

Economics 350:

Two Interpretations of the Mincer Equation

Learning-by-doing vs. On-the-job Training

Based in part on James Heckman, Lance Lochner, and Ricardo Cossa's

“Learning-by-doing vs. on-the-job training: Using variation induced by the EITC to distinguish between models of skill formation,” in Phelps, Edmund S. *Designing inclusion: tools to raise low-end pay and employment in private enterprise*. Cambridge Univ Press, 2003, pp. 74–130.

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- 1 Is learning rivalrous with or complementary with working?
Rivalrous with or complementary with earning?
- 2 Do people pay for their learning? What is the form of the payment? Foregone earnings? Foregone leisure? Both?
- 3 What is the correct price of time to include in a labor supply equation? Is the measured average wage the correct price of time?
- 4 What is the correct interpretation of empirical Mincer earnings equations? What do we learn from cross-section estimates?
- 5 Should we correct our estimates of inequality in wage income for consequences of human capital investments?

Point of Departure:

- Two observationally equivalent interpretations of

$$\ln W = \alpha_0 + \alpha_1 S + \alpha_2 x + \alpha_3 x^2$$

- S = schooling
- x = work experience
- α_1 = “average rate of return” to schooling
- α_2, α_3 = “returns to experience”

Mincer's Justification

- OJT model: Becker-Ben Porath
- Learning comes at the expense of earning.
- $k(x)$: earnings forgone as % of potential earnings.
- Mincer assumes:
 - ① Constant rates of return to post school investment r_p (If heterogeneous assumed to be independent of $k(x)$).
 - ② $k(x) = 1 - \frac{x}{T}$
 - ③ T : maximum possible amount of experience.
 - ④ Effect of OJT on log earnings, additively separable from schooling.
 - ⑤ T functionally independent of S . (Each year of schooling adds one year to effective working life.)
 - ⑥ $r(x)$ same for all x .
- Then (1), (2), (3), (4) and (5) \Rightarrow Mincer model. (See Mincer handout.)

- $\alpha_1 = r_s$; average “rate of return to schooling.”
- $\alpha_2, \alpha_3 \Rightarrow r_p$; average rate of return to post school investment.
- Can show:

$$\left(\alpha_2 = \left(r_p + \frac{r_p}{2T} \right); \alpha_3 = -\frac{r_p}{2T} \right)$$

(see “Mincer” notes).

Second Model

- Empirically indistinguishable from first model.
- Work produces current wages and future wage growth.
- x = cumulated work experience.
- The only cost of x is forgone leisure.
- $\ln W = \alpha_1 + \alpha_2 S + \alpha_3 x + \alpha_4 x^2$.
- Keane and Wolpin (1997, 2001) and many successor models.
- Keane, 2016, *EJ*, on reading list.

Question: can we distinguish the two models?

General model and special cases: 2 period analysis: Worker Problem

- (C_0, L_0) : Consumption and leisure in “0”
- (C_1, L_1) : Consumption and leisure in “1”

$$\text{Preferences: } U(C_0, L_0) + \frac{1}{1 + \rho} U(C_1, L_1) \quad (1)$$

- r is the borrowing rate; perfect certainty.

- H_0 = initial human capital; H_1 = final human capital
- Production function of human capital:
$$H_1 = H_0 + F(\theta_0, H_0, 1 - L_0)$$
$$F_{\theta_0} \geq 0, F_{H_0} \geq 0, F_{1-L_0} \geq 0.$$
- Depreciation implicit.
- θ_0 = learning “quality” of a job in period 0.
- As $\theta_0 \uparrow$ $H_1 \uparrow$ ($F_{\theta_0} > 0$)
- Learning quality irrelevant in period “1” because there is no period 2.
- Assume $\rho = r = 0$.
- θ_0 is valuable.
- It helps produce human capital.
- However, you have to be at a firm to realize its value.
- Does it have a price? Do people pay for learning opportunities?

- Assume all learning takes place at firms.
- Earnings in "0": $W(H_0, 1 - L_0, \theta_0)$
- Earnings in "1": $W(H_1, 1 - L_1)$
- Budget Constraint:

$$C_0 + C_1 = \underbrace{W(H_0, 1 - L_0, \theta_0)}_{\text{Earnings in period 0}} + \underbrace{W(H_1, 1 - L_1)}_{\text{Earnings in period 1}} \quad (2)$$

Pricing of human capital services in final output:

- R : rental rate on a unit of human capital: (efficiency units model).
- $W(H_0, 1 - L_0, \theta_0) = \underbrace{RH_0(1 - L_0)}_{\text{potential earnings}} - \underbrace{P(\theta_0, 1 - L_0, H_0)}_{\text{amount "paid" to the firm by agent to access } \theta_0}$
- $W(H_1, 1 - L_1) = RH_1(1 - L_1)$
- $P(\theta_0, 1 - L_0, H_0)$ is the cost of learning quality θ_0 with $1 - L_0$ hours of work and with the agent having H_0 amount of human capital.

Consider Becker-Ben Porath Model

- Leisure fixed: $L_0 = L_1 = \bar{L}$
- Jobs priced out in a special way
- Price of learning content θ_0 in a job: $P(\theta_0, 1 - \bar{L}, H_0) = P(\theta_0)$
- Production function: $H_1 = F(\theta_0, H_0) + H_0$
- $\theta_0 = I$ (time spent investing): In this model, “learning” is through investment time I_0 spent at work.
- $P(\theta_0) = RH_0I$ (cost of investment)
- $W(H_0, 1 - L_0, \theta_0) = RH_0(1 - \bar{L}) - RH_0I$
- Can add leisure (Blinder and Weiss, 1976; Heckman, 1976)

- The Ben-Porath (1967) model has a special functional form

$$\begin{aligned} H_1 &= G(H_0\theta_0) + H_0 \\ &= G(H_0I_0) + H_0 \end{aligned} \tag{3}$$

- **Question: What are the first order conditions for the model (1), (2), and (3) with leisure fixed $L_0 = L_1 = \bar{L}$?**
- **How does investment depend on H_0 and R ?**

Learning by Doing (LBD) Model in the Literature

- Cost of learning is foregone leisure.
- Ignored in Becker-Ben Porath models.
- Investment is a “free good.”

$$\frac{\partial P}{\partial \theta_0} = 0$$

(Imai and Keane, 2004; Keane, 2016)

- Implicitly: θ_0 is the same at all jobs.
- *Free lunch. (No direct cost of learning.)*
- The only cost of learning is *foregone leisure*.
- Other intermediate cases are possible.

Firm Side of the Problem: Firm “Sells” Investment Opportunities

- Firm has a valuable good: training possibilities.
- Firms may be heterogeneous in training opportunities (**but typically ignored**).
- Two sector model of the firm.
- Firms: produce skills in one sector and then use skills for producing final output.
- Profits for a one-worker firm offering opportunity θ_0 :

$$\underbrace{\Pi}_{\text{Profits}} = \underbrace{J((1 - L_0), H_0, \theta_0)}_{\text{Final Goods Output}} + \underbrace{P(\theta_0, (1 - L_0), H_0)}_{\substack{\text{Revenue from selling} \\ \text{training opportunities} \\ \text{to workers}}} - \underbrace{WRH_0(1 - L_0)}_{\text{Labor Costs}}$$

- $J_{\theta_0} \leq 0$ (costly for firm to provide learning opportunities).

$P(\theta_0, (1 - L_0), H_0, R)$ is market clearing pricing function.

- Equates demand and supply across jobs, indexed by θ, L_0 .
- **Question: What is the life cycle mobility of workers across firms?**

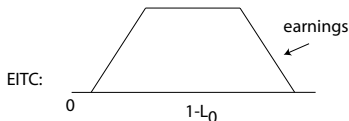
Can One Distinguish Between the Two Models?

- See Cossa, Heckman et al. (2003).

Consider taxes and subsidies in periods “0” and “1” in Two Models

Model 1: OJT (Becker-Ben Porath with Leisure)

- Motivated by analysis of EITC (Earned Income Tax Credit) program (Cossa et al., 2003).



- Assume learning takes place on the job.
- τ_0, τ_1 are proportional taxes or subsidies.
- $R = 1$
- Individuals maximize (1): $U(C_0, L_0) + U(C_1, L_1)$ subject to

$$C_0 + C_1 = \underbrace{(1 + \tau_0)H_0 \overbrace{(1 - l_0 - L_0)}^{\text{work time}}}_{\text{Measured after tax/subsidy earnings in period 0}} + \underbrace{(1 + \tau_1)H_1(1 - L_1)}_{\text{Measured after tax/subsidy earnings in period 1}}$$

- This formulation abstracts from the basic problem of where on the budget set should agents locate.
- $V(L_0, A_0)$ is period zero value fn. $L_0 = \operatorname{argmax} V(L_0, A_0)$.
- **Question: What is the FOC for the Ben Porath version of the model with labor supply?**
- $H_1 = F(l_0 H_0) + H_0$
- $(1 + \tau_0)H_0 \leq (1 + \tau_1)(1 - L_1)G'(l_0 H_0)H_0$
- Neutrality: H_0 raises productivity proportional to opportunity cost.
- If instead, $H_1 = F(l_0) + H_0$ (abstract from self productivity of H_0).
- FOC for l_0 : $(1 + \tau_0)H_0 \leq (1 + \tau_1)(1 - L_1)F'(l_0)$
- Higher H_0 raises the opportunity cost of investment.
- Feature missing in Becker-Ben Porath model with neutrality.

Consider the Impact of Taxes and Subsidies

Compensate for income effects (λ constant or Frisch demands, not Hicks-Slutsky) (see Frisch Demands handout for background),

- $\tau_0 > \tau_1 = 0$: Period 0 subsidy raises MC of l_0 : $l_0 \downarrow \therefore H_1 \downarrow$
- $\tau_1 > \tau_0 = 0$: Period 1 subsidy raises MR of l_0 : $l_0 \uparrow H_1 \uparrow$
- $\tau_0 = \tau_1 > 0$: FOC unchanged.

Digression for Non-Ben Porath Case

Consider an interior solution (local)

- Consider the following Lagrangian:

$$\mathcal{L} = U(C_0, L_0) + U(C_1, L_1) \\ - \lambda [C_0 + C_1 - (1 + \tau_0)H_0(1 - I_0 - L_0) - (1 + \tau_1)H_1(1 - L_1)]$$

- FOC: C_0, C_1

$$U_1(C_0, L_0) = \lambda$$

$$U_1(C_1, L_1) = \lambda$$

- FOC: L_0, L_1

$$U_2(C_0, L_0) = \lambda(1 + \tau_0)H_0$$

$$U_2(C_1, L_1) = \lambda(1 + \tau_1)H_1$$

- FOC: l_0 for the case where
- For $H_1 = F(l_0) + H_0$

$$(1 + \tau_0)H_0 = (1 + \tau_1)F'(l_0)(1 - L_1)$$

λ is Held Constant: Suppose We Relax this Condition?

- Assume (C_0, C_1) and (L_0, L_1) are normal goods.
- If $\tau_0 = \tau_1 \uparrow$, so agent gets a subsidy (or pays less tax) per period $L_0, L_1 \uparrow \therefore I_0 \downarrow, H_1 \downarrow$
- In the general case where $\tau_0 = \tau_1 = \tau$, as $\tau \uparrow$, value of time (price of leisure) increases, agents substitute toward consumption effects reinforced by income effects.
- $(1 - L_1) \downarrow \Rightarrow I_0 \downarrow \Rightarrow H_1 \downarrow$. (Pure wealth effects, more consumption, more leisure, and less work and investment.)
- **Problem:** Verify these claims.

- **Question: For a Ben Porath Technology with labor supply, what is the answer to these questions for these subsidy changes?**

Model 2: Learning By Doing (LBD): Cost of Learning is Same as Cost of Work–Foregone Leisure

- $R = 1$
- Individuals maximize $U(C_0, L_0) + U(C_1, L_1)$ subject to

$$C_0 + C_1 = (1 + \tau_0)H_0(1 - L_0) + (1 + \tau_1)H_1(1 - L_1).$$

and

$$H_1 = H_0 + \phi(1 - L_0) \quad (\text{Period "1" earnings})$$

FOC:

$$\begin{aligned}
 U_2(C_0, L_0) = & \lambda \left[\underbrace{H_0(1 + \tau_0)}_{\text{Marginal effect of a unit of work on current earnings}} + \underbrace{\phi'(1 - L_0)(1 - L_1)(1 + \tau_1)}_{\text{Effect of current hour of work on future earnings}} \right] \\
 U_2(C_1, L_1) = & \lambda \left[\underbrace{H_0 + \phi(1 - L_0)}_{\text{Measured effect of an extra hour of work on after subsidy on earnings}} \right] (1 + \tau_1)
 \end{aligned}$$

Compensate for income effects (λ constant)

- Start from $\tau_0 = 0, \tau_1 = 0$.
- $\tau_0 = \tau_1$: Flat subsidy increases the current and future return to work $h_0 = 1 - L_0$ and $h_1 = 1 - L_1$.
- $\therefore H_1 \uparrow$.
- $\tau_0 > \tau_1$: Period 0 subsidy raises current return to $h_0, (H_1) \uparrow$
- $\tau_1 > \tau_0$: Period 1 subsidy raises future return to $h_1, (H_1) \uparrow$
- Positive wealth and income effects discourage work and reduce learning and investment in all cases.
- $\therefore \lambda$ vary, impacts ambiguous on it.

Model 2': LBD with a Market for Learning Opportunities (No Free Lunch and Heterogeneous Firms)

- Suppose firms offer different learning opportunities indexed by $\theta \in (\underline{\theta}_0, \bar{\theta}_0)$.
- So $H_1 = H_0 + \phi(1 - L_0, \theta_0)$ where $\frac{\partial^2 \phi}{\partial(1-L_0)\partial\theta_0} > 0$.
- With a distribution of firm types, a market for learning will emerge.
- All old workers and young workers who expect high L_1 (low h_1) place little value on learning, θ_0 .
- Pricing function $P(\theta_0)$ may arise with $P'(\theta_0) > 0$. (Worker pays for learning opportunities)
- This adds a new wrinkle to the LBD model.
- Wage earnings:
 - In the first period: $W(H_0, \theta_0) = H_0(1 - L_0) - P(\theta_0)$.
 - In the second period, it is $H_1(1 - L_1)$



- We acquire a new first order condition in the LBD model.
- Individuals choose firm type or learning opportunity (θ) according to:

$$(1 + \tau_0)P'(\theta_0) = (1 + \tau_1)(1 - L_1) \frac{\partial \phi(1 - L_0, \theta_0)}{\partial \theta_0} \quad (*)$$

- **Problem:** Verify this condition.

- Consider an income-compensated change from an initial position: $\tau_0 = \tau_1 = 0$.
- $\uparrow \tau_0 = \tau_1 > 0$: Flat subsidy increases current and future return to h_0 (= period zero hours of work) and raises return to θ_0 by increasing h_0 and h_1 (period 1 hours of work).
- \therefore This is a force for $H_1 \uparrow$.
- But it raises the cost of buying θ_0 , a force for $H_1 \downarrow$ (see *).
- **Problem:** Verify.

- $\tau_0 > \tau_1$: Period 0 subsidy raises current return to h_0 and the MC of θ_0 .
- **Ambiguous on H_1** (everything else constant).
- $\tau_1 > \tau_0$: Period 1 subsidy raises future return to h_0 and return to θ_0 .
- $\therefore H_1 \uparrow$.
- Test of model not clear anymore.
- **Note:** Can equate this model with OJT model if θ_0 equated to l_0 in Ben Porath. Then the two models are indistinguishable.

- Implicit is a theory of life cycle mobility (stepping stone mobility).

Implications for Measured Wages

OJT:

- a First period earnings $<$ potential earnings if investment is paid by foregone earnings (wage rates understated).
- b First period earnings = potential earnings if investment occurs off the job or not paid via earnings.

LBD (free lunch):

- First period earnings $<$ potential earnings. Wage rates understated (true price of time is greater than measured wage).