Labor Supply

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Econ 350, Winter 2023

One period models: $(L < 1)$

$$U(C, L) = \frac{C^{\alpha} - 1}{\alpha} + b\left(\frac{L^{\varphi} - 1}{\varphi}\right) \qquad \alpha, \varphi < 1$$

$b \uparrow \implies$ taste for leisure increases

MRS at zero hours of work (Reservation Wage or Virtual Price):

$$R = \frac{\left(\frac{\partial U}{\partial L}\right)}{\left(\frac{\partial U}{\partial C}\right)} \mid L = 1, C = A$$

$$R = b\frac{L^{\varphi-1}}{C^{\alpha-1}}$$

at $L = 1, C = A$

$$R = \frac{b}{A^{\alpha-1}}$$

$$\ln R = \ln b + (1-\alpha)\ln A$$

Set:

$$\ln b = X\beta + \varepsilon_b$$

Assume:

$$\varepsilon_b \sim N\left(0, \sigma_b^2\right)$$

Assume:

$$\ln W \perp\!\!\!\perp \varepsilon_b$$
$$(X, A, W) \perp\!\!\!\perp \varepsilon_b$$

Assume wage is observed for everyone. Probability that a person with assets A, X, and Wage W works:

$$\begin{aligned} & \Pr\left(\ln R \leq \ln W \mid \; X, A\right) \\ = \; & \Pr(X\beta + (1-\alpha)\ln A + \varepsilon_b \leq \ln W \mid \; X, A) \\ = \; & \Pr\left(\frac{\varepsilon_b}{\sigma_b} \leq \frac{\ln W - X\beta - (1-\alpha)\ln A}{\sigma_b}\right) \\ = \; & \Phi\left(C\right) \text{ where} \\ C \equiv \; & \frac{\ln W - X\beta - (1-\alpha)\ln A}{\sigma_b} \qquad A > 0 \end{aligned}$$

Let

$$\left.\begin{array}{ll} D=1 & \text{if person works} \\ D=0 & \text{otherwise} \end{array}\right\} \Rightarrow D = \mathbf{1}\left[\ln W \geqslant \ln R\right]$$

$$\Pr(\ln R \leq \ln W \mid X, A) = \Pr(D = 1 \mid X, A)$$

Take Grouped Data: Each cell has common values of W_i, X_i and A_i.

$$\widehat{P}_i = \text{ cell proportion working } i$$

Set $\widehat{P}_i = \Phi\left(\widehat{C}_i\right)$

$$C_i = \frac{\ln W_i - X_i\beta - (1-\alpha)\ln A_i}{\sigma_b}$$

inverse exists:

$$\widehat{C}_i = \Phi^{-1}\left(\widehat{P}_i\right) \qquad \text{(table lookup)}$$

Run Regression:

$$\widehat{C}_i \text{ on } \frac{\ln W_i - X_i\beta - (1-\alpha)\ln A_i}{\sigma_b}$$

$$\text{Coefficient on } \ln W_i \text{ is } \frac{1}{\sigma_b}$$

$$\text{Coefficient on } X \text{ is } \frac{\beta}{\sigma_b}$$

$$\text{Coefficient on } \ln A \text{ is } \frac{1-\alpha}{\sigma_b}$$

Do for Logit

$$\Pr\left(\frac{\varepsilon}{\sigma_b} \leq z\right) = \frac{e^z}{1+e^z}$$

Linear Probability Model

$$\Pr\left(\frac{\varepsilon}{\sigma_b} \leq z\right) = \frac{z}{z_U - z_L} \qquad z_L \leq \frac{\varepsilon}{\sigma_b} \leq z_U$$

Micro Data Analogue:
Sample size I, (Assumes we have symmetric ε around zero):

$$\mathcal{L} = \prod_{i=1}^{I} \Phi\left(C_i\left(2D_i - 1\right)\right)$$

$$\left(\widehat{\beta}, \widehat{\sigma}_b, \widehat{\alpha}\right) = \arg\max \ln \mathcal{L}$$

consistent, asymptotically normal. (Likelihood is concave) Assumes we know wage for all persons, including those who work, but we don't.
Can be nonparametric about F_{ε_b} (Cosslett, Manski, Matzkin)

Digression

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$$\Pr(D = 1 \mid Z) = F_V(Z\gamma)$$

$$\frac{\dfrac{\partial \Pr(D = 1 \mid Z\gamma)}{\partial Z_i}}{\dfrac{\partial \Pr(D = 1 \mid Z\gamma)}{\partial Z_j}} = \frac{f_V(Z\gamma)\gamma_i}{f_V(Z\gamma)\gamma_j} = \frac{\gamma_i}{\gamma_j}$$

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- We can identify the coefficients up to scale.
- Back to text.

Method II

Don't know wage, but

$$\begin{aligned} \ln W &= Z\gamma + U \\ \ln R &= X\beta + (1-\alpha)\ln A + \varepsilon \end{aligned}$$

$$\begin{pmatrix} U \\ \varepsilon \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{matrix} \sigma_{UU} & \sigma_{\varepsilon U} \\ \sigma_{\varepsilon U} & \sigma_{\varepsilon\varepsilon} \end{matrix} \right)$$

$$\ln R - \ln W \geq 0 \iff D = 0$$

$$\begin{aligned} Y_1 &\equiv -X\beta - (1-\alpha)\ln A + Z\gamma - (\varepsilon - U) \\ &= \ln W - \ln R \end{aligned}$$

$$(X, \ln A, Z) \perp\!\!\!\perp (\varepsilon - U)$$

$$(\varepsilon - U) \sim N\left(0, \sigma_{\varepsilon\varepsilon} + \sigma_{UU} - 2\sigma_{\varepsilon_U}\right)$$

$$\begin{aligned} \operatorname{Var}(\varepsilon - U) &= (\sigma^*)^2 \\ \sigma^* &\equiv \sqrt{\sigma_{\varepsilon\varepsilon} + \sigma_{UU} - 2\sigma_{\varepsilon_U}} \end{aligned}$$

$$\Pr\left(Y_1 \geq 0 \mid X, A, Z\right) = \Pr\left(D = 1 \mid X, \ln A, Z\right)$$

$$
\begin{aligned}
& \Pr\left(D = 1 \mid X, \ln A, Z\right) \\
= \; & \Pr\left(\frac{-X\beta - (1-\alpha)\ln A + Z\gamma}{\sigma^*} \geq \frac{\varepsilon - U}{\sigma^*}\right) \\
= \; & \Phi\left(\frac{-X\beta - (1-\alpha)\ln A + Z\gamma}{\sigma^*}\right) = \Phi(C) \\
C \equiv \; & \frac{-X\beta - (1-\alpha)\ln A + Z\gamma}{\sigma^*}
\end{aligned}
$$

If Z and X distinct from each other and A, estimate $\frac{\gamma}{\sigma^*}, \frac{\beta}{\sigma^*}, \frac{1-\alpha}{\sigma^*}$, can't estimate σ^*, $\therefore$ get relative values.

Suppose X and Z have some elements in common;

$$X_c = Z_c \qquad \text{elements in common}$$

$$X_d, Z_d \text{ are distinct elements in } X, Z$$

$$\frac{Y_1}{\sigma^*} = -\frac{X_d\beta_d}{\sigma^*} - \frac{X_c\left(\beta_c - \gamma_c\right)}{\sigma^*} + \frac{Z_d\gamma_d}{\sigma^*} + \frac{(1-\alpha)}{\sigma^*}\ln A + \frac{\varepsilon - U}{\sigma^*}$$

$$\therefore \text{ identify } \frac{\beta_d}{\sigma^*}, \frac{\beta_c - \gamma_c}{\sigma^*}, \frac{\gamma_d}{\sigma^*}, \frac{1-\alpha}{\sigma^*}$$

(The leading example of variables in common is education.)
Allows U to be correlated with ε.

(Method II may be required anyway.)

Observe the wage only for working persons

$$\begin{aligned} \ln W &= Z\gamma + U \\ \ln R &= X\beta + (1-\alpha)\ln A + \varepsilon \end{aligned}$$

Assume $(X, Z, A) \perp\!\!\!\perp (\varepsilon, U)$

$$Y_1 = \ln W - \ln R = Z\gamma - X\beta - (1-\alpha)\ln A + U - \varepsilon$$

Letting $\tilde{\lambda}(q) = \dfrac{\phi(q)}{\Phi(q)}$, we have

$$E\left(\ln W \mid \ln W - \ln R \geq 0, X, Z, A\right)$$

$$= E\left(\ln W \,\middle|\, \begin{array}{c} \dfrac{Z\gamma - X\beta - (1-\alpha)\ln A}{\sigma^*} \\ \geq \dfrac{\varepsilon - U}{\sigma^*}, \quad X, Z, A \end{array}\right)$$

$$= Z\gamma + \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*}\tilde{\lambda}\left(\frac{Z\gamma - X\beta - (1-\alpha)\ln A}{\sigma^*}\right)$$

$$C(X, A, Z) = \frac{Z\gamma - X\beta - (1-\alpha)\ln A}{\sigma^*}$$

Remembering the Truncated Normal Random variable:

$$
\begin{aligned}
\text{Let:} & \quad Z \sim N(0,1) \\
E\left(Z|Z \geq q\right) &= \lambda(q); \quad \lambda(q) \equiv \frac{\phi(q)}{1-\Phi(q)} = \frac{\phi(q)}{\Phi(-q)} \\
E\left(Z|q \geq Z\right) &= -E\left(-Z|-Z \geq -q\right) \\
&= -\frac{\phi(-q)}{1-\Phi(-q)} = -\frac{\phi(q)}{\Phi(q)} \\
&\Rightarrow \widetilde{\lambda}(q) \equiv \frac{\phi(q)}{\Phi(q)} = -E\left(Z|Z \leqslant q\right) \\
\text{and} &: \quad E\left(Z|Z \geq q\right) = \frac{\phi(q)}{\Phi(-q)} = \lambda(q) = \widetilde{\lambda}(-q)
\end{aligned}
$$

Two Stage Procedures

(1) Probit on Work participation

$$
\begin{aligned}
& \Pr\left(D = 1 \mid Z, X, A\right) \\
= \; & \Pr\left(\ln W - \ln R \geq 0 \mid Z, X, A\right) \\
= \; & \Pr\left(\frac{Z\gamma - X\beta - (1-\alpha)\ln A}{\sigma^*} \geq \frac{\varepsilon - U}{\sigma^*} \middle| Z, X, A\right) \\
= \; & \Phi\left(\frac{Z\gamma - X\beta - (1-\alpha)\ln A}{\sigma^*}\right)
\end{aligned}
$$

$$
\sigma^* = \left[\operatorname{Var}\left(U - \varepsilon\right)\right]^{\frac{1}{2}}
$$

$\therefore$ we can estimate $C\left(X, A, Z\right)$

(2) Form $\tilde{\lambda}\left(C\right)$

Run Linear Regression
Get Consistent Estimates of

$$\gamma, \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*}$$

With one exclusion restriction (one variable in Z not in X or $\ln A$, say Z_1).

Note that using Probit if X_d, Z_d are distinct elements in X, Z and $X_c = Z_c$ are elements in common we can identify $\frac{\beta_d}{\sigma^*}$, $\frac{\beta_c - \gamma_c}{\sigma^*}$, $\frac{\gamma_d}{\sigma^*}$, $\frac{1-\alpha}{\sigma^*}$.

Say we recover $\frac{\gamma_1}{\sigma^*}$ (by Probit)

Note that we have γ (by Wage Regression on Z and $\widetilde{\lambda}$)

$\therefore$know σ^*

The estimated coefficient on $\widetilde{\lambda}$ is $\dfrac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*}$

$\therefore$know $\sigma_{UU} - \sigma_{U\varepsilon}$

Look at the residuals from equations

$$
\begin{aligned}
V &\equiv \ln W - \left[Z\gamma + \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*} \tilde{\lambda}\left(C\left(X, A, Z\right)\right) \right] \\
&= U - \left(\frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^* \left(\sigma_{UU}\right)^{\frac{1}{2}}} \right) \left(\sigma_{UU}\right)^{\frac{1}{2}} \tilde{\lambda}\left(C\left(X, A, Z\right)\right) \\
\text{Let} &: \quad \rho \equiv \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\left(\sigma_{UU}\right)^{\frac{1}{2}} \sigma^*} \\
V &= U - \rho \left(\sigma_{UU}\right)^{\frac{1}{2}} \tilde{\lambda}\left(C\left(X, A, Z\right)\right) \\
&= U - E\left(U \middle| \ln W - \ln R \geq 0\right) \\
&\Rightarrow E\left(V\right) = 0 \\
E\left(V^2\right) &= \mathrm{Var}(V) = \mathrm{Var}\left(U \middle| \ln W - \ln R \geq 0\right)
\end{aligned}
$$

$$
\begin{aligned}
E\left(V^2\right) &= \sigma_{UU}\left[\left(1-\rho^2\right)+\rho^2\left(1+\tilde{\lambda}C-\tilde{\lambda}^2\right)\right] \\
&= \sigma_{UU}+\sigma_{UU}\rho^2\left(\tilde{\lambda}C-\tilde{\lambda}^2\right)
\end{aligned}
$$

Regress

$\widehat{V}^2$ on $\left(\tilde{\lambda}C - C^2\right)$ Get σ_{UU} and $\sigma_{UU}\rho^2$

$\therefore$ know ρ^2

Look at model:

1. Wrong variables appear in wage equation
2. Errors heteroskedastic
3. Omitted variables

Recovered Coefficients:

$$\left.\begin{array}{c} \frac{\gamma_1}{\sigma^*} \text{ (Probit)} \\ \gamma \text{ (Wage Regression)} \end{array}\right\} \Rightarrow \sigma^*$$

$$\left.\begin{array}{c} \frac{\sigma_{UU}-\sigma_{U\varepsilon}}{\sigma^*}\text{(Wage Regression)} \\ \sigma^* \end{array}\right\} \Rightarrow \sigma_{UU} - \sigma_{U\varepsilon}$$

$$\left.\begin{array}{c} \sigma_{UU} \text{ (Error}^2 \text{ Regression)} \\ \sigma_{UU} - \sigma_{U\varepsilon} \end{array}\right\} \Rightarrow \sigma_{U\varepsilon}$$

The term $\dfrac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*}$:

$$\frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*} \text{ is a Wage Regression coefficient}$$

$$\left.\begin{array}{c} \rho \equiv \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{(\sigma_{UU})^{\frac{1}{2}}\sigma^*} \text{ (Error}^2 \text{ Regression)} \\ \sigma_{UU} \text{ (Error}^2 \text{ Regression)} \end{array}\right\} \Rightarrow \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*}$$

$$\Rightarrow 2 \text{ estimates of } \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*}$$

The term σ^* :

$$\left.\begin{array}{c} \frac{\gamma_1}{\sigma^*} \text{ (Probit)} \\ \gamma \text{ (Wage Regression)} \end{array}\right\} \Rightarrow \sigma^*$$

$$\left.\begin{array}{c} \sigma_{UU} - \sigma_{U\varepsilon} \text{ (Wage Regression \& } \sigma^* \text{ above)} \\ \rho \equiv \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{(\sigma_{UU})^{\frac{1}{2}} \sigma^*} \text{ (Error}^2 \text{ Regression)} \\ \sigma_{UU} \text{ (Error}^2 \text{ Regression)} \end{array}\right\} \Rightarrow \sigma^*$$

$$\Rightarrow 2 \text{ estimates of } \sigma^*$$

To obtain $\sigma_{\varepsilon\varepsilon}$, we can solve

$$\begin{aligned} (\sigma^*)^2 &= \sigma_{UU} + \sigma_{\varepsilon\varepsilon} - 2\sigma_{U\varepsilon} \\ &\therefore \quad (\sigma^*)^2 + 2\widehat{\sigma}_{U\varepsilon} - \widehat{\sigma}_{UU} = \widehat{\sigma}_{\varepsilon\varepsilon} \end{aligned}$$

Suppose we have no exclusion restriction, just regressors. Then we can still estimate γ, σ_{UU}, $\sigma_{\varepsilon\varepsilon}$ provided we substitute other information for exclusion restrictions.

$$b = \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\sigma^*} = \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\left(\sigma_{UU} + \sigma_{\varepsilon\varepsilon} - 2\sigma_{U\varepsilon}\right)^{\frac{1}{2}}}$$

(coefficient on λ)

$$\begin{aligned} E\left(V^2\right) &= \sigma_{UU} + \sigma_{UU}\rho^2\left(\tilde{\lambda}C - \tilde{\lambda}^2\right) \\ &= \sigma_{UU} + \sigma_{UU}\left(\frac{\sigma_{UU} - \sigma_{U\varepsilon}}{\left(\sigma_{UU}\right)^{\frac{1}{2}}\sigma^*}\right)^2\left(\tilde{\lambda}C - \tilde{\lambda}^2\right) \\ &= \sigma_{UU} + b^2\left(\tilde{\lambda}C - \tilde{\lambda}^2\right) \\ \Rightarrow &\quad \sigma_{UU} = E\left(V^2\right) - b^2\left(\tilde{\lambda}C - \tilde{\lambda}^2\right) \end{aligned}$$

Normalize variables: $\sigma_{\varepsilon\varepsilon} = 1$ or $\sigma_{U\varepsilon} = 0$
Example: $\sigma_{U\varepsilon} = 0$
Then know

$$\frac{\sigma_{UU}}{(\sigma_{UU} + \sigma_{\varepsilon\varepsilon})^{\frac{1}{2}}}$$

$\therefore$ can solve for $\sigma_{\varepsilon\varepsilon}$
Alternatively, if $\sigma_{\varepsilon\varepsilon} = 1$

$$\frac{\sigma_{UU} - \sigma_{U\varepsilon}}{(1 + \sigma_{UU} - 2\sigma_{U\varepsilon})^{\frac{1}{2}}} = \text{known}$$

solve for $\sigma_{U\varepsilon}$, quadratic equation – sometimes get unique root.
Note crucial role of regressor in getting full identification.

Labor Supply – Hours of Work – Single Period Model

More Information:
Direct Utility Function for non workers:

$$V_1(A_1, \varphi) \qquad A_1 = \text{ unearned income if person works}$$

best attainable utility for a person who doesn't work
Indirect Utility Function:

$$V_2(A_2, W, \varphi) \qquad (W = \text{ wage})$$

best available utility given that he "works", (which may be V_1).
A_2 is unearned income net of money costs of work

For person who works:

$$\text{If } V_2 > V_1 \text{ person works}$$

Index Function:

$$\begin{aligned} Y_1 &= V_2 - V_1 \qquad Y_1 \geq 0 \text{ person works} \\ Y_2 &= H = \left(\frac{\partial V_2}{\partial W}\right) \Big/ \left(\frac{\partial V_2}{\partial A}\right) = H\left(A_2, W, \varphi\right) \end{aligned}$$

Roy's Identity:
3 types of labor supply functions:

(a) participation
(b) $E(H|H > 0, W, A)$
(c) $E(H|W, A)$ aggregate labor supply

None estimates a labor supply function (Hicks-Slutsky).
Workers free to choose hours of work. Wage W is independent of hours of work. No fixed costs. Local comparison is global comparison.

Consider a simple example based on Heckman (1974), MRS Function:

$$\ln R = \alpha_0 + \alpha_1 A + \alpha_2 X + \eta H + \varepsilon \tag{1}$$
$$\ln W = Z\gamma + U$$

$\ln R$ defines an equilibrium value of time locus.

Labor supply H is the value that equates $\ln W = \ln R$:

$$\begin{aligned} \ln W &= \alpha_0 + \alpha_1 A + \alpha_2 X + \eta H + \varepsilon \\ \therefore \quad H &= \frac{1}{\eta}\left(\ln W - \alpha_0 - \alpha_1 A - \alpha_2 X - \varepsilon\right) \end{aligned}$$

The "causal effect" of ln (wage) on labor supply is $\frac{1}{\eta}$ (holding A, X and ε constant).

This is a Hicks-Slutsky effect.

E.g.

$$\frac{\partial H}{\partial \ln W} = \underbrace{S}_{\substack{\text{substitution} \\ \text{effect}}} + \underbrace{H \frac{\partial H}{\partial Y}}_{\text{income effect}},$$

$$\frac{1}{\eta} = \frac{\partial H}{\partial \ln W} = WS + (WH)\frac{\partial H}{\partial Y}.$$

If η is constant, then as $H\uparrow$, for a fixed W, $S\uparrow$ (more substitution).

As $W\uparrow$, $S + H\frac{\partial H}{\partial Y}\downarrow$, so the Hicks-Slutsky effect declines (net labor supply becomes more inelastic in this sense).

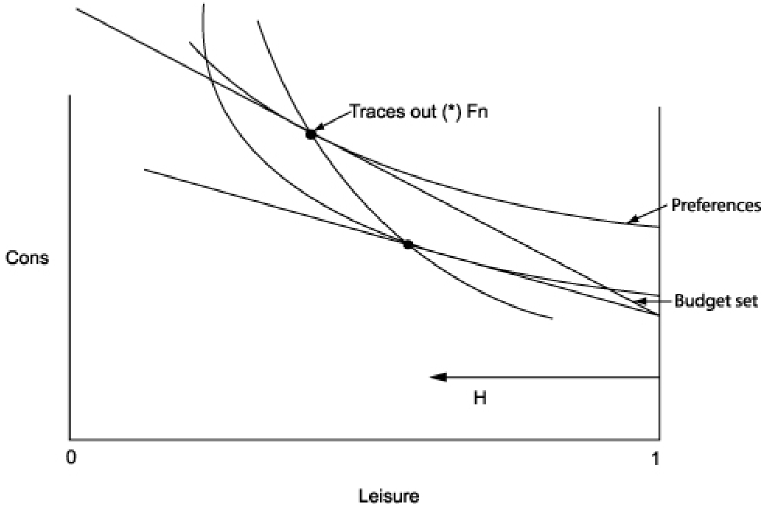

Figure: Value of Time

Define

$$\begin{aligned} Y_1 &= \ln W - \ln R = Z\gamma + U - \alpha_0 - \alpha_1 A - \alpha_2 X - \varepsilon \\ &= (Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X) + (U - \varepsilon). \end{aligned}$$

Hours of work then are:

$$\begin{aligned} Y_2 &= H = \frac{1}{\eta} Y_1 \quad \text{if } Y_1 \geq 0 \\ Y_3 &= \ln W = Z\gamma + U \end{aligned}$$

$$\text{Var}(U - \varepsilon) = (\sigma^*)^2$$

Population Labor Supply is generated from Y_2

$$
\begin{aligned}
& E\left(H \middle| Y_1 > 0, Z, A, X\right) \\
= \ & \frac{1}{\eta} E\left(\ln W - \ln R \middle| \ln W - \ln R > 0, Z, A, X\right) \\
= \ & \frac{Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X}{\eta} \\
& + \frac{1}{\eta} E\left(U - \varepsilon \mid U - \varepsilon > -(Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X)\right) \\
= \ & \frac{Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X}{\eta} \\
& + \frac{1}{\eta} E\left(\frac{U - \varepsilon}{\sigma^*} \mid \frac{U - \varepsilon}{\sigma^*} > -\frac{(Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X)}{\sigma^*}\right)
\end{aligned}
$$

Let

$$C \equiv \frac{Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X}{\sigma^*}$$

$$\begin{aligned}
& E\left(H \middle| Y_1 > 0, Z, A, X\right) \\
= \quad & \frac{1}{\eta} E\left(Y_1 \middle| Y_1 > 0, Z, A, X\right) \\
= \quad & \frac{(Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X)}{\eta} \\
& + \frac{1}{\eta} E\left(U - \varepsilon \middle| U - \varepsilon > -(Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X), Z, A, X\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{C}{\eta}\sigma^* + \frac{\sigma^*}{\eta} E\left(\frac{U-\varepsilon}{\sigma^*} | \frac{U-\varepsilon}{\sigma^*} > -C, Z, A, X\right) \\
&= \frac{C}{\eta}\sigma^* + \frac{1}{\eta}\frac{\text{Cov}(U-\varepsilon, U-\varepsilon)}{\sigma^*}\tilde{\lambda}(C) \\
&= \frac{C}{\eta}\sigma^* + \frac{1}{\eta}\sigma^*\tilde{\lambda}(C) \\
&= \frac{\sigma^*}{\eta}\left(C + \tilde{\lambda}(C)\right)
\end{aligned}
$$

$$
\begin{aligned}
E(\ln W | Y_1 > 0, Z) &= E(\ln W | \ln W - \ln R > 0, Z) \\
&= Z\gamma + \sigma^* E(\frac{U}{\sigma^*} | \frac{U-\varepsilon}{\sigma^*} > -C, Z) \\
&= Z\gamma + \frac{\text{Cov}(U, U-\varepsilon)}{\sigma^*}\tilde{\lambda}(C)
\end{aligned}
$$

Assume Regressors are available:
$\therefore$ We can estimate γ from linear regression $\ln W$ on $Z\gamma$ and $\tilde{\lambda}(C)$ using the known steps:
(1) From participation equation, we can use probit to estimate

$$\begin{aligned} \Pr(D = 1 \mid Z, A, X) &= \Pr(Y_1 > 0 \mid Z, A, X) \\ &= \Phi\left(\frac{Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X}{\sigma^*}\right) \\ &= \Phi(C) \end{aligned}$$

$\therefore$ We know $\frac{\gamma}{\sigma^*}, \frac{\alpha_0}{\sigma^*}, \frac{\alpha_1}{\sigma^*}, \frac{\alpha_2}{\sigma^*}$ if $X \neq A \neq Z$ or set of common and distinct coefficients depending on X, A, Z elements.
$\therefore$ We know C.
(2) Form $\widetilde{\lambda}(C)$.
(3) From the Wage Regression of $\ln W$ on Z and $\tilde{\lambda}(C)$.

$\therefore$we know γ, $\frac{\text{Cov}(U,U-\varepsilon)}{\sigma^*}$.
Thus we know

$$\frac{\text{Cov}(U, U-\varepsilon)}{\sigma^*} = \frac{\sigma_{UU} - \sigma_{U\varepsilon}}{(\sigma_{UU} + \sigma_{\varepsilon\varepsilon} - 2\sigma_{U\varepsilon})^{1/2}}.$$

(4) From Error Regression : $\widehat{V}^2$ on constant and $\left(\tilde{\lambda}C - C^2\right)$, we estimate:

$E\left(V^2\right) = \sigma_{UU} + \sigma_{UU}\rho^2(\tilde{\lambda}C - \tilde{\lambda}^2)$

$\therefore$ know σ_{UU}, ρ^2

Same position as before. Further identification of parameters is possible due to hours of work:

(5) From hours of work data we have a proportionality restriction

$$E(H|Y_1 > 0, Z, A, X) = \left(\frac{\sigma^*}{\eta}\right) C + \frac{\sigma^*}{\eta}\tilde{\lambda}(C)$$

but from employment (participation) equation we know C

$$\therefore \text{can estimate } \frac{\sigma^*}{\eta}$$

(6) Using $\frac{\text{Cov}(U,U-\varepsilon)}{\sigma^*}$ from Wage Regression and covariance assumptions, we obtain:

$$\begin{aligned} \text{if} \quad \sigma_{U\varepsilon} &= 0 \Rightarrow \frac{\text{Cov}(U, U-\varepsilon)}{\sigma^*} = \frac{\sigma_{UU}}{(\sigma_{UU}+\sigma_{\varepsilon\varepsilon})^{1/2}} \\ & \quad \text{but } \sigma_{UU} \text{ was obtained by Error Regression} \\ \therefore & \quad \text{know } \sigma^* \text{ and } \sigma_{\varepsilon\varepsilon} \\ & \quad \text{By The hours of Work Regression (5) we obtain } \frac{\sigma^*}{\eta} \\ \therefore & \quad \text{know } \eta \end{aligned}$$

Similarly if $\sigma_{\varepsilon\varepsilon} = 1 \Rightarrow \sigma_{U\varepsilon}$ known
$\sigma_{U\varepsilon}$ known (sometimes; multiple roots)
$\therefore$ we have that all parameters are identified.

(7) If there is one variable in Z not in (X, A), say Z_1, from coefficient on Z_1 in $E(H|Y_1 > 0, Z, A, X)$, we obtain:

$$
\begin{aligned}
& E\left(H|Y_1 > 0, Z, A, X\right) \\
= & \left(\frac{\sigma^*}{\eta}\right) C + \frac{\sigma^*}{\eta}\tilde{\lambda}(C) \\
= & \left(\frac{\sigma^*}{\eta}\right)\left(C + \tilde{\lambda}(C)\right) \\
= & \left(\frac{\sigma^*}{\eta}\right)\left(\frac{Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X}{\sigma^*} + \tilde{\lambda}(C)\right) \\
= & Z\frac{\gamma}{\eta} + \frac{-\alpha_0 - \alpha_1 A - \alpha_2 X}{\eta} + \left(\frac{\sigma^*}{\eta}\right)\tilde{\lambda}(C)
\end{aligned}
$$

$$\begin{aligned} & \text{but from Wage Regression (3), we obtain } \gamma \\ \therefore \quad & \text{can estimate } \eta \\ & \text{but the coefficient of } \tilde{\lambda}(C) \text{ is } \left(\frac{\sigma^*}{\eta}\right) \\ \therefore \quad & \text{can estimate } \sigma^*. \end{aligned}$$

Alternatively, we can determine η if

$$\mathrm{Cov}(U, \varepsilon) = 0 \quad \text{or} \quad \mathrm{Var}(\varepsilon) = 1.$$

Selection Bias in Labor Supply

Assume $\gamma_j > 0$

$$\frac{\partial E(H \mid Y_1 > 0, Z, A, X)}{\partial Z_j} = \frac{\partial \left[\left(\frac{\sigma^*}{\eta} \right) \left(C + \tilde{\lambda}(C) \right) \right]}{\partial Z_j}$$

$$= \frac{\partial \left[Z \frac{\gamma}{\eta} + \frac{-\alpha_0 - \alpha_1 A - \alpha_2 X}{\eta} + \left(\frac{\sigma^*}{\eta} \right) \tilde{\lambda}\left(\frac{Z \frac{\gamma}{\eta} + \frac{-\alpha_0 - \alpha_1 A - \alpha_2 X}{\eta}}{\sigma^*} \right) \right]}{\partial Z_j}$$

But

$$\begin{aligned}
\frac{\partial \tilde{\lambda}(q)}{\partial q} &= \frac{\partial \frac{\phi(q)}{\Phi(q)}}{\partial q} = -q\frac{\phi(q)}{\Phi(q)} - \frac{\phi^2(q)}{\Phi^2(q)} = -\tilde{\lambda}(q)\left(q + \tilde{\lambda}(q)\right) \\
&= \frac{\gamma_j}{\eta} - \frac{1}{\eta}\widetilde{\lambda}(\widetilde{\lambda} + C)\gamma_j \\
&= \left(\frac{\gamma_j}{\eta}\right)(1 - \widetilde{\lambda}(\widetilde{\lambda} + C)) \\
0 &< 1 - \widetilde{\lambda}(\widetilde{\lambda} + C) < 1 \therefore \quad < \frac{\gamma_j}{\eta} \text{ downward bias.}
\end{aligned}$$

$$\frac{\frac{\partial H}{\partial Z_j}}{\frac{\partial W}{\partial Z_j}} = \frac{\partial E(H \mid Y_1 > 0, Z, A, X)}{\partial \ln W} \leq \frac{1}{\eta}$$

$\therefore$ downward biased

$$
\begin{aligned}
& E\left(H \middle| Y_1 > 0, Z, A, X\right) \\
= & \left(\frac{\sigma^*}{\eta}\right) C + \frac{\sigma^*}{\eta}\tilde{\lambda}(C) \\
= & \left(\frac{\sigma^*}{\eta}\right)\left(C + \tilde{\lambda}(C)\right) \\
= & \left(\frac{\sigma^*}{\eta}\right)\left(\frac{Z\gamma - \alpha_0 - \alpha_1 A - \alpha_2 X}{\sigma^*} + \tilde{\lambda}(C)\right) \\
= & Z\frac{\gamma}{\eta} + \frac{-\alpha_0 - \alpha_1 A - \alpha_2 X}{\eta} + \left(\frac{\sigma^*}{\eta}\right)\tilde{\lambda}(C)
\end{aligned}
$$

but from Wage Regression (3), we obtain γ

$\therefore$ can estimate η

but the coefficient of $\tilde{\lambda}(C)$ is $\left(\dfrac{\sigma^*}{\eta}\right)$

$\therefore$ can estimate σ^*.

Aggregate Labor Supply

$$\begin{aligned} \text{ALS} &\equiv \Phi(C) \cdot E\left(H \middle| Y_1 > 0\right) \\ &\quad + \Phi(-C) \cdot \underbrace{E\left(H \middle| Y_1 < 0\right)}_{0} \\ &= \Phi(C)\left[\left(\frac{\sigma^*}{\eta}\right)\left(C + \tilde{\lambda}(C)\right)\right] + \Phi(-C) \cdot [0] \\ &= \left(\frac{\sigma^*}{\eta}\right)\left[\Phi(C)C + \frac{1}{\sqrt{2\pi}} e^{-C^2/2}\right] \end{aligned}$$

$$
\begin{aligned}
\frac{\partial E(H_{\mathrm{ALS}}|Z,A,X)}{\partial C} &= \frac{\sigma^*}{\eta}\left[\Phi(C) + \frac{Ce^{-\frac{1}{2}C^2/2}}{\sqrt{2\pi}} - \frac{Ce^{-C^2/2}}{\sqrt{2\pi}}\right] \\
&= \frac{\sigma^*}{\eta}\left[\Phi(C)\right] \\
\frac{\partial E(H_{\mathrm{ALS}}|Z,A,X)}{\partial Z_j} &= \frac{\partial E(H|Z,A,X)}{\partial C}\frac{\partial C}{\partial Z_j} = \frac{\gamma_j}{\eta}\Phi(C)
\end{aligned}
$$

Obviously aggregate labor supply more elastic because of entry or exit:

$$\frac{1}{E(H|Y_1>0,W)P(Y_1>0|W)} \cdot \frac{\partial\left\{E(H|Y_1>0,W)P(Y_1>0|W)\right\}}{\partial \ln W} \geq \left(\frac{\partial E(H|Y_1>0,W)}{\partial \ln W}\right)\frac{1}{E(H|Y_1>0,W)}, \text{ and}$$

$$\frac{\partial \ln E(H|Y_1>0,W)}{\partial \ln W} + \frac{\partial \ln P(Y_1>0|W)}{\partial \ln W} > \frac{\partial \ln E(H|Y_1>0,W)}{\partial \ln W}$$

Many ways to estimate model.

Labor Supply with Optimal Wage-Hours Contracts (Lewis, 1969; Rosen, 1974; Tinbergen, 1951, 1956)

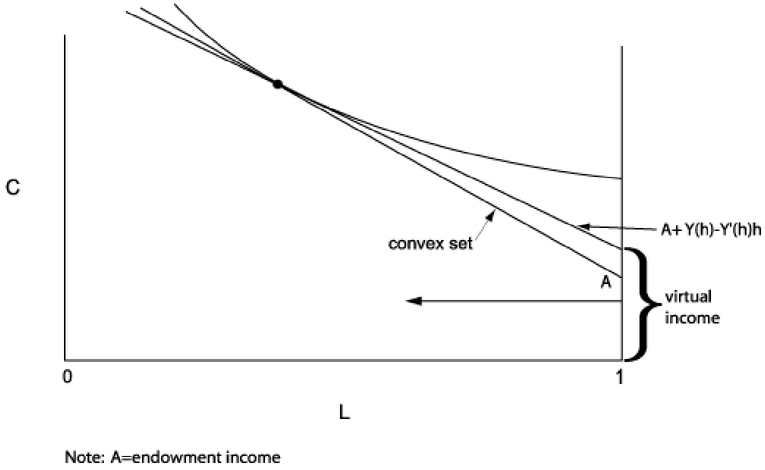

Figure: Optimal Wage

If $Y(h)$ is earnings, and $Y'(h)$ is marginal wage,
Virtual Income $= Y(h) - Y'(h)h + A$,
where $h = h(Y(h), Y(h) - Y'(h)h + A, \nu)$
Any equilibrium calculation use slope at zero hours of work.

$$\ln M(0, A) \leq \ln W(0) \qquad \text{doesn't work}$$

Equilibrium:

$$\ln M(h, A) = \ln W(h) \qquad \text{person works}$$

Can use estimated $\ln M(0, A)$ to price out goods that previously were not purchased.

Fixed Cost Models: Fixed Money Cost (Cogan; 1981 Econometrica)

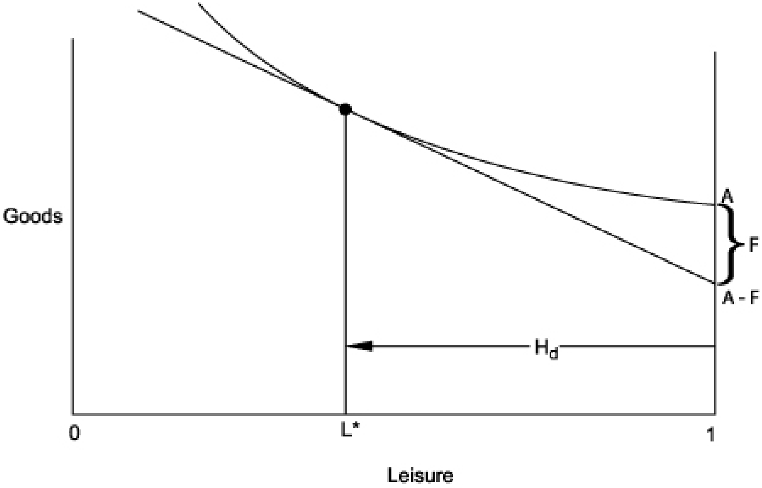

Figure: Fixed Cost Models

Introduce fixed money cost: given the wage, the worker selects a minimum number of hours.

(1) Solve for W_d and H_d that causes the worker to be indifferent between work and no work

$$V(A - F, W_d, \varphi) = U(A, 1, \varphi)$$

Solve for W_d
If no solution, person doesn't work

(2) Minimum number of hours $H_d = V_W / V_A$

$$\begin{aligned} H_d &= H_d(A - F, W_d, \varphi) \\ W_d &= W_d(A - F, \varphi) \\ H &= H(A - F, W, \varphi) \end{aligned}$$

Index Function Model.

$$\begin{aligned} Y_1 &= H - H_d \\ Y_2 &= H \end{aligned}$$

Observe Y_2 only when $Y_1 > 0$

Example: (assume wage is known).

$$\begin{aligned} H &= X\beta + W\eta + \varepsilon_1 \quad \text{functional form assumptions} \\ H_d &= X\tau + \varepsilon_2 \end{aligned}$$

Pr (consumer works) =

$$\begin{aligned} & \Pr\left(H - H_d > 0 | X, W\right) \\ &= \Pr(X\beta + W\eta + \varepsilon_1 - X\tau - \varepsilon_2 > 0) \\ &= \Pr(X(\beta - \tau) + W\eta > \varepsilon_2 - \varepsilon_1) \\ \sigma^* &\equiv \sqrt{\text{Var}(\varepsilon_2 - \varepsilon_1)} \end{aligned}$$

Assume normality and we can identify

$$\frac{\beta - \tau}{\sigma^*} \text{and} \frac{\eta}{\sigma^*}$$

From hours of work equation we know η
$\therefore$ know σ^*

$$\begin{aligned} & E\left(H | H - H_d > 0, X, W\right) \\ = & X\beta + W\eta + E\left(\varepsilon_1 | X(\beta - \tau) + W\eta > \varepsilon_2 - \varepsilon_1, X, W\right) \\ = & X\beta + W\eta + \frac{\text{Cov}(\varepsilon_{1,}\varepsilon_2)}{\sigma^*}\widetilde{\lambda}(C) \end{aligned}$$

$$C = \left(\frac{X(\beta, \tau) + W\eta}{\sigma^*}\right)$$

Know η, β $\therefore$ know τ

Know Var(ε_1), know σ^*
$\therefore$ know Cov($\varepsilon_1, \varepsilon_2$)
$\therefore$ know Var(ε_2) $= (\sigma^*)^2 - \text{Var}(\varepsilon_1) + 2\,\text{Cov}(\varepsilon_1, \varepsilon_2)$

Cogan doesn't measure fixed costs

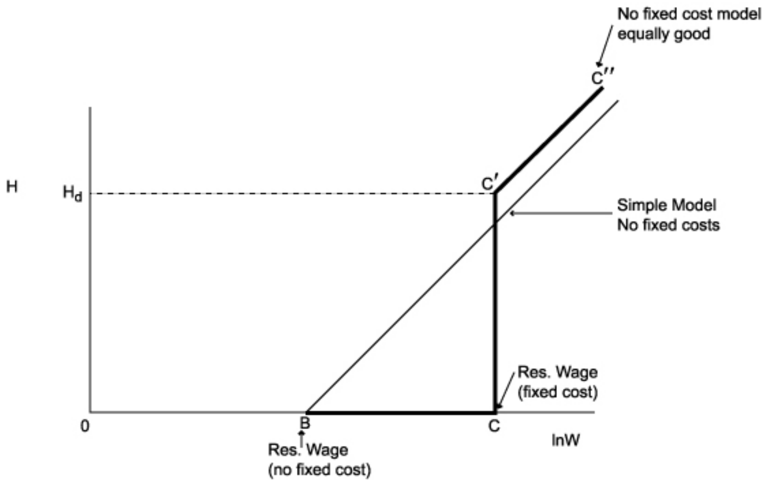

Figure: Fixed vs. Not Fixed

Null: Departure from simple proportionality model Cogan's test conditional on function form.

Broken line Budget Constraint (2 part prices; negative income tax data)

Two cases:

A. Know which interval person is in (no measurement error for hours)

B. Don't know which branch (income tax data)

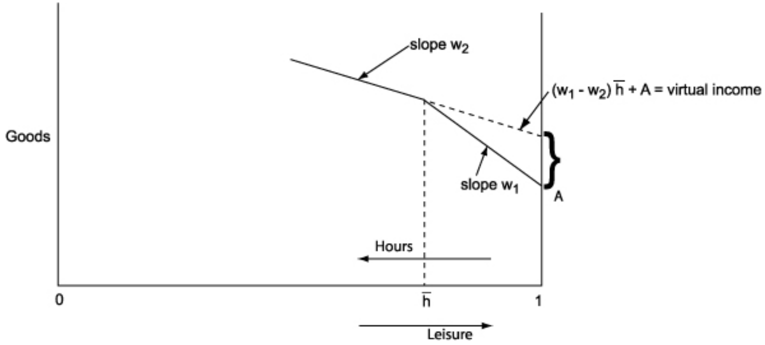

Figure: Which Branch?

Take case 1:

(1) $M(A, 1, \varepsilon) \geq W_1$ does a person work?

(2) Person is interior in interval $(0, \overline{h})$ if

$$\begin{aligned} M(A + W_1 \overline{h}, 1 - \overline{h}, \varepsilon) &\geq W_1 \\ M(A, 1, \varepsilon) &< W_1 \end{aligned}$$

(3) In equilibrium at $\overline{h}$ if $W_2 \leq M(A + W_1 \overline{h}, 1 - \overline{h}, \varepsilon) \leq W_1$

(4) Works beyond $\overline{h}$ if $M(A + W_1 \overline{h}, 1 - \overline{h}, \varepsilon) \leq W_2$

Example:

$$U = \frac{C^\alpha - 1}{\alpha} + b\left(\frac{L^\varphi - 1}{\varphi}\right) \qquad \alpha < 1, \varphi < 1$$

$$b\frac{L^{\varphi-1}}{C^{\alpha-1}} = MRS$$

at zero hours work $(L = 1)$

$\ln b = X\beta + \varepsilon$

$\ln b + (\varphi - 1)\ln(1) - (\alpha - 1)\ln A \geq \ln W_1$ (doesn't work)

$$X\beta + \varepsilon - (\alpha - 1)\ln A \geq \ln W_1$$

$$\varepsilon \geq \ln W_1 + (\alpha - 1)\ln A - X\beta$$

$E(\varepsilon^2) = \sigma_\varepsilon^2$

$$\frac{\varepsilon}{\sigma_\varepsilon} \geq \frac{\ln W_1 + (\alpha - 1)\ln A - X\beta}{\sigma_\varepsilon} \qquad \text{condition for not working}$$

estimate: $\sigma_\varepsilon, \alpha, \beta$

(2) Interior in the first branch $(0, \bar{h})$

$$\frac{X\beta + (\varphi - 1)\ln(1 - \bar{h}) - (\alpha - 1)\ln(A + W_1\bar{h}) - \ln W_1}{\sigma_\varepsilon} \geq \frac{-\varepsilon}{\sigma_\varepsilon}$$

$$\frac{X\beta + (\varphi - 1)\ln(1) - (\alpha - 1)\ln(A) - \ln W_1}{\sigma_\varepsilon} \leq \frac{-\varepsilon}{\sigma_\varepsilon}$$

Use principle of index function, with variation in $\bar{h}$, we identify φ and β.

(3) Kink Equilibrium:

$$\ln W_2 \leq X\beta + \varepsilon + (\varphi - 1)\ln(1 - \overline{h}) - (\alpha - 1)\ln(A + W_1\overline{h}) \leq \ln W_1$$

$$\frac{\ln W_2 - X\beta - (\varphi - 1)\ln(1 - \overline{h}) + (\alpha - 1)\ln(A + W_1\overline{h})}{\sigma_\varepsilon} \leq \frac{\varepsilon}{\sigma_\varepsilon}$$

$$\leq \frac{\ln W_1 - X\beta - (\varphi - 1)\ln(1 - \overline{h}) + (\alpha - 1)\ln(A + W_1\overline{h})}{\sigma_\varepsilon}$$

$$\Pr(h = \bar{h} | W_1, W_2, X, A) =$$

$$\Phi\left(\frac{\ln W_1 - X\beta - (\varphi - 1)\ln(1 - \bar{h}) - (\alpha - 1)\ln(A + W_1\bar{h})}{\sigma_\varepsilon}\right) -$$

$$\Phi\left(\frac{\ln W_2 - X\beta - (\varphi - 1)\ln(1 - \bar{h}) + (\alpha - 1)\ln(A + W_1\bar{h})}{\sigma_\varepsilon}\right)$$

(4) In second branch interior

$$\Pr\left(\frac{X\beta + (\eta - 1)\ln(1-\bar{h}) - (\alpha - 1)\ln(A + W_1\bar{h}) - \ln W_2}{\sigma_\varepsilon} \leq \frac{\varepsilon}{\sigma_\varepsilon}\right)$$

$$= \Phi\left(\frac{\ln W_2 - X\beta - (\eta - 1)\ln(1-\bar{h}) + (\alpha - 1)\ln(A + W_1\bar{h})}{\sigma_\varepsilon}\right)$$

$\therefore$ solve out hours of work.

Hours of work (standard case)

$$U = \frac{C^{\alpha} - 1}{\alpha} + b\left(\frac{L^{\varphi} - 1}{\varphi}\right)$$

$A =$ nonmarket income
$W =$ wage
$C = W(1 - L) + A$

Set $\varphi = \alpha$,
Set

$$h = \frac{\left(\frac{b}{W}\right)^{\frac{1}{\alpha-1}} - A}{\left(\frac{b}{W}\right)^{\frac{1}{\alpha-1}} + W}$$

estimating equation:

$$\ln\left(\frac{Wh + A}{1 - h}\right) = \frac{\ln W}{1 - \alpha} - \frac{X\beta}{1 - \alpha} - \frac{\varepsilon}{1 - \alpha}$$

Interior in Branch 1:

$$E\left[\ln\left(\frac{W_1 h + A}{1-h}\right) \middle| W_1, A\right] = \frac{\ln W_1}{1-\alpha} - \frac{X\beta}{1-\alpha}$$

$$-\frac{1}{1-\alpha} E\left(\varepsilon \,\middle|\, \begin{array}{c} \dfrac{\left(\begin{array}{l} X\beta + (\eta - 1)\ln(1-\bar{h}) \\ -(\alpha - 1)\ln(A + W_1\bar{h}) - \ln W_1 \end{array} \right)}{\sigma_\varepsilon} \\ \\ \geq \dfrac{-\varepsilon}{\sigma_\varepsilon} \geq \dfrac{\left(\begin{array}{l} X\beta + (\alpha - 1)\ln(1) \\ -(\alpha - 1)\ln A - \ln W_1 \end{array} \right)}{\sigma_\varepsilon} \end{array} \right)$$

Last term is:

$$\frac{-1}{1-\alpha}\left\{\frac{\frac{1}{\sqrt{2\pi}}\left(e^{-C_1^2/\sigma_\varepsilon^2}-e^{-C_2^2/\sigma_\varepsilon^2}\right)}{\Phi\left(\frac{C_1}{\sigma_\varepsilon}\right)-\Phi\left(\frac{C_2}{\sigma_\varepsilon}\right)}\right\}$$

$$C_1 = \frac{\left(\begin{array}{l} X\beta+(\alpha-1)\ln(1-\overline{h}) \\ -(\alpha-1)\ln(A+(W_1-W_2)\overline{h})-\ln W_1 \end{array}\right)}{\sigma_\varepsilon}$$

$$C_2 = \frac{X\beta+(\alpha-1)\ln(1)-(\alpha-1)\ln A-\ln W_1}{\sigma_\varepsilon}$$

At $h = \bar{h}$ (with probability $P = P_3$)Q

$$E(h|h = \bar{h}) = \bar{h}P_3$$

$$P_3 = \Pr\left(\begin{array}{c} \frac{1}{\sigma_\varepsilon}\left(\begin{array}{c} \ln W_2 - X\beta - (\eta - 1)\ln(1 - \bar{h}) \\ +(\alpha - 1)\ln(A + W_1\bar{h}) \end{array}\right) \leq \frac{\varepsilon}{\sigma_\varepsilon} \\ \\ \leq \frac{1}{\sigma_\varepsilon}\left(\begin{array}{c} \ln W_1 - X\beta - (\eta - 1)\ln(1 - \bar{h}) \\ +(\alpha - 1)\ln(A + W_1\bar{h}) \end{array}\right) \end{array}\right)$$

etc.

Branch 2 labor supply

$$\ln\left(\frac{W_2 h + (W_1 - W_2)h + A}{1-h}\right) = \frac{\ln W_2}{1-\alpha} - \frac{X\beta}{1-\alpha} - \frac{\varepsilon}{1-\alpha}$$

$$E\left(\left.\frac{(W_1 - W_2)h + A}{1-h}\right| h > \bar{h}\right) = \frac{\ln W_2}{1-\alpha} - \frac{X\beta}{1-\alpha}$$

$$-\frac{1}{1-\alpha} E\left(\varepsilon \left| \frac{\begin{pmatrix} \ln W_2 + (\alpha-1)\ln(A + W_1\bar{h}) \\ -X\beta - (\alpha-1)\ln(1-\bar{h}) \end{pmatrix}}{\sigma_\varepsilon} \geq \frac{\varepsilon}{\sigma_\varepsilon} \right.\right)$$

Let

$$Z^{(1)} = \ln\left(\frac{W_1 h + A}{1-h}\right)$$

$$Z^{(2)} = \ln\left(\frac{W_2 h + (W_1 - W_2)h + A}{1-h}\right)$$

$$E(Z^{(1)}|W_1, A, 0 < h < \overline{h}) = \frac{\ln W_1}{1-\alpha} - \frac{X\beta}{1-\alpha} - \frac{1}{1-\alpha}E(\varepsilon|0 < h < \overline{h})$$

$$E(Z^{(2)}|W_2, A, h > \overline{h}) = \frac{\ln W_2}{1-\alpha} - \frac{X\beta}{1-\alpha} - \frac{1}{1-\alpha}E(\varepsilon|h > \overline{h})$$

Can estimate by 2 stage methods.