How Large is the Stock Component of Human Capital?

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- An agent solves Problem P1.
- Lifetime utility $U(c)$ is determined by a consumption plan $c = (c_1, ..., c_J)$.
- Consumption at age j is given by a function $c_j : Z^j \rightarrow R^1_+$ that maps shock histories $z^j = (z_1, ..., z_j) \in Z^j$ into consumption.
- All variables analyzed are functions of these shocks.

$$\text{Problem P1: } \max U(c) \text{ subject to}$$
$$(1)\ c_j + \sum_{i \in \mathcal{I}} a^i_{j+1} = \sum_{i \in \mathcal{I}} a^i_j R^i_j + e_j \text{ and } c_j \geq 0, \forall j$$
$$(3)\ a^i_{J+1} = 0, \forall i \in \mathcal{I}$$

- Period resources are divided between consumption c_j and savings $\sum_{i \in \mathcal{I}} a^i_{j+1}$.
- Period resources: exogenous earnings process e_j; the value of financial assets brought into the period $\sum_{i \in \mathcal{I}} a^i_j R^i_j$.

2.2 Value and Return Concepts

- The value of human capital v_j = expected discounted dividends (i.e. net earnings) at a solution $(c^*, a^*) = ((c_1^*, ..., c_J^*), \{(a_1^{*,i}, ..., a_{J+1}^{*,i})\}_{i \in \mathcal{I}})$ to Problem P1.
- Discounting: done using the agent's stochastic discount factor $m_{j,k}$ from the solution to Problem P1.
- Agent's marginal valuation of an extra period k consumption good in terms of the period j consumption good.
- $P(z^k | z^j)$ arises because human capital values are stochastic.

$$v_j(z^j) \equiv E[\sum_{k=j+1}^{J} m_{j,k} e_k | z^j]$$

$$m_{j,k}(z^k) \equiv \frac{\partial U(c^*)/\partial c_k(z^k)}{\partial U(c^*)/\partial c_j(z^j)} \frac{1}{P(z^k|z^j)}$$

- Gross return R^h_{j+1} to human capital: next period's value and dividend divided by this period's value: $R^h_{j+1} = \frac{v_{j+1} + e_{j+1}}{v_j}$.
- The return to human capital is then well integrated into standard asset pricing theory.
- Off corners, all returns R_{j+1} satisfy the same type of restriction: $E[m_{j,j+1} R_{j+1} | z^j] = 1$.

2.3 An Interpretation

- Value v_j is the price at which an agent would be willing to sell a marginal share of a claim to their future earnings stream.
- It is thus the value of all the shares (total shares are normalized to 1) in the future earnings stream.
- The price process $\{v_j\}_{j=1}^{J}$ has the property that if the agent were allowed to change share holdings in this earnings stream at any age at these prices, then the agent would optimally decide not to change share holdings and would make exactly the same consumption and asset choices (c^*, a^*) that were optimal in Problem P1.
- The value v_j is not the market or social value of future earnings.
- **It is the shadow price of a unit of human capital (a private evaluation).**

2.4 A Decomposition

- Decompose human capital values into financial asset components and a residual-value component.
- Project next period's human capital payout $v_{j+1} + e_{j+1}$ onto the space of conditional asset returns.
- The decomposition is carried out in the two equations below.
- The human capital payout contains a component $(\sum_{i \in \mathcal{I}} \alpha_j^i R_{j+1}^i)$ spanned by gross asset returns and a component (ϵ_{j+1}) orthogonal to asset returns, where α_j^i are the projection coefficients.
- The orthogonal component ϵ_{j+1} will be mean zero when one of the financial assets is riskless.

$$v_j = E[m_{j,j}(v_{j+1} + e_{j+1})|z^j] = E[m_{j,j}(\sum_{i \in \mathcal{I}} \alpha_j^i R_{j+1}^i + \epsilon_{j+1})|z^j]$$

$$v_j = \sum_{i \in \mathcal{I}} \alpha_j^i E[m_{j,j+1} R_{j+1}^i|z^j] + E[m_{j,j+1}\epsilon_{j+1}|z^j]$$

2.5 A Simple Example

- An agent's preferences are given by a constant relative risk aversion utility function.
- Earnings follow an exogenous Markov process.
- There is a single, risk-free financial asset.

Utility: $U(c) = E[\sum_{j=1}^{J} \beta^{j-1} u(c_j) | z^1]$, where

$$u(c_j) = \begin{cases} \frac{c_j^{1-\rho}}{(1-\rho)} & : \quad \rho > 0, \rho \neq 1 \\ \log(c_j) & : \quad \rho = 1 \end{cases}$$

Earnings: $e_j = \prod_{k=1}^{j} z_k$, where $\ln z_k \sim N(\mu, \sigma^2)$ is i.i.d.

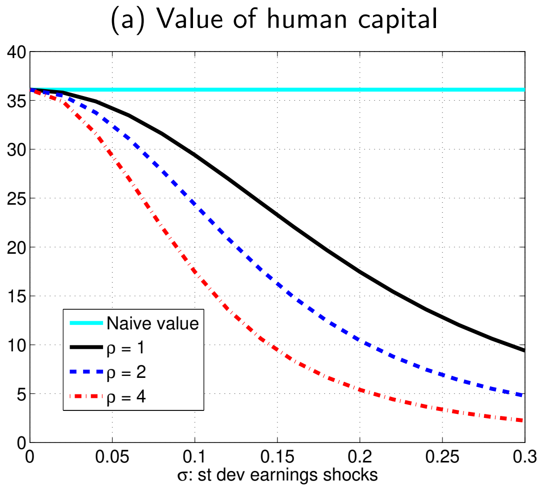
Decision Problem: $\max U(c)$ subject to

(1) $c_j + a_{j+1} \leq a_j(1+r) + e_j$, (2) $c_j \geq 0, a_{J+1} \geq 0$

- When $1 + r = \frac{1}{\beta} \exp(\rho\mu - \frac{\rho^2\sigma^2}{2})$ and initial financial assets are zero, then setting consumption equal to earnings each period is optimal.
- The stochastic discount factor equals $m_{j,k}(z^k) \equiv \frac{\partial U(c^*)/\partial c_k(z^k)}{\partial U(c^*)/\partial c_j(z^j)} \frac{1}{P(z^k|z^j)} = \frac{\beta^{k-j} u'(e_k(z^k))}{u'(e_j(z^j))}$.
- This example leads to a closed-form formula where v_j is proportional to earnings e_j and where R_j^h is a time-invariant function of the period shock z_j.

- Figure 1 illustrates some quantitative properties.
- The parameter σ, governing the standard deviation of earnings shocks, varies over the interval $[0, 0.3]$ and $\mu = -\sigma^2/2$.
- As all agents start with earnings equal to 1, the expected earnings profile over the lifetime is flat and equals 1 in all periods.
- The lifetime is $J = 46$ periods which can be viewed as covering real-life ages $20 - 65$.
- The interest rate is fixed at $r = .01$.
- Thus, the discount factor β is adjusted to be consistent with this interest rate given the remaining parameters: $1 + r = \frac{1}{\beta} \exp(\rho\mu - \frac{\rho^2\sigma^2}{2})$.

Figure 1: Human capital values and returns: a simple example



- Value used in traditional literature.

Figure 1: Human capital values and returns: a simple example, Cont'd

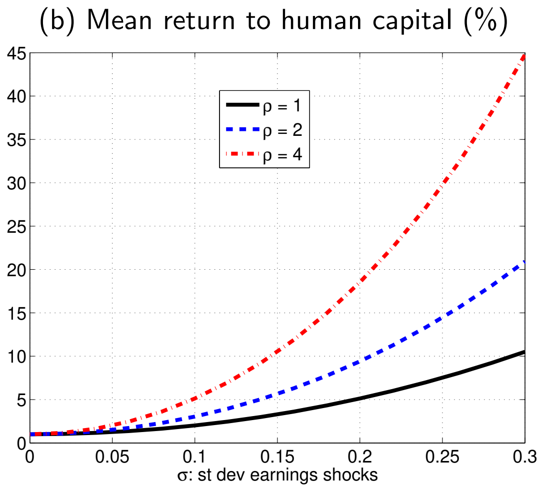
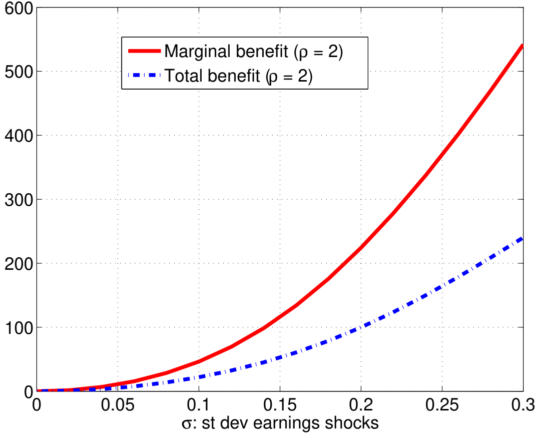


Figure 1: Human capital values and returns: a simple example, Cont'd

(c) Benefit of moving to a smooth consumption plan



- Figure 1 shows that the value v_1 of an age 1 agent's human capital falls and that the mean return in any period rises as the shock standard deviation increases.
- Thus, a high mean return on human capital is the flip side of a low value attached to future earnings.
- These patterns are amplified as the preference parameter ρ increases.

- Figure 1 also plots the "naive value."
- The naive value equals earnings discounted at a constant interest rate r that we set equal to the risk-free rate (i.e. $v_1^{naive} = E[\sum_{j=2}^{J} \frac{e_j}{(1+r)^{j-1}} | z^1])$.
- This follows a traditional empirical procedure that is employed in the literature as was mentioned in the introduction.
- The naive value is exactly the same in each economy in Figure 1 because the risk-free interest rate and the mean earnings profile are unchanged across economies.
- Our notion of value v_1 differs from v_1^{naive} because the agent's stochastic discount factor covaries with earnings.
- Figure 1 shows that negative covariation can be substantial.

- Figure 1 plots the total benefit and the marginal benefit of moving from the model consumption plan c to a smooth consumption plan where $c_j^{smooth} = E_1[c_j] = E_1[e_j] = 1$.
- The benefit function Ω is defined, following Alvarez and Jermann (2004), by the first equation:

$$U((1+\Omega(\gamma))c) = U((1-\gamma)c + \gamma c^{smooth})$$

- The total benefit is $\Omega(1)$ and the marginal benefit is $\Omega'(0)$.
- The marginal benefit in Figure 1 increases as the standard deviation of the period earnings shock increases.

$$\begin{aligned}\Omega'(0) =& \frac{\sum_{j=1}^{J} \sum_{z^j} \frac{\partial U(c)}{\partial c_j(z^j)} (c_j^{smooth}(z^j) - c_j(z^j))}{\sum_{j=1}^{J} \sum_{z^j} \frac{\partial U(c)}{\partial c_j(z^j)} c_j(z^j)} \\ =& \frac{E[\sum_{j=1}^{J} m_{1,j} c_j^{smooth} | z^1]}{v_1(z^1) + e_1(z^1) + a_1(z^1)(1+r)} - 1\end{aligned}$$

- The marginal benefit is tightly connected to the value v_1 of human capital.
- To see this, differentiate the first equation above with respect to γ.
- This implies the leftmost equality in the second equation above.
- The rightmost equality holds by rearrangement because the individual solves Problem P1.

- The numerator term in the second equation is pinned down by asset prices so that $E[\sum_{j=1}^{J} m_{1,j} c_j^{smooth} | z^1] = \sum_{j=1}^{J} (\frac{1}{1+r})^{j-1}$, whereas the denominator is determined by the value of human capital plus initial earnings and initial wealth.
- The only unobservable is the value of human capital.
- The theory then implies that a high marginal benefit of moving towards perfect consumption smoothing coincides with a low value of human capital.
- Point is new.

3. Empirics: Earnings and Asset Returns

- We outline an empirical framework for idiosyncratic earnings shocks, aggregate earnings shocks and stochastic stock returns.
- Let $e_{i,j,t}$ denote real pretax annual earnings for individual i of age j in year t.
- We assume that the natural logarithm of earnings consists of an aggregate component (u^1) and an idiosyncratic component (u^2) and

$$\log e_{i,j,t} = u_t^1 + u_{i,j,t}^2. \tag{1}$$

- In Section 25 we describe the structure of the idiosyncratic component of earnings, our estimation procedure and the fit of the estimated model. In Section 42 we describe the structure and estimation of the joint process for the aggregate component of earnings and stock returns.

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3.1 Idiosyncratic Component of Earnings

- The idiosyncratic component of earnings is the sum of four orthogonal components: a common age effect κ_j, an individual-specific fixed effect ξ, a persistent component ζ and a transitory component υ.

$$
\begin{aligned}
u^2_{i,j,t} &= \kappa_j + \xi_i + \zeta_{i,j,t} + \upsilon_{i,j,t} \\
\zeta_{i,j,t} &= \rho\zeta_{i,j-1,t-1} + \eta_{i,j,t} \\
\zeta_{i,0,t} &= 0.
\end{aligned}
\tag{2}
$$

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- The common age effect is modeled as a quartic polynomial.
- The individual fixed effects are assumed to be normally distributed with a constant variance σ_ξ^2.
- The transitory idiosyncratic shocks are assumed to be distributed according to a distribution with zero mean, variance $\sigma_{\upsilon,j}^2$, and third central moment $\mu_{3,\upsilon,j}$.
- In order to capture life-cycle properties of the variance and skewness of earnings we allow the moments of the transitory component to be age-dependent and model this as a quartic polynomial.

- Persistent idiosyncratic shocks are assumed to be distributed according to a distribution with zero mean, variance $\sigma^2_{\eta,t}(X_t)$ and third central moment $\mu_{3,\eta,t}(X_t)$.
- The variance and skewness have a linear trend, in order to capture low frequency trends over the sample period, and are state dependent via the variable X_t.
- We model X_t as a two-state process. Specifically, we set $X_t = 1_{\Delta u^1_t > 0}$ so that X_t is an indicator function taking on the value 1 in booms and 0 in recessions.
- Thus, aside from a trend term, the variance and skewness of the persistent innovations take on different values in expansions and contractions.

- We estimate the idiosyncratic earnings process using data on male annual labor earnings from the Panel Study of Income Dynamics (PSID) from 1967 to 1996.
- We focus on male heads of households between ages 22 and 60 with real annual earnings of at least $\$1,000$.
- Our measure of annual gross labor earnings includes pre-tax wages and salaries from all jobs, plus commission, tips, bonuses and overtime, as well as the labor part of income from self-employment.
- Labor earnings are inflated to 2008 dollars using the CPI All Urban series.
- We also consider two sub-samples.
- Individuals with 12 or fewer years of education are included in the High School sub-sample, while those with at least 16 years or a Bachelor's degree are included in the College sub-sample.

- Estimation is done in two stages.
- In the first stage we estimate κ_j by regressing log real annual earnings on a quartic polynomial in age and a full set of year dummies.
- This is done separately for the three education samples.
- On the basis of the first-stage results for the PSID, and related results for the Current Population Survey data and NIPA data, we set the contraction years over the time interval 1967-1996 to be 1970, 1974-5, 1979-82, 1989-91 and 1993.
- Residuals from this first-stage regression are then used to estimate the remaining parameters of the individual earnings equation: $\left(\rho, \sigma^2_{\xi}, \sigma^2_{\eta,t}(X_t), \mu_{3,\eta,t}(X_t), \sigma^2_{\nu,j}, \mu_{3,\nu,j}\right)$.
- A GMM estimator is then used to estimate the parameters, using the full set of second and third-order autocovariances as moments.

- The estimated process delivers a good fit to the variance and third central moment of the earnings distribution as a function of both age and time.
- The fit of these and other moments for the full sample is displayed in Figure 2.
- Corresponding results for the College and High School samples are contained in the Appendix.

Figure 2: Fit of estimated idiosyncratic earnings model for the full sample

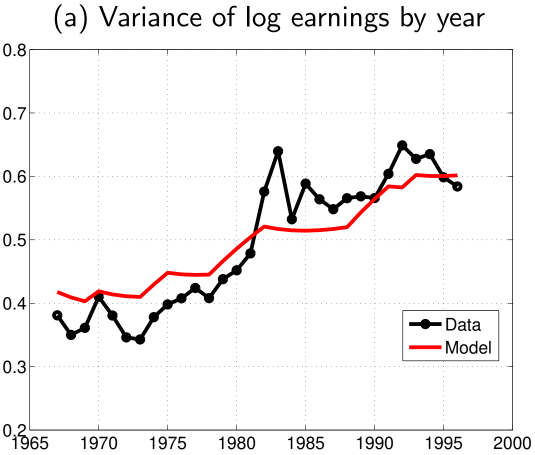
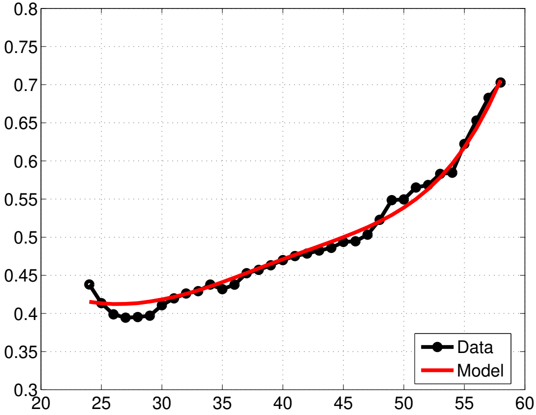


Figure 2: Fit of estimated idiosyncratic earnings model for the full sample, Cont'd



(b) Variance of log earnings by age

Figure 2: Fit of estimated idiosyncratic earnings model for the full sample, Cont'd

(c) Third central moment of log earnings by year

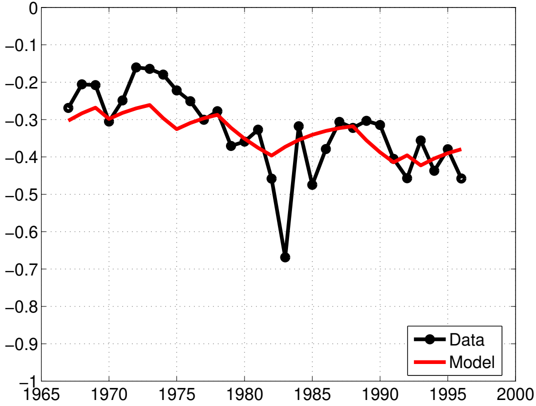


Figure 2: Fit of estimated idiosyncratic earnings model for the full sample, Cont'd

(d) Third central moment of log earnings by age

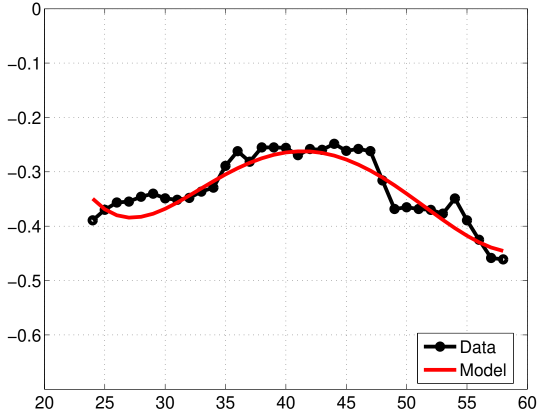


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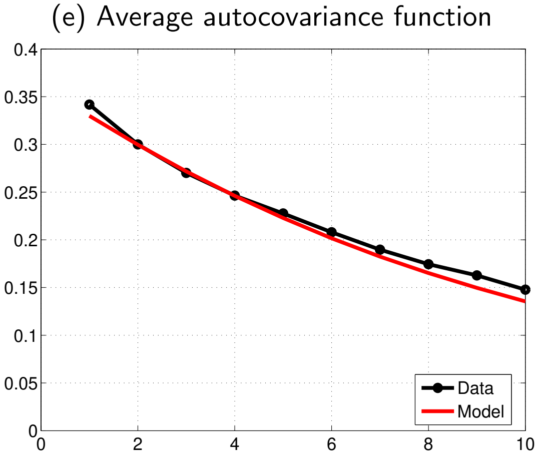
Figure 2: Fit of estimated idiosyncratic earnings model for the full sample, Cont'd



- We highlight three findings from Table 1.
- First, transitory shocks are left skewed for the full sample and the college sample.
- Left skewness is needed to match the left skewness of the first-stage residuals as documented in Figure 2.
- Guvenen, Ozkan and Song (2014) document that male earnings growth rates are left skewed in administrative data.
- Second, the variance and left skewness of persistent shocks is higher in recessions than in booms.

Table 1: Parameter Estimates for the Idiosyncratic Earnings Process

	Full Sample	College Sample	High School Sample
Fixed Effect			
σ^2_ξ	0.082	0.064	0.137
	(0.013)	(0.015)	(0.032)
Persistent Component			
ρ	0.942	0.951	0.843
	(0.012)	(0.015)	(0.097)
σ^2_η: boom	0.038	0.037	0.047
	(0.005)	(0.006)	(0.023)
σ^2_η: recession	0.058	0.048	0.072
	(0.005)	(0.008)	(0.020)
σ^2_η: linear trend	0.001	0.001	0.002
	(0.000)	(0.000)	(0.002)
σ^2_η: recession	0.058	0.048	0.072
	(0.005)	(0.008)	(0.020)
σ^2_η: linear trend	0.001	0.001	0.002
	(0.000)	(0.000)	(0.002)

Notes: Models of the moments of the transitory shock include a fourth-order polynomial in age. The reported moments for transitory shocks are averages over the age range. Standard errors are computed by block bootstrap with 39 repetitions.

Table 1: Parameter Estimates for the Idiosyncratic Earnings Process, Cont'd

	Full Sample	College Sample	High School Sample
Fixed Effect			
σ_ξ^2	0.082	0.064	0.137
	(0.013)	(0.015)	(0.032)
Persistent Component			
$\mu_{3,\eta}$: boom	-0.020	-0.006	-0.149
	(0.014)	(0.015)	(0.181)
$\mu_{3,\eta}$: recession	-0.061	-0.040	-0.190
	(0.013)	(0.019)	(0.170)
$\mu_{3,\eta}$: linear trend	-0.001	-0.001	-0.003
	(0.001)	0.001)	(0.002)
Transitory Component			
σ_υ^2	0.132	0.139	0.128
	(0.005)	(0.006)	(0.023)
$\mu_{3,\upsilon}$	-0.161	-0.162	-0.002
	(0.027)	(0.029)	(0.172)

Notes: Models of the moments of the transitory shock include a fourth-order polynomial in age. The reported moments for transitory shocks are averages over the age range. Standard errors are computed by block bootstrap with 39 repetitions.

- Consistent with the findings in Storesletten, Telmer and Yaron (2004), there is evidence for counter-cyclical variance even when the framework is generalized to account for skewness and a time trend.
- However, the cyclical variation that we estimate is less dramatic than their findings.
- Third, the autoregression parameter ρ is higher for the full sample and the college sample compared to the high school sample.
- Thus, persistent innovations of a given magnitude will be of greater proportional importance for those with a college than a high school education.

- The parameter estimates are broadly consistent with those from related specifications (that do not account for skewness), that have been estimated elsewhere in the literature and summarized in Meghir and Pistaferri (2010).
- We note that our estimate of the variance of the transitory component is approximately 0.1 larger than what has been estimated by others (see for example, Guvenen (2009)).
- The source of this difference is due entirely to our broader sample selection.
- Since it is likely that a substantial fraction of this variance is due to measurement error, we make an adjustment when using these estimates as parameters in the structural model.

3.2 Aggregate Component of Earnings and Stock Returns

- Our benchmark model is a standard, first-order VAR for $\Delta y_t = \begin{pmatrix} \Delta u_t^1 & \log R_t^s \end{pmatrix}'$.
- The error term ε_t is a vector of zero mean IID random variables with covariance matrix Σ.

$$\Delta y_t = \gamma + \Gamma \Delta y_{t-1} + \varepsilon_t \tag{3}$$

- We estimate (3) using data on male annual labor earnings from the the Current Population Survey (CPS) from 1967 to 2008.
- Our sample selection criteria and definition of earnings are the same as those used for the PSID, descibed in section 3.1.
- We construct an empirical counterpart to u_t^1 by estimating a median regression (Least Absolute Deviations) of earnings on a full set of age and time dummies.
- We use a median regression rather than OLS since it is more robust to the effects of changes in top coding in the CPS over our sample period.
- We use the estimates of $\hat{u}_t^1$ from our CPS sample, rather than corresponding estimates from the PSID, as input to the estimation because CPS data has both a longer time dimension and a larger cross-section sample each year compared to PSID data.

- Data on equity and bond returns are annual returns.
- Equity returns are based on a value-weighted portfolio of all NYSE, AMEX and NASDAQ stocks including dividends.
- Bond returns are based on 1-month Treasury bill returns.
- Real returns are calculated by adjusting for realized inflation using the CPI All Urban series.

- Table 2 reports the estimation results.
- The parameter estimates reveal a moderate degree of persistence in aggregate earnings growth.
- The covariance between innovations to earnings growth and innovations to stock returns is positive in all models.
- Thus, the estimated models imply a positive conditional correlation between earnings growth and stock returns.
- This is one feature, among others, that will later produce a positive conditional correlation between stock and human capital returns and a positive stock component of the value of human capital.

Table 2: Parameter Estimates for the Aggregate Stochastic Process

		No Cointegration			With Cointegration		
		Full Sample	College Sample	High School Sample	Full Sample	College Sample	High School Sample
Equation 1: Δu_t^1							
Δu_{t-1}^1	Γ_{11}	0.383	0.260	0.348	0.364	0.12	0.295
		(0.14)	(0.15)	(0.14)	(0.19)	(0.15)	(0.18)
$\log R_{t-1}^s$	Γ_{12}	0.044	0.04	0.057	0.045	0.016	0.058
		(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
Constant	γ_1	-0.004	-0.003	-0.009	-0.004	-0.005	-0.008
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Equation 2: $\log R_t^s$							
Δu_{t-1}^1	Γ_{21}	-2.149	-2.203	-1.731	0.473	-2.248	0.236
		(1.15)	(1.29)	(0.97)	(1.42)	(1.45)	(1.18)
$\log R_{t-1}^s$	Γ_{22}	0.106	0.153	0.101	0.054	0.145	0.072
		(0.17)	(0.18)	(0.17)	(0.16)	(0.21)	(0.17)
Constant	γ_2	0.032	0.031	0.024	0.00	0.029	0.00
		(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)

Notes: Standard errors in parentheses.

Table 2: Parameter Estimates for the Aggregate Stochastic Process, Cont'd

		No Cointegration			With Cointegration		
		Full Sample	College Sample	High School Sample	Full Sample	College Sample	High School Sample
Var-Cov Matrix							
$var\ (\varepsilon_{1,t}) \times 10^{-4}$		4.42	4.24	6.49	4.42	3.37	6.44
$var\ (\varepsilon_{2,t}) \times 10^{-2}$		3.2	3.24	3.23	2.57	3.24	2.92
$cov\ (\varepsilon_{1,t}, \varepsilon_{2,t}) \times 10^{-3}$		1.23	1.24	1.52	1.28	1.21	2.00
Cointegrating Vector							
$\log R_t^s$	β_2				0.309	-0.211	0.469
					(0.10)	(0.06)	(0.15)
Trend	ρ				-0.019	0.016	-0.026
					(0.01)	(0.00)	(0.01)
Constant	μ				-0.67	0.343	-0.976
Adjustment Parameters							
Δu_t^1	α_1				0.007	-0.196	0.017
					(0.05)	(0.07)	(0.04)
$\log R_t^s$	α_2				-1.04	-0.063	-0.651
					(0.36)	(0.64)	(0.23)

Notes: Standard errors in parentheses.

- The implied steady-state dynamics are reported in Table 3.
- The estimated model matches the observed correlation structure well.
- When we input the estimated process into our economic model, we adjust the constants (γ_1, γ_2) estimated in Table 2 so that all models produce in steady state $E[\log R^s] = .041$ and $E[\Delta u^1] = 0$.
- This facilitates comparisons of human capital value and return properties across models.

Table 3: Implied Steady-State Statistics for the Aggregate Stochastic Process

	Full Sample		
	Data	No Cointegration	With Cointegration
$E\left(logR_t^b\right)$	0.012	0.012	0.012
$E\left(\log R_t^s\right)$	0.041	0.045	0.070
$E\left(\Delta u_t^1\right)$	-0.002	-0.004	-0.002
$sd\left(\Delta u_t^1\right)$	0.025	0.025	0.025
$sd\left(\log R_t^s\right)$:	0.187	0.187	0.187
$corr\left(\Delta u_t^1, \log R_t^s\right)$	0.184	0.177	0.156
$corr\left(\Delta u_t^1, \Delta u_{t-1}^1\right)$	0.425	0.441	0.435
$corr\left(\log R_t^s, \log R_{t-1}^s\right)$	0.057	0.055	0.005
$corr\left(\Delta u_t^1 \log R_{t-1}^s\right)$	0.372	0.398	0.394
$corr\left(\log R_t^s, \Delta u_{t-1}^1\right)$	-0.292	-0.270	-0.289

Notes: Table shows average moments in the data, together with implied steady-state statistics from the corresponding estimated model. Data cover the period 1967-2008. When implementing the estimated processes in the structural model, we adjust the constants (γ_1, γ_2) estimated in Table 2 so that all models have $E[\log R_t^s] = 0.041$ and $E[\Delta u_t^1] = 0$.

Table 3: Implied Steady-State Statistics for the Aggregate Stochastic Process, Cont'd

	Full Sample		
	Data	No Cointegration	With Cointegration
	College Sub-sample		
	Data	No Cointegration	With Cointegration
$E\left(logR_t^b\right)$	0.012	0.012	0.012
$E\left(\log R_t^s\right)$	0.041	0.040	0.045
$E\left(\Delta u_t^1\right)$	0.000	-0.001	-0.001
$sd\left(\Delta u_t^1\right)$	0.023	0.023	0.023
$sd\left(\log R_t^s\right)$:	0.187	0.187	0.186
$corr\left(\Delta u_t^1, \log R_t^s\right)$	0.248	0.251	0.243
$corr\left(\Delta u_t^1, \Delta u_{t-1}^1\right)$	0.346	0.341	0.342
$corr\left(\log R_t^s, \log R_{t-1}^s\right)$	0.057	0.084	0.050
$corr\left(\Delta u_t^1 \log R_{t-1}^s\right)$	0.377	0.387	0.367
$corr\left(\log R_t^s, \Delta u_{t-1}^1\right)$	-0.225	-0.235	-0.229

Notes: Table shows average moments in the data, together with implied steady-state statistics from the corresponding estimated model. Data cover the period 1967-2008. When implementing the estimated processes in the structural model, we adjust the constants (γ_1, γ_2) estimated in Table 2 so that all models have $E[\log R_t^s] = 0.041$ and $E[\Delta u_t^1] = 0$.

Table 3: Implied Steady-State Statistics for the Aggregate Stochastic Process, Cont'd

	Full Sample		
	Data	No Cointegration	With Cointegration
	High School Sub-sample		
	Data	No Cointegration	With Cointegration
$E\left(logR_t^b\right)$	0.012	0.012	0.012
$E\left(\log R_t^s\right)$	0.041	0.045	0.074
$E\left(\Delta u_t^1\right)$	-0.007	-0.010	-0.008
$sd\left(\Delta u_t^1\right)$	0.030	0.030	0.030
$sd\left(\log R_t^s\right)$:	0.187	0.187	0.186
$corr\left(\Delta u_t^1, \log R_t^s\right)$	0.207	0.194	0.175
$corr\left(\Delta u_t^1, \Delta u_{t-1}^1\right)$	0.386	0.416	0.411
$corr\left(\log R_t^s, \log R_{t-1}^s\right)$	0.057	0.047	0.003
$corr\left(\Delta u_t^1 \log R_{t-1}^s\right)$	0.387	0.420	0.420
$corr\left(\log R_t^s, \Delta u_{t-1}^1\right)$	-0.289	-0.261	-0.276

Notes: Table shows average moments in the data, together with implied steady-state statistics from the corresponding estimated model. Data cover the period 1967-2008. When implementing the estimated processes in the structural model, we adjust the constants (γ_1, γ_2) estimated in Table 2 so that all models have $E[\log R_t^s] = 0.041$ and $E[\Delta u_t^1] = 0$.

- In Appendix A.3 we consider a generalization of (3).
- The generalization allows us to address the possibility that earnings u_t^1 and a process generating log stock returns are cointegrated.
- We assume that $y_t = \begin{pmatrix} u_t^1 & P_t \end{pmatrix}'$ follows a p-th order VAR, where P_t is a process generating stock returns (i.e. $\Delta P_t = \log R_t^s$).
- We show that this VAR can be written as a Vector Error Correction Mechanism (VECM), a useful tool in the cointegration literature.

- We present lag order selection tests that suggest the presence of two lags, i.e. $p = 2$.
- We also present tests of the cointegrating rank of this system.
- We interpret these test findings as providing only weak evidence for cointegration.
- This leads us to adopt (3) as the benchmark process.
- However, since these tests may all have relatively little power given the short time series, we also estimate the model with cointegration and assess their implications within the portfolio choice model.

4. The Benchmark Model

- We now use the theoretical framework and the empirical results to quantify the value and return to human capital.
- The benchmark model has two financial assets.
- Asset $i = 1$ is riskless and asset $i = 2$ is risky.
- The agent cannot go short on either financial asset.
- Initial resources at age $j = 1$ are denoted by x which describes the sum of initial earnings and financial wealth.

- Benchmark Model: $\max U(c)$ subject to $c \in \Gamma_1(x, z^1)$

$$\Gamma_1(x, z^1) = \{c = (c_1, ..., c_J) : \exists (a^1, a^2) \; s.t. \; 1-2 \; holds \; \forall j\}$$

1. $c_j + \sum_{i \in \mathcal{I}} a^i_{j+1} \leq x$ for $j = 1$ and
   $c_j + \sum_{i \in \mathcal{I}} a^i_{j+1} \leq \sum_{i \in \mathcal{I}} a^i_j R^i_j + e_j$ for $j > 1$
2. $c_j \geq 0$ and $a^1_{j+1}, a^2_{j+1} \geq 0, \forall j$

- The utility function $U(c) = U^1(c_1, ..., c_J)$ is of the type employed by Epstein and Zin (1991).
- It is defined recursively by applying an aggregator W and a certainty equivalent F.
- The certainty equivalent encodes attitudes towards risk with α governing risk-aversion.
- The aggregator encodes attitudes towards intertemporal substitution where γ is the inverse of the intertemporal elasticity of substitution.

- We allow for mortality risk via the one-period-ahead survival probability ψ_{j+1}.

$$U^j(c_j, ...c_J) = W(c_j, F(U^{j+1}(c_{j+1}, ..., c_J)), j)$$

$W(a, b, j) = [(1-\beta)a^{1-\gamma} + \beta\psi_{j+1}b^{1-\gamma}]^{1/(1-\gamma)}$ and
$F(x) = (E[x^{1-\alpha}])^{1/(1-\alpha)}$

- Table 4 summarizes the parameters in the benchmark model.

Table 4: Parameter Values for the Benchmark Model

Category	Symbol	Parameter Value
Demographics	J, Ret	$(J, Ret) = (69, 40)$
	ψ_{j+1} Survival Probability	U.S. Life Table
Preferences	α Risk Aversion	$\alpha \in \{4, 6, 10\}$
	$1/\gamma$ Intertemporal Substitution	$1/\gamma = 1.17$
	β Discount Factor	see Notes
Returns	R^s, R^b	Table 2 - 3
Earnings	$e_j(z) = \begin{cases} z_1 \exp(\kappa_j + \xi + \zeta + \upsilon)(1-\tau) & \text{if} \quad j < Ret \\ z_1 b(\xi)(1-\tau) & \text{if} \quad j \geq Ret \end{cases}$	$\tau = .27$
	$\zeta' = \rho\zeta + \eta'$ and $\eta' \sim GN(0, \sigma^2_\eta(X), \mu_{3,\eta}(X))$	$b(\cdot)$ see text
	$\xi \sim N(0, \sigma^2_\xi)$ and $\upsilon \sim GN(0, \sigma^2_{\upsilon,j}, \mu_{3,\upsilon,j})$	Table 1-2
Initial Wealth	$\sum_{i \in \mathcal{I}} a^i_1 R^i_1$	$\sum_{i \in \mathcal{I}} a^i_1 R^i_1 = 0.3E[e_1]$

Notes: β is calibrated to generate a steady-state ratio of wealth to income equal to 4.1. All sensitivity analyses are performed by re-calibrating β to generate the same ratio. Survival probabilities are smoothed versions of male values from the 1989-91 US Decennial Life Tables in NCHS (1992). Smoothing is done using a nine point moving average. $E[e_1]$ denotes mean earnings at age 1 in the model.

Demographics:

- Agents start economic life at real-life age 22, retire at age 61 and live at most up to age 90.
- Thus, we set $J = 69$ and $Ret = 40$.
- Agents face a conditional probability ψ_{j+1} of surviving from period j to period $j+1$ that is set to estimates for males from the 1989-91 US Decennial Life Tables in NCHS (1992).

Preferences:

- We set the preference parameters to values estimated from Euler equation restrictions.
- Vissing-Jorgensen and Attanasio (2003) estimate $1/\gamma = 1.17$ for a prefered specification and conclude that the risk aversion parameter α in the interval $[5, 10]$ can be obtained under realistic assumptions, based on household-level data.
- Thus, the special case of constant-relative-risk-aversion (CRRA) preferences, where $\gamma = \alpha$, is not the parameter configuration that best fits the Euler equation restrictions.
- We examine model implications for $1/\gamma = 1.17$ and $\alpha \in \{4, 6, 10\}$.

Preferences (Cont'd):

- We later investigate CRRA preferences to determine whether our main conclusions are robust across many parameter configurations.
- We set the discount factor β so that, given all other model parameters, the model estimated based on the full sample produces an average wealth-income ratio equal to 4.1.
- This is the 2010 US value documented by Piketty and Zucman (2014).

Initial Wealth:

- Initial wealth is set to equal 30 percent of mean earnings at age 22.

Earnings and Asset Returns:

- Earnings and asset returns in the benchmark model are based on the estimates in Tables 1-3 for the case of no cointegration.
- The earnings that enter the model are earnings after taxes and transfers.
- Before the retirement age, model earnings $e_j(z)$ are the process estimated in Tables 1-3 times $(1-\tau)$, where $\tau = .27$.
- After the retirement age, model earnings $e_j(z)$ equal model social security benefits after taxes.

$$e_j(z) = z_1 g_j(z_2) = \begin{cases} \exp(u^1)\exp(\kappa_j + \xi + \zeta + \upsilon)(1-\tau) & j < \mathit{Retire} \\ \exp(u^1) b(\xi)(1-\tau) & \mathit{otherwise} \end{cases}$$

- We group the variables from the statistical model into a state variable $z = (z_1, z_2)$, where $z_1 = \exp(u^1)$ captures the aggregate component of earnings and $z_2 = (\xi, \zeta, \upsilon, \Delta u^1, \log R^s)$ captures the idiosyncratic components (ξ, ζ, υ), the growth in the aggregate component of earnings and the stock return.
- The aggregate innovations governing earnings and stock returns are assumed to be jointly normally distributed with the covariance matrix estimated in Table 2.
- Shocks are discretized following the method described in Appendix A.2.
- Since shock innovations are jointly normal, they are not drawn from a fat-tailed or time varying distribution.

- The fixed effect ξ is normally distributed with the variance given in Table 1.
- The transitory shock υ and the persistent shock innovations η follow a Generalized Normal distribution, determined by the first three central moments.
- The second and third central moments of the persistent shock innovations are state dependent as described in Table 1.
- The age-dependent second and third moments of the transitory shock distribution are scaled as discussed in footnote 12.
- See Hosking and Wallis (1997, Appendix A.8) for a discussion of the Generalized Normal distribution.

Social Security:

- The nature of social security benefits is potentially of great importance for how people value future earnings flows after taxes and transfers.
- Social security wealth is by some calculations the single most important asset type for many older households.
- Social security benefits in the model are an annuity payment which is determined by the aggregate earnings level z_1 when the agent reaches the retirement age and by a concave benefit function b.
- We adopt the benefit function employed by Huggett and Parra (2010) which captures the bend-point structure of old-age benefits in the U.S. social security system.

- We employ the computationally-useful assumption that the benefit function applies only to an agent's idiosyncratic fixed effect rather than to an average of the agent's past earnings as in the U.S. system.
- Thus, the model benefit is risky after entering the labor market only because the aggregate component of earnings at the time of retirement is risky.
- Old-age benefit payments in the U.S. system are indexed to average economy-wide earnings when an individual hits age 60.
- This is captured within the model by the fact that benefits are proportional to $z_1 \equiv \exp(u^1)$.

- Properties of the benchmark model are displayed in Figure 3.
- It is constructed by simulating many shock histories, calculating decisions along these histories and then taking averages at each age.
- Figure 3 shows that mean consumption, wealth and earnings net of taxes and transfers are hump shaped.
- Financial wealth is exhausted before the end of life whereupon agents live off social security.

- Equity participation rates in Figure 3 are above US values at all ages for all the risk aversion values we analyze.
- Participation rates are calculated among agents with strictly positive financial assets as is done in the literature.
- Chang et al. (2015) provide a useful review of findings in the literature and document properties in data from the Survey of Consumer Finances (SCF).
- They find an average participation rate of 55 percent in the SCF 1998-2007 for risky financial assets that include stock, trusts, mutual funds and retirement accounts with exposure to risky assets.

- They find a hump-shaped participation rate with age where the rate is roughly 30 percent within the 21-25 age group.
- Equity participation rates in the model are just as high or higher at all ages as the patterns in Figure 3 when we analyze constant-relative-risk-aversion (CRRA) preferences for CRRA values ranging from 1 to 6.

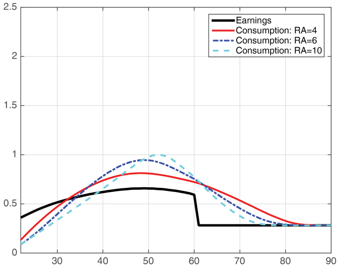
- The results summarized above lead us to conclude that the benchmark model is far away from producing the low equity partcipation rates early in the working lifetime found in US data.
- This holds for a wide range of preference parameter configurations.
- Thus, we provide support for one view within the portfolio literature: the explanation of these portfolio facts is likely to rest on model properties beyond those encompassed by enriching the Samuelson (1969) model with empirically realistic earnings and financial asset return properties.
- This negative view is based on the robust result that the stock share of the value of human capital is not particularly large when earnings and asset returns are driven by a process estimated from US data.

5. Human Capital Values and Returns: Benchmark Model

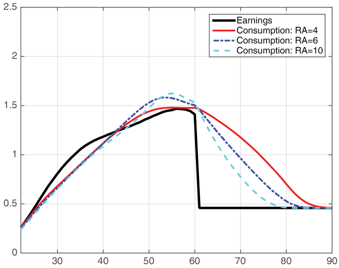
5.1. Human Capital Values

$$v_j = E[m_{j,j+1}(v_{j+1} + e_{j+1})|z^j] = E[m_{j,j+1}(\sum_{i \in \mathcal{I}} \alpha_j^i R_{j+1}^i + \epsilon_{j+1})|z^j] \quad (4)$$

Figure 3: Life-cycle profiles in the benchmark model



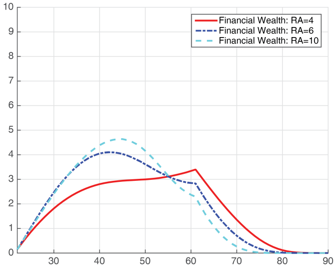
(a) Mean earnings, consumption



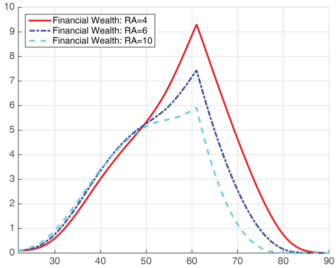
(b) Mean earnings, consumption

Notes: Left panels refer to high-school sub-sample, right panels refer to college sub-sample. The vertical scale in (a)-(b) is in units of 100,000 dollars in year 2008. Participation rates are conditional on having positive financial wealth. Equity shares are conditional on having positive equity.

Figure 3: Life-cycle profiles in the benchmark model



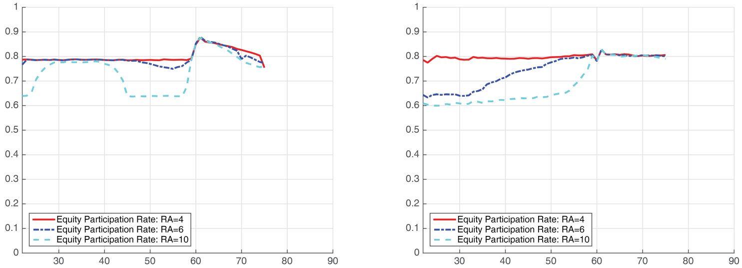
(a) Mean financial wealth



(b) Mean financial wealth

Notes: Left panels refer to high-school sub-sample, right panels refer to college sub-sample. The vertical scale in (a)-(b) is in units of 100,000 dollars in year 2008. Participation rates are conditional on having positive financial wealth. Equity shares are conditional on having positive equity.

Figure 3: Life-cycle profiles in the benchmark model

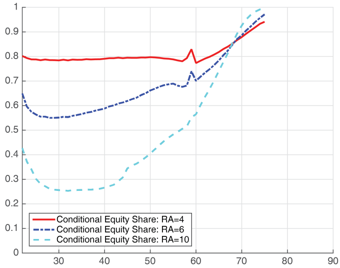


(a) Equity participation rate

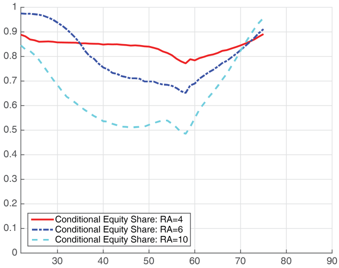
(b) Equity participation rate

Notes: Left panels refer to high-school sub-sample, right panels refer to college sub-sample. The vertical scale in (a)-(b) is in units of 100,000 dollars in year 2008. Participation rates are conditional on having positive financial wealth. Equity shares are conditional on having positive equity.

Figure 3: Life-cycle profiles in the benchmark model



(a) Conditional equity share



(b) Conditional equity share

Notes: Left panels refer to high-school sub-sample, right panels refer to college sub-sample. The vertical scale in (a)-(b) is in units of 100,000 dollars in year 2008. Participation rates are conditional on having positive financial wealth. Equity shares are conditional on having positive equity.

5.2. Human Capital Returns

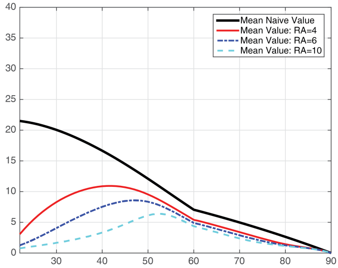
- Figure 5 plots properties of human capital returns.
- Mean returns are very large early in the working lifetime.
- To understand what drives the mean human capital returns, it is useful to return to the main ideas used in the value decomposition.

- The first equation below decomposes gross returns by decomposing the future payout into a bond, a stock and an orthogonal component.
- The second equation shows that the conditional mean human capital return always equals the weighted sum of the conditional mean of the bond and stock return.

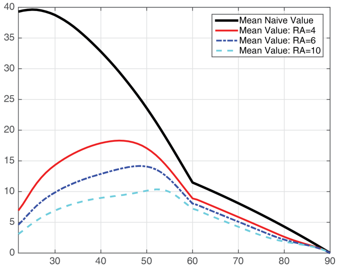
$$R^h_{j+1} \equiv \frac{v_{j+1} + e_{j+1}}{v_j} = \frac{\alpha^b_j R^b_{j+1} + \alpha^s_j R^s_{j+1} + \epsilon}{v_j} \tag{5}$$

$$E[R^h_{j+1}|z^j] = \frac{\alpha^b_j}{v_j} E[R^b_{j+1}|z^j] + \frac{\alpha^s_j}{v_j} E[R^s_{j+1}|z^j] \tag{6}$$

Figure 4: Human capital values and a decomposition

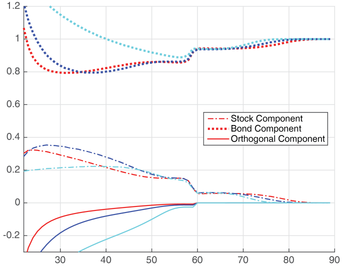


(a) Human capital values (high school)

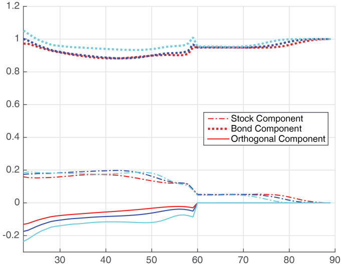


(b) Human capital values (college)

Figure 4: Human capital values and a decomposition



(a) Decomposition (high school)



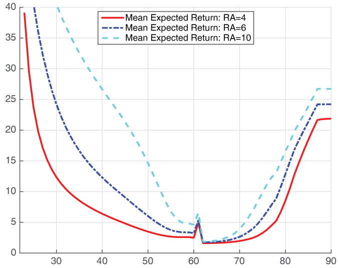
(b) Decomposition (college)

- The weights on the bond and stock return do not always sum to one.
- When the agent's Euler equation for both stock and bonds hold with equality, then these weights will sum to more than one exactly when the value of the orthogonal component is negative.
- The value of the orthogonal component of human capital payouts is negative early in the working lifetime.
- Human capital returns can vastly exceed a convex combination of stock and bond returns when the weights sum to more than one.

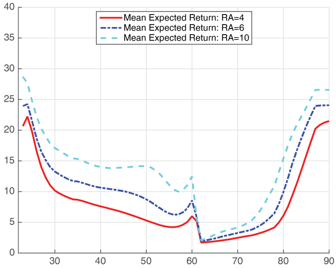
- The mean return to human capital is near the risk-free rate immediately after retirement but subsequently increases.
- The high return towards the end of the lifetime might at first seem odd.
- This should not be surprising, however, as in the penultimate period
  $v_{J-1} = E[m_{J-1,J}e_J]$ and $1 = E[m_{J-1,J}e_J/v_{J-1}]$.
- As the payment e_J, conditional on surviving to the last period, is certain, the return is
  $R^h_J = e_J/v_{J-1} = 1/E[m_{J-1,J}]$.
- Thus, the return equals the risk-free bond rate when the agent is off the corner (i.e. $R^h_J = 1/E[m_{J-1,J}] = R^b$) but can exceed the risk-free rate when the agent is on the corner (i.e. $R^h_J = 1/E[m_{J-1,J}] \geq R^b$).
- Towards the end of the lifetime an increasing fraction of agents in the model are on this corner consistent with the wealth profile in Figure 3.

- The positive correlation between human capital returns and stock returns in Figure 5 is based in part on two properties.
- First, innovations to the aggregate component of earnings growth and to stock returns are positively correlated.
- This implies that the component of human capital returns related directly to the earnings payout next period covaries positively with stock returns.
- Second, the old-age transfer benefit formula in the benchmark model is proportional to the aggregate component of earnings at the retirement age.
- The U.S. social security system has a similar feature as old-age benefits are proportional to a measure of average earnings in the economy when the worker turns age 60, other things equal.

Figure 5: Properties of human capital returns

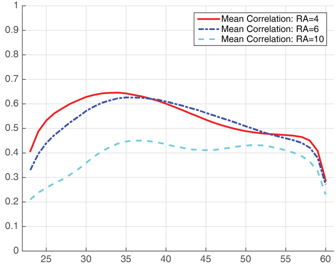


(a) Human capital returns (percentage) (high school)

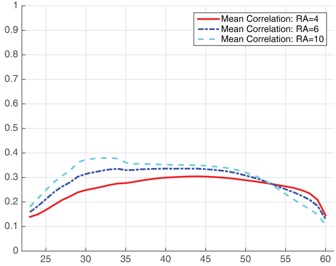


(b) Human capital returns (percentage) (college)

Figure 5: Properties of human capital returns



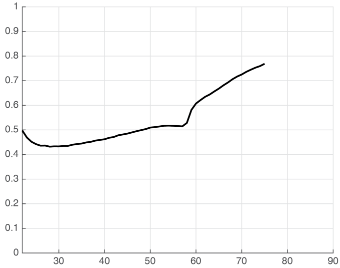
(a) Correlation: HC, stock returns (high school)



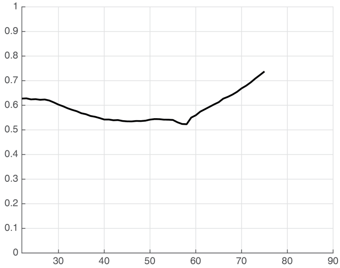
(b) Correlation: HC, stock returns (college)

5.3 Portfolio Allocation

Figure 6: Portfolio shares in the benchmark model



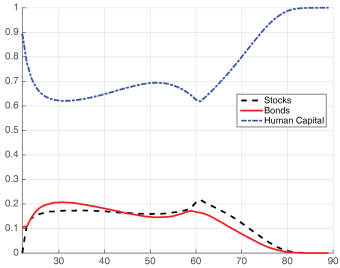
(a) Stock share of financial wealth (high school)



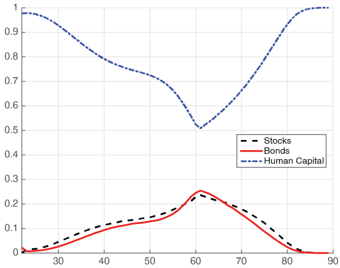
(b) Stock share of financial wealth (college)

Notes: Financial portfolio shares in panels (a)-(b) are averages over the sub-population with positive asset holdings. All results are for risk aversion equal to 6.

Figure 6: Portfolio shares in the benchmark model



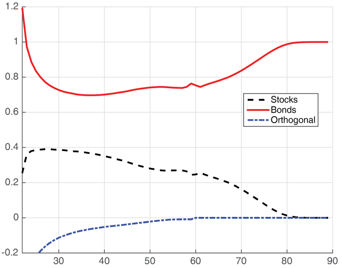
(a) Portfolio shares (high school)



(b) Portfolio shares (college)

Notes: Financial portfolio shares in panels (a)-(b) are averages over the sub-population with positive asset holdings. All results are for risk aversion equal to 6.

Figure 6: Portfolio shares in the benchmark model



(a) Overall portfolio shares (high school)

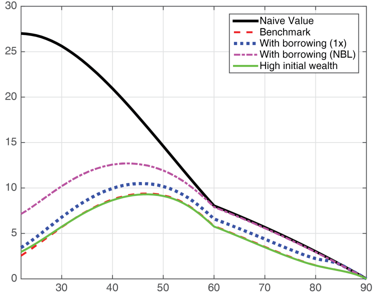
(b) Overall portfolio shares (college)

Notes: Financial portfolio shares in panels (a)-(b) are averages over the sub-population with positive asset holdings. All results are for risk aversion equal to 6.

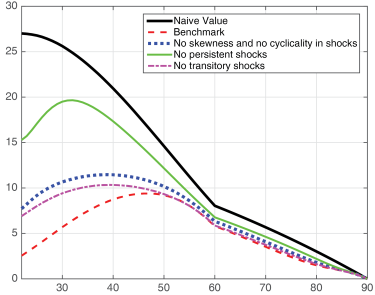
- We find that the bond share of overall wealth exceeds the stock share at all ages.
- The stock share for the college group averages between 20 to 30 percent over the working lifetime, whereas for high school it average between 20 to 40 percent over the working lifetime.
- The overall stock share early in life is largely determined by the decomposition analysis presented earlier in Figure 4.
- This is because financial assets are small in value compared to the value of human capital and negative positions in either financial asset are not allowed.

6. Discussion: Robustness and Drivers

Figure 7: What drives the value of human capital?



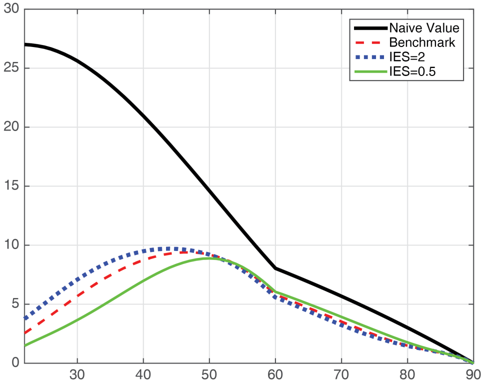
(a) Human capital values (borrowing, initial wealth)



(b) Human capital values (idiosyncratic risk)

Notes: In panel (a) "With borrowing (1x)" refers to the model that allows borrowing up to 1 times average annual earnings, and "With borrowing (NBL)" refers to model that allows borrowing up to the "Natural Borrowing Limits" i.e. limits that impose only that the agent must be able to repay his debt in all states of the world.

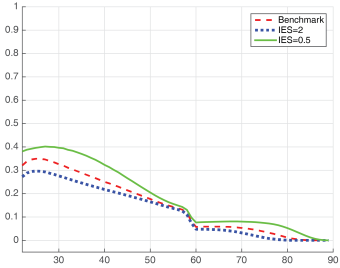
Figure 7: What drives the value of human capital?



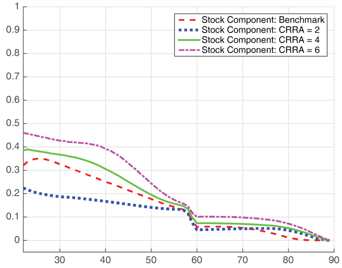
(a) Human capital values (IES)

Notes: In panel (a) "With borrowing (1x)" refers to the model that allows borrowing up to 1 times average annual earnings, and "With borrowing (NBL)" refers to model that allows borrowing up to the "Natural Borrowing Limits" i.e. limits that impose only that the agent must be able to repay his debt in all states of the world.

Figure 8: What drives the stock component of human capital?



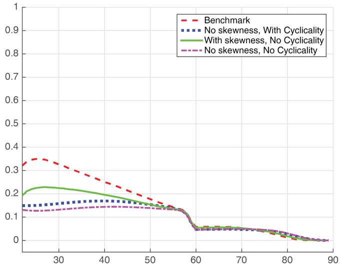
(a) Stock component of human capital – IES



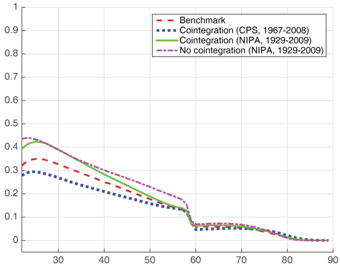
(b) Stock component of human capital – CRRA

Notes: When comparing different models estimated on the same data set or the same model estimated on different data sets, the constants in all models are reset so that $E[\Delta u_t^i] = 0$ and $E[\log R_t^s] = 0.041$ as previously noted in Table 3. The benchmark model is the model from Table 4 with $\alpha = 6$ and without cointegration.

Figure 8: What drives the stock component of human capital?



(a) Stock component of human capital – counter-cyclical risk

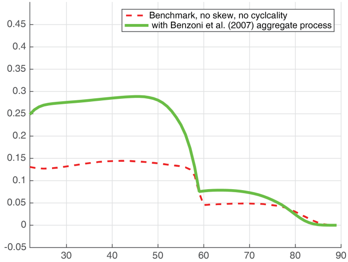


(b) Stock component of human capital – cointegration

Notes: When comparing different models estimated on the same data set or the same model estimated on different data sets, the constants in all models are reset so that $E[\Delta u_t^i] = 0$ and $E[\log R_t^s] = 0.041$ as previously noted in Table 3. The benchmark model is the model from Table 4 with $\alpha = 6$ and without cointegration.

Figure 9: Analysis of Benzoni et al. (2007)

(a) Stock share of human capital



(b) Stock share of human capital

7. Conclusion

- Our analysis highlights two main properties of human capital values based on an analysis of U.S. data on males earnings and financial asset returns:
    1. the value of human capital is far below the value implied by discounting future earnings at the risk-free rate and
    2. the stock component of the value of human capital averages less than 35 percent at each age over the working lifetime.
- These properties hold for
    - (i) different educational groups,
    - (ii) a wide range of parameters characterizing risk aversion and intertemporal substitution,
    - (iii) a range of assumptions on borrowing constraints and
    - (iv) two different statistical models for earnings estimated using male earnings data.

- We investigate the main drivers of these two findings.
- Persistent idiosyncratic shocks and the left skewness of idiosyncratic shocks are two key drivers of low human capital values.
- In our model framework, an agent's stochastic discount factor falls for larger realizations of idiosyncratic shocks, other things equal.

- A number of model features lead to a positive stock component of the value of human capital including
  1. social security benefits linked to average earnings,
  2. positive conditional correlation between the aggregate component of earnings and stock returns and
  3. left-skewed idiosyncratic earnings shocks.
- We provide support for these features in US data.
- We do not find much support in US data for the idea that cointegration between the aggregate component of earnings and stock returns is a key factor driving the size of the stock share of the value of human capital.