Firms and Labor Market Inequality: Evidence and Some Theory

by David Card, Ana Rute Cardoso, Joerg Heining, and Patrick Kline

James J. Heckman

THE UNIVERSITY OF CHICAGO

Econ 350, Winter 2023

I. Introduction

II. Productivity, Wages, and Rent Sharing

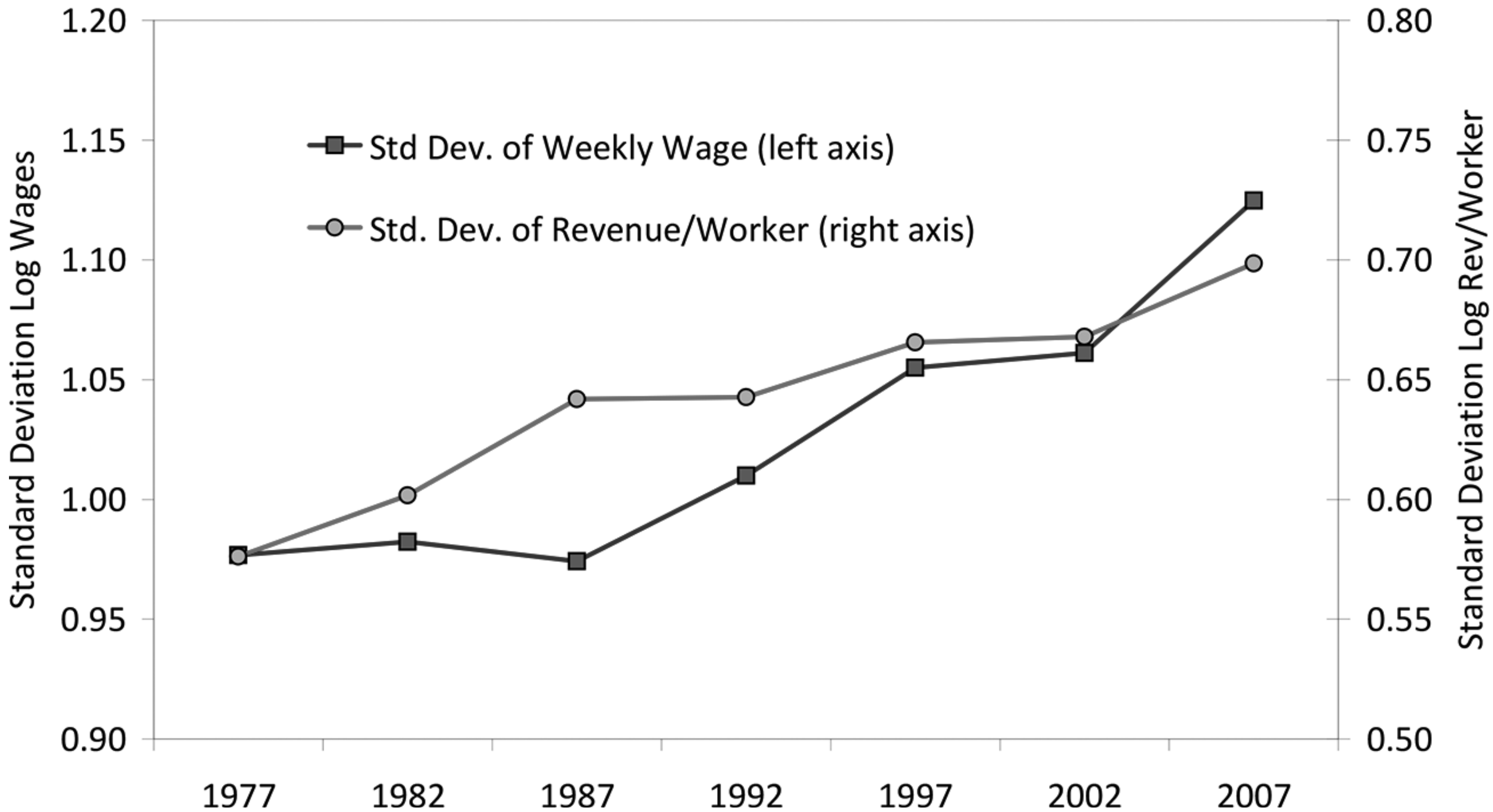
- Figure 1 plots results from Barth et al. (2016), showing remarkably similar trends in the dispersion of wages and productivity across business establishments in the United States.

- Taken at face value, these parallel trends are consistent with a roughly unit elasticity of establishment wages with respect to productivity (see Barth et al. 2016, S71).

- Of course, figure 1 does not tell us whether the composition of the workforce employed at these establishments is changing over time.

- What appear to be more productive establishments may simply be establishments that hire more skilled workers, which is fully consistent with the competitive labor market model in which all firms pay the same wages for any given worker.

FIG. 1.—Trends in between-establishment dispersion in wages and productivity.



Source: Barth et al. (2016). A color version of this figure is available online.

A. Measuring Rents

- The empirical rent-sharing literature is motivated by an assumed structural relationship between wages and either profit per worker or a measure of quasi rent per worker.

- To facilitate discussion, suppose that there is a single type of labor at a firm j and that the wage (w_j) is determined by a structural relationship of the form

$$w_j = b + \frac{\gamma Q_j}{N_j}, \tag{1}$$

- where b represents an alternative wage, N_j is employment at the firm, Q_j represents quasi rents, and γ is a rent-sharing parameter.

- The firm combines labor inputs and capital (K_j) and faces an exogenous rental rate r on capital, yielding the quasi rent

$$Q_j = \mathrm{VA}_j - bN_j - rK_j,$$

- where VA_j is value added (revenue net of materials costs). Value added is related to labor and capital inputs by

$$\mathrm{VA}_j = P_j T_j f(N_j, K_j),$$

- where P_j is a potentially firm-specific selling price index, T_j is an index of technical efficiency, and f is a standard production function.

- Here $P_j T_j$ represents total factor productivity (TFP_j), which, in the terminology of Foster et al. (2008), is also referred to as revenue productivity because it is the product of physical productivity T_j and product price P_j.

- We assume that TFP_j is the driving source of variation that researchers are implicitly trying to model in the rent-sharing literature.

- Under this interpretation, firm-specific TFP shocks lead to changes in quasi rent per worker that cause wages to fall or rise relative to the alternative wage.

- The elasticity of wages with respect to an exogenous change in quasi rent per worker is

$$\xi_{Qj} = \frac{\gamma(Q_j/N_j)}{b_j + \gamma(Q_j/N_j)}, \tag{2}$$

- which corresponds to the share of rents in wages.

- The elasticity of wages with respect to profit per worker (π_j/N_j) should be of comparable magnitude.

- Indeed, under the usual bargaining interpretation of equation (1), profits per worker are a constant share of quasi rents per worker:

$$\frac{\pi_j}{N_j} = (1-\gamma)\frac{Q_j}{N_j}.$$

- Rather than measure quasi rents, a majority of studies relate wages to value added per worker.

- The elasticity of wages with respect to value added per worker is

$$\xi_j = \xi_{Qj} \times \frac{\text{VA}_j}{Q_j},$$

- which will be bigger than ξ_{Qj}, since $Q_j < VA_j$.

- For example, data reported by Card et al. (2014) suggest that the ratio of value added to quasi rent for firms in Northeast Italy is typically around 2.

B. A Summary of the Rent-Sharing Literature

- Table 1 synthesizes the estimated rent-sharing elasticities from the 22 studies listed in table A1, extracting one or two preferred specifications from each study and adjusting all elasticities to an approximate value added per worker basis.

- We divide the studies into three broad generations on the basis of the level of aggregation in the measures of rents and wages.

- The first group of studies, which includes two influential papers from the early 1990s, uses industry-wide measures of productivity and either individual-level or firm-wide average wages.

- A second generation of studies includes five papers, mostly from the mid-1990s, that use firm- or establishment-specific measures of rents but measure average wages of employees at the workplace level.

- A third generation of studies consists of 18 relatively recent papers that study the link between firm- or establishment-specific measures of rents and individual-specific wages.

Table 1
Summary of Estimated Rent-Sharing Elasticities from the Recent Literature (Preferred Specification, Adjusted to Total Factor Productivity Basis)

Study	Country/Industry	Estimated Elasticity	Standard Error
Group 1—Industry-level profit measure:			
Christofides and Oswald 1992	Canadian manufacturing	.140	.035
Blanchflower, Oswald, and Sanfey 1996	US manufacturing	.060	.024
Estevao and Tevlin 2003	US manufacturing	.290	.100
Group 2—Firm-level profit measure, mean firm wage:			
Abowd and Lemieux 1993	Canadian manufacturing	.220	.081
Van Reenen 1996	UK manufacturing	.290	.089
Hildreth and Oswald 1997	United Kingdom	.040	.010
Hildreth 1998	UK manufacturing	.030	.010
Barth et al. 2016	United States	.160	.002
Group 3—Firm-level profit measure, individual-specific wage:			
Margolis and Salvanes 2001	French manufacturing	.062	.041
Margolis and Salvanes 2001	Norwegian manufacturing	.024	.006
Arai 2003	Sweden	.020	.004
Guiso et al. 2005	Italy	.069	.025
Fakhfakh and FitzRoy 2004	French manufacturing	.120	.045
Du Caju et al. 2011	Belgium	.080	.010
Martins 2009	Portuguese manufacturing	.039	.021
Gürtzgen 2009	Germany	.048	.002
Cardoso and Portela 2009	Portugal	.092	.045
Arai and Heyman 2009	Sweden	.068	.002
Card et al. 2014	Italy (Veneto region)	.073	.031
Carlsson et al. 2014	Swedish manufacturing	.149	.057
Card et al. 2016	Portugal, between firm	.156	.006
Card et al. 2016	Portugal, within job	.049	.007
Bagger et al. 2014	Danish manufacturing	.090	.020

NOTE.—For a more complete description of each study, see table A1.

C. Specification Issues: A Replication in Portuguese Data

- Panel A of table 2 presents a series of specifications in which we relate the log hourly wage observed for a worker in a given year (between 2005 and 2009) to mean log value added per worker or mean log sales per worker at his employer, averaged over the sample period.

- These are simple crosssectional rent-sharing models in which we use an averaged measure of rents at the employer to smooth out the transitory fluctuations and measurement errors in the financial data.

- In row 1 we present models using mean log value added per worker as the measure of rents; in row 2 we use mean log sales per worker; and in row 3 we use mean log value added per worker over the 2005–2009 period but instrument this with mean log sales per worker over a slightly wider window (2004–2010).

- For each choice we show a basic specification (with only basic human capital controls) in column 1, a richer specification with controls for major industry and city in column 2, and a full specification with dummies for 202 detailed industries and 29 regions in column 3.

Table 2
Cross-Sectional and Within-Job Models of Rent Sharing for Portuguese Male Workers

	Basic Specification (1)	Basic + Major Industry/City (2)	Basic + Detailed Industry/City (3)
A. Cross-sectional models (worker-year observations, 2005–9):			
OLS: rent measure = mean log value added per worker, 2005–9	.270	.241	.207
	(.017)	(.015)	(.011)
OLS: rent measure = mean log sales per worker, 2005–9	.153	.171	.159
	(.009)	(.007)	(.004)
IV: rent measure = mean log value added per worker, 2005–9; instrument = mean log sales per worker, 2004–10	.327	.324	.292
	(.014)	(.011)	(.008)
First-stage coefficient	.475	.541	.562
	(t = 26.19)	(t = 40.72)	(t = 64.38)
B. Within-job models (change in wages from 2005 to 2009 for stayers):			
OLS: rent measure = change in log value added per worker from 2005 to 2009	.041	.039	.034
	(.006)	(.005)	(.003)
OLS: rent measure = change in log sales per worker from 2005 to 2009	.015	.014	.013
	(.005)	(.004)	(.003)
IV: rent measure = change in log value added per worker from 2005 to 2009; instrument = change in log sales per worker, 2004–10	.061	.059	.056
	(.018)	(.017)	(.016)
First-stage coefficient	.221	.217	.209
	(t = 11.82)	(t = 13.98)	(t = 18.63)

NOTE.—The sample in panel A is 2,503,336 person-year observations from Quadros de Pessoal (QP) for males working in 2005–9 between the ages of 19 and 65 years with at least 2 years of potential experience employed at a firm with complete value-added data (from Sistema de Analisis de Balances Ibericos [SABI]) for 2005–9 and sales data (from QP) for 2004 and 2010. The sample in panel B is 284,071 males ages 19–61 years in 2005 who worked every year from 2005 to 2009 at a firm with complete value-added data (from SABI) for 2005–9 and sales data from QP) for 2004 and 2010. Standard errors are clustered by firm (62,845 firms in panel A, 44,661 firms in panel B). Models in panel A control for cubic in experience and unrestricted education*year dummies. Models in panel B control for a quadratic in experience and education. Models in col. 2 also control for 20 major industries and two major cities (Lisbon and Porto). Models in col. 3 also control for 202 detailed industry dummies and 29 Nomenclature of Territorial Units for Statistics region 3 location dummies. IV 5 instrumental variables; OLS 5 ordinary least squares.

III. Firm Switching

A. AKM Models

- In their seminal study of the French labor market, Abowd et al. (1999) specified a model for log wages that includes additive effects for workers and firms.

- Specifically, their model for the log wage of person i in year t takes the form

$$\ln w_{it} = \alpha_i + \psi_{J(i,t)} + X'_{it}\beta + \varepsilon_{it},$$

- where X_{it} is a vector of time-varying controls (e.g., year effects and controls for experience), α_i is a person effect capturing the (time-invariant) portable component of earnings ability, the $\{\psi_j\}_{j=1}^{J}$ are firm-specific relative pay premiums, $J(i,t)$ is a function indicating the employer of worker i in year t, and ε_{it} is an unobserved time-varying error capturing shocks to human capital, person-specific job match effects, and other factors.

- If different firms pay different wage premiums, the pattern of sorting of workers to firms will also matter for overall wage inequality.

- In particular, the variance of log wages is

$$\begin{aligned}\text{var}(\ln w_{it}) \;=\; & \text{var}(\alpha_i) + \text{var}(\psi_{J(i,t)}) + \text{var}(X_{it}'\beta) + \text{var}(\varepsilon_{it}) \\ & + 2\text{cov}(\alpha_i, \psi_{J(i,t)}) + 2\text{cov}(\alpha_i, X_{it}'\beta) + 2\text{cov}(\psi_{J(i,t)}, X_{it}'\beta), \qquad (3)\end{aligned}$$

- which includes both the variance of the firm-specific wage premiums and a term reflecting the covariance of the worker and firm effects.

- If workers with a higher earning capacity are more likely to work at higher-premium firms, then this covariance term will be positive, and any inequality effects from the presence of the firm premiums will be amplified.

- An alternative decomposition uses the fact that

$$\text{var}(\ln w_{it}) = \text{cov}(\ln w_{it}, \alpha_i) + \text{cov}(\ln w_{it}, \psi_{J(i,t)}) + \text{cov}(\ln w_{it}, X'_{it}\beta) + \text{cov}(\ln w_{it}, \varepsilon_{it}). \quad (4)$$

- This yields an ensemble assessment of the importance of each variance component to wage dispersion that includes the contribution of the covariance between wage components.

- For example, under this decomposition, the contribution of the firm component to total wage variation would be $\text{cov}(\ln w_{it}, \psi_{J(i,t)}) = var(\psi_{J(i,t)}) + \text{cov}(\alpha_i, \psi_{J(i,t)}) + \text{cov}(X'_{it}\beta, \psi_{J(i,t)})$.

- One way to think about this decomposition is that one-half of the firm covariance terms in equation (3) are attributed to the firm-specific wage premiums.

B. Identifying Age and Time Effects

- Table 3 examines the sensitivity of the results of Card et al. (2016) to four alternate normalizations of the age effects.

- The first column shows the baseline normalization, which attributes a relatively small fraction of the overall variance of wages to the time-varying individual component of wages.

- Renormalizing the age profile to be flat at age 50 (col. 2) has little effect on this conclusion, whereas renormalizing the profile to be flat at age 30 leads to a slightly larger variance share for the time-varying component and also implies a relatively strong negative correlation between the person effects and the index $X'_{it}\beta$.

- Normalizing the age profile to be flat at age 0—which is what is being done by simply omitting the linear term from an uncentered age polynomial—exacerbates this pattern and leads to a decomposition that suggests that the variances of α_i and $X'_{it}\beta$ are both very large and that the two components are strongly negatively correlated.

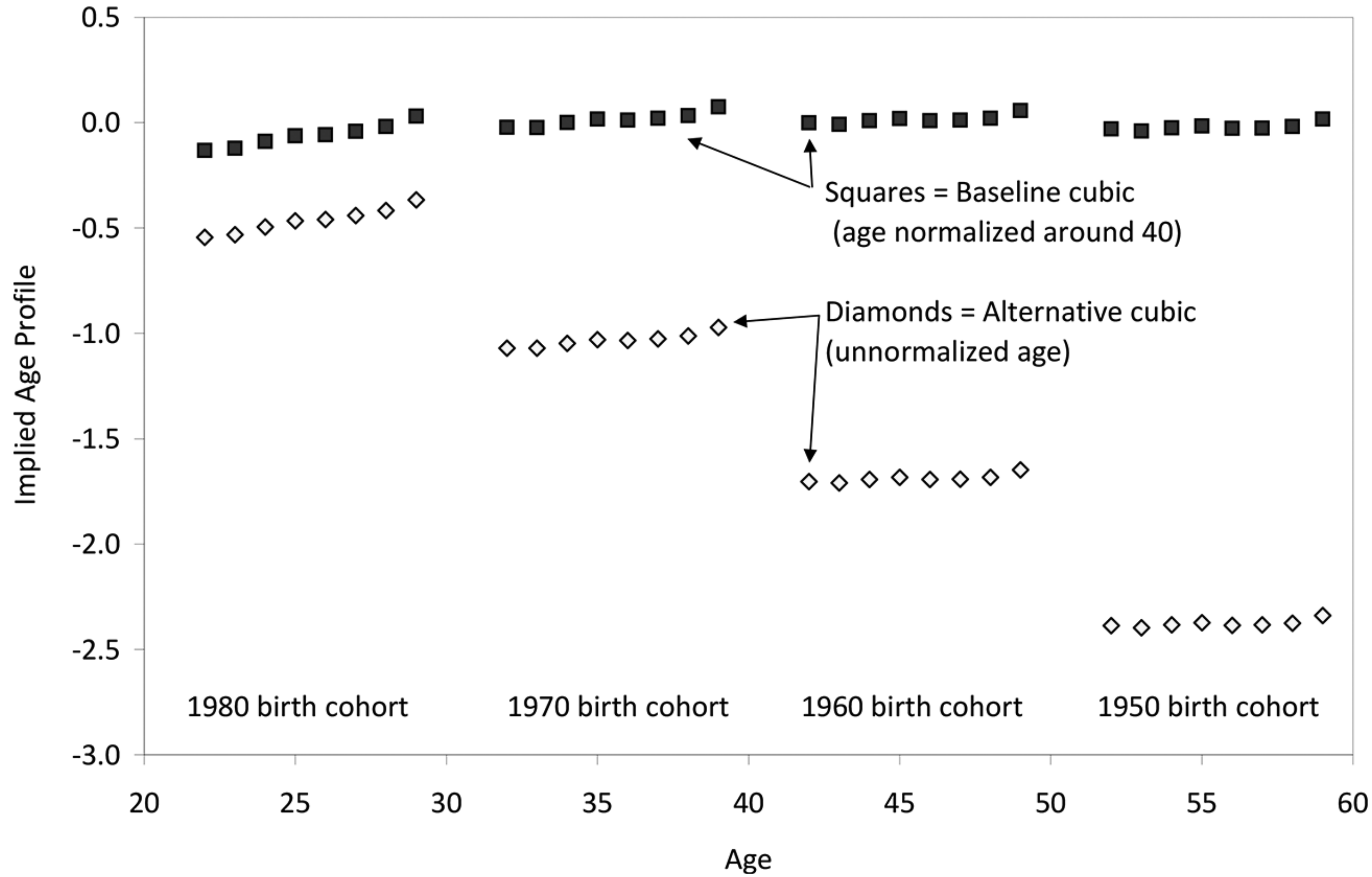
Table 3
Summary of Estimated Abowd, Kramarz, and Margolis (1999) Models for Portuguese Men, Alternative Normalizations of Age Function

	Cubic Age Function Flat				
	Age 40 (Baseline) (1)	Age 50 (2)	Age 30 (3)	Age 0 (4)	Gaussian Basis Function (5)
SD of person effects (across person-year observations)	.42	.41	.46	.93	.44
SD of firm effects (across person-year observations)	.25	.25	.25	.25	.25
SD of Xb (across person-year observations)	.07	.10	.12	.74	.08
Correlation of person/firm effects	.17	.16	.17	.14	.17
Correlation of person effects and covariate index	.19	.19	−.32	−.89	−.06
Correlation of firm effects and covariate index	.11	.14	−.03	−.08	.04
Inequality decomposition (percentage of variance of log wage explained):					
Person effects + covariate index	63	63	63	63	63
Person effects	58	54	70	282	62
Covariate index	2	3	4	180	2
Covariate of person effects and covariate index	3	5	−11	−399	−1
Firm effects	20	20	20	20	20
Covariance of firm effects with person effect + covariate index	12	12	12	12	12
Covariance of firm effects with person effects	11	10	13	21	12
Covariance of firm effects with covariate index	1	2	−1	−9	0
Residual	5	5	5	5	5

NOTE.—The sample includes 8,225,752 person-year observations for male workers in the largest connected set of QP in 2005–9. Sample and baseline specifications are the same as in the study by Card et al. (2016). Models include 1,889,366 dummies for individual workers and 216,459 dummies for individual firms, year dummies interacted with education dummies, and function of age interacted with education dummies. The age function in models in cols. 1–4 includes quadratic and cubic terms, with age deviated from 40, 50, 30, and 0 for models in cols. 1–4, respectively. The age function in model in col. 5 is a Gaussian basis function with five equally spaced spline points. All models have the same fit; root mean square error of the model is 0.143, and the adjusted R2 is 0.934. SD 5 standard deviation; Xb = fitted covariate index.

- Figure 2 contrasts the implied age profiles for four single year-of-birth cohorts of low-education men from this naive specification, with the implied profiles for the same groups under the baseline normalization.
- Evidently, the strong negative correlation between the person effects and the covariate index reported in column 4 of table 3 is driven by implausibly large cohort effects, which trend in a way to offset the imposed assumption that the cubic age profile is flat at age 0.
- Rather than restricting the age profile to be flat at a point, we can also achieve identification by assuming that the true profile is everywhere nonlinear.
- Column 5 shows the results of using a linear combination of normal density functions in age (with 5-year bandwidths) to approximate the age profile.
- Because each Gaussian component is nonlinear, we do not need restrictions on the parameters to avoid collinearity with cohort and time effects.

FIG. 2.—Implied age profiles from Abowd, Kramarz, and Margolis (1999) models with alternative normalizations of the age profile (men with primary education only).



Note: A color version of this figure is available online.

- Nevertheless, using Gaussian basis functions will solve the identification problem only if the true age profile has no linear segments.

- As shown in column 5, the Gaussian approximation yields results somewhere between our baseline normalization and the specification in column 3: although the estimated variability of the worker, firm, and time-varying components is very close to baseline, the correlation of the person effects and $X'_{it}\beta$ becomes slightly negative.

- Fortunately, the covariance of the person and firm effects is essentially the same under our baseline normalization and the Gaussian specification, leading us to conclude that most of the statistics of interest in this literature found under an age 40 normalization are robust to alternate identifying assumptions.

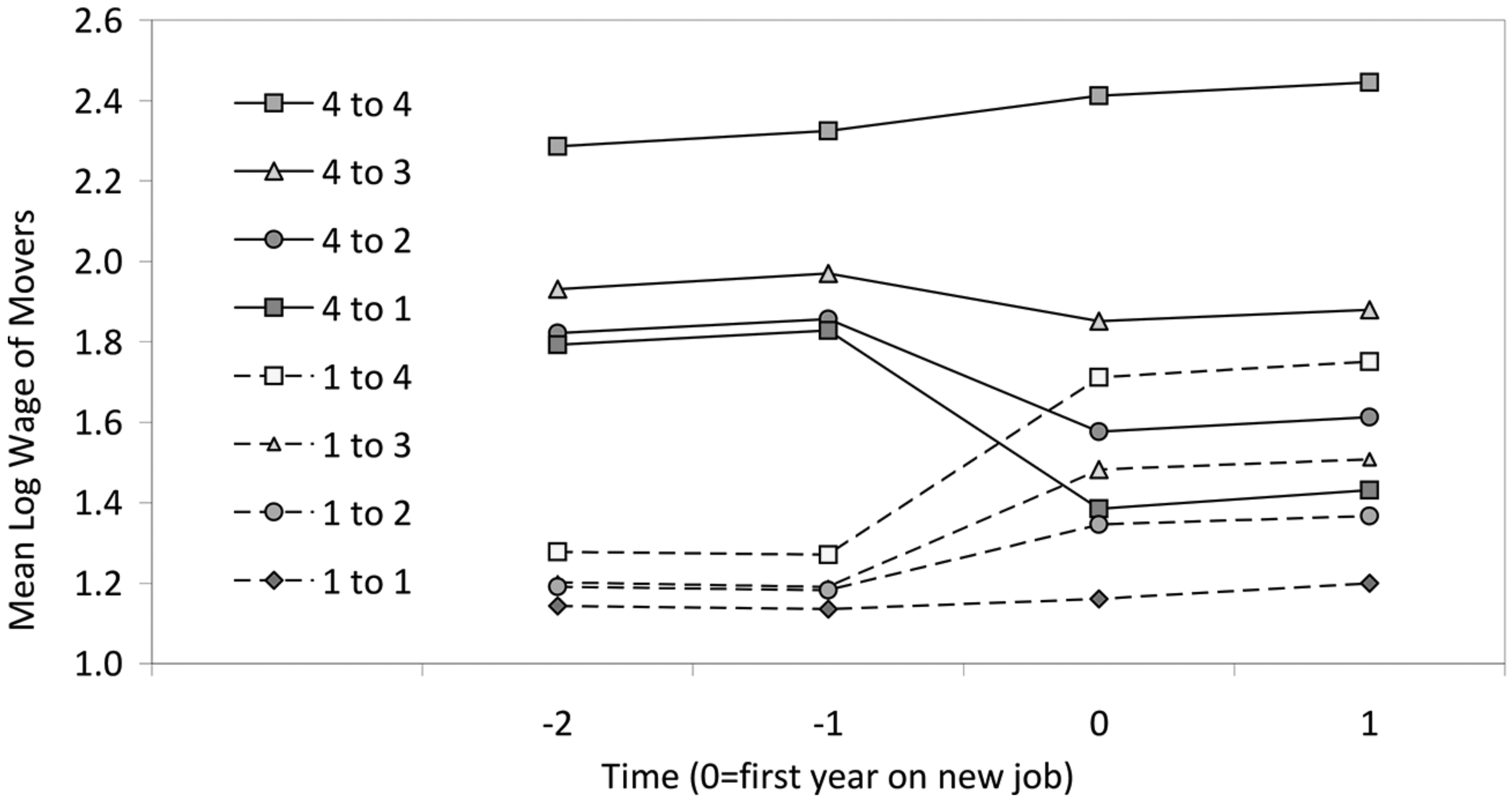
C. Worker-Firm Sorting and Limited Mobility Bias

- This is a version of the incidental parameters problem.
- We only observe workers at firms for a limited period.
- Short panels ⇒ incidental parameters problem
- $(\widehat{\alpha}_i, \widehat{\Psi}_{J(i,t)})$ inconsistent

D. Exogenous Mobility

- Figures 3 and 4 present the results of this analysis using data for male and female workers in Portugal, taken from Card et al. (2016).
- The samples are restricted to workers who switch establishments and have at least 2 years of tenure at both the origin and destination firm.
- Firms are grouped into coworker pay quartiles (using data on male and female coworkers).
- For clarity, only the wage profiles of workers who move from jobs in quartile 1 and quartile 4 are shown in the figures.
- The wage profiles exhibit clear steplike patterns: when workers move to higher-paying establishments, their wages rise; when they move to lower-paying establishments, their wages fall.

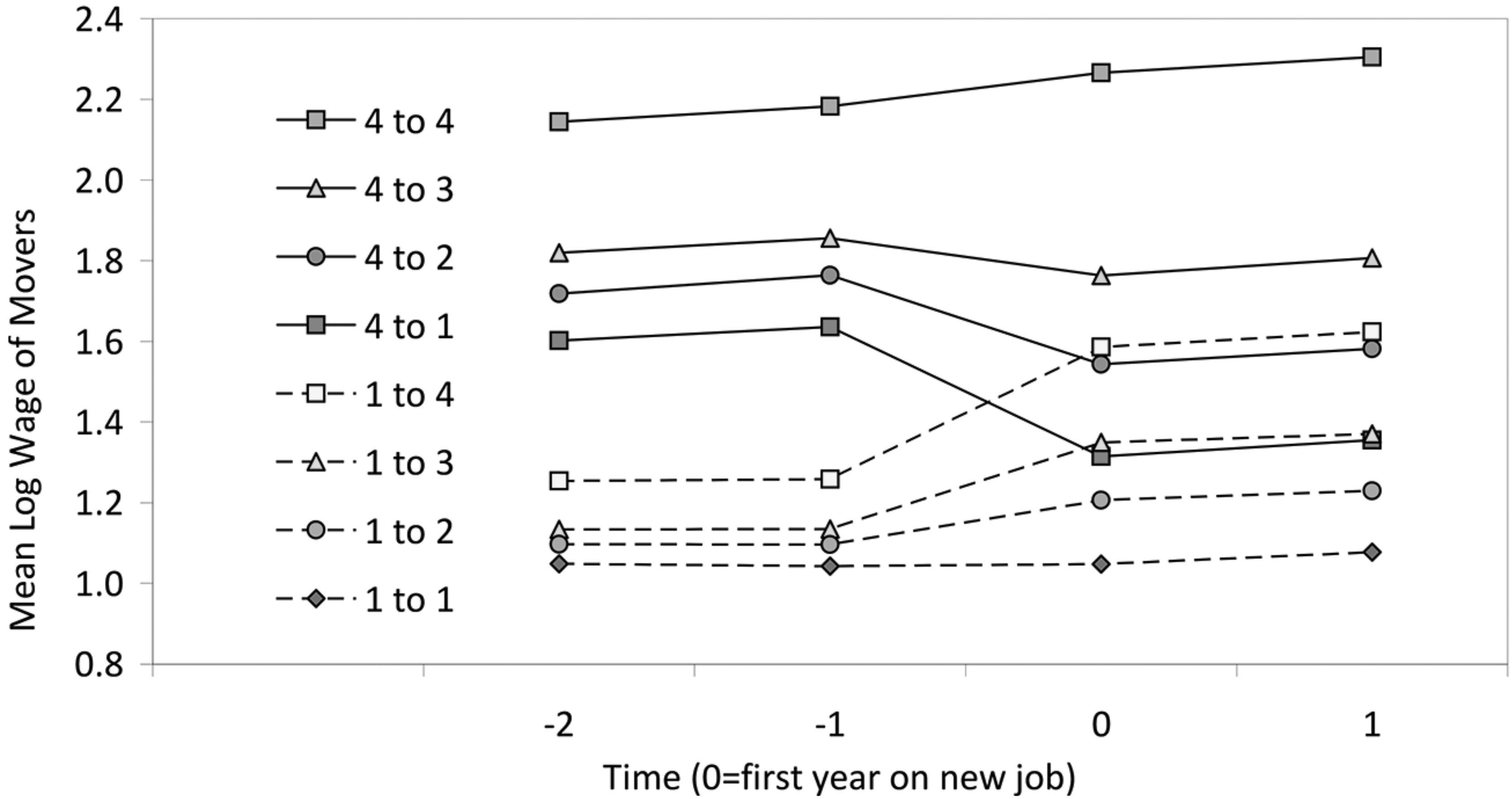
FIG. 3.—Mean log wages of Portuguese male job changers classified by quartile of coworker wages at origin and destination.



Notes: The figure shows mean wages of male workers at mixed-gender firms who changed jobs in 2004–7 and held the preceding job for 2 years or more and the new job for 2 years or more. Jobs are classified into quartiles based on mean log wage of coworkers of both genders.

Source: Card et al. (2016, fig. I). A color version of this figure is available online.

FIG. 4.—Mean wages of Portuguese female job changers classified by quartile of coworker wages at origin and destination.



Notes: The figure shows mean wages of female workers at mixed-gender firms who changed jobs in 2004–7 and held the preceding job for 2 years or more and the new job for 2 years or more. Jobs are classified into quartiles based on mean log wage of coworkers of both genders.

Source: Card et al. (2016, fig. II). A color version of this figure is available online.

- For example, males who start at a firm in the lowest quartile group and move to a firm in the top quartile have average wage gains of 39 log points, while those who move in the opposite direction have average wage losses of 43 log points.

- The gains and losses for other matched pairs of moves are also roughly symmetric, while the wage changes for people who stay in the same coworker pay group are close to 0.

- Another important feature of the wage profiles in figures 3 and 4 is that wages of the various groups are all relatively stable in the years before and after a job move.

- Workers who are about to experience a major wage loss by moving to a firm in a lower coworker pay group show no obvious trend in wages beforehand.

- Similarly, workers who are about to experience a major wage gain by moving to a firm in a higher pay group show no evidence of a pretrend.

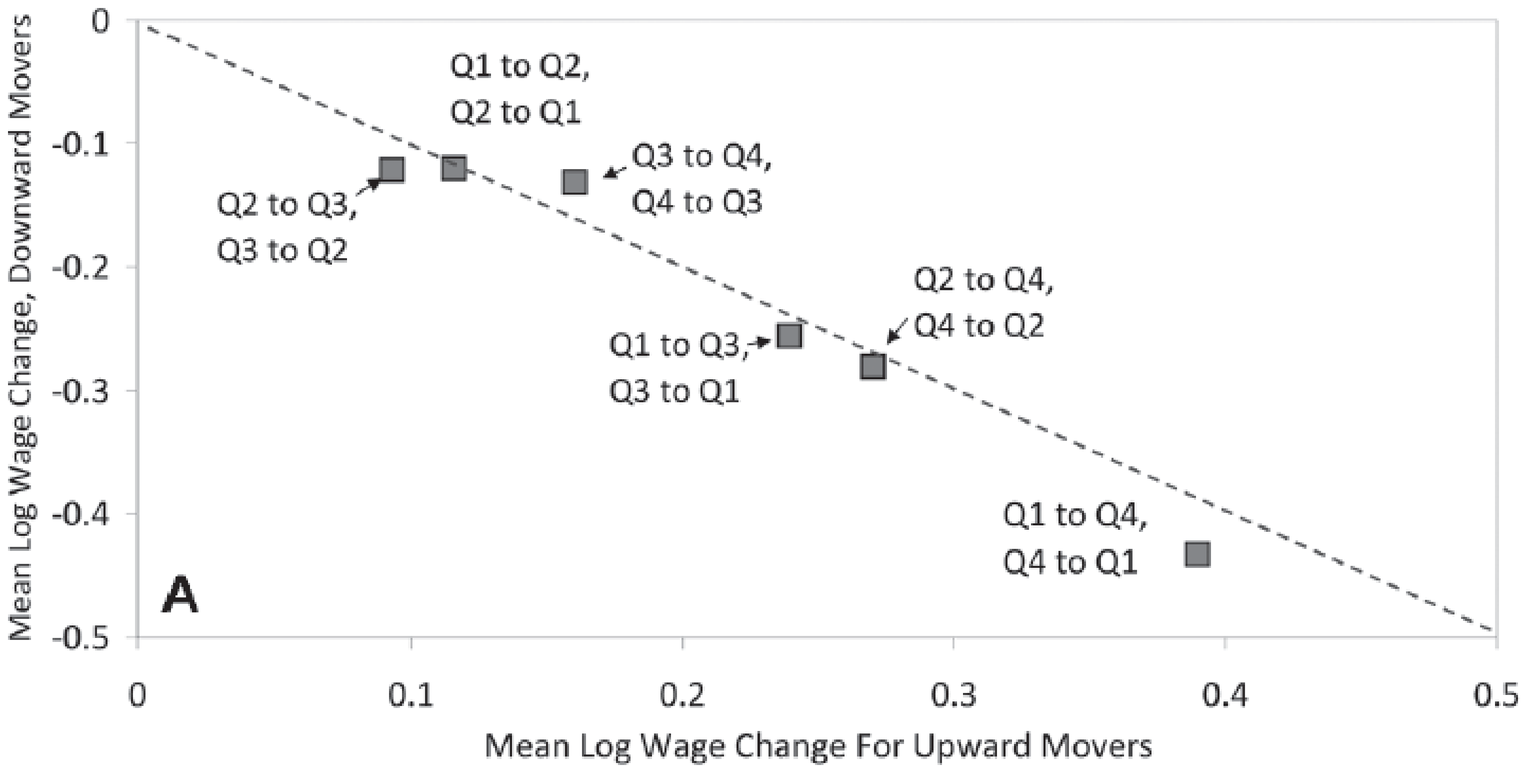
- By contrast, if worker mobility were driven by gradual employer learning, we would expect wage changes to precede moves between firm quality groups over the time horizons examined (Lange 2007).

- This analysis assumes a constant unit model: quartiles measure “skills.”

- Card et al. (2016) also present simple tests of the symmetry restrictions imposed by the AKM specification, using regression-adjusted wage changes of males and females moving between firms in the four coworker pay groups.

- Comparisons of upward and downward movers are displayed visually in figure 5 and show that the matched pairs of adjusted wage changes are roughly scattered along a line with slope of 21, consistent with the symmetry restriction.

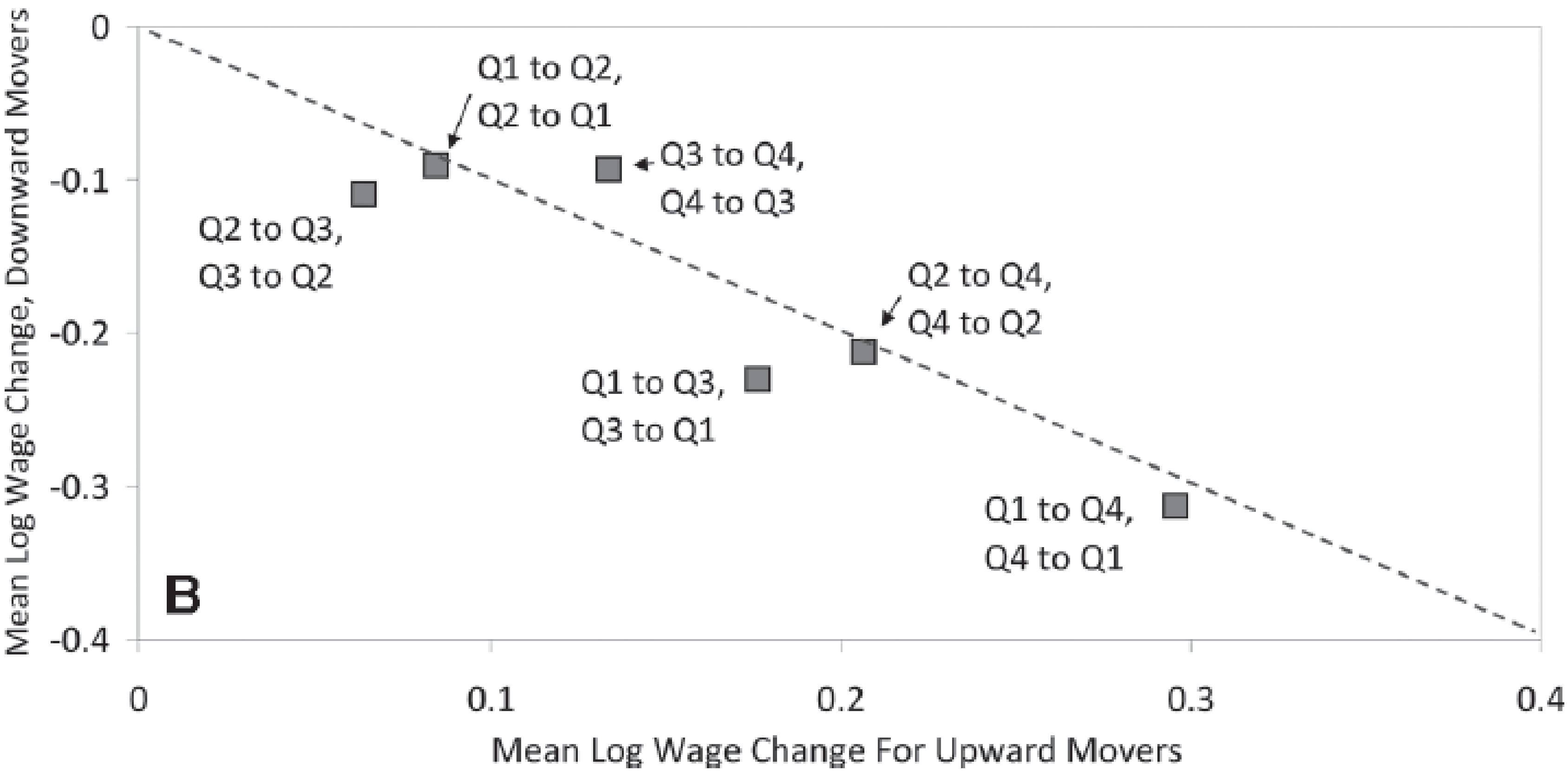
FIG. 5.—A, Test for symmetry of regression-adjusted wage changes of Portuguese male movers across coworker wage quartiles.



Notes: The figure plots regression adjusted mean wage changes over a 4-year interval for job changers who move across the coworker wage quartile groups indicated. The dashed line represents symmetric changes for upward and downward movers.

Source: Card et al. (2016, fig. B3). A color version of this figure is available online.

FIG. 5.—B, Test for symmetry of regression-adjusted wage changes of Portuguese female movers across coworker wage quartiles.



Notes: The figure plots regression-adjusted mean wage changes over a 4-year interval for job changers who move across the coworker wage quartile groups indicated. The dashed line represents symmetric changes for upward and downward movers.
Source: Card et al. (2016, fig. B4). A color version of this figure is available online.

E. Additive Separability

- Figures 6 and 7, taken from Card et al. (2016), show the mean residuals for 100 cells on the basis of deciles of the estimated worker effects and deciles of the estimated firm effects.

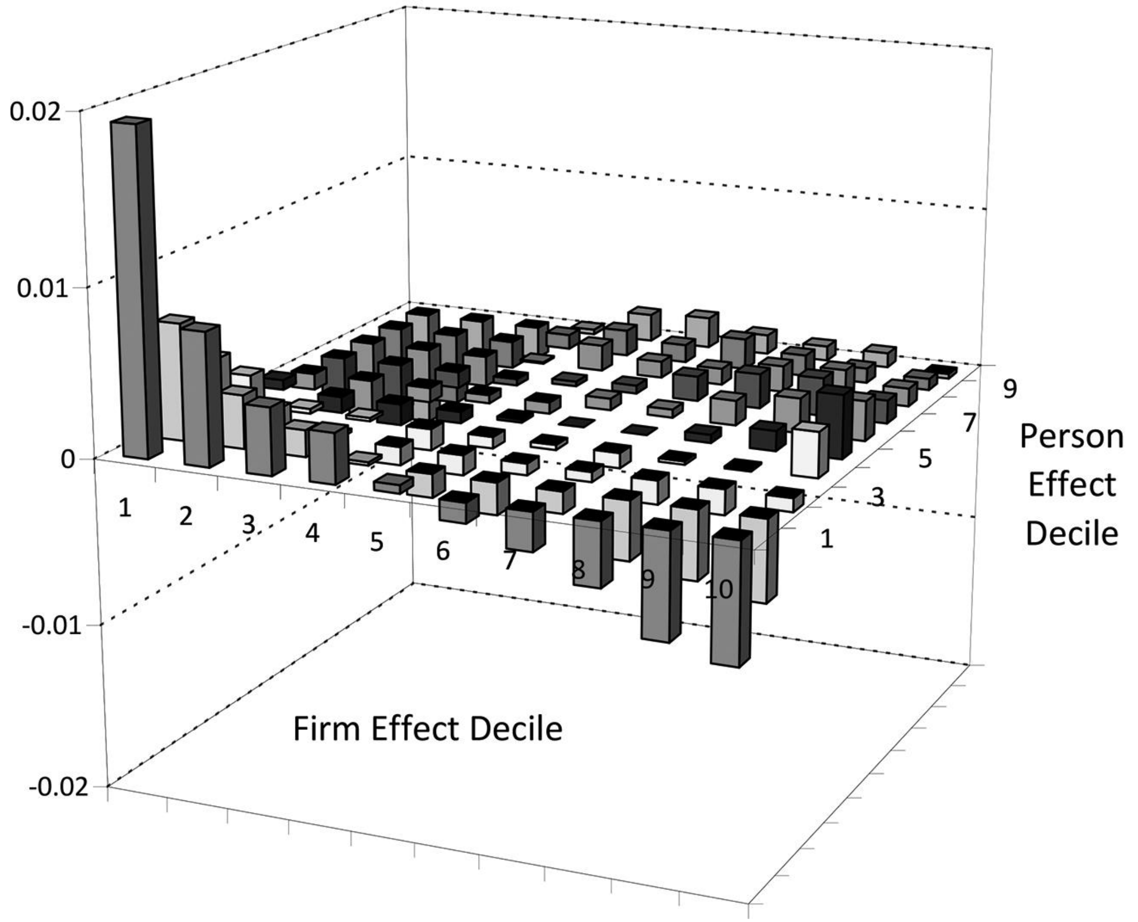
- If the additive model is correct, the residuals should have mean 0 for matches composed of any grouping of worker and firm effects, while if the firm effects vary systematically with worker skill, we expect departures from 0.

- Reassuringly, the mean residuals are all relatively close to 0.

- In particular, there is no evidence that the most able workers (in the 10th decile of the distribution of estimated person effects) earn higher premiums at the highest-paying firms (in the 10th decile of the distribution of estimated firm effects).

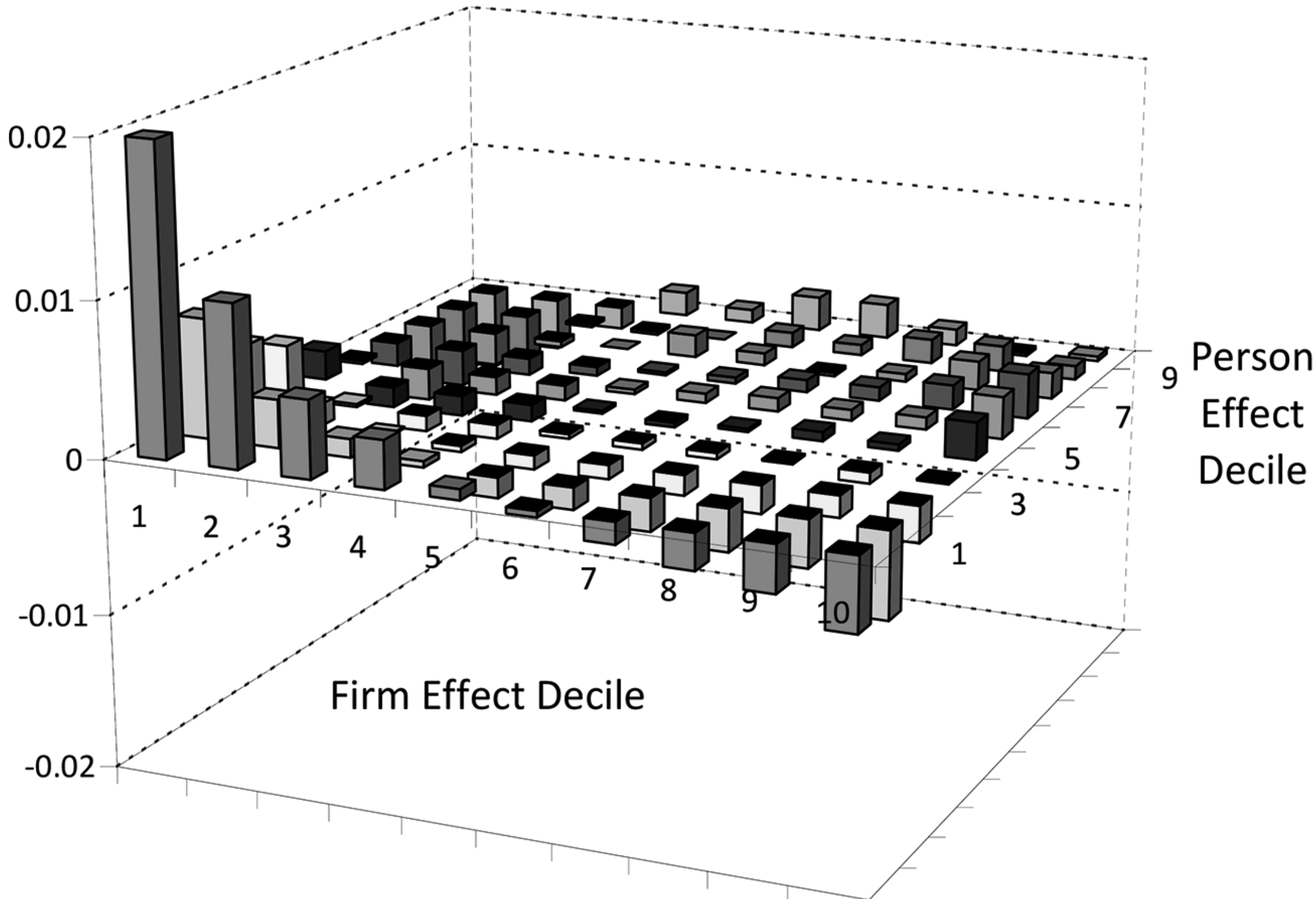
- The largest mean residuals are for the lowest-ability workers in the lowest paying firms, an effect that may reflect the impact of the minimum wage in Portugal.

FIG. 6.—Mean residuals by person/firm deciles for Portuguese male workers.



Notes: The figure shows mean residuals from an estimated Abowd, Kramarz, and Margolis (1999) model with cells defined by decile of estimated firm effects interacted with decile of estimated person effect.
Source: Card et al. (2016, fig. B5). A color version of this figure is available online.

FIG. 7.—Mean residuals by person/firm deciles for Portuguese female workers.



Notes: The figure shows mean residuals from an estimated Abowd, Kramarz, and Margolis (1999) model with cells defined by decile of estimated firm effects interacted with decile of estimated person effect.
Source: Card et al. (2016, fig. B6). A color version of this figure is available online.

IV. Reconciling Rent-Sharing Estimates with Results from Studies of Firm Switching

- The AKM model posits that the log of the wage of a given worker in a given year can be decomposed into the sum of a person effect, a firm or establishment effect, a time-varying index of person characteristics, and a residual that is orthogonal to the firm and person effects.

- It follows that the rent-sharing elasticity obtained from a regression of wages on a time-invariant measure of rents at the current employer (γ_w) can be decomposed into the sum of three components reflecting the regression on firm-specific rents of the estimated worker effects (γ_a), the estimated firm effects (γ_ψ), and the time-varying covariate index ($\gamma_{X\beta}$):

$$\gamma_w = \gamma_\alpha + \gamma_\psi + \gamma_{X\beta}.$$

- The regression coefficients γ_α and $\gamma_{X\beta}$ represent sorting effects.
- To the extent that firms with larger measured rents hire older workers or workers with greater permanent skills, γ_α and/or $\gamma_{X\beta}$ will be positive.
- The coefficient γ_ψ, on the other hand, is arguably a clean measure of the rent-sharing elasticity, since $\psi_{J(i,t)}$ represents a firm-specific wage premium that is paid on top of any reward for individual-specific skills.
- To implement this idea, we use the estimated AKM parameters from Card et al. (2016), which were estimated on a sample that includes virtually all the observations used for the cross-sectional models in panel A of table 2.

- The results are presented in panel A of table 4.
- Row 1 of the table reports estimated rent-sharing elasticities using the log hourly wage of each worker as a dependent variable.
- As in table 2, we report three specifications corresponding to models with only simple human capital controls (col. 1), controls for major industry and city (col. 2), and controls for detailed industry and location (col. 3).
- The estimated rent-sharing elasticities in row 1 are qualitatively similar to the estimates in row 1 of table 2 but differ slightly because the AKM model estimates are not available for all workers/firms.
- Rows 2–4 show how the overall rent-sharing elasticities in row 1 can be decomposed into a worker quality effect (row 2), a firm wage premium effect (row 3), and an experience-related sorting effect (row 4), which is close to 0.

Table 4
Relationship between Components of Wages and Mean Log Value Added per Worker

	Basic Specification (1)	Basic + Major Industry/City (2)	Basic + Detailed Industry/City (3)
A. Combined sample (n = 2,252,436 person-year observations at 41,120 firms):			
Log hourly wage	.250	.222	.187
	(.018)	(.016)	(.012)
Estimated person effect	.107	.093	.074
	(.010)	(.009)	(.006)
Estimated firm effect	.137	.123	.107
	(.011)	(.009)	(.008)
Estimated covariate index	.001	.001	.001
	(.000)	(.000)	(.000)
B. Less educated workers (n = 1,674,676 person-year observations at 36,179 firms):			
Log hourly wage	.239	.211	.181
	(.017)	(.016)	(.011)
Estimated person effect	.089	.072	.069
	(.009)	(.009)	(.005)
Estimated firm effect	.144	.133	.107
	(.015)	(.013)	(.008)
Estimated covariate index	.000	.000	.000
	(.000)	(.000)	(.000)
C. More educated workers (n = 577,760 person-year observations at 17,615 firms):			
Log hourly wage	.275	.247	.196
	(.024)	(.020)	(.017)
Estimated person effect	.137	.130	.094
	(.016)	(.013)	(.009)
Estimated firm effect	.131	.113	.099
	(.012)	(.009)	(.010)
Estimated covariate index	−.001	−.001	−.001
	(.000)	(.000)	(.000)

NOTES.—Entries are coefficients of mean log value added per worker (at current firm) in regression models with dependent variables listed in the row headings. Standard errors are clustered by firm (in parentheses). The sample in panel B includes males with less than completed secondary education at firms in the connected set for less educated workers. The sample in panel C includes males with a high school education or more at firms in the connected set for more educated workers. The sample in panel A includes males in either the panel B or the panel C sample. All models control for cubic in experience and unrestricted education*year dummies. Models in col. 2 also control for 20 major industries and two major cities (Lisbon and Porto). Models in col. 3 also control for 202 detailed industry dummies and 29 Nomenclature of Territorial Units for Statistics region 3 location dummies.

- A key conclusion from these estimates is that rent-sharing elasticities estimated from a cross-sectional specification incorporate a sizable worker quality bias.

- In each column of table 4, roughly 40% of the overall wage elasticity in row 1 is due to the correlation of worker quality (measured by the person effect component of wages) with firm-specific quality.

- Adjusting for worker quality, the estimates in row 3 point to a rent-sharing elasticity in the range of 0.10–0.15, large enough to create a Lester range of wage variation of 16–24 log points associated with the differences between firms at the 90th and 10th percentiles of log value added per worker.

A. Differential Rent Sharing

- We can use the AKM framework to examine another interesting question: to what extent do different groups of workers receive larger or smaller shares of the rents at different firms?
- To do this, we fit separate AKM models for less educated men (with less than a high school education) and more educated men (with a high school education or more) to our Portuguese wage sample.
- We then reestimated the same rent-sharing specifications reported in panel A of table 4 separately for the two groups.
- The results are reported in panels B and C of table 4.
- The estimates reveal several interesting patterns.

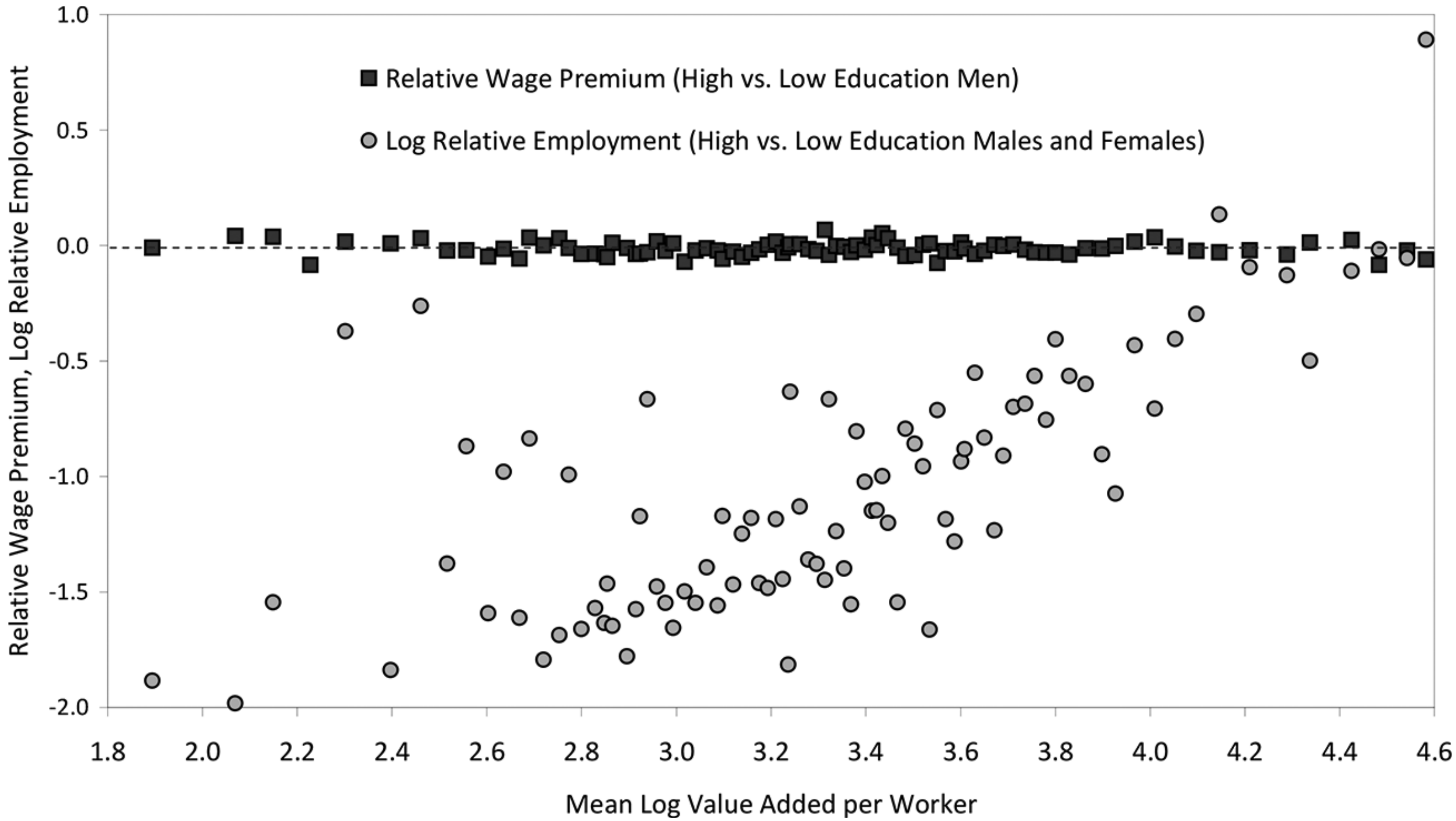
- Most importantly, although the correlation between wages and value added per worker is a little higher for the more educated men, virtually all of this gap is due to a stronger correlation between the worker quality component of wages and value added.

- The correlations with the firm-specific pay premiums are very similar for the two education groups.

- Thus, we see no evidence of differential rent sharing.

- This finding is illustrated in figure 8, which shows a binned scatterplot of mean log value added per worker at different firms (on the horizontal axis) versus the relative wage premium for high-educated versus low-educated men at these firms.

FIG. 8.—Relative wage premium and relative employment of high- versus loweducation workers.



Notes: Firms are divided into 100 cells on the basis of mean log value added per worker in 2005–9, with equal numbers of person-year observations per cell.
A color version of this figure is available online.

- We also superimpose a bin-scatter of the relative share of higher-education workers at different firms (including both men and women in the employment counts for the two education groups).

- The relative wage premium is virtually flat, consistent with the regression coefficients in rows 7 and 11 of table 4, which show nearly the same effect of value added per worker on the wage premiums for the two education groups.

- In contrast, the relative share of highly educated workers is increasing with value added per worker, a pattern we interpret as largely driven by the labor quality component in value added per worker.

V. Imperfectly Competitive Labor Markets and Inequality

A. Market Structure

- There are J firms and two types of workers: lower skilled (L) and higher skilled (H).

- Each firm $j \in \{1, \dots, J_g\}$ posts a pair (w_{Lj}, w_{Hj}) of skill-specific wages that all workers costlessly observe.

- Firms exhibit differentiated work environments over which workers have heterogeneous preferences.

- For worker i in skill group $S \in \{L, H\}$, the indirect utility of working at firm j is

$$u_{iSj} = \beta_S \ln(w_{Sj} - b_S) + a_{Sj} + \epsilon_{iSj},$$

- where b_S is a skill group–specific reference wage level (e.g., arising from wages paid in an outside competitive sector), Sj is a firm-specific amenity common to all workers in group S, and ϵ_{iSj} captures idiosyncratic preferences for working at firm j, arising, for example, from nonpecuniary match factors such as distance to work or interactions with coworkers and supervisors.

- Weassume that the $\{\epsilon_{iSj}\}$ are independent draws from a type I extreme value distribution.

- Given posted wages, workers are free to work at any firm they wish.

- Hence, by standard arguments (McFadden 1973), workers have logit choice probabilities of the form

$$p_{Sj} \equiv P\left(\arg\max_{k\in\{1,\ldots,J\}}\{u_{iSk}\} = j\right) = \frac{\exp\left(\beta_S(\ln(w_{Sj} - b_S) + a_{Sj}\right)}{\sum_{k=1}^{J}\exp(\beta_S \ln(w_{Sk} - b_S) + a_{Sk})}.$$

- To simplify the analysis and abstract from strategic interactions in wage setting, we assume that the number of firms J is very large, in which case the logit probabilities are closely approximated by exponential probabilities

$$p_{Sj} \approx \lambda_S \exp(\beta_S \ln(w_{Sj} - b_S) + a_{Sj}),$$

- where $\{\lambda_H, \lambda_L\}$ are constants common to all firms in the market.

- Thus, for large J, the approximate firm-specific supply functions are

$$\ln L_j(w_{Lj}) = \ln(\mathcal{L}\lambda_L) + \beta_L \ln(w_{Lj} - b_L) + a_{Lj}, \tag{5}$$

$$\ln H_j(w_{Hj}) = \ln(\mathcal{H}\lambda_H) + \beta_H \ln(w_{Hj} - b_H) + a_{Hj}, \tag{6}$$

- where $\mathcal{L}$ and $\mathcal{H}$ give the total numbers of lower-skilled and higher-skilled workers in the market.

- Note that as $\beta_L, \beta_H \to \infty$ these supply functions become perfectly elastic, and we approach a competitive labor market with exogenous wages β_L and β_H.

B. Firm Optimization

- Firms have production functions of the form where T_j is a firm-specific productivity shifter.

$$Y_j = T_j f(L_j, H_j), \tag{7}$$

- We assume that $f(\dots)$ is twice differentiable and exhibits constant returns to scale with respect to L_j and H_j. For simplicity, we also ignore capital and intermediate inputs.

- The firm's problem is to post a pair of skill-specific wages that minimize the cost of labor services given knowledge of the supply functions (5) and (6).

- Firms cannot observe workers' preference shocks $\{\epsilon_{iSj}\}$, which prevents them from perfectly price discriminating against workers according to their idiosyncratic reservation values.

- The firm's optimal wage choices solve the cost minimization problem

$$\min_{w_{Lj}, w_{Hj}} w_{Lj} L_j(w_{Lj}) + w_{Hj} H_j(w_{Hj}) \text{ such that } T_j f(L_j(w_{Lj}), H_j(w_{Hj})) \geq Y.$$

- The associated first-order conditions can be written as

$$w_{Lj}\frac{1+e_{Lj}}{e_{Lj}} = T_j f_L \mu_j, \qquad (8)$$

$$w_{Hj}\frac{1+e_{Hj}}{e_{Hj}} = T_j f_H \mu_j, \qquad (9)$$

- where e_{Lj} and e_{Hj} represent the elasticities of supply of L and H workers at the optimal choice of wages and μ_j represents the marginal cost of production, which the firm will equate to marginal revenue at an optimal choice for Y.

- Thus, the terms $T_j f_L \mu_j$ and $T_j f_H \mu_j$ on the right-hand sides of equations (8) and (9) represent the marginal revenue products of the two types of labor, while the terms on the left-hand sides represent their marginal factor costs.

- Using equations (5) and (6), the elasticities of supply are

$$e_{Lj} = \frac{\beta_L w_{Lj}}{w_{Lj} - b_L},$$

$$e_{Hj} = \frac{\beta_H w_{Hj}}{w_{Hj} - b_H}.$$

- Note that for both groups, labor supply to the firm becomes infinitely elastic as wages approach the reference wage level b_S.

- Using these expressions, the firm's first-order conditions can be rewritten as

$$w_{Lj} = \frac{1}{1 + \beta_L} b_L + \frac{\beta_L}{1 + \beta_L} T_j f_L \mu_j, \tag{10}$$

$$w_{Hj} = \frac{1}{1 + \beta_H} b_H + \frac{\beta_H}{1 + \beta_H} T_j f_H \mu_j. \tag{11}$$

C. Baseline Case: Linear Production Function and Fixed Output Price

- To develop intuition, we begin with the simplest possible example, where the firm faces a fixed output price P_j^0 and has a linear production function

$$Y_j = T_j N_j = T_j\big((1-\theta)L_j + \theta H_j\big).$$

- Here Nj represents the efficiency units of labor at the firm and the parameter $\theta \in (0.5,1)$, which we assume is common to all firms, governs the relative productivity of the two types of labor.

- Crucial: perfect substitutability.

- Under this specification of technology and market structure, the first-order conditions (10) and (11) evaluate to

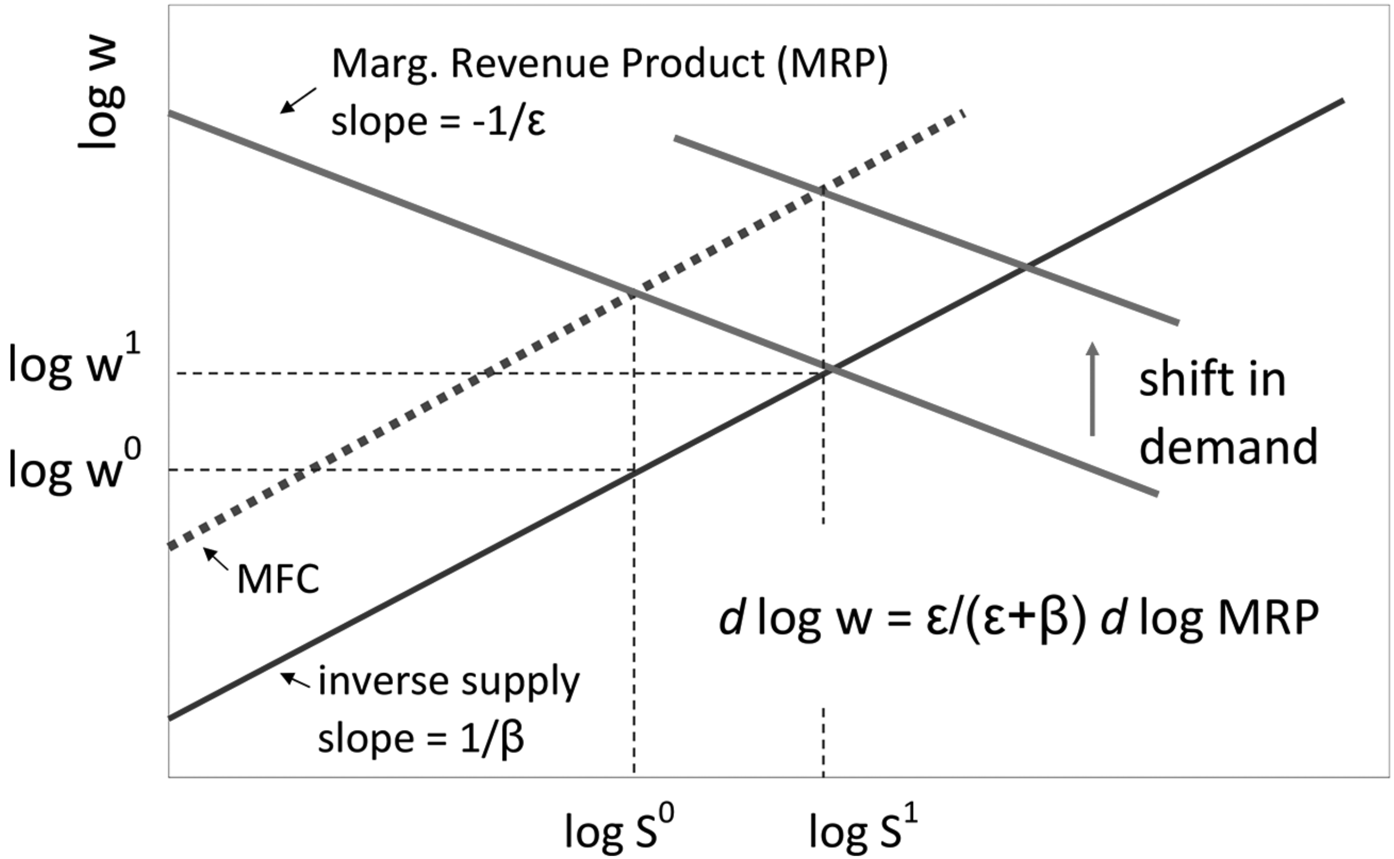
$$w_{Lj} = \frac{1}{1+\beta_L} b_L + \frac{\beta_L}{1+\beta_L} T_j P_j^0 (1-\theta),$$

$$w_{Hj} = \frac{1}{1+\beta_H} b_H + \frac{\beta_H}{1+\beta_H} T_j P_j^0 \theta\,.$$

- P_j^0= price of output for firm j.

- The determination of the optimal wage in the simplified situation where there is only one skill group is illustrated in figure 9.
- The firm faces an upward-sloping inverse labor supply function of the form $w = b + N^{1/\beta}$.
- The associated marginal factor cost is $\mathrm{MFC} = b + [(1+\beta)/\beta)]N^{1/\beta}$.
- The firm equates MFC with marginal revenue product (MRP), leading to an equilibrium wage $w = [1/(1+\beta)]b + [\beta/(1+\beta)]MRP$.
- As shown in the figure, if the firm's marginal revenue product increases, both employment and wages will increase at the firm.

FIG. 9.—Effect of total factor productivity shock (single skill group).



Notes: MFC = marginal factor cost. A color version of this figure is available online.

- In contrast to traditional rent-sharing models, however, this positive relationship between wages and productivity does not stem from wage bargaining.

- Firms unilaterally post profit maximizing wages that leave the marginal worker with no surplus on the job.

- The firm shares rents with inframarginal workers only because it lacks the information necessary to price discriminate on the basis of reservation wages.

- To understand the implications of this model for the relative wage structure, suppose that the reference wages of the two skill groups are proportional to their relative productivities, so that

$$b_L = (1-\theta)b,\ b_H = \theta b.$$

- This restriction is natural if one views b_S as an outside wage that can be earned in a fully competitive sector where wages equal marginal products.

- Now the first-order conditions can be rewritten as

$$\ln w_{Lj} = \ln \frac{(1-\theta)b}{1+\beta_L} + \ln(1+\beta_L R_j), \quad (12)$$

$$\ln w_{Hj} = \ln \frac{\theta b}{1+\beta_H} + \ln(1+\beta_H R_j), \quad (13)$$

- where $R_j = T_j P_j^0 / b$ gives the proportional gap in marginal labor productivity at firm j relative to the competitive sector.

- Wages of both skill groups contain a rent-sharing component that depends on R_j and the skill group–specific supply parameter b_S.

- Note that under the linear technology assumption, value added per standardized unit of labor is $v_j \equiv P_j^0 Y_j / N_j = P_j^0 T_j$, so $R_j = v_j / b$ is the ratio of value added per standardized unit of labor to the outside wage for a worker with 1 efficiency unit of labor.

- Equations (12) and (13) therefore imply that the elasticity of wages of skill group S with respect to value added per worker is

$$\xi_{Sj} \equiv \frac{\partial \ln w_{Sj}}{\partial \ln v_j} = \frac{\beta_S R_j}{1 + \beta_S R_j}.$$

- Interestingly, this is the same as the expression for the rent-sharing elasticity (eq. [2]) in a bargaining model where workers are assumed to capture a fixed share of the quasi rents.

- The elasticity of labor supply for skill group S when wages are determined by the first-order conditions (12) and (13) is

$$e_{Sj} = \frac{\beta_L w_{Sj}}{w_{Sj} - b_S} = \frac{1 + \beta_S R_j}{R_j - 1}.$$

- Assuming that $\beta_S R_j = 0.1$, a value of the firm-specific elasticity of supply of around 4 implies that $R_j \approx 1.3$ and $\beta_S \approx 0.08$.

- While many empirical estimates of the elasticity of supply to the firm are lower than 4 (Manning 2011), we consider this a reasonable near-competitive benchmark because it implies an equilibrium markdown of wages relative to marginal products of only 20%.

- A key implication of equations (12) and (13) is that when $\beta_L = \beta_H$, the relative wages of the two skill groups are independent of firm-specific productivity.

- To simplify the discussion, assume that $\beta_L R_j$ and $\beta_H R_j$ are both relatively small (i.e., on the order of 0.10).

- In such a case, the Taylor approximation

$$\ln w_{Lj} = \ln \frac{(1-\theta)b}{1+\beta_L} + \beta_L R_j,$$

$$\ln w_{Hj} = \ln \frac{\theta b}{1+\beta_H} + \beta_H R_j,$$

- will be highly accurate.

- This implies that the log wage gap between high and low-skilled workers at firm j is

$$\ln \frac{w_{Hj}}{w_{Lj}} = \ln \frac{\theta}{1-\theta} + \ln \frac{1+\beta_L}{1+\beta_H} + (\beta_H - \beta_L) R_j \,. \qquad (14)$$

- When $\beta_L = \beta_H = \beta$, wages can be written in the form

$$\ln w_{Sj} = \alpha_S + \psi_j, \tag{15}$$

- where $\alpha_s \equiv \ln(b/(1+\beta)) + 1(S=L) \times \ln 1 - \theta + 1(S=H) \times \ln \theta$ is a skill group–specific constant and $\psi_j = \beta R_j = (\beta/b)v_j$is the firm-specific wage premium paid by firm j.

- This simple model therefore yields a reduced form specification for individual wages that is consistent with the additively separable formulation proposed by Abowd et al. (1999).

- Moreover, the firm effects should be strongly related to value added per worker, something we saw evidence for in table 4.

- When one group has a higher value of the supply parameter β, the log wage gap between workers in different skill groups will be higher at more profitable firms.

- In this case, the data will be described by an AKM-style model with skill group–specific firm effects.

- The wage premium for skill group S at firm j will be

$$\psi_j^S = \beta_S R_j.$$

1. Between-Firm Sorting

- Even when $\beta_L = \beta_H$ and the wage gap between workers in the two skill groups is constant at any given firm, the market-wide average wage for each skill group will depend on their relative distribution across firms.

- In particular, equation (15) implies that the expected log wage for workers in skill group S is

$$E[\ln w_{Si}] = \alpha_S + \sum_j \psi_j \pi_{Sj},$$

- where π_{sj} is the share of workers in skill group S employed at firm j.

- Thus, the market-wide wage differential between high- and low-skilled workers depends on their relative productivity, their relative supply elasticities, and the relative shares of the two groups employed at firms with higher or lower wage premiums:

$$E[\ln w_{Hi}] - E[\ln w_{Li}] = \alpha_H - \alpha_L + \sum_j \psi_j (\pi_{Hj} - \pi_{Lj}). \qquad (16)$$

- Some simple evidence on the importance of the sorting component for the structure of wages for Portuguese male workers is presented in figure 10.

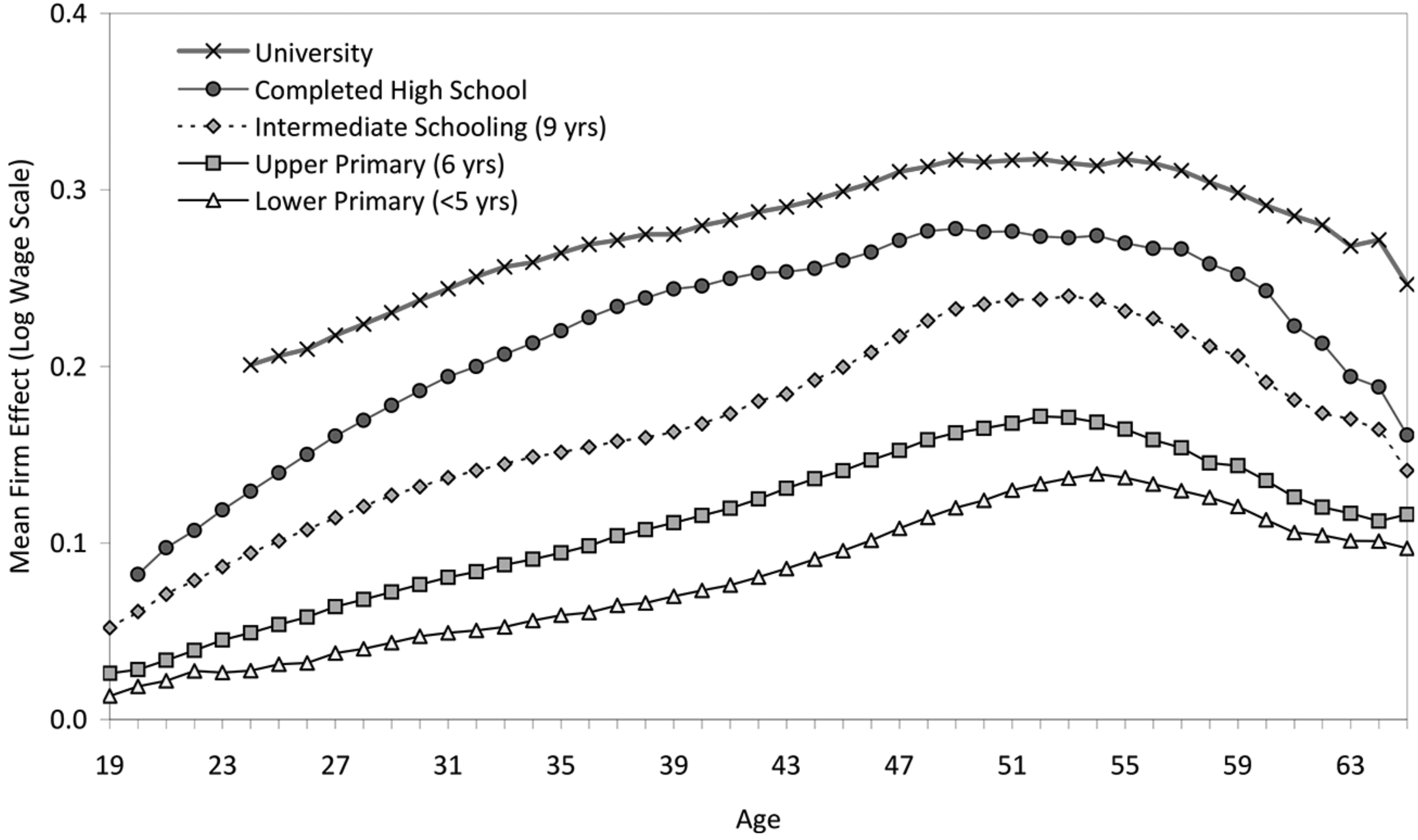
- Here, we plot the mean firm effects by age for Portuguese men in five different education groups. We normalize the estimated firm effects using the procedure described by Card et al. (2016), which sets the average firm effect to 0 for firms in (roughly) the bottom 15% of the distribution of log value added per worker.

- The figure shows two important features.

- First, within each education group, the mean firm effect associated with the jobs held by workers at different ages is increasing until about age 50 and then slightly decreasing. Thus, the life-cycle pattern of between-firm sorting contributes to the well-known shape of the life-cycle wage profile.

- Second, at all ages more highly educated workers are more likely to work at firms that pay higher wage premiums to all their workers. A significant share of the wage gap between men with different education levels is therefore attributable to differential sorting.

FIG. 10.—Mean firm effects by age and education group for Portuguese males.



Notes: Firm effects are normalized using the method of Card et al. (2016).

- When the supply parameter β varies across groups, the wage decomposition will contain an additional term, reflecting a weighted average across firms of the rent-sharing components of the two skill groups:

$$\begin{aligned} E[\ln w_{Hi}] - E[\ln w_{Li}] &= \alpha_H - \alpha_L + \sum_j \psi_j^L (\pi_{Hj} - \pi_{Lj}) + \sum_j (\psi_j^H - \psi_j^L) \pi_{Hj} \\ &= \alpha_H - \alpha_L + \sum_j \psi_j^H (\pi_{Hj} - \pi_{Lj}) + \sum_j (\psi_j^H - \psi_j^L) \pi_{Lj}. \end{aligned}$$

D. Downward-Sloping Firm-Specific Product Demand

- So far we have assumed that the firm is a price taker in its output market.
- Suppose now that the firm faces an inverse demand function $P_j = P_j^0 Y_j^{-1/\varepsilon}$, with $\varepsilon > 1$ giving the elasticity of product demand.
- This yields the marginal revenue function

$$\mathrm{MR}_j = \left(\frac{\varepsilon - 1}{\varepsilon}\right) P_j^0 Y_j^{-1/\varepsilon}.$$

- In this case, assuming as above that $b_L = (1-\theta)b$ and $b_H = \theta b$, the first order conditions (10) and (11) evaluate to

$$w_{Lj} = \frac{b(1-\theta)}{1+\beta_L}\left[1 + \beta_L\left(\frac{\varepsilon - 1}{\varepsilon}\right) T_j P_j^0 Y_j^{-1/\varepsilon}\right], \qquad (17)$$

$$w_{Hj} = \frac{b\theta}{1+\beta_H}\left[1 + \beta_H\left(\frac{\varepsilon - 1}{\varepsilon}\right) T_j P_j^0 Y_j^{-1/\varepsilon}\right]. \qquad (18)$$

- These equations can be simplified by noting that value-added per efficiency unit of labor is

$$v_j \equiv \frac{P_j Y_j}{N_j} = P_j^0 T_j Y_j^{-1/\varepsilon}.$$

- Thus, the optimal choices for wages can be written

$$\ln w_{Lj} = \ln \frac{(1-\theta)b}{1+\beta_L} + \ln(1 + \beta_L R_j'),$$

$$\ln w_{Hj} = \ln \frac{\theta b}{1+\beta_H} + \ln(1 + \beta_H R_j'),$$

- Where $R_j' = [(\varepsilon - 1)/\varepsilon] v_j / b$.

- Note that as $\varepsilon \rightarrow \infty$, these reduce to equations (12) and (13).

- Moreover, regardless of the value of ε, if $\beta_L = \beta_H$, then relative wages are constant across firms, and the AKM model of the wage structure remains valid, with the firm effects being monotone functions of value added per worker.

- The implied elasticity of wages of skill group S with respect to value added per standardized unit of labor is

$$\xi_{Sj} = \frac{\partial \ln w_{Sj}}{\partial \ln v_j} = \frac{\beta_S R_j'}{1 + \beta_S R_j'}.$$

- Assuming that this elasticity is approximately 0.10 suggests that $\beta_S R_j' \approx 0.10$. Moreover, the elasticity of labor supply of skill group S to the firm is

$$e_{Sj} = \frac{\beta_L w_{Sj}}{w_{Sj} - b_S} = \frac{1 + \beta_S R_j'}{R_j' - 1},$$

- so calibrating this elasticity to a value of 4 would suggest that $R_j' = 1.28$, again pointing to a value of $\beta_s \approx 0.08$.

- Finally, note that the elasticity of employment of skill group S with respect to a change in v_j is

$$e_{Sj}\xi_{Sj} = \frac{\beta_S R_j'}{R_j' - 1},$$

- which has a value of approximately 4 under the preceding assumptions.

- When the firm faces a downward-sloping product demand, value added per efficiency unit of labor (v_j) depends on the endogenous choice of output.

- In the appendix, we show that the elasticities of v_j with respect to an exogenous shift in output demand (indexed by P_j^0) or an exogenous increase in productivity (indexed by T_j) are

$$\frac{\partial \ln v_j}{\partial \ln P_j^0} = \frac{\varepsilon}{\varepsilon + m_j},$$

$$\frac{\partial \ln v_j}{\partial \ln T_j} = \frac{\varepsilon - 1}{\varepsilon + m_j},$$

- where

$$m_j \equiv \frac{\partial \ln N_j}{\partial \ln v_j} = \frac{R_j'}{R_j' - 1}\left[\beta_L(1-\theta)\left(\frac{L_j}{N_j}\right) + \beta_H \theta \frac{H_j}{N_j}\right]$$

- measures the rate at which overall efficiency units of labor expand when there is an exogenously driven increase in value added.

- From these expressions, it follows that the elasticities of the wages of skill group S with respect to demand shocks and productivity shocks are

$$\frac{\partial \ln w_{Sj}}{\partial \ln P_j^0} = \frac{\varepsilon}{\varepsilon + m_j} \times \xi_{Sj},$$

$$\frac{\partial \ln w_{Sj}}{\partial \ln T_j} = \frac{\varepsilon - 1}{\varepsilon + m_j} \times \xi_{Sj}.$$

- Under the calibrations above, m_j is approximately 4.

- Assuming that the firm-specific product demand elasticity is between 3 and 10, the elasticity of wages with respect to a shift in the firm's demand curve will be between 0.04 and 0.07, and the elasticity with respect to a shift in technological efficiency will be between 0.035 and 0.065.

E. Imperfect Substitution between Skill Groups

- A limitation of our baseline model is that it assumes perfect substitutability between the two skill groups.

- We now extend the model by assuming that the firm's output is a CES aggregate of high- and low-skilled labor:

$$Y_j = T_j N_j = T_j f(L_j, H_j),$$

$$f(L_j, H_j) = \left[(1-\theta)L_j^\rho + \theta H_j^\rho\right]^{1/\rho}, \qquad (19)$$

- where $r \in (-\infty, 1]$ and $\sigma = (1-\rho)^{-1}$ are the elasticity of substitution between the types of labor.

- The marginal productivities of the two groups take the form

$$T_j f_L = T_j (1-\theta) L_j^{\rho-1} N_j^{1-\rho},$$

$$T_j f_H = T_j \theta H_j^{\rho-1} N_j^{1-\rho}.$$

- Assuming that the firm faces a constant price P_j^0 for its output, that $b_L = (1-\theta)b$, and that $b_H = \theta b$, the first-order conditions (10) and (11) evaluate to

$$w_{Lj} = \frac{b(1-\theta)}{1+\beta_L}\left[1 + \beta_L R_j \left(\frac{L_j}{N_j}\right)^{-1/\sigma}\right], \tag{20}$$

$$w_{Hj} = \frac{b\theta}{1+\beta_H}\left[1 + \beta_H R_j \left(\frac{H_j}{N_j}\right)^{-1/\sigma}\right], \tag{21}$$

- where $R_j = T_j P_j^0 / b = Y_j P_j^0 / bN_j = v_j / b$ is value added per standardized unit of labor relative to the reference wage.

- These differ from the corresponding equations with a linear technology (eqq. [12], [13]) by the terms $(L_j/N_j)^{-1/\sigma}$ and $(H_j/N_j)^{-1/\sigma}$, which adjust the marginal productivities of L and H workers on the basis of their relative employment shares.

- These terms disappear when $L_j = H_j$ or when j is large.

- If skill types were observable, it would be natural to estimate such a model via nonlinear least squares using data on firm value added.

- With unobserved skill types, an interactive fixed effects specification would be required that allows the firm effects to depend on the unobserved skill ratio at the firm.

- To derive the rent-sharing elasticities in this model, we define

$$\tau_{Lj} = \frac{\beta_L R_j \left(L_j/N_j\right)^{-1/\sigma}}{1 + \beta_L R_j \left(L_j/N_j\right)^{-1/\sigma}},$$

$$\tau_{Hj} = \frac{\beta_H R_j \left(H_j/N_j\right)^{-1/\sigma}}{1 + \beta_H R_j \left(H_j/N_j\right)^{-1/\sigma}}.$$

- These are the elasticities of wages with respect to v_j, ignoring any adjustment to the relative input of L and H labor.

- They also represent the proportional wage premiums for L and H workers associated with working at a firm with $R = R_j$ relative to a marginal firm with R close to 1.

- With this notation, we show in the appendix that the elasticities of wages with respect to value added per labor input can be expressed as

$$\xi_{Lj} \equiv \frac{\partial \ln w_{Lj}}{\partial \ln v_j} = \frac{\tau_{Lj}\left[1 + \left(\tau_{Hj}e_{Hj}/\sigma\right)\right]}{1 + (1/\sigma)\left[\left(1-\kappa_j\right)\tau_{Lj}e_{Lj} + \kappa_j\tau_{Hj}e_{Hj}\right]},$$

$$\xi_{Hj} \equiv \frac{\partial \ln w_{Hj}}{\partial \ln v_j} = \frac{\tau_{Hj}\left[1 + \left(\tau_{Lj}e_{Lj}/\sigma\right)\right]}{1 + (1/\sigma)\left[\left(1-\kappa_j\right)\tau_{Lj}e_{Lj} + \kappa_j\tau_{Hj}e_{Hj}\right]},$$

- where (as above) e_{Lj} and e_{Hj} are the elasticities of labor supply of L and H workers to the firm and

$$\kappa_j \equiv \frac{(1-\theta)L_j^\rho}{(1-\theta)L_j^\rho + \theta H_j^\rho} = \frac{\partial \ln f}{\partial \ln L_j} = 1 - \frac{\partial \ln f}{\partial \ln H_j}.$$

- Notice that

$$\lim_{\sigma \to \infty} \xi_{Sj} = \frac{\beta_S R_j}{1 + \beta_S R_j},$$

- which is the expression derived above for our baseline case with a linear technology.

- With imperfect substitution between groups, the value-added elasticities of the two skill groups, ξ_{Lj} and ξ_{Hj}, will depend on τ_{Lj} and τ_{Hj} and on the labor supply elasticities of the two groups.

F. Relationship to Other Models and Open Questions

VI. Conclusions