Lifecycle Labor Supply: The Intertemporal Substitution Hypothesis (ISH) and the "λ Constant" Functions

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Model

Assume Intertemporal Additive Separability

$$\max U(C_t, L_t) + E_t \sum_{j=1}^{\infty} \beta^j U(C_{t+j}, L_{t+j}) \qquad 0 < \beta < 1$$

U is concave. Assume Inada conditions.

- Assets at time $t+1$ are

$$A_{t+1} = (1 + r_{t+1})(A_t + W_t(1 - L_t) - P_t C_t)$$

where $P_0 = 1$ (normalization).

- Assume perfect credit markets. At time t, W_t, P_t are known: future values of W_t, P_t and r_t are *not* known.
- Optimality

$$(1) \qquad U_C = \lambda_t P_t$$
$$(2) \qquad U_L \geq \lambda_t W_t \quad \text{(Possible corner solution)}$$

- $\lambda_t = \frac{\partial V_t}{\partial A_t}$ where V_t is value function associated with the program.

$$\lambda_t = E_t \left[\frac{\partial V}{\partial A_{t+1}} \left(A_{t+1}\right) \delta_{t+1} \right]$$

or

$$(3) \qquad \lambda_t = E_t \left(\lambda_{t+1} \delta_{t+1}\right)$$

where $\delta_{t+1} = \frac{1}{1+r_{t+1}}$.

- We can invert (1) and (2) to write

$$C_t = C\left(\lambda_t P_t, \lambda_t W_t\right) \tag{4}$$

$$L_t = L\left(\lambda_t P_t, \lambda_t W_t\right) \tag{5}$$

Substitution Matrix:

$$\begin{bmatrix} C_1 & C_2 \\ L_1 & L_2 \end{bmatrix} \qquad C_2 = L_1$$

negative definite from concavity of U.

- Then we have

$$\lambda_t = E_t\left(\lambda_{t+1}\delta_{t+1}\right) = F(\sim) \quad \text{(a general functional form)}$$

in general

1. ful. form of λ_t is messy
2. arguments required to solves λ_t out are not known to the econometrician

- Requires writing explicit functional forms about how future values of variables enter as well as making strong statements about expectations processes. (examples Lucas Rapping, 1969; Hall, 1979, Ashenfelter-Altonji, 1980).

3. Ghez-Becker (1975), Smith (1977), and Heckman (1974) use synthetic cohorts and the assumption of perfect certainty to absorb λ into intercepts.

Solution: To estimate only part of the Model

Goal: To somehow eliminate λ_t from the model.

Approach I

- Exploit FOC (1) and (2) assuming interior solutions so that

(6) $$\frac{U_L}{U_C} = \frac{W_t}{P_t}$$

use subsets of excluded variables (from λ_t) as instruments for C_t and L_t and can accomodate general forms of heterogeneity. This procedure is pursued by MaCurdy (1981) and in Altonji (1983).

- What can be estimated? Within period MRS functions.
- But obviously we can also estimate any montone transformation of U. Thus

$$G\left(U\left(C_t, L_t\right)\right)$$

also solves (6).

- To pin down G, we require intertemporal information.

- Altonji method (1984) is consistent with

$$U(C_t, L_t) = \frac{C_t^{\alpha-1}}{\alpha} + b\frac{L_t^{\gamma-1}}{\gamma} \tag{7}$$

$$\frac{U_L}{U_C} = \frac{bL_t^{\gamma-1}}{C_t^{\alpha-1}} = \frac{W_t}{P_t}$$

so that we have

$$\ln L_t = \frac{1}{1-\gamma}\left(\ln b + (1-\alpha)\ln C_t - \ln\frac{W_t}{P_t}\right)$$

- Therefore, we can estimate α, γ and b. But obviously not period specific shifters.

- For more general functions, *e.g.* log linear (4) and (5)

$$(8) \qquad \ln C_t = \alpha_0 + \alpha_1 \ln W_t + \alpha_2 \ln P_t - (\alpha_1 + \alpha_2) \ln \lambda_t$$
$$\ln L_t = \epsilon_0 + \epsilon_1 \ln W_t + \epsilon_2 \ln P_t - (\epsilon_1 + \epsilon_2) \ln \lambda_t$$

(note constraint imposed)

- Normality of goods:

$$(\alpha_1 + \alpha_2) < 0 \qquad\qquad (\epsilon_1 + \epsilon_2) < 0$$

- We can solve out to reach the equations

$$(9) \qquad \ln L_t = \text{intercept } + \left[\epsilon_1 - \alpha_1 \frac{\epsilon_1 + \epsilon_2}{\alpha_1 + \alpha_2}\right] \ln W_t + \left[\alpha_1 - \alpha_2 \frac{\epsilon_1 + \epsilon_2}{\alpha_1 + \alpha_2}\right] \ln C_t$$

- Altonji (1983) assumes $\alpha_1 = 0$. Empirical evidence strongly suggests $\alpha_1 > 0$. (See Heckman, 1974; Ghez-Becker, 1975; Smith 1977.)
- Therefore given valid instruments, leisure demands understated.
- Labor supply response overstated.
- These functions are still sensitive to monotone transformation argument as well.
- Therefore *cannot* isolate the intertemporal substitution terms without some intertemporal data.
- An identification problem. Resolved by (7) as functional formand not considering any G except $G = I$

- In principle, these parameters can only determine allocations within branch t and *not* interbranch allocations.
- Using utility tree notation: with functional form assumptions (*e.g.* equation (7) these determine the utility function except we cannot estimate β.

Approach II: Use of Intertemporal Identifying Information

- In the perfect certainty case Heckman-MaCurdy (1980) or MaCurdy (1981), use (1) and (2) to solve out for λ_t as a function of C_t and L_t
$$\lambda_t = \frac{U_L}{W_t}.$$
- Note that (3) becomes $\lambda_t = (\delta_{t+1})\,\lambda_{t+1}$.
- Substitute into (4) and (5) to get C_t and L_t as functions of lagged C_t and L_t.
- Note we assume $G = I$ in this set up (*i.e.* we take an explicit position or preferences).

- Thus for demand functions given by (8) we have that

$$\text{(10)} \qquad \ln L_t - \ln L_{t-1} = \text{intercept} \ + \epsilon_1(\ln W_t - \ln W_{t-1}) + \epsilon_2(\ln P_t - \ln P_{t-1})$$

- Therefore, we can identify ϵ_1, ϵ_2 and by the same approach with C_t we can estimate α_1 and α_2.
- But taking differences raises the well known econometric problem of increasing measurement error to true components.
- Perfect certainty is key. Suppose that we assume an uncertain environment, but δ_{t+1} is known with certainty, then we have that

$$\lambda_t = \delta_{t+1} E_t \lambda_{t+1}$$
$$\ln \lambda_t = \ln \delta_{t+1} + \ln E_t(\lambda_{t+1})$$

- Assuming innovation variance is "small", we have the *approximate martingale property*

$$\ln \lambda_t \doteq \ln \delta_{t+1} + E_t \ln \lambda_{t+1} \doteq \ln \delta_{t+1} + \ln \lambda_{t+1} - \psi_{t-1} \tag{11}$$

where

$$\psi_{t+1} = \ln \lambda_{t+1} - E_t \ln \lambda_{t+1}$$

- This sort of approximation made by MaCurdy (1977) and Hall (1978).
- Then we can make similar substitutions and reach

$$\ln L_t - \ln L_{t-1} = \text{intercept} \; + \epsilon_1(\ln W_t - \ln W_{t-1}) + \epsilon_2(\ln P_t - \ln P_{t-1}) \tag{12}$$

where ψ_t is (approximately) uncorrelated with information available at $t-1$ (but not exactly).

- Must take position on whether W_t and P_t are known at $t-1$ to determine exogeneity of W_t and P_t.
- Note that the distribution of ψ_t will depend on exogenous forcing variables (past history, current shocks in forcing variables, *etc.*).
- Altonji (1984) claims to permit δ_{t+1} to be random as of date t.
- Then he claims, without formal justification that he can write out expressions like (11).

- Altonji's approximation is that

$$\lambda_t = E_t(\delta_{t+1}\lambda_{t+1})$$

can be written as

$$\begin{aligned} \ln \lambda_t &= \ln E_t(\delta_{t+1}\lambda_{t+1}) \\ &\doteq E_t \ln \delta_{t+1} + E_t \ln \lambda_{t+1} \end{aligned}$$

or

$$\ln \lambda_t = \ln \delta_{t+1} + \ln \lambda_{t+1} + \tilde{\psi}_{t+1}$$

where $\tilde{\psi}_{t+1}$ is innovation.

- Like ψ_t, its distribution is generated by exogenous variables including innovations in wages, prices and the like—see Altonji's equation (3).
- For forcing variables to be exogenous in the equation, we require that they be known at $t - 1$.
- To achieve this, the variance in the δ_{t+1} is "small" and the variance in other shocks "small".
- Unknown is the validity of the approximation.
- It is certainly *not* plausible over business cycles and in presence of large macro shocks.

- Actually, it is *not* necessary to assume $G = I$ and for simple G *e.g.* β (time preference) it is possible to use intertemporal data to estimate β, or determine more general G.
- But Altonji does *not* exploit this source of information.
- *In general,* λ *constant functions do not estimate economically interesting parameters.*
- For special functional forms, we have seen that with either method, we can estimate γ in (7) which is the McFadden DES between leisure in any two periods.

- In general, though

$$\frac{d\left(\ln(L_{t+1}/L_t)\right)}{d\ln W_{t+1}} = W_{t+1}\left\{\frac{L_1(t+1)W_{t+1}}{L(t+1)} + \frac{\partial\lambda_{t+1}}{\partial W_{t+1}}\left(\frac{L_1(t+1)W_{t+1}+L_2(t+1)P_{t+1}}{L(t+1)}\right) - \frac{\partial\lambda_t}{\partial W_t}\left(\frac{L_1(t)W_t+L_2(t)P_t}{L(t)}\right)\right\}$$

- Except for special log linear forms like (8),knowledge of the λ constant functions does not enable us to directly address the intertemporal substitution questions.
- It is log linearity together with λ constant functions that makes λ constant approach attractive.