{T}_{1i}, \ldots, \mathcal{T}_{Gi}\}$, *equilibrium wages, labor supply, industry prices, and output solve the system*

$$\text{(B-7)} \qquad w_g = y^{\frac{1}{\lambda+\varsigma}} \cdot A_g^{\frac{\lambda-1}{\lambda+\varsigma}} \cdot \left(\sum_{i \in \mathcal{I}} s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{gi} \right)^{\frac{1}{\lambda+\varsigma}}$$

$$\text{(B-8)} \qquad \ell_g = y^{\frac{\varsigma}{\lambda+\varsigma}} \cdot A_g^{\frac{\varsigma\cdot(\lambda-1)}{\lambda+\varsigma}} \cdot \left(\sum_{i \in \mathcal{I}} s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{gi} \right)^{\frac{\varsigma}{\lambda+\varsigma}}$$

$$\text{(B-9)} \qquad p_i = \frac{1}{A_i} \left(A_k^{\lambda-1} \cdot \Gamma_{ki} + \sum_{g \in \mathcal{G}} w_g^{1-\lambda} \cdot A_g^{\lambda-1} \cdot \Gamma_{gi} \right)^{\frac{1}{1-\lambda}}$$

$$\text{(B-10)} \qquad c = \left(1 - A_k^{\lambda-1} \cdot \sum_{i \in \mathcal{I}} s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{ki} \right) \cdot y$$

$$\text{(B-11)} \qquad 1 = \sum_{i \in \mathcal{I}} s_i^Y(p).$$

- ***Proposition B-4 (Cont'd)*** *Moreover, the effect of task displacement on wages and employment is given by*

$$d\ln w_g = \frac{\varepsilon_g}{\lambda+\varsigma}\cdot d\ln y + \frac{1}{\lambda+\varsigma}\Theta_g \cdot d\ln\zeta - \frac{1}{\lambda+\varsigma}\Theta_g\cdot d\ln \mathbf{\Gamma}^{disp} \quad for\ all\ g \in \mathcal{G},$$

$$d\ln \ell_g = \frac{\varepsilon_g\cdot\varsigma}{\lambda+\varsigma}\cdot d\ln y + \frac{\varsigma}{\lambda+\varsigma}\Theta_g \cdot d\ln\zeta - \frac{\varsigma}{\lambda+\varsigma}\Theta_g\cdot d\ln \mathbf{\Gamma}^{disp} \quad for\ all\ g \in \mathcal{G},$$

where the propagation matrix now becomes

$$\Theta = \left(\mathbb{1} - \frac{1}{\lambda+\varsigma}\frac{\partial \ln\Gamma(w,\zeta,\Psi)}{\partial\ln w}\right)^{-1}$$

- **Proof**. The household problem gives the labor-supply curve

(B-12) $$\ell_g = w_g^{\varsigma}.$$

- Plugging this labor-supply curve into the expression for wages in Equation (8) yields

$$w_g = \left(\frac{y}{w_g^{\varsigma}}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\sum_{i\in\mathcal{I}} s_i^Y(p) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{gi}\right)^{\frac{1}{\lambda}}.$$

- Using this equation to solve for w_g yields Equation (B-7).

- In turn, plugging (B-7) into Equation (B-12) yields (B-8).

- The derivations of the remaining expressions in the proposition are identical to those in the proof of Proposition 3.

- Turning to the effect of technologies on wage changes, and following the same steps as in the derivation of Proposition 4, we obtain

$$d\ln w_g = \frac{1}{\lambda} d\ln y - \frac{1}{\lambda} d\ln \ell_g - \frac{1}{\lambda} d\ln \Gamma_g^{\text{disp}} + \frac{1}{\lambda} \sum_{i\in\mathcal{I}} \omega_{gi} \cdot d\ln \zeta_i + \frac{1}{\lambda} \frac{\partial \ln \Gamma_g}{\partial \ln w} \cdot d\ln w.$$

- Using the fact that $d\ln \ell_g = \varsigma \cdot d\ln w_g$ (from the labor-supply curve in B-12), we can rewrite this as

$$d\ln w_g = \frac{1}{\lambda+\varsigma} d\ln y - \frac{1}{\lambda+\varsigma} d\ln \Gamma_g^{\text{disp}} + \frac{1}{\lambda+\varsigma} \sum_{i\in\mathcal{I}} \omega_{gi} \cdot d\ln \zeta_i + \frac{1}{\lambda+\varsigma} \frac{\partial \ln \Gamma_g}{\partial \ln w} \cdot d\ln w.$$

- Solving this system for wage changes gives the formula for the propagation matrix in the proposition. ■

- **Propagation Matrix and Elasticities of Substitution**

- This section provides additional properties of the propagation matrix and relates it to traditional definitions of elasticities of substitution.

- First, let us recall that the *Morishima elasticity of substitution* between capital and labor of type g can be defined as

$$\sigma_{k,\ell_g} = \frac{1}{1 + \left.\frac{\partial \ln(s_g^L / s^k)}{\partial \ln A_k}\right|_k}.$$

- Similarly, the *Morishima elasticity of substitution* between capital and labor can be defined as

$$\sigma_{k,\ell} = \frac{1}{1 + \left.\dfrac{\partial \ln(s^L/s^k)}{\partial \ln A_k}\right|_k},$$

and the *Morishima elasticity of substitution* between labor of type g' and g can be defined as

$$\sigma_{\ell_{g'},\ell_g} = \frac{1}{1 + \left.\dfrac{\partial \ln(s_g^L/s_{g'}^L)}{\partial \ln \ell_{g'}}\right|_k}.$$

- The Morishima elasticities tell us about changes in factor shares as one factor becomes more abundant or productive.
- In the presence of multiple factors, these elasticities need not be symmetric, as is the case with only two factors of production.
- Also, define the q*−elasticity of substitution* between capital and labor of type g by the identity

$$\sigma^{Q}_{k,\ell_g} = \frac{1}{\left.\frac{1}{s^k}\frac{\partial \ln w_g}{\partial \ln A_k}\right|_k},$$

and the q*−elasticity of substitution* between labor of type g' and g by

$$\sigma^{Q}_{\ell_{g'},\ell_g} = \frac{1}{\left.\frac{1}{s^L_{g'}}\frac{\partial \ln w_g}{\partial \ln \ell_{g'}}\right|_k}.$$

- The q−elasticities of substitution tell us whether factors are q−complements (a positive elasticity) or q−substitutes (a negative elasticity), and are symmetric in a competitive economy by definition (a corollary of Young's theorem).

- Note that in all these definitions we are holding k—the resources devoted to produce capital—constant.

- ***Proposition B-5 (Elasticities of substitution and Θ)*** *The Morishima elasticity of substitution between capital and labor is*

$$\sigma_{k,\ell} = \frac{1}{\frac{\bar{\varepsilon}}{\lambda} + \frac{1}{s^k} \cdot (\bar{\varepsilon} - 1)} \qquad \textit{where: } \bar{\varepsilon} := \sum_{g \in \mathcal{G}} \frac{s_g^L}{s^L} \varepsilon_g \in (0,1).$$

Moreover, the Morishima elasticities of substitution between pairs of factors are

$$\sigma_{k,\ell_g} = \frac{1}{\frac{\varepsilon_g}{\lambda} s^k + \frac{\bar{\varepsilon}}{\lambda} s^L + (\varepsilon_g - 1) + \frac{s^L}{s^k}(\bar{\varepsilon} - 1)} \qquad \sigma_{\ell_{g'},\ell_g} = \frac{1}{1 + \frac{s_{g'}^L}{\lambda} \cdot \left(\varepsilon_g - \varepsilon_{g'} - \left(\frac{\theta_{gg'}}{s_{g'}^L} - \frac{\theta_{g'g'}}{s_{g'}^L}\right)\right)},$$

and the q−elasticities of substitution are

$$\sigma_{k,\ell_g}^Q = \frac{1}{\frac{\varepsilon_g}{\lambda} + \frac{1}{s^k} \cdot (\varepsilon_g - 1)} \qquad \sigma_{\ell_{g'},\ell_g}^Q = \frac{1}{\frac{1}{\lambda} \cdot \left(\varepsilon_g - \frac{\theta_{gg'}}{s_{g'}^L}\right)}.$$

- **Proof**. First, note that we can rewrite the definition of the set $\mathcal{T}_g$ as

$$\mathcal{T}_g = \left\{ x : \frac{1}{\psi_g(x)} \cdot \frac{w_g \cdot A_k}{A_g} \leq \frac{1}{\psi_{g'}(x)} \cdot \frac{w_{g'} \cdot A_k}{A_{g'}}, \frac{1}{\psi_k(x) \cdot q(x)} \forall g' \right\}$$

$$\mathcal{T}_k = \left\{ x : \frac{1}{\psi_k(x) \cdot q(x)} \leq \frac{1}{\psi_{g'}(x)} \cdot \frac{w_{g'} \cdot A_k}{A_{g'}} \forall g' \right\}.$$

- These expressions imply that the effect of an increase in A_k on the allocation of tasks is equivalent to a uniform rise in wages.

- That is:

$$\frac{\partial \ln \Gamma_g}{\partial \ln A_k} = \sum_{g'} \frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}}.$$

- Using this property, we can compute the change in wages as

$$d\ln w_g = \frac{1}{\lambda} d\ln y + \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln \boldsymbol{w}} \cdot d\ln \boldsymbol{w} + \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln \boldsymbol{w}} \cdot d\ln \boldsymbol{A}_k.$$

- We can then solve for the change in wages as

$$d\ln w = \frac{1}{\lambda}\Theta d\ln y + \Theta \frac{1}{\lambda}\Sigma \cdot d\ln \boldsymbol{A}_k.$$

- Moreover, using the definition of Θ, we get

$$\Theta \frac{1}{\lambda}\Sigma = \Theta - \mathbb{1}.$$

- Plugging this into the expression for wages, we obtain

$$d \ln w_g = \frac{\varepsilon_g}{\lambda} \cdot d \ln y + (\varepsilon_g - 1) d \ln A_k.$$

- Finally, holding k constant, we have that $d \ln y = s^K \cdot d \ln A_k$. Therefore

(B-13) $$\frac{1}{\sigma^Q_{k,\ell_g}} = \frac{1}{s^k} \frac{\partial \ln w_g}{\partial \ln A_k}\bigg|_k = \frac{\varepsilon_g}{\lambda} + \frac{1}{s^k} \cdot (\varepsilon_g - 1).$$

- In addition, we also have that

(B-14) $$\frac{\partial \ln s^L_g}{\partial \ln A_k}\bigg|_k = \left(\frac{\varepsilon_g}{\lambda} - 1\right) \cdot s^k + (\varepsilon_g - 1)$$

- Using Equation (B-14), we can compute the Morishima elasticity of substitution between capital and labor as

$$\begin{aligned}
\frac{1}{\sigma_{k,\ell}} &= 1 + \left.\frac{\partial \ln(s^L/s^k)}{\partial \ln A_k}\right|_k \\
&= 1 + \frac{1}{s^k} \cdot \left.\frac{\partial \ln s^L}{\partial \ln A_k}\right|_k \\
&= 1 + \frac{1}{s^k} \sum_{g \in \mathcal{G}} \frac{s_g^L}{s^L} \left.\frac{\partial \ln s_g^L}{\partial \ln A_k}\right|_k \\
&= 1 + \frac{1}{s^k} \sum_{g \in \mathcal{G}} \frac{s_g^L}{s^L} \cdot \left(\left(\frac{\varepsilon_g}{\lambda} - 1\right) \cdot s^k + (\varepsilon_g - 1)\right) \\
&= 1 + \frac{1}{s^k} \left(\left(\frac{\bar{\varepsilon}}{\lambda} - 1\right) \cdot s^k + (\bar{\varepsilon} - 1)\right) \\
&= \frac{\bar{\varepsilon}}{\lambda} + \frac{1}{s^k} \cdot (\bar{\varepsilon} - 1)
\end{aligned}$$

- Similarly, using Equation (B-14), we can compute the Morishima elasticity of substitution between capital and labor of type g as

$$\begin{aligned}\frac{1}{\sigma_{k,\ell_g}} &= 1 + \left.\frac{\partial \ln(s_g^L/s^k)}{\partial \ln A_k}\right|_k \\ &= 1 + \left.\frac{\partial \ln s_g^L}{\partial \ln A_k}\right|_k + \frac{s^L}{s^k}\left.\frac{\partial \ln s^L}{\partial \ln A_k}\right|_k \\ &= 1 + \left(\frac{\varepsilon_g}{\lambda} - 1\right)\cdot s^k + (\varepsilon_g - 1) + \frac{s^L}{s^k}\left(\left(\frac{\bar{\varepsilon}}{\lambda} - 1\right)\cdot s^k + (\bar{\varepsilon} - 1)\right) \\ &= \frac{\varepsilon_g}{\lambda}s^k + \frac{\bar{\varepsilon}}{\lambda}s^L + (\varepsilon_g - 1) + \frac{s^L}{s^k}(\bar{\varepsilon} - 1).\end{aligned}$$

- We now turn to the elasticities involving changes in $\ell_{g'}$.

- Following a change in $\ell_{g'}$, we have:

$$d\ln w_g = \frac{\varepsilon_g}{\lambda}d\ln y - \frac{\theta_{gg'}}{\lambda}d\ln \ell_{g'}. \tag{B-15}$$

- Holding k constant, $d \ln y = s^L_{g'}$.
- Therefore,

$$\frac{1}{\sigma^Q_{\ell_{g'},\ell_g}} = \frac{1}{s^L_{g'}} \frac{\partial \ln w_g}{\partial \ln \ell_{g'}}\bigg|_k = \frac{1}{\lambda} \cdot \left(\varepsilon_g - \frac{\theta_{gg'}}{s^L_{g'}} \right).$$

- Finally, we can write the Morishima elasticity of substitution between labor of type g' and g as

$$\frac{1}{\sigma_{\ell_{g'},\ell_g}} = 1 + \frac{\partial \ln (s^L_g / s^L_{g'})}{\partial \ln \ell_{g'}}\bigg|_k = 1 + \frac{\partial \ln w_g}{\partial \ln \ell_{g'}}\bigg|_k - \frac{\partial \ln w_{g'}}{\partial \ln \ell_{g'}}\bigg|_k .$$

- Using the formula for the change in wages in Equation (B-15), we obtain

$$\frac{1}{\sigma_{\ell_{g'},\ell_g}} = 1 + \frac{s_{g'}^L}{\lambda} \cdot \left(\varepsilon_g - \varepsilon_{g'} - \left(\frac{\theta_{gg'}}{s_{g'}^L} - \frac{\theta_{g'g'}}{s_{g'}^L} \right) \right),$$

which completes proof of the proposition. ■

- ***Proposition B-6 (Quasi-symmetry of the propagation matrix)*** *The propagation matrix satisfies the symmetry property*

$$\text{(B-16)} \qquad \varepsilon_g - \frac{\theta_{gg'}}{s^L_{g'}} = \varepsilon_{g'} - \frac{\theta_{g'g}}{s^L_g}.$$

- **Proof**. By definition $\sigma^Q_{\ell_{g'},\ell_g} = \sigma^Q_{\ell_g,\ell_{g'}}$, which implies the symmetry property in (B-16). ■

B-2 Estimating the Propagation Matrix

- This appendix provides additional details regarding the estimation of the propagation matrix.

- Our estimation strategy makes two assumptions:

    1. The propagation matrix has a common diagonal term $\theta_{gg} = \theta \geq 0$. This is motivated by the strong reduced form evidence between task displacement and the observed change in real wages.

    2. The extent of competition for tasks between groups is determined by their similarity across a set of characteristics $\mathcal{N}$. We operationalize this by assuming that

$$\frac{\theta_{gg'}}{s^L_{g'}} + \frac{\theta_{g'g}}{s^L_g} = 2 \sum_{n \in \mathcal{N}} \beta_n \cdot f(d^n_{g,g'}).$$

- Using these two assumptions, and combining them with the theoretical restriction that

$$\varepsilon_g - \frac{\theta_{gg'}}{s^L_{g'}} = \varepsilon_{g'} - \frac{\theta_{g'g}}{s^L_g},$$

yields the parametrization used in the main text.

B-3 Measuring Task Displacement

- **Theoretical derivations**

- This section derives our measures of task displacement in the extended version of our model that allows for markups.

- We assume that tasks can be partitioned into routine tasks $\mathcal{R}_i$ and non-routine tasks $\mathcal{N}_i$, whose union equals $\mathcal{T}_i$.

- Moreover, let $\mathcal{R}_{gi}$ and $\mathcal{N}_{gi}$ denote the (disjoint) sets of routine and non-routine tasks allocated to workers of type g.

- Assumption 2 is equivalent to:

  i. Only routine tasks have been automated, which implies that $\mathcal{D}_{gi} \subset \mathcal{R}_{gi}$

  ii. Routine tasks in a given industry have been automated at the same rate for all workers, which implies that

$$\frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1} dx}{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1} dx} = \vartheta_i \geq 0 \text{ for all } g.$$

- Before continuing with our derivations, we introduce some notation that we will use throughout this appendix.

- Define by ω_X^Y the share of wages in some cell X earned within another sub-cell Y.

- For example, define ω_g^i as the share of wages earned by members of group g in industry i as a fraction of their total wage income:

$$\omega_g^i = \frac{s_i^Y(\,p) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{gi}}{\sum_{g'} s_{g'}^Y(\,p) \cdot (A_{g'} p_{g'})^{\lambda-1} \cdot \Gamma_{gg'}}.$$

- Define ω_{gi}^{R} as the share of wages earned by members of group g in industry i in routine jobs as a fraction of the total wage income earned by workers of group g in industry i:

$$\omega_{gi}^{R} = \frac{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1} dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} dx}.$$

- And define ω_{i}^{R} as the share of wages earned by workers in industry i in routine jobs as a fraction of the total wage income earned by workers in industry i:

$$\omega_{i}^{R} = \frac{\sum_{g \in \mathcal{G}} w_g^{1-\lambda} \int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1} dx}{\sum_{g \in \mathcal{G}} w_g^{1-\lambda} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} dx}.$$

- We next define the average cost-saving gains from automating tasks in sector i as

$$\pi_i = \sum_{g \in \mathcal{G}} \frac{\omega_i^{Rg}}{\omega_i^R} \cdot \pi_{gi},$$

- where ω_i^{Rg} is the share of wages in industry i paid to g workers in routine jobs, and ω_i^R is the share of wages in industry i paid to workers in routine jobs.

- Finally, for each type of worker g, define the elasticity σ_{gi}^L by

$$\frac{1}{\omega_i^g} \cdot \frac{\partial \ln s_i^L}{\partial \ln w_g} = s^k \cdot (1 - \sigma_{gi}^L).$$

- When $\sigma_{gi}^L > 1$, an increase in w_g reduces the labor share.

- Instead, when $\sigma_{gi}^L < 1$, an increase in w_g increases the labor share.

- The following proposition characterizes the change in the labor share as a function of various driving forces:

1. Task displacement generated by automation or offshoring, $d \ln \Gamma_{gi}^{\text{disp}}$ generating productivity gains $\pi_{gi} > 0$;

2. Productivity deepening and factor augmenting technologies taking place in that industry, and denoted by $d \ln \Gamma_{gi}^{\text{deep}}$ and $d \ln A_{gi}$. Note that, in this proposition, factor-augmenting technologies may vary by industry;

3. Changes in markups at the industry level, denoted by $d \ln \mu_i$;

4. Changes in wages, $d \ln w_g$ due to other shocks in the economy or changes in factor supplies;

5. And changes in the user cost of capital. In particular, we assume there are two types of technologies increasing the productivity of capital or reducing its price. On the one hand we have the task displacement technologies introduced above. And on the other hand, we have uniform declines in the user cost of capital driven by lower capital prices at all tasks or cheap access to credit. Formally, we write $q(x) = \frac{1}{R_i} \cdot q_0(x)$ and consider changes in $q_0(x)$ leading to task displacement or changes in R_i common to all uses of capital in a given industry.

- **Proposition B-7 (Industry labor shares)** Let s_i^L denote the labor share in industry i. Also, let $q(x) = \frac{1}{R_i} \cdot q_0(x)$, where R_i captures uniform changes in the price of capital at all tasks in industry i, and let $w_{gi}^e = w_g / A_{gi}$ denote the wages per efficiency unit of labor paid in industry i for workers of type g. Following a change in technology (task displacement, productivity deepening, A_{ki}, A_{gi}, and A_i) factor prices (w_g, R_i), and markups μ_i, we have

$$
\begin{aligned}
d\ln s_i^L = & -d\ln \mu_i - \left(1 + (\lambda - 1) \cdot s_i^L \cdot \pi_i\right) \cdot \omega_i^R \cdot \vartheta_i \\
& + (1 - s_i^L) \cdot (\lambda - 1) \cdot \left(\sum_{g \in \mathcal{G}} \omega_i^g \cdot d\ln \Gamma_{gi}^{deep} - d\ln \Gamma_{ki}^{deep} \right) \\
& + (1 - s_i^L) \cdot (1 - \sigma_i^L) \cdot d\ln w_i^e - (1 - s_i^L) \cdot (1 - \sigma_i^K) \cdot d\ln(R_i / A_{ki}),
\end{aligned}
$$

- **Proposition B-7 (Cont'd)**

where

$$\sigma_i^L := \sum_{g \in \mathcal{G}} \frac{\omega_i^g \cdot d \ln w_g^e}{\sum_{g'} \omega_i^{g'} \cdot d \ln w_{g'}^e} \cdot \sigma_{gi}^L \qquad \sigma_i^K := \sum_{g \in \mathcal{G}} \omega_i^g \cdot \sigma_{gi}^L,$$

and

$$d \ln w_i^e = \sum_{g \in \mathcal{G}} \omega_i^g \cdot d \ln w_{gi}^e.$$

- **Proof**. Given a vector of wages and technologies, we can write the labor share as

$$s_i^L = \frac{1}{\mu_i} \cdot \frac{\sum_{g \in \mathcal{G}} {w_{gi}^e}^{1-\lambda} \cdot \Gamma_{gi}}{A_k^{\lambda-1} \cdot \Gamma_{ki} + \sum_{g \in \mathcal{G}} {w_{gi}^e}^{1-\lambda} \cdot \Gamma_{gi}}, \tag{B-17}$$

where recall that the denominator is also equal to

$$p_i^{1-\lambda} = A_k^{\lambda-1} \cdot \Gamma_{ki} + \sum_{g \in \mathcal{G}} {w_{gi}^e}^{1-\lambda} \cdot \Gamma_{gi}.$$

- We can decompose changes in the labor share into four terms:

$$d \ln s_i^L = \begin{array}{l}\text{contribution of} \\ \text{markups}\end{array} + \begin{array}{l}\text{contribution of} \\ \text{task displacement}\end{array} + \begin{array}{l}\text{contribution of} \\ \text{prod. deepening}\end{array}$$

$$+ \begin{array}{l}\text{contribution of} \\ \text{eff. wage changes}\end{array} + \begin{array}{l}\text{contribution of} \\ \text{price of capital}\end{array},$$

- which we now derive in detail.

1. **Contribution of markups**: this is simply given by $-d \ln \mu_i$.

2. **Contribution of task displacement**: we can compute this as

$$\begin{array}{l}\text{contribution of} \\ \text{task displacement}\end{array} = -\sum_{g \in \mathcal{G}} \omega_i^{Rg} \cdot \vartheta_i + (1-\lambda) \cdot s_i^L \cdot \sum_{g \in \mathcal{G}} \omega_i^{Rg} \cdot \vartheta_i \cdot \pi_{gi}$$

where the first term captures the effect of task displacement on the numerator and the second term the effect on the denominator of the labor share expression in Equation (B-17). Using the definition of π_i, this can be simplified as

$$\begin{array}{l}\text{contribution of} \\ \text{task displacement}\end{array} = -\left(1 + (\lambda - 1) \cdot s_i^L \cdot \pi_i\right) \cdot \omega_i^R \cdot \vartheta_i.$$

3. **Contribution of productivity deepening**: we can compute this as

$$\begin{matrix}\text{contribution of} \\ \text{prod. deepening}\end{matrix} = (\lambda - 1) \cdot \sum_{g \in \mathcal{G}} \omega_i^g \cdot d\ln \Gamma_{gi}^{\text{deep}} - (\lambda - 1) \cdot \left(s_g^L \cdot \sum_{g \in \mathcal{G}} \omega_i^g \cdot d\ln \Gamma_{gi}^{\text{deep}} + s_i^k \cdot d\ln \Gamma_{ki}^{\text{deep}} \right),$$

where the first term captures the effect of task displacement on the numerator and the second term the effect on the denominator of the labor share expression in equation (B-17). We can rewrite this as

$$\begin{matrix}\text{contribution of} \\ \text{prod. deepening}\end{matrix} = (\lambda - 1) \cdot (1 - s_i^L) \cdot \left(\sum_{g \in \mathcal{G}} \omega_i^g \cdot d\ln \Gamma_{gi}^{\text{deep}} - d\ln \Gamma_{ki}^{\text{deep}} \right)$$

4. **Contribution of wages per efficiency unit of labor**: We now turn to the contribution of wages per efficiency unit of labor. Using the definition of σ_{gi}^{L}, we can compute their effect as

$$\begin{array}{l}\text{contribution of} \\ \text{wage changes}\end{array} = \sum_{g \in \mathcal{G}} \omega_i^g \cdot (1 - s_i^L) \cdot (1 - \sigma_{gi}^L) \cdot d \ln w_{gi}^e.$$

Using the definition of σ_i^L and $d \ln w_i$, we obtain

$$\begin{array}{l}\text{contribution of} \\ \text{wage changes}\end{array} = (1 - s_i^L) \cdot (1 - \sigma_i^L) \cdot d \ln w_i.$$

5. **Contribution of price of capital**: To compute the effects of a uniform change in capital prices, we first provide explicit formulas for σ_{gi}^{L}, which we will use in our derivations below. We have that

$$\frac{1}{\omega_i^g} \cdot \frac{\partial \ln s_i^L}{\partial \ln w_g} = \frac{1}{\omega_i^g} \cdot \left(\omega_i^g \cdot (1-\lambda) + \sum_{g'} \omega_i^{g'} \cdot \frac{\partial \ln \Gamma_{g'i}}{\partial \ln w_{g'i}} - s^L \cdot \omega_i^g \cdot (1-\lambda) \right),$$

where the first two terms capture the effect of task displacement on the numerator and the third term the effect on the denominator of the labor share expression in Equation (B-17).

- Here, we used the fact that the effect of wages on the denominator equals the direct effect holding the task allocation constant—an implication of the envelope theorem. We can rewrite this expression as

$$\frac{1}{\omega_i^g} \cdot \frac{\partial \ln s_i^L}{\partial \ln w_g} = (1 - s_i^L) \cdot (1 - \lambda) + \sum_{g'} \frac{\omega_i^{g'}}{\omega_i^g} \cdot \frac{\partial \ln \Gamma_{g'i}}{\partial \ln w_{g'i}},$$

which implies that

$$\sigma_{gi}^L = \lambda - \frac{1}{1 - s_i^L} \cdot \sum_{g'} \frac{\omega_i^{g'}}{\omega_i^g} \cdot \frac{\partial \ln \Gamma_{g'i}}{\partial \ln w_{g'i}},$$

and

$$(1 - s_i^L) \cdot (\lambda - \sigma_{gi}^L) = \sum \sum_{g'} \frac{\omega_i^{g'}}{\omega_i^g} \cdot \frac{\partial \ln \Gamma_{g'i}}{\partial \ln w_{g'i}}. \tag{B-18}$$

- Consider a uniform change in the cost per efficiency unit of capital $d\ln(R_i/A_{ki})$ on the labor share of industry i.

- The effect of this change in the allocation of tasks is the same as a uniform reduction in wages of $d\ln(R_i/A_{ki})$.

- Moreover, the effect of $d\ln(R_i/A_{ki})$ on the denominator of the labor share is just its direct effect—an application of the envelope theorem.

- Thus, we get

$$\begin{array}{l}\text{contribution of}\\ \text{price of capital}\end{array} = -\sum_{g \in \mathcal{G}} \sum_{g'} \omega_i^{g'} \cdot \frac{\partial \ln \Gamma_{g'i}}{\partial \ln w_g} \cdot d\ln(R_i/A_{ki}) - s_i^k \cdot (1-\lambda) \cdot d\ln(R_i/A_{ki}),$$

where the first term captures the effect of task changes on the numerator and the second term the effect on the denominator of the labor share expression in Equation (B-17).

- Using Equation (B-18), we can rewrite this expression as

$$\begin{array}{l}\text{contribution of}\\ \text{price of capital}\end{array} = -\sum_{g \in \mathcal{G}} \omega_i^{g} \cdot (1-s_i^L) \cdot (\lambda - \sigma_{gi}^L) \cdot d\ln(R_i/A_{ki}) - s_i^k \cdot (1-\lambda) \cdot d\ln(R_i/A_{ki}).$$

- Finally, using the definition of σ_i^K, we can rewrite this as

$$\begin{matrix}\text{contribution of} \\ \text{price of capital}\end{matrix} = -(1 - s_i^L) \cdot (1 - \sigma_i^K) \cdot d\ln(R_i/A_{ki}),$$

  which completes the proof of the proposition. ■

- We are now in a position to derive the measures of task displacement used in the text.

- We start with the case with no ripple effects, no change in markups, and $\lambda = 1$, which gives the baseline measure in Equation (12).

- ***Proposition B-8*** *Suppose that Assumptions 1 and 2 hold. Suppose also that* $\lambda = 1$ *and there are no markups. Then* $\sigma_i^L = \sigma_i^K = 1$ *and task displacement can be computed as*

$$d\ln\Gamma_g^{disp} = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot (-d\ln s_i^L) \quad and \quad d\ln\Gamma_{gi}^{disp} = \frac{\omega_{gi}^R}{\omega_i^R} \cdot (-d\ln s_i^L).$$

Moreover, total task displacement taking place in industry i *is given by*

$$d\ln\Gamma_i^{disp} = \sum_{i\in\mathcal{I}} \omega_i^g \cdot d\ln\Gamma_{gi}^{disp} = (-d\ln s_i^L).$$

- **Proof**. Proposition B-7 implies that

$$d\ln s_i^L = -\omega_i^R \cdot \vartheta_i \Rightarrow \vartheta_i = \frac{(-d\ln s_i^L)}{\omega_i^R}.$$

- Moreover, by definition

$$d\ln \Gamma_{gi}^{\text{disp}} = \frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}dx} = \frac{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}dx} \cdot \frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}dx} = \omega_{gi}^R \cdot \vartheta_i = \frac{\omega_{gi}^R}{\omega_i^R} \cdot (-d\ln s_i^L).$$

and task displacement for worker groups is given by

$$d\ln \Gamma_g^{\text{disp}} = \sum_{i\in\mathcal{I}} \omega_g^i \cdot d\ln \Gamma_{gi}^{\text{disp}} = \sum_{i\in\mathcal{I}} \omega_g^i \frac{\omega_{gi}^R}{\omega_i^R} \cdot (-d\ln s_i^L).$$

■

- ***Proposition B-9** Suppose that Assumptions 1 and 2 hold. Then, $\sigma_i^L = \sigma_i^K = \lambda$. In the absence of productivity deepening or factor-augmenting technologies affecting the labor share, task displacement can be computed as*

$$d\ln\Gamma_g^{disp} = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\lambda)\cdot(d\ln w_i - d\ln R_i)}{1+(\lambda-1)\cdot s_i^L\cdot\pi_i}$$

$$d\ln\Gamma_{gi}^{disp} = \frac{\omega_{gi}^R}{\omega_i^R} \cdot \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\lambda)\cdot(d\ln w_i - d\ln R_i)}{1+(\lambda-1)\cdot s_i^L\cdot\pi_i}.$$

Moreover, total task displacement taking place in industry i is given by

$$d\ln\Gamma_i^{disp} = \sum_{i\in\mathcal{I}} \omega_i^g \cdot d\ln\Gamma_{gi}^{disp} = \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\lambda)\cdot(d\ln w_i - d\ln R_i)}{1+(\lambda-1)\cdot s_i^L\cdot\pi_i}.$$

- **Proof**. Proposition B-7 implies that

$$d \ln s_i^L = -d \ln \mu_i - (1 + (\lambda - 1) \cdot s_i^L \cdot \pi_i) \cdot \omega_i^R \cdot \vartheta_i + (1 - s_i^L) \cdot (1 - \lambda) \cdot (d \ln w_i - d \ln R_i),$$

which gives

$$\vartheta_i = \frac{-d \ln s_i^L - d \ln \mu_i + (1 - s_i^L) \cdot (1 - \lambda) \cdot (d \ln w_i - d \ln q_i)}{(1 + (\lambda - 1) \cdot s_i^L \cdot \pi_i) \cdot \omega_i^R}.$$

- Moreover, following the same steps as in the proof of Proposition B-8, we get

$$
\begin{aligned}
d\ln\Gamma_{gi}^{\text{disp}} &= \frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}dx} \\
&= \frac{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}dx} \cdot \frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}dx} \\
&= \omega_{gi}^R \cdot \vartheta_i \\
&= \frac{\omega_{gi}^R}{\omega_i^R} \cdot \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\lambda)\cdot(d\ln w_i - d\ln R_i)}{1+(\lambda-1)\cdot s_i^L\cdot\pi_i}.
\end{aligned}
$$

and task displacement for worker groups is given by

$$
d\ln\Gamma_g^{\text{disp}} = \sum_{i\in\mathcal{I}} \omega_g^i \cdot d\ln\Gamma_{gi}^{\text{disp}} = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\lambda)\cdot(d\ln w_i - d\ln R_i)}{1+(\lambda-1)\cdot s_i^L\cdot\pi_i}.
$$

■

- Our final Proposition derives a version of our measure of task displacement that allows for ripple effects.

- Equation (16) corresponds to the special case of this formula when there are no changes in markups and $\sigma_i^L = \sigma_i^K$.

- ***Proposition B-10*** *Suppose that Assumptions 2 holds. In the absence of productivity deepening or factor-augmenting technologies affecting the labor share, task displacement can be computed as*

$$d\ln\Gamma_g^{disp} = \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^R}{\omega_i^R} \cdot \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\sigma_i^L)\cdot d\ln w_i - (1-s_i^L)\cdot(1-\sigma_i^K)\cdot d\ln R_i}{1+(\lambda-1)\cdot s_i^L\cdot\pi_i}$$

$$d\ln\Gamma_{gi}^{disp} = \frac{\omega_{gi}^R}{\omega_i^R} \cdot \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\sigma_i^L)\cdot d\ln w_i - (1-s_i^L)\cdot(1-\sigma_i^K)\cdot d\ln R_i}{1+(\lambda-1)\cdot s_i^L\cdot\pi_i}.$$

Moreover, total task displacement taking place in industry i *is given by*

$$d\ln\Gamma_i^{disp} = \sum_{i\in\mathcal{I}} \omega_i^g \cdot d\ln\Gamma_{gi}^{disp}$$

$$= \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\sigma_i^L)\cdot d\ln w_i - (1-s_i^L)\cdot(1-\sigma_i^K)\cdot d\ln R_i}{1+(\lambda-1)\cdot s_i^L\cdot\pi_i}.$$

- **Proof**. Proposition B-7 implies that

$$d\ln s_i^L = -d\ln\mu_i - (1+(\lambda-1)\cdot s_i^L\cdot\pi_i)\cdot\omega_i^R\cdot\vartheta_i + (1-s_i^L)\cdot(1-\sigma_i^L)\cdot d\ln w_i - (1-s_i^L)\cdot(1-\sigma_i^K)\cdot d\ln R_i,$$

which gives

$$\vartheta_i = \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\sigma_i^L)\cdot d\ln w_i - (1-s_i^L)\cdot(1-\sigma_i^K)\cdot d\ln R_i}{(1+(\lambda-1)\cdot s_i^L\cdot\pi_i)\cdot\omega_i^R}.$$

- Following the same steps as in the proof of Proposition B-8, we get

$$
\begin{aligned}
d\ln\Gamma_{gi}^{\text{disp}} &= \frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}dx} \\
&= \frac{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1}dx} \cdot \frac{\int_{\mathcal{D}_{gi}} \psi_g(x)^{\lambda-1}dx}{\int_{\mathcal{R}_{gi}} \psi_g(x)^{\lambda-1}dx} \\
&= \omega_{gi}^{R} \cdot \vartheta_i \\
&= \frac{\omega_{gi}^{R}}{\omega_i^{R}} \cdot \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\sigma_i^L)\cdot d\ln w_i - (1-s_i^L)\cdot(1-\sigma_i^K)\cdot d\ln R_i}{1+(\lambda-1)\cdot s_i^L \cdot \pi_i}.
\end{aligned}
$$

and task displacement for worker groups is given by

$$
\begin{aligned}
d\ln\Gamma_{g}^{\text{disp}} &= \sum_{i\in\mathcal{I}} \omega_g^i \cdot d\ln\Gamma_{gi}^{\text{disp}} \\
&= \sum_{i\in\mathcal{I}} \omega_g^i \cdot \frac{\omega_{gi}^{R}}{\omega_i^{R}} \cdot \frac{-d\ln s_i^L - d\ln\mu_i + (1-s_i^L)\cdot(1-\sigma_i^L)\cdot d\ln w_i - (1-s_i^L)\cdot(1-\sigma_i^K)\cdot d\ln R_i}{1+(\lambda-1)\cdot s_i^L \cdot \pi_i}.
\end{aligned}
$$

■

- Empirical implementation and bounding exercise

- The empirical implementation of our measures of task displacement in Propositions B-8 and B-9 is straightforward.

- However, the formulas in Proposition B-10 depend on two elasticities of
- substitution, σ_i^K and σ_i^L, which may differ due to the fact that we have different types of workers, and that when wages rise in one industry, we may be capturing the substitution of different worker groups for capital in marginal tasks.

- When implementing these formulas, we will assume that $\sigma_i^K = \sigma_i^L$, and use empirical estimates of the elasticity of substitution between capital and labor at the industry level, σ_i, to discipline their common value.

- This is motivated by the fact that empirical estimates of the elasticity of substitution between capital and labor are also estimating some combination of the group-specific elasticities, σ_{gi}^L's.

- In addition, when computing task displacement, we will use empirical estimates of $d \ln w_i$ and $d \ln R_i$ from the BLS, which account for changes in wages, the quality of workers, and quality-adjusted prices of capital used in an industry.

- We note also that, although our model has common wages for a given skill across industries, the expressions in Propositions B-9 and B-10 apply without modification to the case in which wages are industry-specific.

- In addition, our formula is not affected by factor-neutral improvements in TFP in industry i, since these do not affect an industry's labor share.

- While our formulas incorporate the effects of changes in factor prices, they miss the contribution of general factor-augmenting technologies.

- Now we provide upper bounds on the effects of this type of technological change on our estimates of task displacement, which will reveal that this type of technological change tends to have a very small effect on our inferred task displacement measures.

- We focus on our measures in Proposition B-10 obtained for $\lambda = 0.5$ and σ_i ranging from 0.8 to 1.2 (these technologies do not affect our measures if $\sigma_i = 1$).

- In particular, for $\sigma_i < 1$, the contribution of factor-augmenting technologies to the change in the labor share is between $-s_i^K \cdot (1 - \sigma_i) \cdot d \ln A_{\ell i}$ (where $d \ln A_{\ell i}$ is a weighted average of $d \ln A_{gi}$ across workers) and $s_i^K \cdot (1 - \sigma_i) \cdot d \ln A_{ki}$.

- Moreover, assuming no technological regress, we have that the total increase in (gross output) TFP in industry i must exceed both $\tilde{s}_i^L \cdot d \ln A_{Li}$ and $\tilde{s}_i^K \cdot d \ln A_{ki}$, where now $\tilde{s}_i^L$ and $\tilde{s}_i^K$ denote the share of labor and capital in gross output (an application of Hulten's theorem).

- As a result, we can bound the contribution of factor-augmenting technologies to lie in the interval

$$\left[-\frac{s_i^K}{\tilde{s}_i^L}\cdot(1-\sigma_i)\cdot d\ln \mathrm{TFP}_i, \frac{s_i^K}{\tilde{s}_i^K}\cdot(1-\sigma_i)\cdot d\ln \mathrm{TFP}_i\right].$$

- Likewise, for $\sigma_i > 1$, the contribution of factor-augmenting technologies to the change in the labor share is between $-s_i^K \cdot (\sigma_i - 1) \cdot d \ln A_{ki}$ and $s_i^K \cdot (\sigma_i - 1) \cdot d \ln A_{\ell i}$, which we can bound by

$$\left[-\frac{s_i^K}{\tilde{s}_i^K}\cdot(\sigma_i-1)\cdot d\ln \mathrm{TFP}_i, \frac{s_i^K}{\tilde{s}_i^L}\cdot(\sigma_i-1)\cdot d\ln \mathrm{TFP}_i\right].$$

- Figure B-4 presents our measures of task displacement across industries and worker groups using Equation (16) for $\sigma_i = 0.8$ and $\sigma_i = 1.2$, depicting the bounds on the contribution of factor-augmenting technologies using the whiskers.

- When constructing these bounds, we assume that industries with declining TFP between 1987 and 2016, experienced no factor-augmenting improvements.

- Except for a handful of IT-intensive industries with vast increases in TFP (electronics, computers, and communications), our bounds exclude anything other than very small effects of factor-augmenting technologies on the decline in labor shares and our task displacement measure.

- This is because these technologies have limited distributional effects but generate large TFP gains.

- Through the lens of our model, and given the pervasive lack of productivity growth observed across industries, these technologies cannot play a key role in explaining the decline in the labor share.

B-4 Data Appendix

- **Industry data**: Our main source of industry-level data are the BEA Integrated industry accounts for 1987-2016.
- These data contain information on industry value added, labor compensation, industry prices and factor prices for 61 NAICS industries.
- We aggregated these data to the 49 industries used in our analysis, which we could track consistently both in the BEA and the worker-level data from the 1980 US Census.
- Finally, when computing changes in industry's labor shares, we winsorized labor shares in value added at 20% to reduce noise in our measures of task displacement coming from industries with low and volatile labor shares.
- Besides the BEA data, we also used data from the BLS multifactor productivity tables for 1987-2016.
- These data are also available for 61 NAICS industries which we aggregated to the 49 industries used in our analysis.

- We complement the industry data with proxies for the adoption of automation technologies across industries.

- First, we use the measure of adjusted penetration of robots from Acemoglu and Restrepo (2020), which is available for 1993-2014.

- This measure is constructed using data from the International Federation of Robotics, and is defined for each industry i as

$$\mathrm{APR}_i = \frac{1}{5}\sum_{e=1}^{5}\left[\frac{\mathrm{robots}_{e,i,2014} - \mathrm{robots}_{e,i,1993}}{\ell_{e,i,1993}} - \mathrm{output\ growth}_{e,i,2004-1993} \cdot \frac{\mathrm{robots}_{e,i,1993}}{\ell_{e,i,1993}}\right],$$

where the right-hand side is computed as an average among five European countries, e, leading the US in the adoption of industrial robots.

- Finally, we also use the share of software and specialized machinery in value added from the BLS multifactor productivity tables.
- For software, we add custom-made software or software developed in house—which are more relevant for automation than pre-packaged software like Stata or Word.
- For specialized machinery, we add metalworking machinery (typically numerically controlled machines capable of automatically producing a pre-specified task), agricultural machinery other than tractors, specialized machinery used in the service sector, specialized machinery used in industry applications (which should also include industrial robots), construction machinery, and material handling equipment used in industrial applications.
- For offshoring, we use a measure from Feenstra and Hanson (1999) recently updated by Wright (2014) for 1990-2007.

- Turning to our proxies for changes in market structure, we use changes in sales concentration and several estimates of markups aggregated at the industry level.
- Our data for sales concentration comes from the Census Statistics of U.S. Businesses (SUSB) and is only available for 1997-2016.
- Using these data, we computed the tail index of the sales distribution for all the industries in our sample.
- The SUSB data can also be used to compute tail indices for the employment distribution going back to 1992.
- Using this alternative proxy of concentration available over a longer period didn't alter our findings.

- For markups, we provide three different estimates.

- First, we compute markups in a given industry using an accounting approach, which measures markups by the ratio of output to costs:

$$\mu_i = \frac{\text{gross output}_i}{R_i K_i + \text{Variable inputs}_i}.$$

- This approach requires constant returns to scale and assumes there are no adjustment costs.

- This approach also requires a measurement of the unobserved user cost of capital R_i.

- Second, we compute the change in markups by looking at the percent decline in the share of materials in gross output.

- That is:

$$\Delta \ln \mu_i = -\Delta \ln \text{share materials}_i .$$

- Finally, we compute markups using a production function approach as in De Loecker, Eeckhout and Unger (2020).

- In this approach, markups are computed for firms in industry i as

$$\mu_{i,f} = \frac{\text{elasticity variable inputs}_{i,f}}{\text{share variable inputs}_{i,f}}.$$

- **Census data:** We use the 1980 US Census to measure group-level outcomes and specialization patterns by industry and routine occupations.

- In addition, we also use the 2000 US Census to measure group-level outcomes for the year 2000.

- Finally, and to maximize our sample size, we use data from the pooled 2014-2018 American Community Survey to measure outcomes around the year 2016.

- **Regional variation:** Our estimates in Section 4.7 also exploit variation in specialization patterns across regions.

- In particular, we use two different groupings. First, we look at workers in 300 different demographic groups across 9 Census regions.

- To maintain a reasonable cell size, in this exercise we define demographic groups by gender, education, age (now defined by 16-30 years of age, 31-50 years, and 51-65 years) and race.

- Second, we look at workers in 54 different demographic groups across 722 commuting zones.

- **Routine occupations:** Following Acemoglu and Autor (2011), we use ONET to define routine jobs.

- In particular, for each Census occupation o, we compute a routine index given by

$$\text{routine index}_o = \text{routine manual input}_o + \text{routine cognitive input}_o - \text{average task input}_o.$$

- Here, routine manual input_o denotes the intensity of routine manual tasks in occupation o, the term routine cognitive input_o denotes the intensity of routine cognitive tasks, and the term average task input_o denotes the average task intensity (capturing the extent to which workers also conduct manual and analytical tasks).

- As is common practice in the literature, we define an occupation as routine if it is the top 33% of the routine index distribution.

- Table A-10 explores the robustness of our results to using different thresholds and alternative formulations of the routine index.
- In particular, in Panel A we define an occupation as routine if it is the top 40% of the routine index distribution, and In Panel B we use an alternative index of the form

$$\text{routine index}_o = \text{routine manual input}_o + \text{routine cognitive input}_o.$$

- Panels C-E probed the robustness of our results to using Webb (2020) indices of suitability for automation via robots and software and a combination of both of them.
- These measures provide a ranking of occupations depending on their suitability for automation, and we define an occupation as routine if it lies in the top 33% of each measure.

- **Other covariates**: Table 5 uses additional covariates.

- These include industries exposure to rising Chinese imports for 1990-2011, which we obtained from Acemoglu et al. (2016); the decline in the unionization rates by industry, which we computed for 1984-2016 using union membership by industry from the CPS; and industry-level changes in the quantity of capital per worker and TFP from the BEA Integrated Industry Accounts.

B-5 Additional Figures and Tables

Figure B-1: Labor share decline for 1987-2016 across industries from the 1987-2016 BEA Integrated Industry Accounts

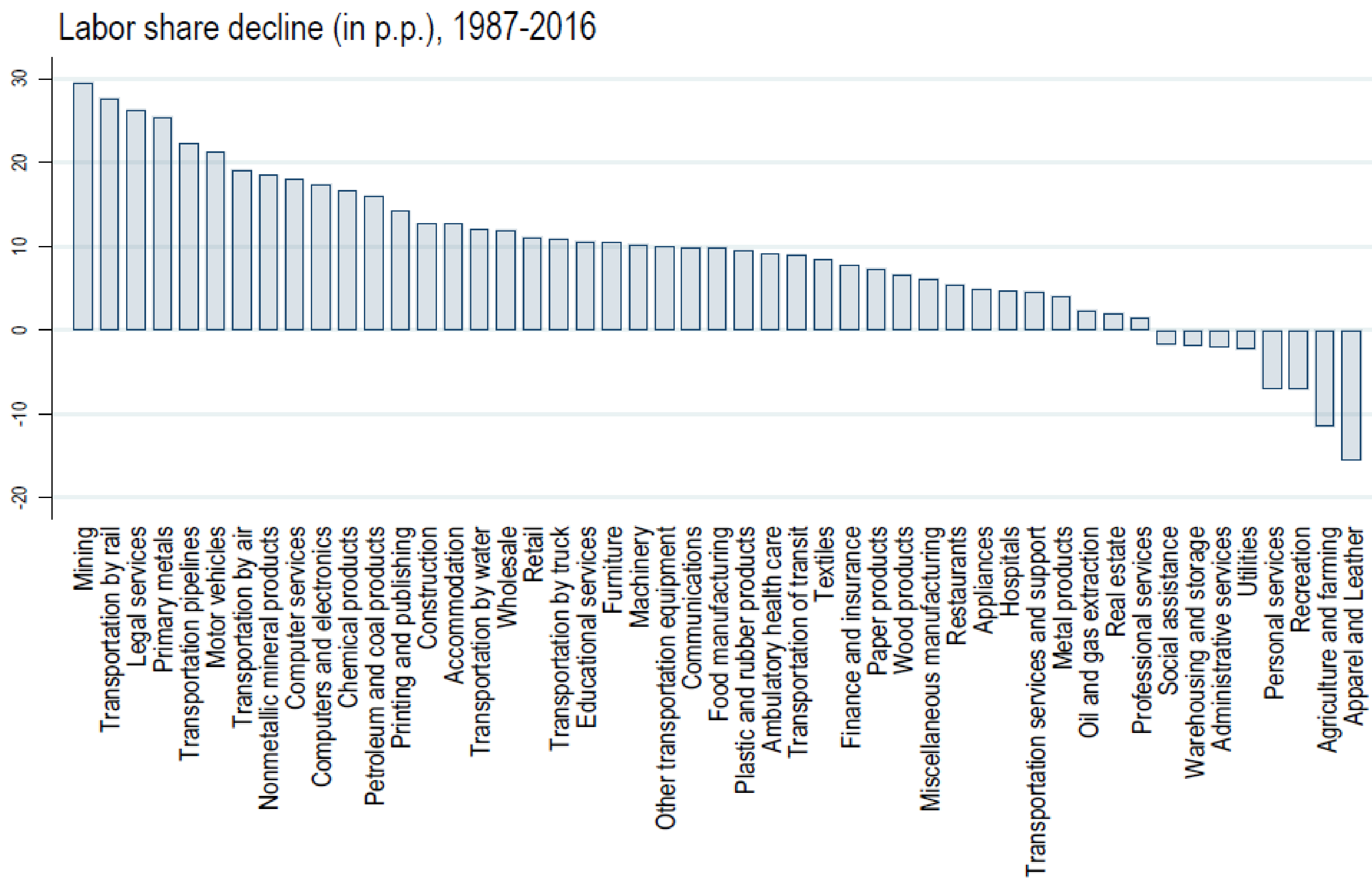


Figure B-2: Relationship between automation technologies and labor share declines across industries

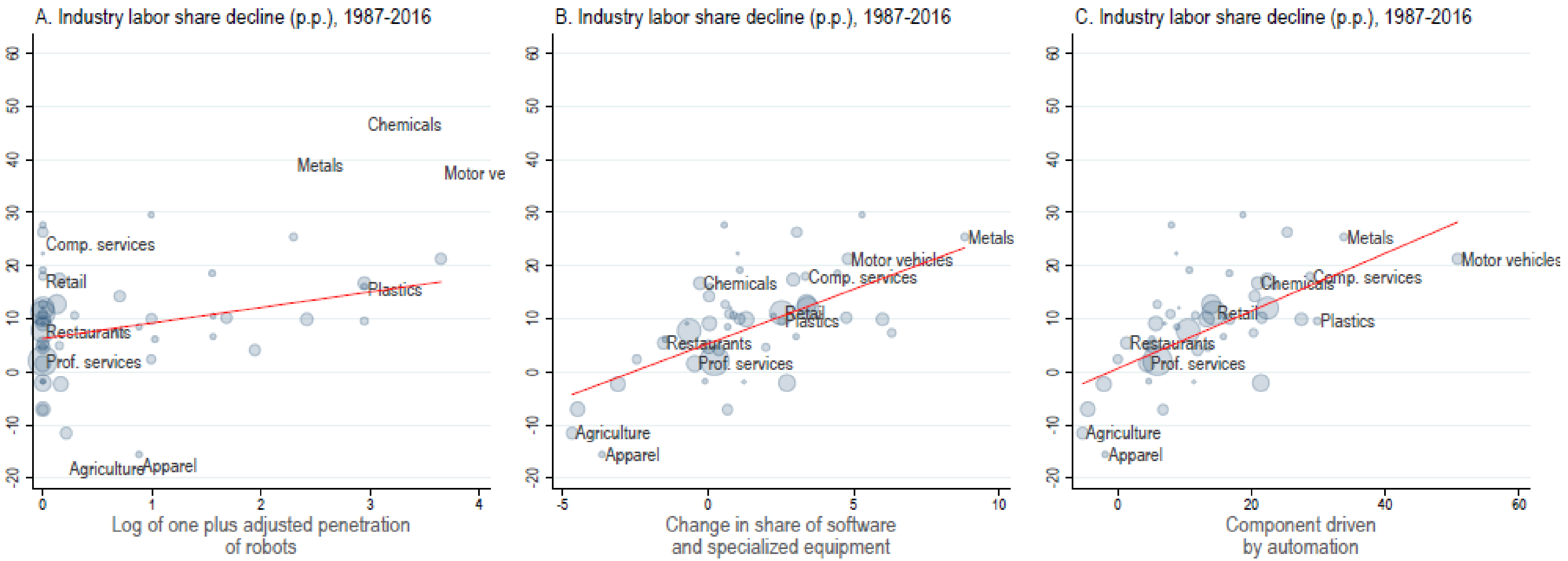


Figure B-3: Relationship between labor share declines and reductions in the demand for routine jobs across industries

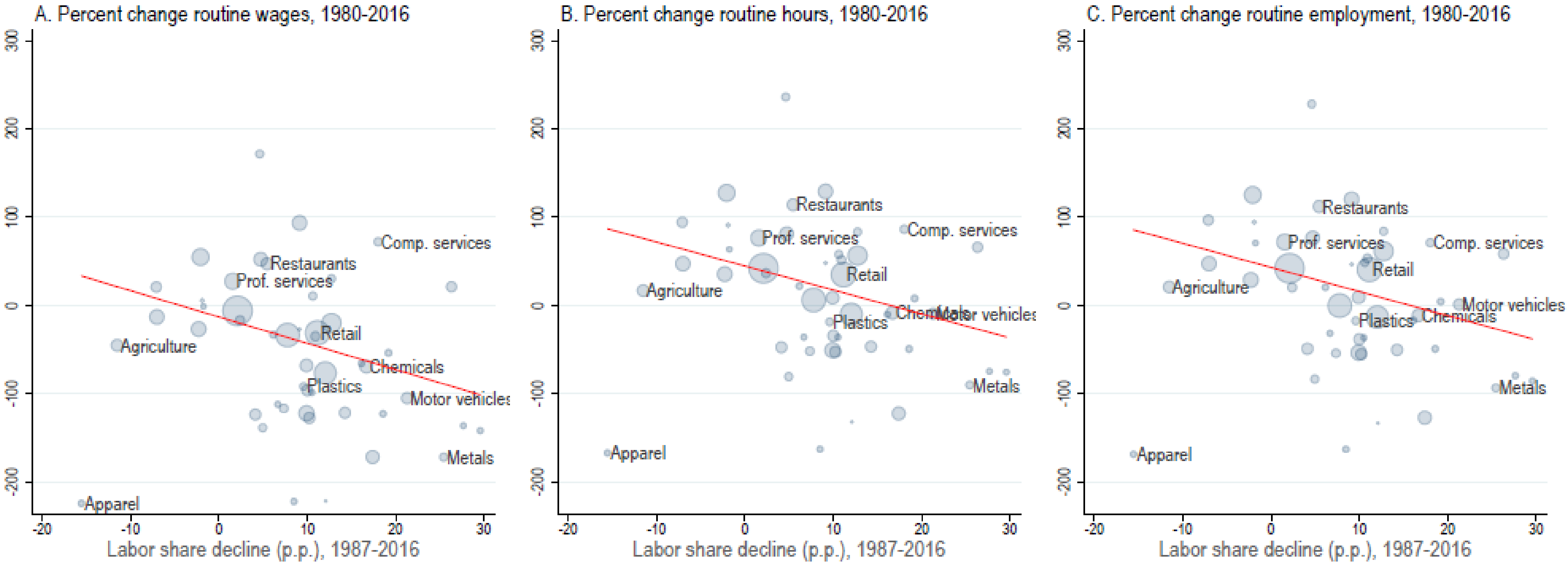


Figure B-4: Bounds on measures of task displacement for $\sigma_i = 0.8$

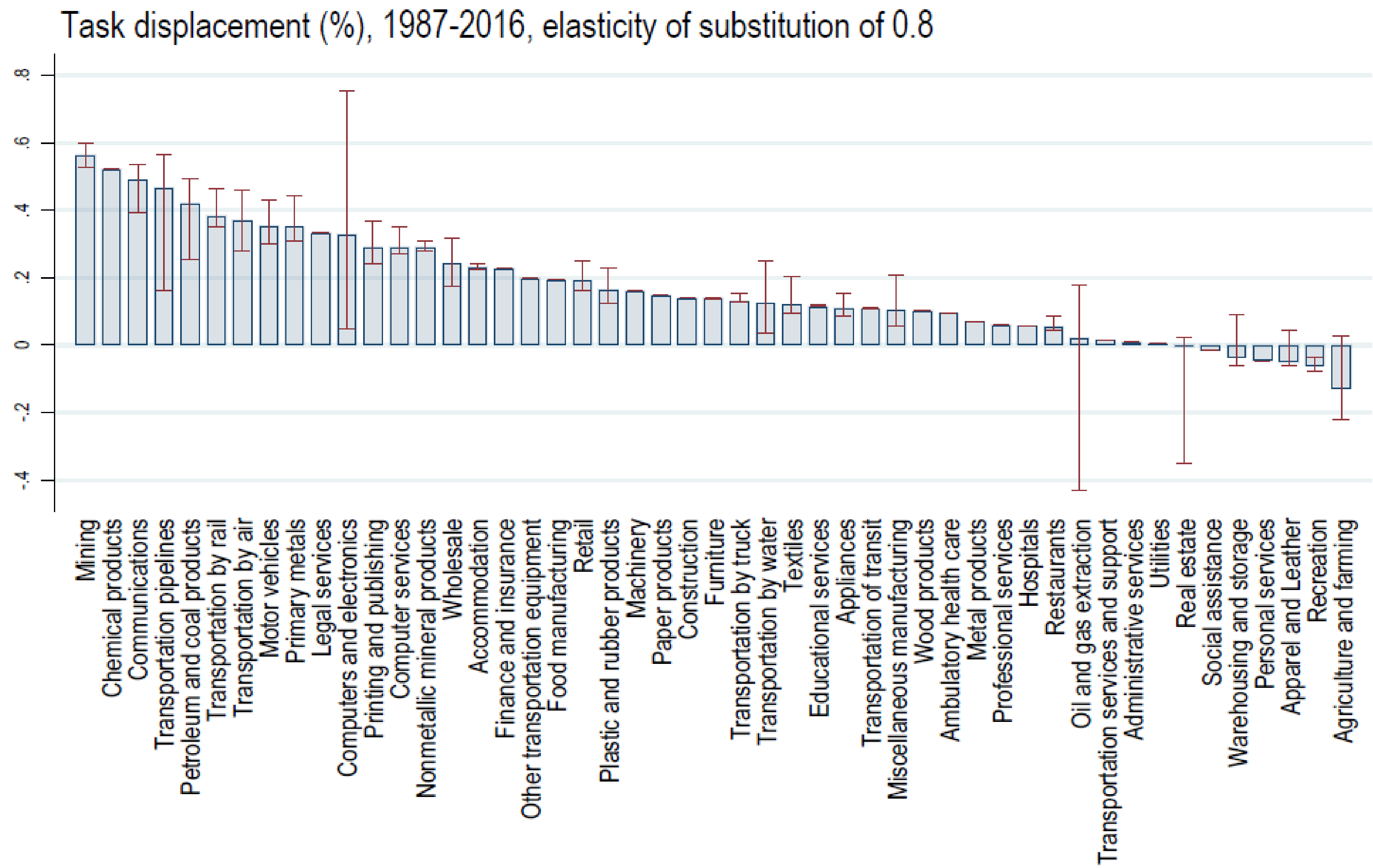


Figure B-4, Cont'd: Bounds on measures of task displacement for $\sigma_i = 1.2$

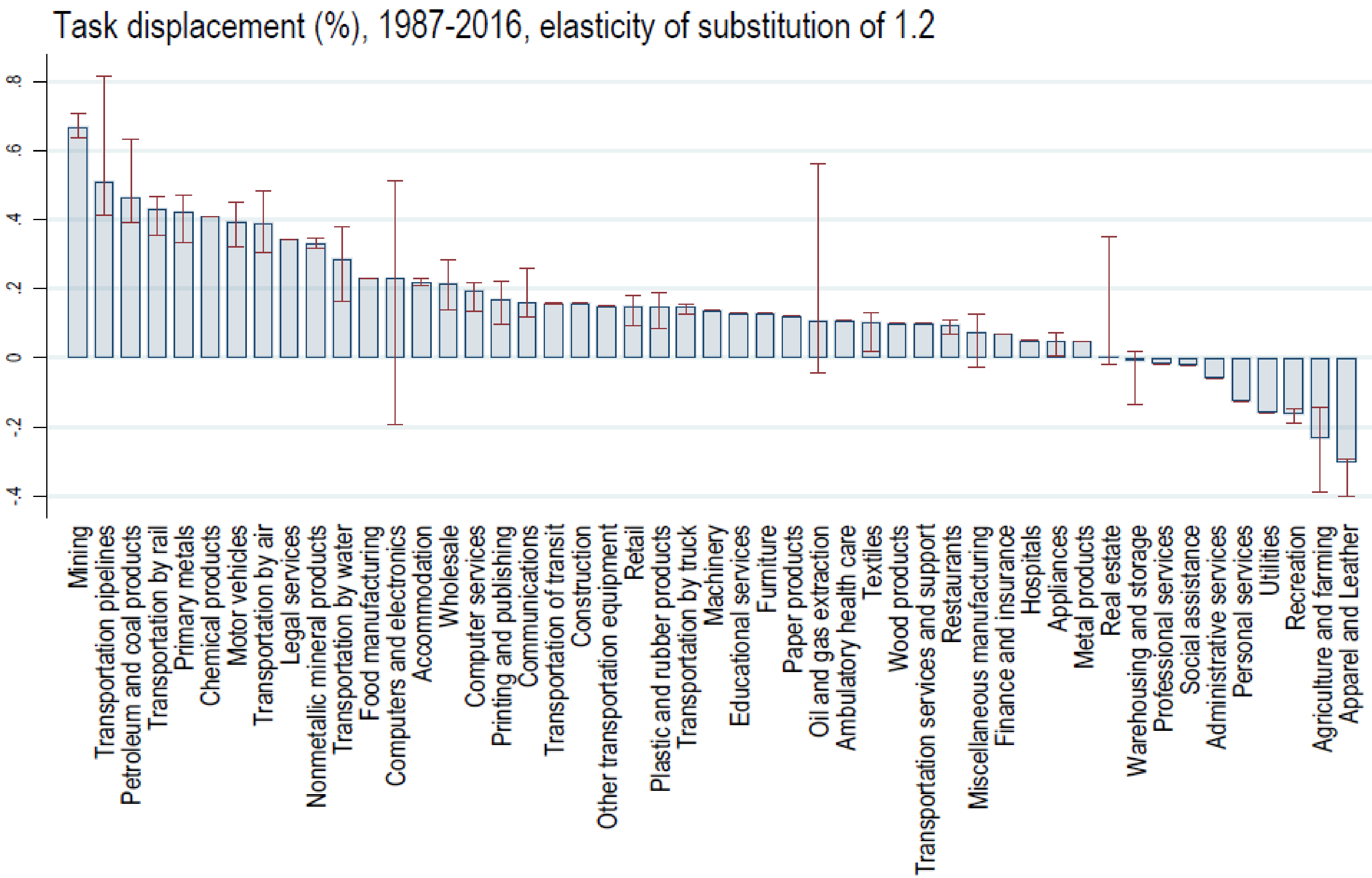


Table B-1: Relationship between task displacement and the decline of routine jobs across industries

	OLS Estimates			IV Estimates		
Dependent variable:	Change in log wages in routine jobs 1980–2016	Change in log hours in routine jobs 1980–2016	Change in log employment in routine jobs 1980–2016	Change in log wages in routine jobs 1980–2016	Change in log hours in routine jobs 1980–2016	Change in log employment in routine jobs 1980–2016
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Labor share declines, 1987–2016						
Labor share decline	-2.981	-2.715	-2.735	-4.427	-3.275	-3.135
	(1.187)	(1.152)	(1.150)	(1.471)	(1.292)	(1.288)
R-squared	0.12	0.12	0.12	0.09	0.11	0.12
First-stage F				32.92	32.92	32.92
Observations	48	48	48	48	48	48
Panel B: Task displacement, 1987–2016						
Task displacement	-2.113	-1.910	-1.931	-2.403	-1.778	-1.702
	(0.537)	(0.518)	(0.517)	(0.744)	(0.673)	(0.674)
R-squared	0.21	0.20	0.20	0.20	0.20	0.20
First-stage F				37.47	37.47	37.47
Observations	48	48	48	48	48	48
Panel C: Task displacement with elasticity of substitution 0.8, 1987–2016						
Task displacement	-2.250	-2.092	-2.129	-2.559	-1.893	-1.812
	(0.470)	(0.469)	(0.464)	(0.777)	(0.698)	(0.693)
R-squared	0.26	0.26	0.27	0.25	0.25	0.26
First-stage F				27.63	27.63	27.63
Observations	48	48	48	48	48	48
Panel D: Task displacement with elasticity of substitution 1.2, 1987–2016						
Task displacement	-1.641	-1.423	-1.426	-2.266	-1.676	-1.604
	(0.518)	(0.498)	(0.501)	(0.732)	(0.660)	(0.664)
R-squared	0.14	0.12	0.12	0.12	0.11	0.12
First-stage F				42.39	42.39	42.39
Observations	48	48	48	48	48	48

Table B-2: Task displacement and hours per worker and unemployment rates, 1980-2016

	Dependent variable: Labor market outcomes 1980–2016					
	OLS Estimates			IV Estimates		
	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A. Unemployment rate					
Task displacement	0.113	0.171	0.024	0.120	0.183	0.018
	(0.019)	(0.044)	(0.097)	(0.021)	(0.049)	(0.107)
Share variance explained by:						
- task displacement	0.18	0.27	0.04	0.19	0.29	0.03
- educational dummies		0.00	-0.01		-0.00	-0.02
R-squared	0.18	0.28	0.29	0.18	0.28	0.29
First-stage F				3246.45	785.80	156.33
Observations	500	500	500	500	500	500
	Panel B. log hours per worker					
Task displacement	-0.862	-0.581	0.790	-0.896	-0.611	0.693
	(0.180)	(0.292)	(0.619)	(0.186)	(0.299)	(0.665)
Share variance explained by:						
- task displacement	0.31	0.21	-0.28	0.32	0.22	-0.25
- educational dummies		0.13	-0.01		0.11	-0.03
R-squared	0.31	0.47	0.50	0.31	0.47	0.50
First-stage F				3246.45	785.80	156.33
Observations	500	500	500	500	500	500
Covariates:						
Industry shifters, manufacturing share, education and gender dummies		✓	✓		✓	✓
Exposure to labor share declines and relative specialization in routine jobs			✓			✓

Table B-3: Task displacement and changes in real hourly wages, 1980-2007

	Dependent variables: Change in wages and wage declines, 1980–2007			
	(1)	(2)	(3)	(4)
	Panel A. Change in real wages 1980–2007			
Task displacement	-1.777	-1.371	-0.920	-0.333
	(0.110)	(0.136)	(0.179)	(0.558)
Industry shifters		0.322	0.505	0.492
		(0.088)	(0.143)	(0.208)
Exposure to industry labor share decline				-0.784
				(0.832)
Relative specialization in routine jobs				-0.085
				(0.075)
Share variance explained by task displacement	0.69	0.53	0.36	0.13
R-squared	0.69	0.74	0.82	0.83
Observations	500	500	500	500
	Panel B. Real wage declines, 1980–2007			
Task displacement	-0.467	-0.486	-0.488	-0.896
	(0.057)	(0.070)	(0.098)	(0.182)
Industry shifters		-0.016	0.137	0.121
		(0.019)	(0.078)	(0.088)
Exposure to industry labor share decline				0.618
				(0.233)
Relative specialization in routine jobs				0.056
				(0.015)
Share variance explained by task displacement	0.65	0.68	0.68	1.26
R-squared	0.65	0.66	0.77	0.79
Observations	500	500	500	500
Other covariates:				
Manufacturing share, and education and gender dummies			✓	✓

Table B-4: Task displacement and changes in real hourly wages—Alternative price adjustments for task displacement

	Dependent variable: Change in real wages 1980–2016			
	(1)	(2)	(3)	(4)
	Panel A. Task displacement for $\lambda = 1$ and $\sigma_i = 0.8$			
Task displacement	-1.349 (0.118)	-1.016 (0.152)	-1.188 (0.173)	-2.050 (0.381)
Share variance explained by task displacement	0.57	0.43	0.51	0.87
R-squared	0.57	0.65	0.84	0.84
Observations	500	500	500	500
	Panel B. Task displacement for $\lambda = 1$ and $\sigma_i = 1.2$			
Task displacement	-1.729 (0.086)	-1.527 (0.152)	-1.263 (0.175)	-0.734 (0.541)
Share variance explained by task displacement	0.71	0.63	0.52	0.30
R-squared	0.71	0.73	0.83	0.83
Observations	500	500	500	500
	Panel C. Task displacement for $\lambda = 0.5$ and $\sigma_i = 0.8$			
Task displacement	-1.220 (0.104)	-0.924 (0.136)	-1.074 (0.156)	-1.858 (0.347)
Share variance explained by task displacement	0.58	0.44	0.51	0.88
R-squared	0.58	0.65	0.84	0.84
Observations	500	500	500	500
	Panel D. Task displacement for $\lambda = 0.5$ and $\sigma_i = 1$			
Task displacement	-1.436 (0.083)	-1.192 (0.141)	-1.172 (0.168)	-1.468 (0.402)
Share variance explained by task displacement	0.67	0.56	0.55	0.69
R-squared	0.67	0.70	0.84	0.84
Observations	500	500	500	500
	Panel E. Task displacement for $\lambda = 0.5$ and $\sigma_i = 1.2$			
Task displacement	-1.545 (0.077)	-1.362 (0.135)	-1.125 (0.156)	-0.631 (0.487)
Share variance explained by task displacement	0.71	0.63	0.52	0.29
R-squared	0.71	0.73	0.83	0.83
Observations	500	500	500	500
Covariates:				
Industry shifters		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓

Table B-5: Task displacement and changes in real hourly wages—Alternative labor share measures

	Dependent variable: Change in real wages 1980–2016			
	(1)	(2)	(3)	(4)
Panel A. Excluding commodities				
Task displacement	-1.675 (0.120)	-1.323 (0.174)	-1.394 (0.201)	-2.144 (0.456)
Share variance explained by task displacement	0.63	0.50	0.52	0.80
R-squared	0.63	0.67	0.83	0.84
Observations	500	500	500	500
Panel B. Winsorized labor share changes				
Task displacement	-1.592 (0.098)	-1.312 (0.165)	-1.345 (0.195)	-1.891 (0.444)
Share variance explained by task displacement	0.66	0.54	0.56	0.78
R-squared	0.66	0.69	0.84	0.84
Observations	500	500	500	500
Panel C. Excluding industries with rising labor shares				
Task displacement	-1.491 (0.090)	-1.250 (0.163)	-1.322 (0.196)	-1.959 (0.419)
Share variance explained by task displacement	0.66	0.55	0.58	0.86
R-squared	0.66	0.68	0.84	0.84
Observations	500	500	500	500
Panel D. Gross labor share changes				
Task displacement	-1.393 (0.082)	-1.113 (0.105)	-0.909 (0.126)	-1.190 (0.310)
Share variance explained by task displacement	0.66	0.53	0.43	0.57
R-squared	0.66	0.74	0.83	0.83
Observations	500	500	500	500
Covariates:				
Industry shifters		✓	✓	✓
Manufacturing share, gender and education dummies			✓	✓
Exposure to labor share declines and relative specialization in routine jobs				✓

Table B-6: Alternative estimates of the propagation matrix

	Dependent variable: Change in wages 1980–2016					
	GMM estimates			GMM using automation index IVs		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Decay parameter $\kappa = 1$.						
Own effect, θ/λ	0.875	0.872	0.806	0.867	0.860	0.786
	(0.049)	(0.052)	(0.054)	(0.050)	(0.052)	(0.055)
Contribution of ripple effects via	0.646	0.631	0.496	0.661	0.647	0.526
occupational similarity	(0.175)	(0.181)	(0.184)	(0.175)	(0.182)	(0.184)
Contribution of ripple effects via	0.241	0.245	0.553	0.237	0.238	0.547
industry similarity	(0.192)	(0.192)	(0.210)	(0.192)	(0.193)	(0.210)
Contribution of ripple effects via	0.194	0.194	0.186	0.194	0.194	0.182
education–age groups	(0.023)	(0.023)	(0.023)	(0.024)	(0.023)	(0.023)
Observations	500	500	500	500	500	500
Panel B. Decay parameter $\kappa = 5$.						
Own effect, θ/λ	0.910	0.900	0.849	0.904	0.888	0.828
	(0.046)	(0.049)	(0.052)	(0.047)	(0.050)	(0.053)
Contribution of ripple effects via	0.250	0.244	0.233	0.250	0.244	0.238
occupational similarity	(0.048)	(0.050)	(0.050)	(0.049)	(0.050)	(0.050)
Contribution of ripple effects via	0.184	0.182	0.236	0.191	0.187	0.242
industry similarity	(0.060)	(0.059)	(0.062)	(0.060)	(0.059)	(0.062)
Contribution of ripple effects via	0.160	0.160	0.152	0.159	0.160	0.149
education–age groups	(0.025)	(0.025)	(0.025)	(0.025)	(0.025)	(0.025)
Observations	500	500	500	500	500	500
Panel C. Decay parameter $\kappa = 2$ and setting $\sigma_\iota = 0.8$.						
Own effect, θ/λ	0.682	0.668	0.615	0.706	0.692	0.639
	(0.040)	(0.042)	(0.045)	(0.042)	(0.043)	(0.046)
Contribution of ripple effects via	0.509	0.477	0.426	0.537	0.519	0.475
occupational similarity	(0.079)	(0.082)	(0.084)	(0.079)	(0.083)	(0.084)
Contribution of ripple effects via	0.080	0.083	0.216	0.076	0.077	0.200
industry similarity	(0.096)	(0.096)	(0.104)	(0.096)	(0.096)	(0.104)
Contribution of ripple effects via	0.198	0.197	0.189	0.182	0.182	0.171
education–age groups	(0.022)	(0.022)	(0.022)	(0.022)	(0.022)	(0.022)
Observations	500	500	500	500	500	500
Panel D. Decay parameter $\kappa = 2$ and setting $\sigma_\iota = 1.2$.						
Own effect, θ/λ	1.045	1.038	0.949	1.121	1.125	1.022
	(0.058)	(0.061)	(0.064)	(0.061)	(0.064)	(0.067)
Contribution of ripple effects via	0.195	0.184	0.130	0.081	0.084	0.057
occupational similarity	(0.104)	(0.107)	(0.106)	(0.106)	(0.108)	(0.107)
Contribution of ripple effects via	0.391	0.391	0.592	0.460	0.461	0.627
industry similarity	(0.123)	(0.123)	(0.130)	(0.124)	(0.124)	(0.130)
Contribution of ripple effects via	0.152	0.153	0.141	0.169	0.169	0.156
education–age groups	(0.028)	(0.028)	(0.027)	(0.028)	(0.028)	(0.027)
Observations	500	500	500	500	500	500
Covariates:						
Industry shifters		✓	✓		✓	✓
Manufacturing share			✓			✓