# **Dynamic Female Labor Supply**

Zvi Eckstein and Osnat Lifshitz

*Econometrica*, Vol. 79, No. 6 (November, 2011), 1675–1726

James J. Heckman



Econ 350, Winter 2023

# **1. Introduction**

Eckstein and Lifshitz **Eckstein and Lifshitz** Dynamic Female Labor Supply



FIGURE 1.—United States per capita GDP (2006 prices).



FIGURE 2.—Employment rates by marital status: Women (aged 22–65; proportion of women working 10+ weekly hours).

# **2. Main Facts and the Literature**

# *Schooling*



FIGURE 3.—Employment rates by level of education: married women (ages 22–65; proportion of women working  $10+$  weekly hours).



FIGURE 4.—Breakdown of married women by level of education (ages 22–65).

# *Earnings*



FIGURE 5.—Annual wages of full-time workers (ages 22–65; full-time full-year workers with nonzero wages; 2006 prices).

# *Fertility*



FIGURE 6.—Number of children per married woman (ages 22–65; extrapolated data for number of young children during 1968-1975).

Eckstein and Lifshitz **Eckstein and Lifshitz** Dynamic Female Labor Supply

# *Marriage and Divorce*



FIGURE 7.—Breakdown of women by marital status (ages 22-65).

*Female Employment by Cohort: Other Explanations*



FIGURE 8.—Married female employment rates by cohort (years 1962–2007; proportion of women working 10+ weekly hours; see definitions of cohorts in Section 2).

# **3. A Dynamic Female Employment Model**

A married female is indicated by  $M_t = 1$ , a single or divorced woman is denoted by  $M_t = 0$ , and the number of children is denoted by  $N_t$ . The objective of each female is to choose  $p_t$  from period t (the year she completes her education) until retirement, to maximize

$$
(3.1) \t E_t\left[\sum_{k=0}^{T-t} \delta^j U(p_{t+k}, x_{t+k}, K_{t+k-1}, N_{t+k,j} (j=1,\ldots,J), S, M_{t+k}, v_t)\right],
$$

where  $x_t$  is consumption,  $K_{t-1}$  is the number of periods that the woman has worked such that  $K_t = K_{t-1} + p_t$ ,  $N_{tj}$  is the number of children in year t of age group j, S is the predetermined level of schooling,  $\delta$  is the subjective discount factor, and  $T$  is the length of the decision horizon.

The female budget constraint is given by

(3.2) 
$$
((1 - \alpha)(1 - M_t) + \alpha)(y_t^w p_t + y_t^h M_t)
$$

$$
= x_t + \sum_{j=1}^J (c_j + c_{jm}(1 - M_t))N_{tj} + (b + b_m(1 - M_t))p_t,
$$

where  $\alpha$  is a fraction that denotes the share of a married woman in household income,  $y_t^h$  denotes the husband's earnings,  $y_t^w$  denotes the female's earnings,  $c_i + c_{im}(1 - M_i)$  is the cost in goods per child of age j, and  $b + b_m(1 - M_i)$  is an additional cost for maintaining the household if the woman works.

We also adopt the standard Mincer/Ben-Porath earning function

$$
(3.3) \qquad \ln y_t^w = \beta_0 + \beta_1 K_{t-1} + \beta_2 K_{t-1}^2 + \beta_3 S + \beta_4 t + \varepsilon_t,
$$

where  $t$  is a time trend that captures aggregate growth in labor productivity and  $\varepsilon$ <sub>*t*</sub> is the standard zero-mean, finite-variance, serially independent error that is uncorrelated with  $K$  and  $S$ . The number of children of age group  $j$ evolves according to

$$
(3.4) \t N_{ij} = N_{t-1,j} + n_{ij} - d_{ij},
$$

where  $n_{ij} = 1$  if a child enters the age group j at t and is zero otherwise, and  $d_{ij} = 1$  if a child leaves the age group j at t and is zero otherwise.

Following EW, we adopt the per period specification of utility

(3.5) 
$$
U_{t} = (\alpha_{1} + v_{t})p_{t} + x_{t} + \alpha_{2} p_{t}x_{t} + \alpha_{3} p_{t}K_{t-1} + \sum_{j=1}^{J} \alpha_{4j}N_{tj}p_{t} + \alpha_{5} p_{t}S + f(N_{tj}),
$$

where  $v_t$  is a preferences shock and  $f(N_{tj}) = \gamma_0 N_{tj} - (\gamma_1 + \gamma_2 S_{tj}) N_{tj}^2$  is a specific functional form that is meant to capture the way in which children enter the utility function.

Following the standard dynamic programming procedure, the value function is defined as

$$
(3.6) \tV_t(K_{t-1}, \varepsilon_t, \Omega_t) = \max[V_t^1(K_{t-1}, \varepsilon_t, \Omega_t), V_t^0(K_{t-1}, \Omega_t)],
$$

where  $V_t^1(\cdot)$  and  $V_t^0(\cdot)$  represent maximum expected discounted utility when the female is working at time  $t$  ( $p_t = 1$ ) and when she is not ( $p_t = 0$ ), respectively. That is,

$$
(3.7) \qquad V_t^1(\Omega_t, \varepsilon_t, v_t, t) = U_t^1(K_{t-1}, \varepsilon_t, \Omega_t, v_t) + \delta \cdot E(V_{t+1}(K_t, \varepsilon_{t+1}, v_{t+1}, \Omega_{t+1}) | \Omega_t, p_t = 1), V_t^0(\Omega_t, t) = U_t^0(K_{t-1}, \Omega_t) + \delta \cdot E(V_{t+1}(K_t, \varepsilon_{t+1}, v_{t+1}, \Omega_{t+1}) | \Omega_t, p_t = 0),
$$

where current utility is derived from insertion of the budget constraint (3.2) into  $(3.5)$  such that

(3.8) 
$$
U_t^1(K_{t-1}, \varepsilon_t, v_t, \Omega_t)
$$
  
\n
$$
= \alpha_1 + v_t - (b + b_m M_t) + \alpha_3 K_{t-1} + \sum_{j=1}^J \alpha_{4j} N_{tj} + \alpha_5 S + f(N_{tj})
$$
  
\n
$$
+ (1 + \alpha_2) \Bigg( ((1 - \alpha)(1 - M_t) + \alpha)
$$
  
\n
$$
\times (\exp{\{\beta_0 + \beta_1 K_{t-1} + \beta_2 K_{t-1}^2 + \beta_3 S + \beta_4 t + \varepsilon_t\} + \bar{y}_t^h M_t)}
$$
  
\n
$$
- \sum_{j=1}^J (c_j + c_{jm} M_t) N_{tj} \Bigg)
$$

and

$$
U_t^0(K_{t-1}, \Omega_t) = \alpha \bar{y}_t^h - \sum_{j=1}^J c_j N_{tj} + f(N_{tj}).
$$

We adopt the logistic form for job-offer probability

(3.9) 
$$
\Pr_{t} = \frac{\exp(\rho_0 + \rho_1 \cdot S + \rho_2 \cdot K_{t-1} + \rho_3 \cdot K_{t-1}^2 + \rho_4 \cdot p_{t-1})}{1 + \exp(\rho_0 + \rho_1 \cdot S + \rho_2 \cdot K_{t-1} + \rho_3 \cdot K_{t-1}^2 + \rho_4 \cdot p_{t-1})}.
$$

The probability of having another child is a function of the female's employment state in the previous period, age, education, marital status, and the current number of children (see Van der Klaauw (1996)), and is given by

(3.10) 
$$
\Pr(N_t = N_{t-1} + 1) = \Phi(\lambda_0 + \lambda_1 \cdot \text{AGE}_t + \lambda_2 \cdot (\text{AGE}_t)^2 + \lambda_3 \cdot S + \lambda_4 p_{t-1} + \lambda_5 \cdot N_{t-1} + \lambda_6 \cdot N_{t-1}^2 + \lambda_7 M_t),
$$

where  $\Phi(\cdot)$  is the standard normal distribution function.

(3.12) 
$$
\Pr(M_t = 0 | M_{t-1} = 1)
$$

$$
= \Phi(\xi_0 + \xi_1 \cdot \text{MT} + \xi_2 \cdot \text{MT}^2 + \xi_3 \cdot N_t + \xi_4 \cdot S + \xi_5 \cdot p_t + \xi_6 y_t^h).
$$

The model is solved backward from the terminal period  $T$  (age 65) assuming that  $V_T(\Omega_T, T + 1) = 0$ .

A special case of the model is a static model where  $\delta = 0$  and the female chooses to work if

(3.13)  $U_t^1(K_{t-1}, \varepsilon_t, v_t, \Omega_t) > U_t^0(K_{t-1}, \Omega_t).$ 

# *Discussion: The Choice of Models*

# **4. Data and Estimation**

The difference between these two vectors is given by the vector

$$
g'(\theta) = [m_1^A - m_1^S(\theta), \dots, m_j^A - m_j^S(\theta), \dots, m_j^A - m_j^S(\theta)].
$$

# *Identification*

# **5. Estimation Results for the 1955 Cohort**

## *Parameters*

# *Quality of Fit*



FIGURE 9.—Actual and predicted employment rates: 1955 cohort (1953-1957 cohorts for the period 1964-2007).



FIGURE 10.-Actual and predicted employment rates: 1955 cohort; HSD, PC, and SC (1953–1957 cohorts for the period 1964–2007).



FIGURE 11.- Actual and predicted employment rates: 1955 cohort; HSG and CG (1953-1957 cohorts for the period 1964-2007).

## **TABLE II**

## **GOODNESS OF FIT TESTS FOR THE THREE MODELS**



<sup>a</sup>Pearson's test statistic is given by

$$
\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i},
$$

where  $\chi^2$  is the Pearson cumulative test statistic,  $O_i$  is an observed frequency,  $E_i$  is an expected (theoretical) frequency, and *n* is the number of cells in the table. The critical values are:  $\chi^2_{(31,0.05)} = 18.5$ ,  $\chi^2_{(31,0.01)} = 14.9$  (all groups, 77.9, 70.1).

<sup>b</sup>Sum of squared differences.

# **6. Accounting for the Increase in Female Employment**

## **TABLE IIIA** FEMALE EMPLOYMENT RATES BY COHORTS, AGES, AND CHARACTERISTICS USING THE DYNAMIC MODEL



(Continues)



## TABLE IIIA-Continued



### **TABLE IIIB**

## FEMALE EMPLOYMENT RATES BY COHORTS, AGES, AND CHARACTERISTICS USING THE ESTIMATED STATIC MODEL

(Continues)



## TABLE IIIB-Continued



## **TABLE IIIC**

## FEMALE EMPLOYMENT RATES BY COHORTS, AGES, AND CHARACTERISTICS USING THE HECKMAN MODEL

(Continues)



## TABLE IIIC-Continued

### **TABLE IV**

## AVERAGE SHARE OF CHANGE IN FEMALE EMPLOYMENT RATES FOR THE COHORTS OF 1925-1975 BY EACH MODEL



## *Robustness*

# **7. Changes by Cohort and Aggregate Fit**

## **TABLE V**

## CHANGE IN ESTIMATED UTILITY/COST OF LEISURE AND YOUNG CHILDREN BY COHORT: DYNAMIC MODEL<sup>a</sup>



<sup>a</sup>To interpret  $\alpha_1$  we divided the difference between the value of the parameter in the specific cohort and the value of the parameter in 1955 by 2000 (number of hours worked per year). To interpret  $\alpha_{41}$  we divided the difference between the value of the parameter in the specific cohort and the value of the parameter in 1955 by the value of  $(1 + \alpha_2)$  and then by 2000 (number of hours worked per year).



FIGURE 12A.—Actual and predicted employment rates: 1940 cohort ( $\alpha_1 = -15,658.1$ ,  $\alpha_{41} = -2733.4$  (estimated from the 1955 cohort);  $\alpha_{41} = -8980.1$  (estimated when this parameter was unconstrained for this cohort)).



FIGURE 12B.—Actual and predicted employment rates: 1930 cohort ( $\alpha_1 = -15,658.1$ ,  $\alpha_{41} = -2733.4$  (estimated from the 1955 cohort);  $\alpha_1 = -25,360.5$ ,  $\alpha_{41} = -8818.8$  (estimated when the two parameters were unconstrained for this cohort)).

# *Aggregate Fit*



FIGURE 13. - Aggregate employment rates of females, aged 83-54.



FIGURE 13. - Aggregate employment rates of females, aged 83-54.

# **8. Concluding Remarks**

## **TABLE VI**

## MARRIED AND UNMARRIED, ACTUAL AND FITTED FEMALE EMPLOYMENT RATES BY COHORT AND AGE: DYNAMIC MODEL

