Integrating Financial Markets with Human Capital Markets

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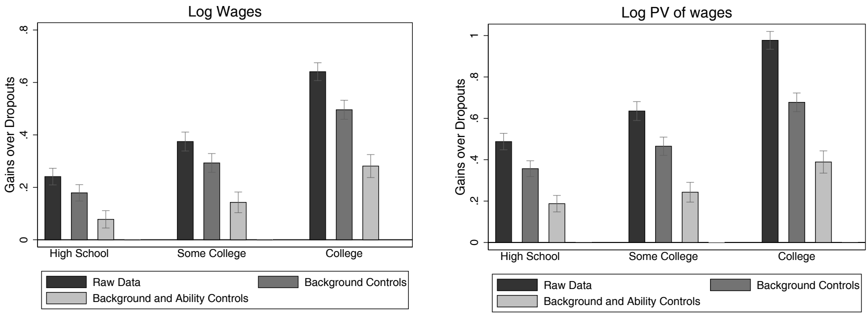
Major Research Challenge: Integrating Study of Human Capital and Financial Capital Markets

Why Relevant?

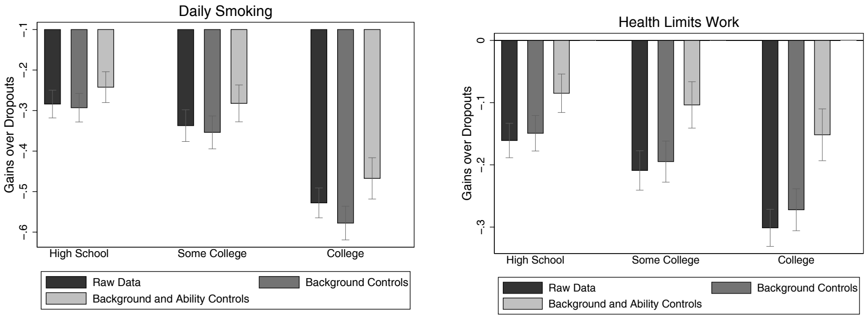
Returns to Education and Trends

Figure 1: Observed and Adjusted Benefits from Education



Source: Heckman, Humphries and Veramendi (2016)

Figure 1: Observed and Adjusted Benefits from Education



Source: Heckman, Humphries and Veramendi (2016)

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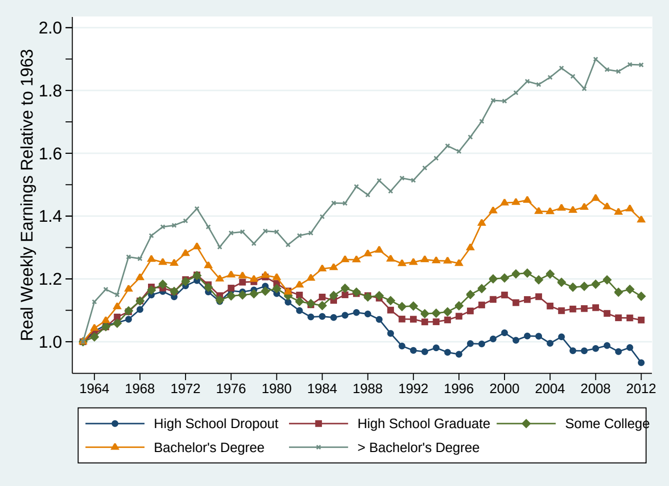
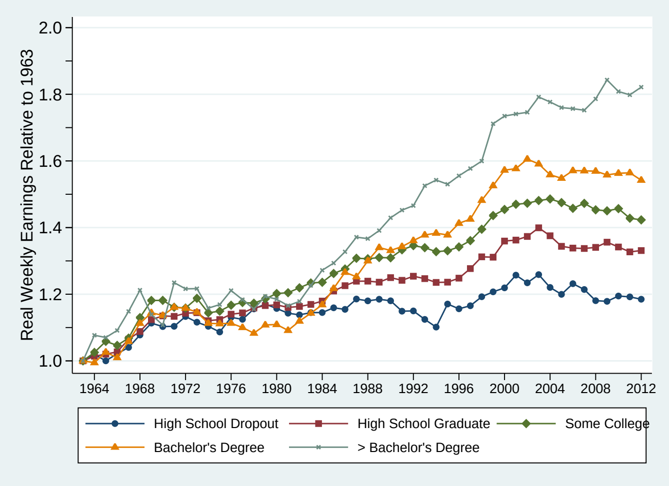
Figure 2: Changes in Real Wage Levels of Full-time U.S. Workers by Sex and Education, 1963–2012, Males



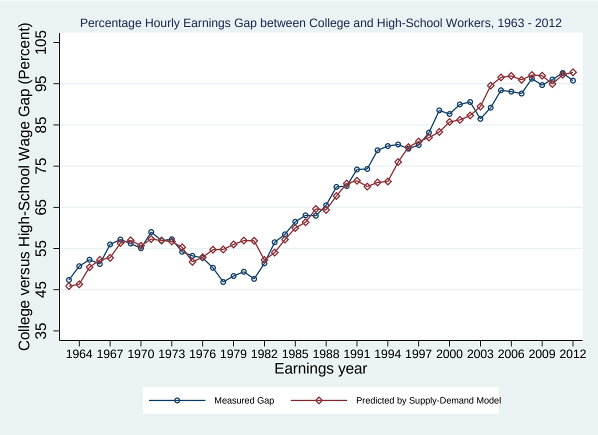
Source: Autor 2014, Skills, Education, and the Rise of Earnings Inequality Among the "Other 99 Percent"

Figure 3: Changes in Real Wage Levels of Full-time U.S. Workers by Sex and Education, 1963–2012, Females



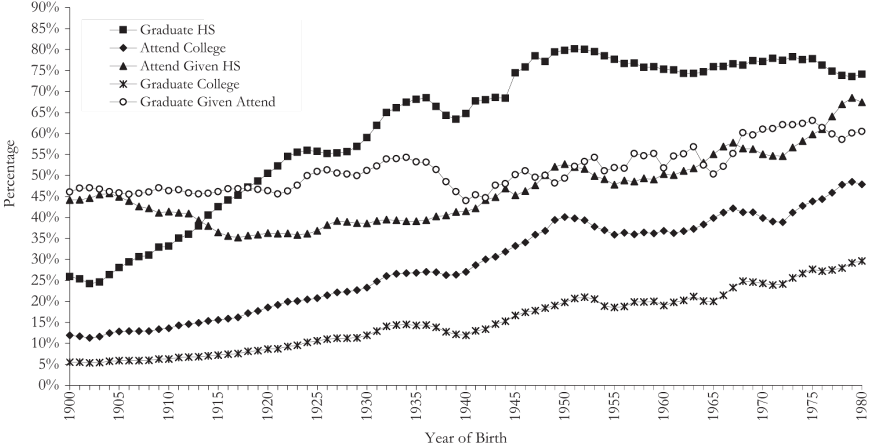
Source: Autor 2014, Skills, Education, and the Rise of Earnings Inequality Among the "Other 99 Percent"

Figure 4: The U.S. College/High School Premium, 1963–2012



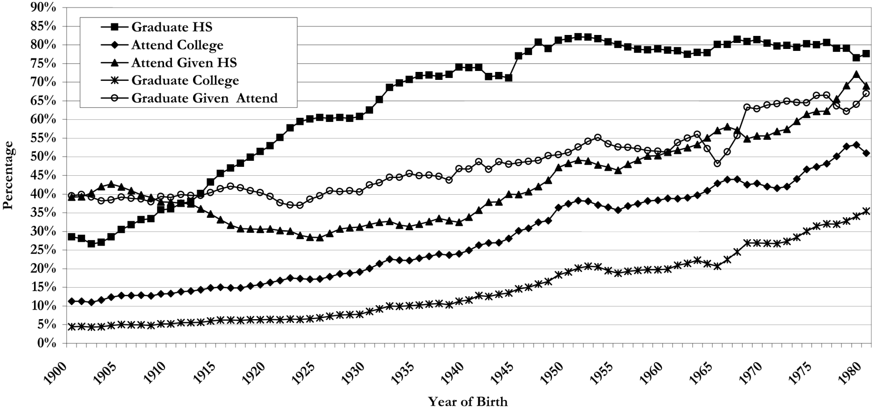
Source: Autor 2014, Skills, Education, and the Rise of Earnings Inequality Among the "Other 99 Percent"
Note: College versus high school wage gap. Figure uses March CPS data for earnings years 1963 to 2012.The series labeled "Measured Gap" is constructed by calculating the mean of the natural logarithm of weekly wages for college graduates and non–college graduates, and plotting the (exponentiated) ratio of these means for each year. This calculation holds constant the labor market experience and gender composition within each education group. The series labeled "Predicted by Supply-Demand Model" plots the (exponentiated) predicted values from a regression of the log college/noncollege wage gap on a quadratic polynomial in calendar years and the natural log of college/noncollege relative supply.

Figure 5: Educational Attainment Decompositions, Males and Females, 1900-1980 Birth Cohorts



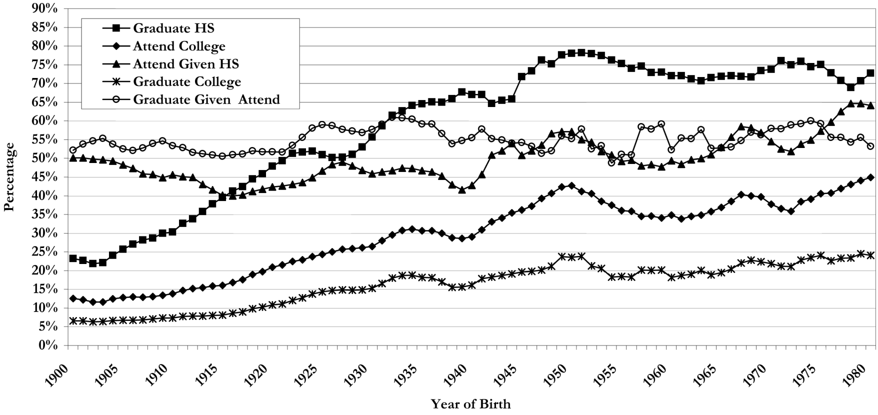
Source: Heckman and LaFontaine (2010).

Figure 6: Educational Attainment Decompositions, Females 1900-1980 Birth Cohorts



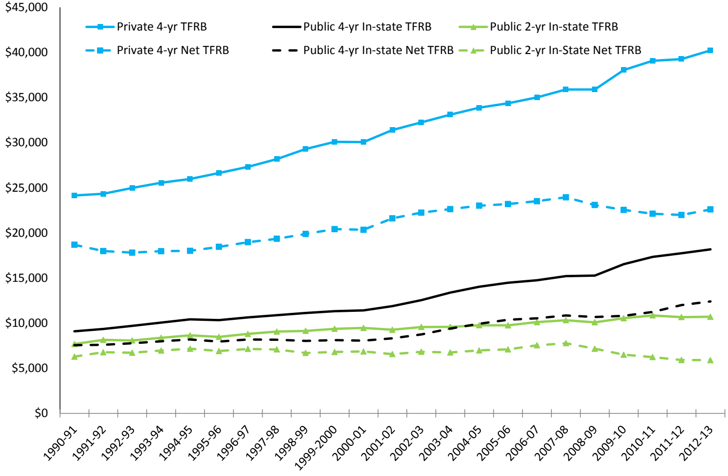
Source: Heckman and LaFontaine (2007).

Figure 7: Educational Attainment Decompositions, Males 1900-1980 Birth Cohorts



Source: Heckman and LaFontaine (2007).

Figure 8: Evolution of Average Tuition, Fees, Room & Board in the U.S. (2013 $)



Source: College Board (Online Tables 7 and 8), *Trends in College Pricing*, 2013.

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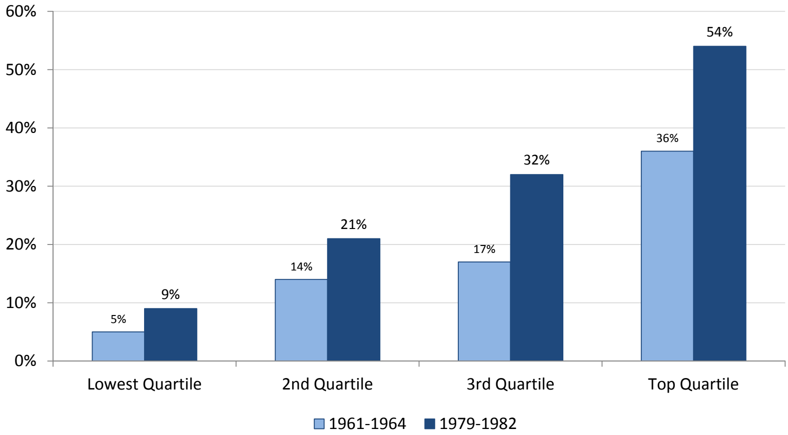
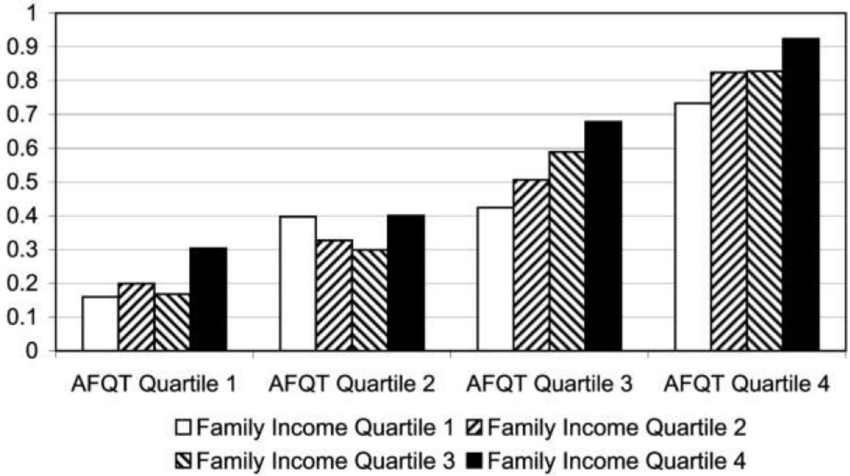
Figure 9: Fraction of Students Completing BA Degree by Age 25, by Income Quartile and Year of Birth



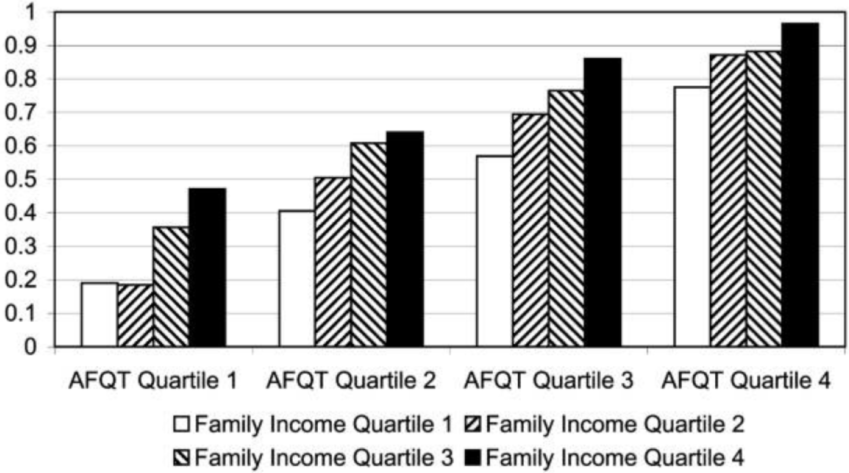
Source: Recreated from Bailey and Dynarski (2011).

Figure 10: College attendance by AFQT and Family Income Quartiles (1979)



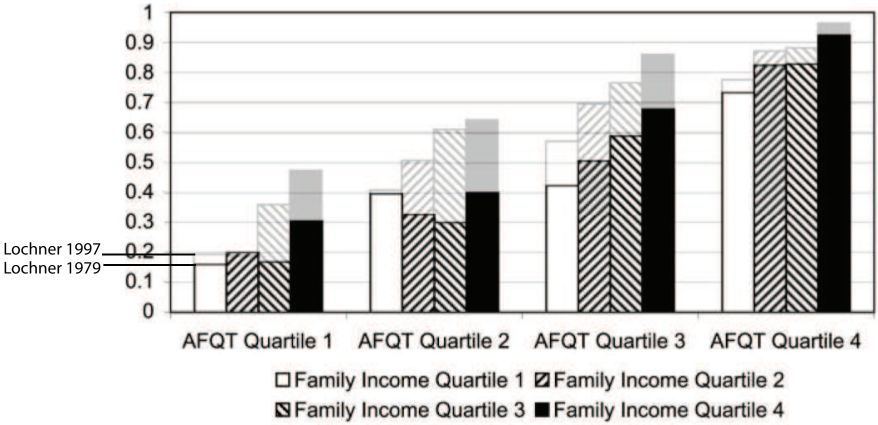
Source: Belley and Lochner (2007).

Figure 11: College attendance by AFQT and Family Income Quartiles (1997)



Source: Belley and Lochner (2007).

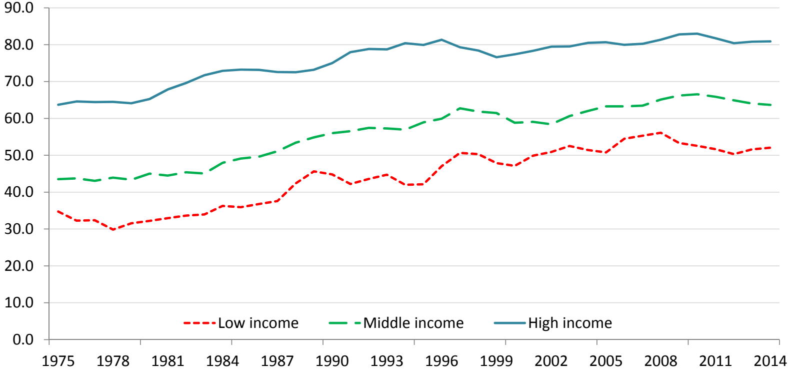
Figure 12: College attendance by AFQT and Family Income Quartiles (1979 and 1997 on one graph)



Source: Belley and Lochner (2007).

Income Gradients in Trends for College Attendance

Figure 13: Percentage of High School Completers who were Enrolled in 2- or 4-year Colleges by the October Immediately Following High School Completion, by Family Income



Source: Digest of Education Statistics 2015, Table 302.30
Note: A 3-year moving average is a weighted average of the year indicated, the year immediately preceding, and the year immediately following. For 1975 and 2014, a 2-year moving average is used: The moving average for income groups in 1975 reflects an average of 1975 and 1976, and the moving average for 2014 reflects an average of 2013 and 2014. Moving averages are used to produce more stable estimates.

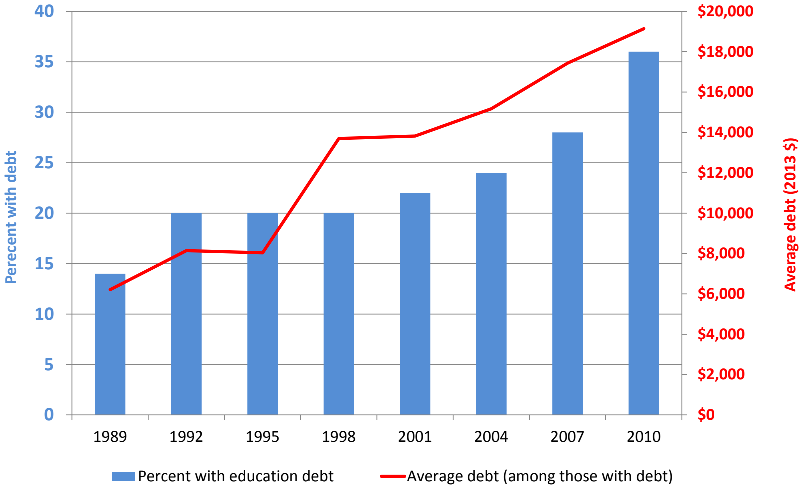
Credit Constraints?

- Little evidence that a lack of borrowing opportunities discouraged schooling 30 years ago
  - Carneiro & Heckman (2002), Cameron & Taber (2004), Keane & Wolpin (2001), Belley & Lochner (2007)
- New evidence suggests that more recent cohorts are unable to borrow as much as they would like:
  - Substantial increase in demand for student loans & rising student debt levels
  - Growth in student debt (switch from grants to loans)
  - Borrowing increased at intensive and extensive margins (Akers & Chingos 2014, Bleemer et al. 2014, Hershbein & Hollenbeck 2014)
  - Government student loan limits declined 50% in value from '93 to '08
  - Sharp increase in fraction of students 'maxing out' their federal student loans (Berkner 2000, Berkner & Wei 2008)
  - Differences in college attendance between youth from high- vs. low-income families have doubled since early 1980s (Belley & Lochner 2007)
  - Most-able low-income students now work much more during school than their high-income counterparts (Belley & Lochner 2007)

Private Sector Has Jumped In

- Share of undergraduate debt from private lenders rose from virtually zero in early 1990s to 25% in 2007-08
- Private lenders scaled back considerably in 2008-09 – credit crisis
- Partially offsetting was an increase in Stafford Loan limits
- Private lenders face different incentives and offer different loan contracts
  - maximizing profits
  - target ‘good investments’
  - generally charge higher interest rates, less payment flexibility
  - often require co-signor
- Important to consider response of private lenders to government policies

Figure 14: Incidence and Amount (in 2013 $) of Household Education Debt for 20-40 Year-Olds in the U.S.



Source: Table 1, Akers and Chingos (2014).

Goal of the Analysis:

Developing and Estimating A Precise Economic and Econometric Framework for Discussing These Issues

- A dynamic model of schooling, post-school human capital investment, borrowing and working
- Agents subject to uninsured human capital risks, and face restrictions on their borrowing possibilities

Main Contributions

- First analysis that extends the natural borrowing limit of Aiyagari to simultaneously encompass endogenous labor supply, consumption, human capital accumulation, and savings in physical capital
  - (i) Model predicts that borrowing limits are lower for individuals who have lower levels of human capital and higher psychic costs of working.
  - (ii) The predicted credit limits vary with age, first increasing, and then decreasing.

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Additional Contributions

- Extend the existing literature by analyzing how cognitive and noncognitive ability affect choices through:
  1. Psychic costs of working and schooling
  2. The technology of human capital production
  3. The discount factor
- Allow our measures of abilities to be fallible.
- Introduce heterogeneity in parental transfers which is not investigated in the existing literature.

Main Findings

- Strong effects of adolescent endowments of cognitive and noncognitive ability on human capital development.
- Tuition costs and family transfers to children play important roles in explaining differences in life outcomes due to human capital investments.

Who Are the Credit Constrained?

- Credit constrained agents fall into two groups:
  a. Those with poor initial endowments and family background who acquire little human capital and have low wage levels and low life cycle wage growth
  b. The very able and those from good family backgrounds who have high levels of human capital, high wage levels, and high life cycle wage growth. They are only constrained early in their life cycles.

- Equalizing cognitive and noncognitive ability (separately and together) has dramatic effects on reducing inequality in education.
- Equalizing parental backgrounds has a much weaker effect on inequality in earnings and consumption.
- Reducing tuition has substantial impacts on schooling, but has only minor effects on measures of inequality (even lifetime utility).
- Enhancing student loan limits has minor effects on all outcomes studied.

2. A Brief Review of the Literature on the Specification of Credit Constraints

Table 1: Leading Structural Models of Educational Choice and Credit Constraints in the Literature

	Human Capital Investment	Labor Supply	Government Student Loans (GSL)	Private Loan Limit
Keane and Wolpin (2001)	Education and work experience	Yes	None	Borrowing limits not observed. They are proxied by a function of age and human capital; the parameters of the borrowing limit are estimated.
Lochner and Monge-Naranjo (2011)	Education and work experience	None	Yes	Endogenous credit limit based on borrowers' cost of default (including temporary exclusion from credit market and wage garnishments), due to private lenders' limited ability to punish default; parameters on the cost of default are calibrated outside the model
Johnson (2013)	Education and work experience	Yes	Yes	Borrowing limits not observed. They are proxied by a function of age and human capital; the parameters of the borrowing limit proxy equation are estimated.

Source: NLSY: National Longitudinal Survey of Youth. BHPS: British Household Panel Survey.

Table 1: Leading Structural Models of Educational Choice and Credit Constraints in the Literature, Cont'd

	CRRA Risk Aversion	Parental Influence	Data
Keane and Wolpin (2001)	Estimate $\gamma = 0.4826$	Parental transfer is a function of parental education and individuals choices	NLSY79 (1979-1992).
Lochner and Monge-Naranjo (2011)	Set $\gamma = 2$	None	NLSY79 (1979-2006)
Johnson (2013)	Set $\gamma = 2$	Parental transfer is a function of parental income and child choices	NLSY97 (1997 to 2007)

Source: NLSY: National Longitudinal Survey of Youth. BHPS: British Household Panel Survey.

Table 1: Leading Structural Models of Educational Choice and Credit Constraints in the Literature, Cont'd

	Human Capital Investment	Labor Supply	Government Student Loans (GSL)	Private Loan Limit
Abbott et al. (2016)	Education	Yes	Yes	Among working-age married households, borrowing limit equals to $75,000 if the most educated spouse is a college graduate, $25,000 if the most educated spouse is a high school graduate, and $15,000 if both spouses are high school dropouts; No borrowing before age 21. Borrowing limits based on self-reported limits on unsecured credit by family type from SCF.
Blundell et al. (2016)	Education and work experience	Yes	Yes	No borrowing permitted except for student loans
This paper	Education and work experience	Yes	Yes	Model-determined natural borrowing limit based on education and labor supply decisions, due to borrowers' limited repayment ability in the presence of uninsurable wage risk; no new auxiliary parameters for borrowing limit is added in estimation, unlike many previous papers.

Source: NLSY: National Longitudinal Survey of Youth. BHPS: British Household Panel Survey.

Table 1: Leading Structural Models of Educational Choice and Credit Constraints in the Literature, Cont'd

	CRRA Risk Aversion	Parental Influence	Data
Abbott et al. (2016)	Set $\gamma = 2$	Parental transfers explicitly modeled	Multiple data, including NLSY79, NLSY97
Blundell et al. (2016)	Set $\gamma = 1.56$	Parental income and background factors affect youth's psychic cost of schooling	BHPS (1991 to 2008)
This paper	Set $\gamma = 2$	Parental transfer is a function of parental education and net worth, and individuals choices; parental education affects youth's psychic cost of schooling	NLSY97 (1997-2013)

Source: NLSY: National Longitudinal Survey of Youth. BHPS: British Household Panel Survey.

3. Model: Specification, Solution Concepts, Initial Conditions, and Measurement System

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Choice Set

- At each age $t \in \{t_0, \dots T\}$ an individual makes decisions on:
  1. Consumption c_t and savings s_{t+1}
  2. Whether to go to school $d_{e,t} \in \{0, 1\}$
  3. Employment $d_{k,t} \in \{0, 0.5, 1\}$, where $d_{k,t} = 0$, $d_{k,t} = 0.5$ and $d_{k,t} = 1$ indicate not working, part-time working, and full-time working, respectively
- An individual cannot go to school and work full-time at the same time, *i.e.* $d_{e,t} + d_{k,t} < 2$
- Individuals can work part-time while in school

3.2 State Variables and Information Set

- At each age t, an individual is characterized by a vector of predetermined state variables that shape preferences, production technology, and outcomes:

$$\overline{\Omega}_t := (t, \boldsymbol{\theta}, e_t, k_t, s_t, d_{e,t-1}, e_p, s_p) \tag{1}$$

- $\boldsymbol{\theta}$: vector that summarizes individual components of unobserved (by the economist) heterogeneity (unobserved cognitive ability and noncognitive ability),
- e_t: individual's years of schooling at t,
- k_t: accumulated years of work experience at t,
- s_t: net worth determined at the end of period $t-1$,
- $d_{e,t-1}$: schooling status in the previous period,
- e_p: parental educational level
- s_p: parental net worth

- **Three sources of uncertainty in the model**
  $\boldsymbol{\epsilon}_t := (\epsilon_{e,t}, \epsilon_{w,t}, \epsilon_{p,t})$
  - Productivity shocks to human capital $\epsilon_{w,t}$
  - Preference shocks to schooling $\epsilon_{e,t}$
  - Shocks to parental transfers $\epsilon_{p,t}$.
- The information set includes all of the predetermined state variables and realized idiosyncratic (serially uncorrelated) shocks at age t:

$$\boldsymbol{\Omega}_t := \{\overline{\boldsymbol{\Omega}}_t, \boldsymbol{\epsilon}_t\}.$$

3.3 Preferences

- An individual has well-defined preferences over consumption c_t and choices on schooling and working $(d_{e,t}, d_{k,t})$:

$$U(c_t, d_{e,t}, d_{k,t}; \Omega_t) = \underbrace{u_c(c_t; \Omega_t)}_{\substack{\text{utility over}\\ \text{consumption}}} + \underbrace{u_e(\Omega_t) \cdot d_{e,t}}_{\substack{\text{Psychic cost}\\ \text{of school}}} + \underbrace{u_k(d_{k,t}, d_{e,t}, \Omega_t)}_{\substack{\text{Working}\\ \text{disutility}}}. \tag{2}$$

- Subjective discount factor: $\exp(-\rho(\boldsymbol{\theta}))$

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3.4 Human Capital Production and Wage Equations

- Human capital $\psi_t \in \mathbb{R}^{++}$ (measured in labor efficiency units):

$$\psi_t = F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t}) \tag{3}$$

  - $\epsilon_{w,t} \geq \underline{\epsilon}_w$: idiosyncratic shock.
  - Two types of human capital: education and work experience
- An individual's hourly wage offer:

$$w_t = \psi_t \cdot \underbrace{F^w(d_{k,t}, d_{e,t})}_{\text{rental price}} \tag{4}$$

  - Rental price differs between a part-time job and a full-time job
  - Normalize the rental price for full-time job: $F^w(1,0) = 1$.

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- After leaving school, the accumulated years of work experience evolves via:

$$k_{t+1} = k_t + \mathbf{1}(d_{k,t} > 0) - \delta_k k_t \mathbf{1}(d_{k,t} = 0) := F^k(k_t, d_{k,t}) \quad (5)$$

- δ_k is the depreciation rate of work experience in each period when the individual does not work.
- Education level at $t+1$, measured by years of schooling, evolves according to the following relationship:

$$e_{t+1} = e_t + d_{e,t}. \quad (6)$$

3.5 Financial Market Frictions and Endogenous Credit Constraints

- The smallest amount of net worth s_{t+1} that an agent can hold at the end of period t is captured by a (potentially negative) lower bound $\underline{S}_{t+1} \in \mathbb{R}^-$, which is determined by both the private loan market borrowing limit and the maximum credit from the government student loan programs as follows:

$$s_{t+1} \geq \underline{S}_{t+1} := -\max\{d_{e,t} \cdot \overline{L}^g(e_t + d_{e,t}),\ \overline{L}^s_t(e_{t+1}, k_{t+1}, \boldsymbol{\theta})\} \quad (7)$$

- $\overline{L}^g(e_t + d_{e,t}) \in \mathbb{R}^+$ is the maximum government student loan credit for schooling level $(e_t + d_{e,t})$ if the individuals choose to enroll in school $(d_{e,t} = 1)$, and $\overline{L}^s_t(e_{t+1}, k_{t+1}, \boldsymbol{\theta}) \in \mathbb{R}^+$ is the natural borrowing limit of an individual in the private debt market.

3.6 Budget Constraint and Transfer Functions

- To finance a youth's college tuition and fees, parents may provide *financial* transfers $tr_{p,t} \geq 0$.
- Parental financial transfers are generated by a stochastic function that depends on
  1. parents' wealth terciles (s_p) and parents' education (e_p);
  2. decisions about schooling and employment $(d_{e,t}, d_{k,t})$;
  3. youth's cognitive ability and noncognitive ability $(\boldsymbol{\theta})$ and current education (e_t) and age t.
- Captured by:

$$tr_{p,t} = tr_p(e_p, s_p, d_{e,t}, d_{k,t}, \boldsymbol{\theta}, e_t, t, \epsilon_{p,t}) \quad (8)$$

- $\epsilon_{p,t}$ is an idiosyncratic shock to parental transfers.

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- **Allow for Imperfections in Lending Rates**:
  Define $r(s_t) := r_l \mathbf{1}(s_t > 0) + r_b \mathbf{1}(s_t < 0)$
- The budget constraint for an individual who chooses to attend college (i.e., $d_{e,t} \cdot \mathbf{1}(e_t + d_{e,t} \geq 13) = 1$) is:

$$
\begin{aligned}
& c_t + (tc(e_t + d_{e,t}) - gr(e_t + d_{e,t}, s_p)) + s_{t+1} && (9) \\
& \quad = (1 + r(s_t)) \cdot s_t + w_t \cdot h(d_{k,t}) + tr_{p,t} c_t - rc(e_t + d_{e,t}) && (10)
\end{aligned}
$$

- $tc(e_t + d_{e,t})$ is the amount of college tuition and fees
- $gr(e_t + d_{e,t}, s_p)$ is the amount of grants and scholarships which depend on schooling level and parental wealth
- $rc(e_t + d_{e,t})$ denotes the cost of college room and board.

- Budget constraint for an individual who is not currently enrolled in college (i.e., $d_{e,t} \cdot \mathbf{1}(e_t + d_{e,t} \geq 13) = 0$):

$$c_t + s_{t+1} = (1 + r(s_t)) \cdot s_t + w_t \cdot h(d_{k,t}) + tr_{p,t} + tr_{c,t} + tr_{g,t} \tag{11}$$

$$c_t \geq tr_{c,t} \tag{12}$$

$$c_t \geq c_{min} \tag{13}$$

- $tr_{c,t} \geq 0$ is the direct consumption subsidy from the parents to their dependent child in the forms of shared housing and meals.

3.7 Model Solution

- The value function $V_t(\cdot)$ for $t = t_0, \ldots, T$ is characterized by the Bellman equation:

$$V_t(\boldsymbol{\Omega}_t) = \max_{d_{e,t}, d_{k,t}, s_{t+1}} \left\{ U(c_t, d_{e,t}, d_{k,t}; \boldsymbol{\Omega}_t) + \exp(-\rho(\boldsymbol{\theta}))\mathbb{E}(V_{t+1}(\boldsymbol{\Omega}_{t+1})|\boldsymbol{\Omega}_t, e_{t+1}, s_{t+1}, k_{t+1}, d_{e,t}) \right\}$$

- Subject to restrictions imposed by wage functions and human capital accumulation functions (Equations (3)-(6)), borrowing constraints (Equation (7)), and state-contingent budget constraints (Equation (9)-(13)).

3.8 Natural Borrowing Limit and Its Extensions

Example: Inelastic Labor Supply

- To illustrate our approach, consider an extreme case where individuals supply their labor inelastically from period t onwards, i.e, $d_{k,\tau} = 1$ for all $\tau \geq t$.
- Case considered by Aiyagari.
- The natural borrowing limit in the private loan market in period $t-1$ in this extreme case is:

$$\overline{L}^s_{t-1}(e, k_t, \boldsymbol{\theta}) = \frac{\overline{L}^s_t(e, k_t + 1, \boldsymbol{\theta}) + \max\{0,\ F^{\psi}(e, k_t, \boldsymbol{\theta}, \underline{\epsilon}_w) \cdot h(1) - c_{min}\}}{1 + r_b}.$$

- $h(1)$: full time annual hours (assume agents can work full time or not at all, for simplicity)

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Elastic Labor Supply

- Credit limit at time t: $\overline{L}_t^s$ for an individual who does not work at t.
- Natural borrowing limit at period $t-1$ (suppressing arguments):

$$\overline{L}_{t-1}^s = \overline{L}_t^s/(1+r_b).$$

- Can be interpreted as saying that individuals borrow new loans at time t, $\overline{L}_t^s$, to pay back debt $(1+r_b)\overline{L}_{t-1}^s$.
- At age t the individual may carry debt $s_{t+1} = -\overline{L}_t^s \leq 0$ and consumes government transfers $c_t^u = tr_{g,t} \geq c_{min}$.

- Define C_t^{ev}: the compensation that makes an individual indifferent between working and not working.
- C_t^{ev} is the solution to the following indifference relationship:

$$\begin{aligned} & u_c(C_t^{ev};\boldsymbol{\Omega}_t) + u_k(d_{k,t}=1,\boldsymbol{\Omega}_t) \\ & + \exp(-\rho(\boldsymbol{\theta}))\mathbb{E}(V_{t+1}(\boldsymbol{\Omega}_{t+1})|\boldsymbol{\Omega}_t, e, s_{t+1} = -\overline{L}_t^s(e, F^k(k_t, d_{k,t}=1), \boldsymbol{\theta}), k_{t+1} = F^k(k_t, d_{k,t}=1)) \\ & = u_c(c_t^u;\boldsymbol{\Omega}_t) + u_k(d_{k,t}=0,\boldsymbol{\Omega}_t) \\ & + \exp(-\rho(\boldsymbol{\theta}))\mathbb{E}(V_{t+1}(\boldsymbol{\Omega}_{t+1})|\boldsymbol{\Omega}_t, e, s_{t+1} = -\overline{L}_t^s(e, F^k(k_t, d_{k,t}=0), \boldsymbol{\theta}), k_{t+1} = F^k(k_t, d_{k,t}=0)). \end{aligned} \tag{14}$$

- Require that the consumption compensation has to be at least equal to the subsistence level. Thus, if $C_t^{ev} < c_{min}$, we set $C_t^{ev} = c_{min}$.

- The minimum consumption compensation C_t^{ev} is higher if
  1. the individual's psychic cost of working is higher,
  2. the government welfare subsidy is higher.
- Equation (14): individual rationality constraint for working.
- The individual chooses to work only if his consumption level under working is at least C_t^{ev}.
- $F^{\psi}(e, k_t, \boldsymbol{\theta}, \underline{\epsilon}_w) \cdot h(1) - C_t^{ev}$: surplus of full-time employment in terms of consumption value under the most unfavorable productivity shock.
- Recall, for simplicity, we assume that labor supply is either full time or none at all.
- Relaxed below.

- In this notation, when an individual can choose between full-time working and not working, the individual's natural borrowing limit is:

$$\overline{L}^{s}_{t-1}(e, k_t, \boldsymbol{\theta}) = \frac{\overline{L}^{s}_{t}(e, k_{t+1}, \boldsymbol{\theta}) + \max\{0, F^{\psi}(e, k_t, \boldsymbol{\theta}, \underline{\epsilon}_w) \cdot h(1) - C^{ev}_t(e, k_t, \boldsymbol{\theta})\}}{1 + r_b} \tag{15}$$

$$k_{t+1} = F^{k}(k_t, d_{k,t}), \quad d_{k,t} = \mathbf{1}(F^{\psi}(e, k_t, \boldsymbol{\theta}, \underline{\epsilon}_w) \cdot h(1) - C^{ev}_t(e, k_t, \boldsymbol{\theta}) \geq 0). \tag{16}$$

- Straightforward to extend this derivation to take into account the part-time employment choices (or any discrete employment choices).

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- Specifically, define the employment specific consumption compensation $C_t^{ev}(d_{k,t}; e, k_t, \boldsymbol{\theta})$ associated with employment status $d_{k,t}$ as follows:

$$
\begin{aligned}
& u_c(C_t^{ev}(d_{k,t}; e, k_t, \boldsymbol{\theta}); \boldsymbol{\Omega}_t) + u_k(d_{k,t}, \boldsymbol{\Omega}_t) \\
& + \exp(-\rho(\boldsymbol{\theta}))\mathbb{E}(V_{t+1}(\boldsymbol{\Omega}_{t+1})|\boldsymbol{\Omega}_t, e, s_{t+1} = -\overline{L}_t^s(e, F^k(k_t, d_{k,t}), \boldsymbol{\theta}), k_{t+1} = F^k(k_t, d_{k,t})) \\
& = u_c(c_t^u; \boldsymbol{\Omega}_t) + u_k(0, \boldsymbol{\Omega}_t) \\
& + \exp(-\rho(\boldsymbol{\theta}))\mathbb{E}(V_{t+1}(\boldsymbol{\Omega}_{t+1})|\boldsymbol{\Omega}_t, e, s_{t+1} = -\overline{L}_t^s(e, F^k(k_t, 0), \boldsymbol{\theta}), k_{t+1} = F^k(k_t, 0)).
\end{aligned} \tag{17}
$$

- Note that when $d_{k,t} = 0$, $C_t^{ev}(d_{k,t} = 0; e, k_t, \boldsymbol{\theta}) = c_t^u > 0$ satisfying Equation (17) automatically.

- Thus, the endogenous borrowing limit is:

$$\overline{L}^{s}_{t-1}(e, k_t, \boldsymbol{\theta}) = \frac{\overline{L}^{s}_{t}(e, k_{t+1}, \boldsymbol{\theta}) + \max\{0, [F^{\psi}(e, k_t, \boldsymbol{\theta}, \underline{\epsilon}_w) F^{w}(\tilde{d}_{k,t}, 0) \cdot h(\tilde{d}_{k,t}) - C^{ev}_{t}(\tilde{d}_{k,t}; e, k_t, \boldsymbol{\theta})]\}}{1 + r_b} \tag{18}$$

$$\tilde{d}_{k,t} = \arg\max_{d_{k,t} \in \{0, 0.5, 1\}} \left\{ \mathbf{1}(d_{k,t} > 0) \left(F^{\psi}(e, k_t, \boldsymbol{\theta}, \underline{\epsilon}_w) F^{w}(d_{k,t}, 0) \cdot h(d_{k,t}) - C^{ev}_{t}(d_{k,t}; e, k_t, \boldsymbol{\theta}) \right) \right\} \tag{19}$$

$$k_{t+1} = F^{k}(k_t, \tilde{d}_{k,t}). \tag{20}$$

Interpreting Equations (18) and (20)

- If $\left(F^{\psi}(e, k_t, \boldsymbol{\theta}, \underline{\epsilon}_w)F^w(d_{k,t}, 0) \cdot h(d_{k,t}) - C_t^{ev}(d_{k,t}; e, k_t, \boldsymbol{\theta})\right) < 0$ for $d_{k,t} > 0$, then $\tilde{d}_{k,t} = 0$, $\overline{L}^s_{t-1}(e, k_t, \boldsymbol{\theta}) = \frac{\overline{L}^s_t(e, k_{t+1}, \boldsymbol{\theta})}{1+r_b}$, and $k_{t+1} = F^k(k_t, 0)$.

- At terminal age T, $\overline{L}_T(\cdot) = 0$, calculate $C_T^{ev}(\cdot)$ using Equation (17).
- Then calculate the natural borrowing limit $\overline{L}_{T-1}(\cdot)$ at $T-1$ based on Equations (18) to (20).
- Using Equations (17)-(20), calculate the natural borrowing limit recursively at any age.

- The natural borrowing limit derived in our model implies that at a given age, an individual's borrowing limit is lower if
  1. the individual has a low level of human capital,
  2. the individual's psychic cost of working is higher,
  3. the returns to work experience are lower, and
  4. the government welfare subsidy for not working is higher.
- An agent's human capital affects his borrowing limit by affecting his future earning capacity $\{F_t^{\psi}(\cdot)\}_t$.
- The individual's psychic cost of working, the future productivity gains of increased work experience, and government welfare policy affect borrowing limits by affecting the minimum consumption compensation level C_t^{ev}.
- This is all predicated on a specific information structure.

3.9 Discussion of the Natural Borrowing Limit

- Concept of the natural borrowing limit first proposed in Aiyagari (1994).
- He defines the natural borrowing limit as the maximum amount an individual can repay with certainty.
- The underlying regime that would generate this constraint is one in which lenders can fully enforce contracts and collect on all resources available to the individual.

- Our notion of the natural borrowing limit extends Aiyagari's borrowing limit by considering endogenous labor supply and human capital investment.
- Following Aiyagari (1994), we assume that lenders can fully enforce contracts and can collect from all resources available to the individual.
- Borrowers must repay as long as they have resources.
- However, different from Aiyagari, who assumes that earnings are exogenous, in our model labor supply is endogenous and lenders cannot force borrowers to work.

- Since borrowers always have the choice not to work and collect welfare covering their minimum consumption requirement, lenders can never drive borrower's utility below that value or they will collect nothing.
- Hence, the lending regime with endogenous labor supply is that lenders can enforce full repayment subject to the restriction that borrowers must be provided a minimum consumption level (C_t^{ev} defined by Equation (17)) that satisfies the borrower's individual rationality constraint of working.

- Our formulation of the borrowing limit is related to studies that assume imperfectly enforceable contracts (see Marcet and Marimon (1992), Kehoe and Levine (1993), Albuquerque and Hopenhayn (2004), and Cooley et al. (2004), Cagetti and De Nardi (2006)).
- Imperfect enforceability of contracts means that the creditors are not able to force the debtors to fully repay their debts as promised and that the debtors fully repay only if it is in their own interest to do so.
- Since both parties are aware of this feature and act rationally, the lender will lend to a given borrower only an amount (possibly zero) that will be in the debtor's interest to repay as promised.

Parental Transfers Do Not Determine the Natural Borrowing Limit

- Our natural borrowing limit does not depend on parental transfers.
- In our model, parental financial transfers $tr_{p,t}$ are governed by a stochastic transfer rule.
- The lowest possible value of parental financial transfers (regardless of the youth's choices) is zero, which consequently implies that the youths cannot credibly promise to pay back positive loans using (possibly zero) parental transfers with certainty.
- Parental consumption transfer $tr_{c,t}$ is in the form of shared housing and meals provided by their parents.
- Youth cannot "cash" such consumption subsidy to pay back their debt.

Government Transfers Cannot be Touch by Lenders

- Regarding government transfers, we assume that private lenders cannot touch government transfers including tuition subsidies and grants.
- Hence the formation of the natural borrowing limit does not take into account government transfers.
- However, both parental and governmental transfers affect labor supply decisions and accumulated work experience, and hence indirectly affect the natural borrowing limit.

No Asymmetric Information

- Implicitly assume that borrowers can costlessly signal their private information to lenders so that there is no asymmetric information in the lending market.
- Examples of such signals include past history of wage earnings, employment, criminal background, test scores on cognitive ability, and FICO scores, etc.
- Any restrictions that reduce an individual's ability to signal his own type will result in asymmetric information in the lending market and may reduce the amount of the borrowing limits for the most able individuals.

- Our extension of the Aiyagari credit constraint does not capture the full array of credit market possibilities facing agents.
- They may go bankrupt with varying penalties ranging from full market exclusion (Alvarez and Jermann, 2001) to a range of other possible penalties (Chatterjee et al., 2007).
- Lenders can monitor and adjust period-by-period loans based on employment and medical histories, and other events realized by agents (see Chatterjee et al., 2007 and Jermann and Quadrini, 2012).

- A variety of financial arrangements are available to lenders and borrowers (see Gertler and Kiyotaki, 2010).
- Lenders can monitor borrowers and demand collateral or some form of enforceable partial repayment conditions.
- The assumption that the only binding constraint facing agents is that they must repay debt in the terminal period (up to some limit) is surely an extreme simplification of a richer set of period-by-period market and default opportunities.
- Moreover, it assumes some implicit mechanism through which agents comply with a no-terminal-default rule.
- Our research is only an opening step.

No Free Parameters

- Our analysis differs from Keane and Wolpin (2001) and Johnson (2013) by not introducing additional free parameters from outside the model to proxy unmeasured credit constraints.
- Ours is a far more stringent approach to estimation.
- Unlike other approaches in the literature, we do not specify *ad hoc* fixed credit limits (see, for example, **?**) or calibrate the model to fit asset distributions.
- Table 1 summarizes how the literature models credit constraints and our distinct approach to modeling them.

3.10 Optimal Decisions

- Envelope condition:

$$\frac{\partial V_t}{\partial s_t} = \lambda_{b,t}(1 + r(s_t)), \quad \text{if } s_t \neq 0, \tag{21}$$

- $r(s_t) = r_l \mathbf{1}(s_t > 0) + r_b \mathbf{1}(s_t < 0)$ and $\lambda_{b,t}$ is the Lagrange multiplier associated with the budget constraint.
- First-order conditions with respect to $c_t > 0$ and $s_{t+1} \neq 0$, $t < T$:

$$\frac{\partial u_c(c_t; \boldsymbol{\Omega}_t)}{\partial c_t} = \lambda_{b,t} \tag{22}$$

$$\exp(-\rho(\boldsymbol{\theta})) \left(\frac{\partial \mathbb{E} V_{t+1}}{\partial s_{t+1}} \right) + \lambda_{s,t} = \lambda_{b,t} \tag{23}$$

- $\lambda_{s,t}$: Kuhn-Tucker multiplier of the borrowing constraint.
- $\lambda_{s,t} > 0 \Rightarrow$ restrictions on intertemporal lending and borrowing.

3.11 Initial Conditions and Our Measurement System

- Complete the specification of our model by defining initial conditions and a set of measurement equations that relate proxied cognitive and noncognitive endowments to a set of observed measures.
- Individuals start life as autonomous agents at age 17 ($t_0 = 17$).
- The age 17 information set, $\overline{\boldsymbol{\Omega}}_{17}$:

$$\overline{\boldsymbol{\Omega}}_{17} := (17, \theta_c, \theta_n, k_{17}, e_{17}, s_{17}, d_{e,16}, e_p, s_p).$$

- The initial condition at age 17 that can be determined from sample information are:

$$\overline{\boldsymbol{\Omega}}_{17}^{\text{observed}} := (17, k_{17}, e_{17}, s_{17}, d_{e,16}, e_p, s_p).$$

- Proxy $\boldsymbol{\theta}$ but do not directly observe it.

- Joint distribution of unobserved ability at initial age 17, conditional on parental background at 17 (X_{17}) is given by:

$$\begin{pmatrix}\theta_c \\ \theta_n\end{pmatrix}\Bigg| X_{17} \sim N\left(\begin{pmatrix}\mu_c(e_p, s_p) \\ \mu_n(e_p, s_p)\end{pmatrix}, \begin{pmatrix}\sigma_c^2 & \sigma_{c,n} \\ \sigma_{c,n} & \sigma_n^2\end{pmatrix}\right)$$

- $\mu_j(e_p, s_p) = \mu_j + \mu_{j,e,1}\mathbf{1}(e_p = 12) + \mu_{j,e,2}\mathbf{1}(e_p > 12 \;\&\; e_p < 16) + \mu_{j,e,3}\mathbf{1}(e_p \geq 16) + \mu_{j,s,1}\mathbf{1}(s_p = \text{2nd Tercile}) + \mu_{j,s,2}\mathbf{1}(s_p = \text{3rd Tercile})$, for $j = c, n$.

- Specifically, we assume that at age 17 there are two sets of dedicated measurement equations for (θ_c, θ_n) given by Equations (24) and (25), respectively

$$Z^*_{c,j} = \mu_{z,c,j} + \alpha_{z,c,j}\theta_c + \epsilon_{z,c,j}, \qquad j \in \{1, \dots, J_c\} \tag{24}$$

$$Z^*_{n,j} = \mu_{z,n,j} + \alpha_{z,n,j}\theta_n + \epsilon_{z,n,j}, \qquad j \in \{1, \dots, J_n\} \tag{25}$$

- Individual control variables, including parental education, parental wealth, and the individual's age in 1997 are omitted from the measurement equations.

- To incorporate both continuous and binary measurements, we assume that the following relationship holds for each measurement at every point of time:

$$Z_{i,j} = \begin{cases} Z^*_{i,j} & \text{if } Z_{i,j} \text{ is continuous} \\ \mathbf{1}(Z^*_{i,j} > 0) & \text{if } Z_{i,j} \text{ is binary.} \end{cases}, \quad i \in \{c, n\} \tag{26}$$

4. Data and Preliminary Regression Analysis

4.1 Variables and Summary Statistics

Table 2: Key Variables over Age

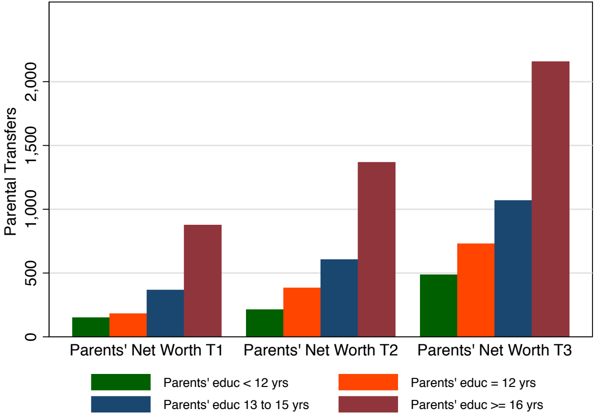
	Age 17	Age 20	Age 25	Age 30
In School	0.87	0.37	0.10	0.01
Full-Time Working	0.04	0.44	0.73	0.78
Part-Time Working	0.49	0.30	0.12	0.06
Part-Time Working While in School	0.46	0.24	0.07	0.03
Education	10.34	12.25	13.43	13.78
Years Worked	0.00	0.77	3.92	8.05
Net Worth	0.00	13467.81	20569.12	34826.70
Full-Time Hourly Wage	6.10	9.55	14.71	18.25
Part-Time Hourly Wage	6.16	8.46	15.28	15.77
Receive Parental Transfers	0.37	0.46	0.18	0.06
Total Parental Transfers	428.53	1766.64	315.89	83.51

Table 3: Measures of Cognitive and Noncognitive Ability (Year 1997)

	mean	sd	min	max	N
ASVAB: Arithmetic Reasoning (1997)	-0.08	0.95	-3.14	2.37	1,786
ASVAB: Mathematics Knowledge (1997)	0.06	0.98	-2.80	2.68	1,781
ASVAB: Paragraph Comprehension (1997)	-0.16	0.93	-2.36	1.83	1,784
ASVAB: Word Knowledge (1997)	-0.28	0.89	-3.15	2.35	1,785
Noncognitive: Violent Behavior (1997)	0.22	0.42	0.00	1.00	2,097
Noncognitive: Had Sex Before Age 15	0.18	0.38	0.00	1.00	2,100
Noncognitive: Theft Behavior (1997)	0.10	0.30	0.00	1.00	2,098

Figure 15: Parental Monetary Transfers By Parental Characteristics

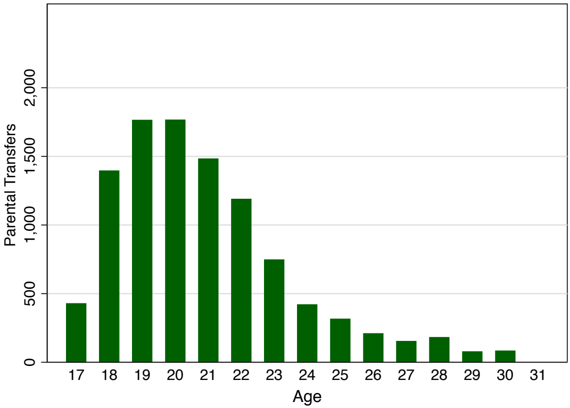
(a) By Parents' Net Worth Terciles & Education



Source: NLSY97. Parental transfer is the total monetary transfers received from parents in each year, including allowance, non-allowance income, college financial aid gift, and inheritance.

Figure 15: Parental Monetary Transfers By Parental Characteristics, Cont'd

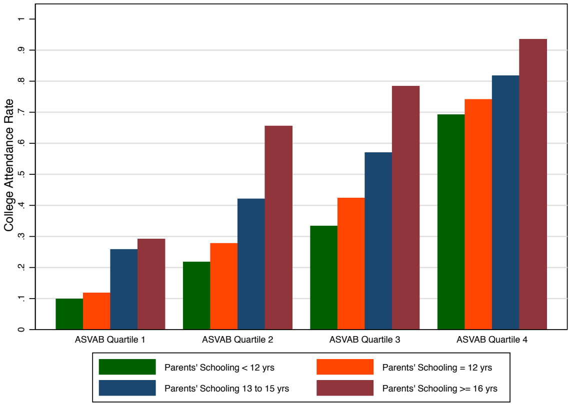
(b) By Youth's Age



Source: NLSY97. Parental transfer is the total monetary transfers received from parents in each year, including allowance, non-allowance income, college financial aid gift, and inheritance.

Figure 16: Relationships Between Early Endowments and Environments and College Choices

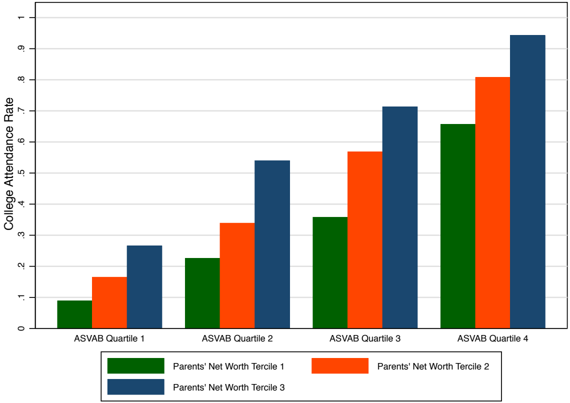
(a) College Attendance by Parental Education



Source: NLSY97 white males. 4-Year college graduate rate is calculate as the fraction of individual whose years of schooling are more than or equal to 16 at age 25.

Figure 16: Relationships Between Early Endowments and Environments and College Choices, Cont'd

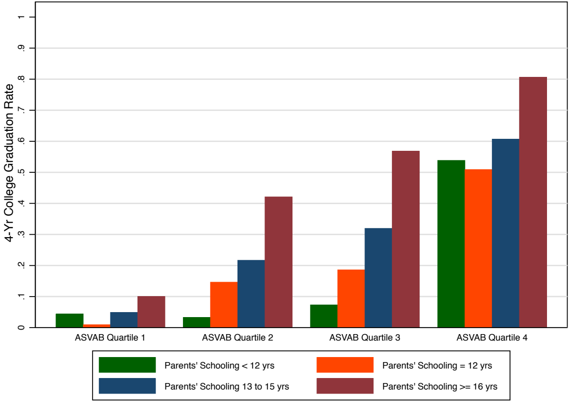
(b) College Attendance by Parental Net Worth



Source: NLSY97 white males. 4-Year college graduate rate is calculate as the fraction of individual whose years of schooling are more than or equal to 16 at age 25.

Figure 16: Relationships Between Early Endowments and Environments and College Choices, Cont'd

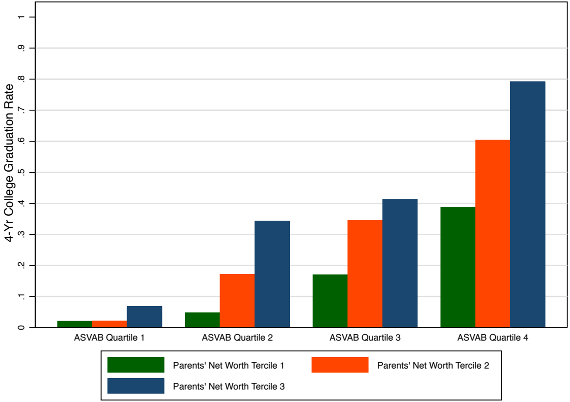
(c) 4-Year College Grad. by Parental Education



Source: NLSY97 white males. 4-Year college graduate rate is calculate as the fraction of individual whose years of schooling are more than or equal to 16 at age 25.

Figure 16: Relationships Between Early Endowments and Environments and College Choices, Cont'd

(d) 4-Year College Grad. by Parental Net Worth



Source: NLSY97 white males. 4-Year college graduate rate is calculate as the fraction of individual whose years of schooling are more than or equal to 16 at age 25.

Table 4: OLS Regression of Adult Educational Outcomes on Early Endowment and Family Influence

	Education	
ASVAB	1.03***	(0.03)
Num of Adverse Behaviors	-0.60***	(0.04)
Parents' Education	0.27***	(0.02)
Parents' Net Worth 2nd Tercile	0.67***	(0.07)
Parents' Net Worth 3rd Tercile	1.12***	(0.07)
Age	0.09***	(0.02)
(R^2)	0.46	
Observations	5354	

Standard errors in parentheses.
Source: NLSY97 white males aged 25 to 30.
$^{*}(p < 0.10)$, $^{**}(p < 0.05)$, $^{***}(p < 0.01)$.

5. Empirical Strategy

5.1 Model Parameterization

- Use the following additively separable current flow utility function:

$$U(c_t, d_{e,t}, d_{k,t}; \boldsymbol{\Omega}_t) = \frac{(c_t/es_{t,e})^{1-\gamma} - 1}{1-\gamma} + u_e(\boldsymbol{\Omega}_t)d_{e,t} + u_k(d_{k,t}, d_{e,t}, \boldsymbol{\Omega}_t) \tag{27}$$

- $es_{t,e}$: the equivalence scales of family size, $u_e(\boldsymbol{\Omega}_t)$ and $u_k(d_{k,t}, d_{e,t}, \boldsymbol{\Omega}_t)$ are flow utility (or disutility if negative) associated with individual choices of schooling and working, respectively:

$$\begin{aligned} u_e(\boldsymbol{\Omega}_t) &= \phi_{e,0}\mathbf{1}(d_{e,t} + e_t \leq 12) + (\phi_{e,1} + \phi_{e,a}\mathbf{1}(t > 22)) \cdot \mathbf{1}(d_{e,t} + e_t > 12 \ \& \ d_{e,t} + e_t \leq 16) \\ &+ \phi_{e,2}\mathbf{1}(d_{e,t} + e_t > 16) + \alpha_{e,c}\theta_c + \alpha_{e,n}\theta_n + \phi_{e,p}\mathbf{1}(e_p \geq 16) - \phi_{e,e}(1 - d_{e,t-1}) + \sigma_e\epsilon_{e,t} \end{aligned} \tag{28}$$

$$\begin{aligned} u_k(d_{k,t}, d_{e,t}, \boldsymbol{\Omega}_t) &= [\phi_{k,e} \cdot \mathbf{1}(d_{k,t} = 0.5 \ \& \ d_{e,t} = 1) + \phi_{k,0} \cdot \mathbf{1}(d_{k,t} = 0.5 \ \& \ d_{e,t} = 0) \\ &+ (\phi_{k,1} + \phi_{k,2}(age - 17)) \cdot \mathbf{1}(d_{k,t} = 1)] \cdot (1 + \alpha_{k,c}\theta_c + \alpha_{k,n}\theta_n) \end{aligned} \tag{29}$$

- The schooling preference shock $\epsilon_{e,t}$ is i.i.d. standard normal distributed.

- Following Gourinchas and Parker (2002) and De Nardi (2004), assume that the terminal value function at age $T+1$ takes the following functional form:

$$V_{T+1}(\Omega_{T+1}) = \phi_s \frac{(s_{T+1}/es_{T,e})^{1-\gamma} - 1}{1-\gamma}, \tag{30}$$

- ϕ_s characterizes the influence of net worth at age $T+1$.
- Allow the subjective discount rate $\rho(\theta_c, \theta_n)$ to depend on cognitive ability and noncognitive ability:

$$\rho(\theta_c, \theta_n) = \rho_0(1 - \rho_c\theta_c - \rho_n\theta_n) \tag{31}$$

- An individual's wage function and human capital function:

$$\log w_t = \log \psi_t + \mathbf{1}(d_{k,t} = 0.5)(\beta_{w,0} + \beta_{w,1} d_{e,t}) \tag{32}$$

where

$$\begin{aligned}\log \psi_t = &\beta_{\psi,0} + \beta_{\psi,k} k_t + \beta_{\psi,kk} k_t^2/100 + \beta_{\psi,e,0}(e_t - 12) \\ &+ \beta_{w,e,1}\mathbf{1}(e_t = 12) + \beta_{w,e,2}\mathbf{1}(e_t > 12 \ \& \ e_t < 16) + \beta_{w,e,3}\mathbf{1}(e_t \geq 16) \\ &+ (\alpha_{\psi,c,0}\theta_c + \alpha_{\psi,n,0}\theta_n) \cdot \mathbf{1}(e_t < 12) \\ &+ (\alpha_{\psi,c,1}\theta_c + \alpha_{\psi,n,1}\theta_n) \cdot \mathbf{1}(e_t \geq 12 \ \& \ e_t < 16) \\ &+ (\alpha_{\psi,c,2}\theta_c + \alpha_{\psi,n,2}\theta_n) \cdot \mathbf{1}(e_t \geq 16) + \epsilon_{w,t} - \mathbb{E}(\epsilon_{w,t})\end{aligned}$$

- $\epsilon_{w,t}$ is the education specific idiosyncratic productivity shock.

- We assume that the productivity shock $\epsilon_{w,t} \geq \underline{\epsilon}_w = 0$ is drawn from a gamma distribution $Gamma(a, b)$ with the following density function:

$$p(\epsilon_{w,t}) = \frac{1}{\Gamma(a) b^a} (\epsilon_{w,t})^{a-1} e^{-(\epsilon_{w,t})/b}. \tag{33}$$

5.2 External Calibration

Table 5: Parameters and Constraints Calibrated Outside the Structural Model

Description	Parameter	Value	Source
College Tuition & Fees	$tc(e = 13, 14)$	$5,073	IPEDS data on average tuition and fees 1999-2006.
	$tc(e \geq 15)$	$10,653	
College Grants and Scholarship	$gr(e = 13, 14, s_p = T1)$	$2,581	NLSY97 data on average grants and scholarship by years of schooling and parental wealth terciles.
	$gr(e = 13, 14, s_p = T2)$	$2,287	
	$gr(e = 13, 14, s_p = T3)$	$2,476	
	$gr(e \geq 15, s_p = T1)$	$3,604	
	$gr(e \geq 15, s_p = T2)$	$2,569	
	$gr(e \geq 15, s_p = T3)$	$2,607	
College Room and Board	$rc(e = 13, 14)$	$4,539	Johnson (2013) room and board for 2-year college and 4-year college.
	$rc(e \geq 15)$	$6,532	
GSL Borrowing Annual Limit	$\bar{l}^g(e = 13)$	$2,625	Annual Stafford Loan Limits 1993 to 2007
	$\bar{l}^g(e = 14)$	$3,500	
	$\bar{l}^g(e = 15, 16)$	$5,500	
	$\bar{l}^g(e > 16)$	$10,500	

Table 5: Parameters and Constraints Calibrated Outside the Structural Model, Cont'd

Description	Parameter	Value	Source
GSL Borrowing Aggregate Limit	$\overline{L}^g(e \geq 13 \ \& \ e \leq 16)$ $\overline{L}^g(e \geq 16)$	\$23,000 \$138,500	Undergraduate Graduate + Undergraduate
Borrowing Interest Rate	r_b	5%	Federal Student Aid
Lending Interest Rate	r_l	1%	Average real interest rate on 1-year U.S. government bonds from 2001 to 2007
Parental Transfer Function	$tr_p(e_p, s_p, d_{e,t}, d_{k,t}, e_t, t)$	Table A5	NLSY97 sample
Parents Consumption Subsidy	$tr_{c,t} = \chi \cdot \mathbf{1}(t < 18)$	\$7,800	Kaplan (2012) & Johnson (2013)
Part-time Annual Hours	$h(0.5)$	1,040	20 hours per week, 52 weeks

Table 5: Parameters and Constraints Calibrated Outside the Structural Model, Cont'd

Description	Parameter	Value	Source
Full-time Annual Hours	$h(1)$	2,080	40 hours per week, 52 weeks
Unemployment Benefit	$b_g(e \leq 12)$ $b_g(e \geq 13 \ \& \ e \leq 16)$ $b_g(e > 16)$	\$540 × 3 \$600 × 3 \$740 × 3	NLSY97 UI benefits
Minimum Consumption Floor	c_{min}	\$2,800	NLSY sample average means-tested transfers among recipients
Risk Aversion Coefficient	γ	2.0	Lochner and Monge-Naranjo (2012) and Johnson (2013)
Terminal Value function	ϕ_s	25.0	PSID 1999-2011: Median(s_{51}/c_{50})=5

5.3 Identification

Dynamic Model and Structural Parameters

- The parameters on the subjective discount rate can in principle be identified by using consumption data formed from the asset data.
- To illustrate, consider the Euler equation under a CRRA utility specification for those who are far away from borrowing constraints (abstracting from uncertainty):

$$\gamma \cdot (\log c_{t+1} - \log c_t) = -\rho(\theta_c, \theta_n) + \log(1 + r),$$

- Use the fact that γ and r are set externally to identify $\rho(\theta_c, \theta_n)$.

5.4 Estimation Method

- Two-step estimation procedure.
- In the first step, estimate the parameters of the measurement system and the joint distribution of cognitive ability and noncognitive ability at age 17, using simulated maximum likelihood:

$$\max \ \Pi_i \int_{\theta_c, \theta_n} f(Z_i; X_i, \theta_c, \theta_n) dF(\theta_c, \theta_n).$$

- Second step, use the method of simulated moments to estimate parameters of individual preferences (15 parameters), human capital production function and wage equation (20 parameters), and discount factors (3 parameters).

Table 6: Targeted Moments for SMM Estimation

Targeted Moments	# Moments
Choice probabilities, state variables, and outcome variables over the lifecycle	
Probabilities of schooling for each age 17 to 30	14
Probabilities of working part-time for each age 17 to 30	14
Probabilities of working full-time for each age 17 to 30	14
Average hourly full-time wage for each age 18 to 30	13
Average hourly part-time wage for each age 18 to 30	13
Average net worth at ages 20, 25, and 30	3
Average negative net worth at ages 20, 25, and 30	3
Average years of schooling at age 30	2
Average years of work experience at age 30	2
Probability of enrolling in college at age 21	1
Probability of graduating from 4-year college at age 25	1
Average years of work experience if working full time at age 30	1
Probability of working part-time while in school at ages 18 to 22	1
Average log wage rate when working in school at ages 18 to 22	1
Probabilities of high school, some college, and 4-year college at ages 25 and 30	3×2
Probabilities of years of school more than 16 years at age 30	1
Variance of log earnings at age 30	1
Variance of log hourly wage at age 30	1
Variance of log years of schooling at age 30	1
Variance of log experience at age 30	1

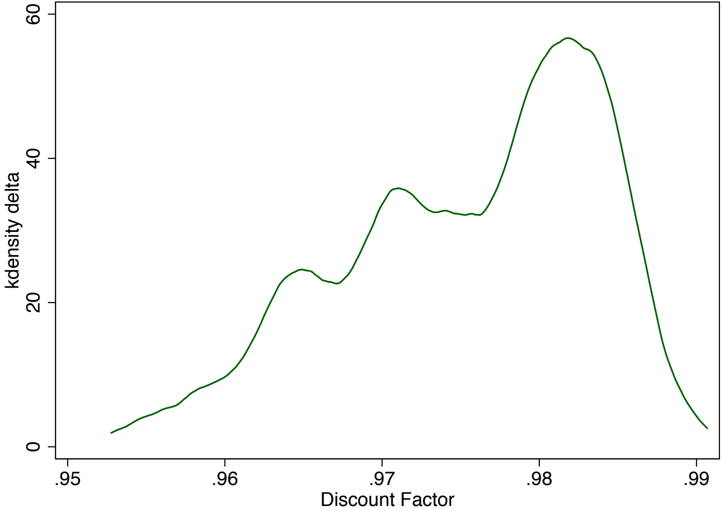
Table 6: Targeted Moments for SMM Estimation, Cont'd

Targeted Moments	# Moments
Covariance terms from auxiliary models (Indirect Inference)	
Regression coefficients of log hourly wage on work experience, work experience squared, years of schooling, HSG, SCL , CLG, cognitive ability $\times$ HSD, noncognitive ability $\times$ HSD, cognitive ability $\times$ HSG, noncognitive ability $\times$ HSG, cognitive ability $\times$ SCL, noncognitive ability $\times$ SCL, cognitive ability $\times$ CLG, noncognitive ability $\times$ CLG, previously not working, constant; and standard deviation of regression residual	17
Regression coefficients of school enrollment on previous period's enrollment status, age, age=17, parents' education, cognitive ability, noncognitive ability	6
Regression coefficients of full-time working on years of schooling, cognitive ability, noncognitive ability	3
Regression coefficients of log net worth on cognitive ability, noncognitive ability, and log wage, age $\geq$ 20, age $\geq$ 25, constant	6
Conditional moments for each of the 4 education categories	
Average net worth by 4 education categories at ages 25 and 30	4×2
Average negative net worth by 4 education categories at ages 25 and 30	4×2
Average log hourly wage by 4 education categories at ages 25 and 30	4×2
Standard deviation of log hourly wage by 4 education categories at ages 25 and 30	4×2
Median log hourly wage by 4 education categories at ages 25 and 30	4×2
Bottom 5 percentile of log hourly wage by 4 education categories at ages 25 and 30	4×2
Top 5 percentile of log hourly wage by 4 education categories at ages 25 and 30	4×2
Average hourly full-time wage by 4 education categories at ages 25 and 30	4×2
Average hourly wage by 4 education categories at ages 25 and 30	4×2
Average years of work experience by 4 education categories at ages 25 and 30	4×2
Probability of working part-time by 4 education categories at ages 25 and 30	4×2
Probability of working full-time by 4 education categories at ages 25 and 30	4×2

6. Estimation Results

6.1 Parameter Estimates

Figure 17: Density of Estimated Discount Factors: $\exp(-\rho(\theta_c, \theta_n))$



6.2 Model Goodness of Fit

Figure 18: Model Fit over the Lifecycle

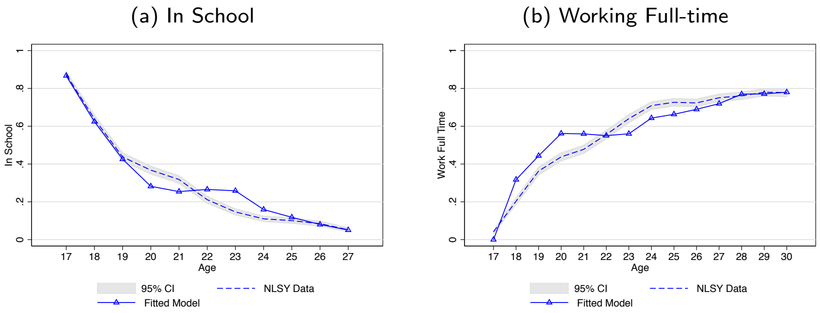
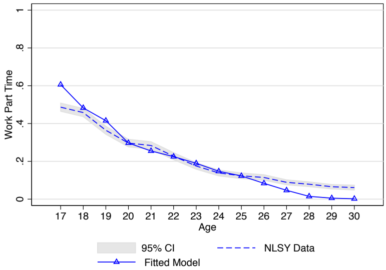


Figure 19: Model Fit over the Lifecycle, Cont'd

(c) Working Part-time



(d) Hourly Wages over Age

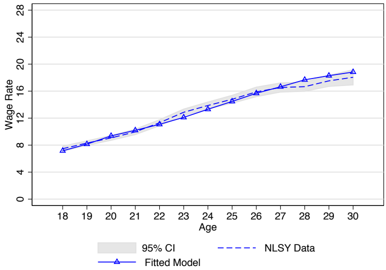
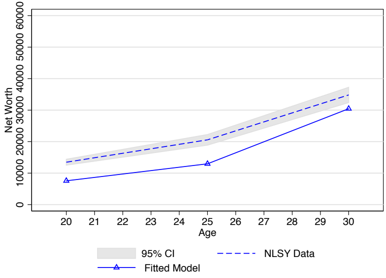


Figure 20: Model Fit over the Lifecycle, Cont'd

(e) Net Worth by Age



(f) Negative Net Worth by Age

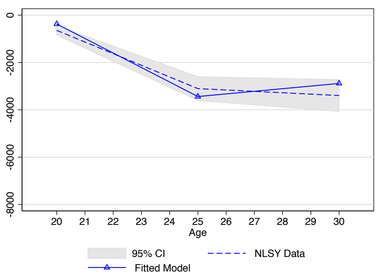
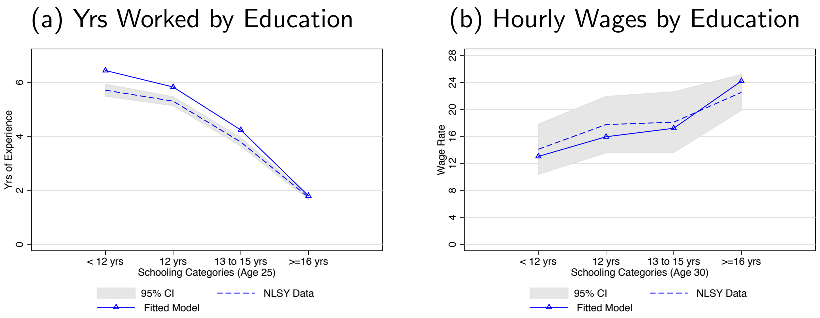


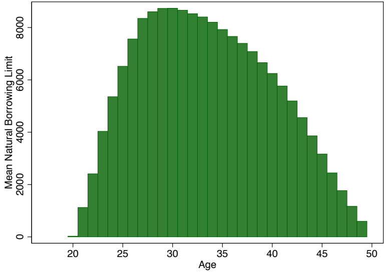
Figure 21: Model Fit by Education



6.3 Natural Borrowing Limit $\bar{L}_t^s$

Figure 22: Mean of Natural Borrowing Limit $\bar{L}_t^s(e_{t+1}, k_{t+1}, \theta)$

(a) Natural Borrowing Limit over Ages 17 to 50



(b) Natural Borrowing Limit *vs* Human Capital at Age 30

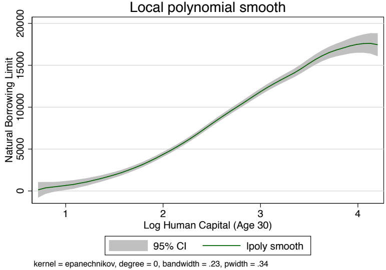
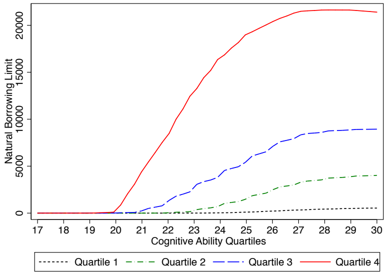
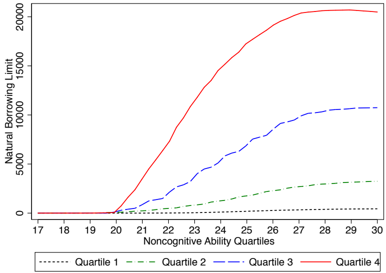


Figure 23: Evolution of Average Natural Borrowing Limit by Ability Endowments

(a) Natural Borrowing Limit *vs* Cognitive Ability



(b) Natural Borrowing Limit *vs* Noncog. Ability



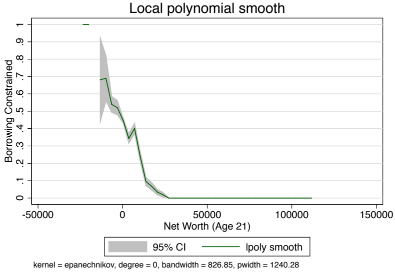
6.4 Borrowing Constrained Youths

- Using the first order conditions from an individual's optimization problem (Equations (22) and (23)), can calculate the Kuhn-Tucker multiplier associated with restrictions on individuals' next period asset decisions as follows:

$$\lambda_{s,t}(c_t, s_{t+1}; \boldsymbol{\Omega}_t) = \frac{\partial u_c(c_t; \boldsymbol{\Omega}_t)}{\partial c_t} - \exp(-\rho(\theta_c, \theta_n)) \left(\frac{\partial \mathbb{E} V_{t+1}}{\partial s_{t+1}} \right). \tag{34}$$

Figure 24: Borrowing Constrained Youths ($s_{t+1} \leq -\bar{L}_t^s(e_{t+1}, k_{t+1}, \boldsymbol{\theta})$ & $\lambda_{s,t} > 0$) at Age 21

(a) Fraction Constrained *vs* s_t



(b) Fraction Constrained *vs* θ_c

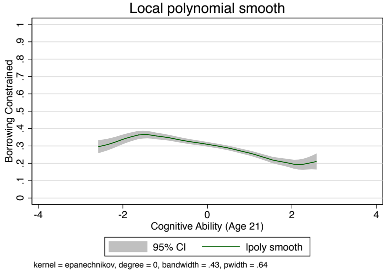
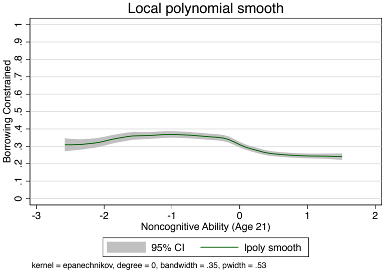


Figure 24: Borrowing Constrained Youths $(s_{t+1} \leq -\bar{L}_t^s(e_{t+1}, k_{t+1}, \boldsymbol{\theta})$ & $\lambda_{s,t} > 0)$ at Age 21, Cont'd

(c) Fraction Constrained *vs* θ_n



(d) Fraction Constrained *vs* $\log F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$

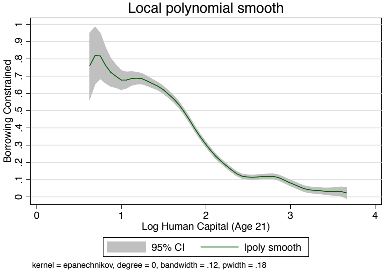
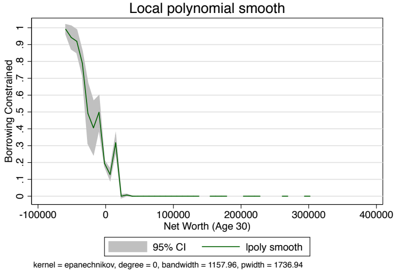


Figure 25: Borrowing Constrained Youths ($s_{t+1} \leq -\bar{L}_t^s(e_{t+1}, k_{t+1}, \boldsymbol{\theta})$ & $\lambda_{s,t} > 0$) at Age 30

(a) Fraction Constrained *vs* s_t



(b) Fraction Constrained *vs* θ_c

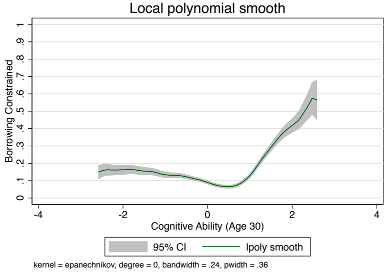
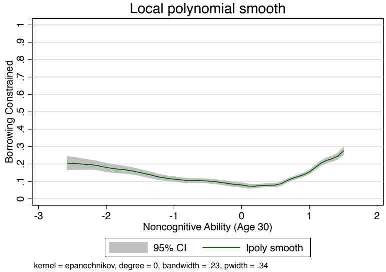


Figure 25: Borrowing Constrained Youths $(s_{t+1} \leq -\bar{L}^s_t(e_{t+1}, k_{t+1}, \boldsymbol{\theta})$ & $\lambda_{s,t} > 0)$ at Age 30, Cont'd

(c) Fraction Constrained *vs* θ_n



(d) Fraction Constrained *vs* $\log F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$

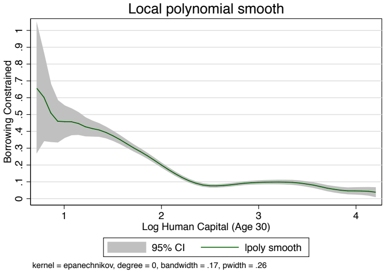


Figure 26: Borrowing Constrained Youths $(s_{t+1} \leq -\bar{L}_t^s(e_{t+1}, k_{t+1}, \boldsymbol{\theta})$ & $\lambda_{s,t} > 0)$ at Age 40

(a) Fraction Constrained *vs* s_t (b) Fraction Constrained *vs* θ_c

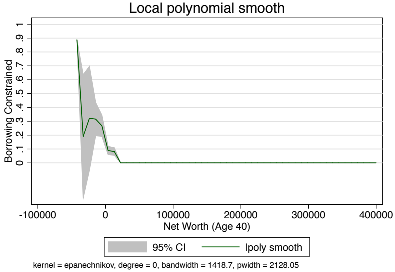
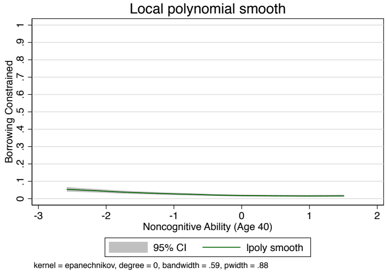


Figure 26: Borrowing Constrained Youths $(s_{t+1} \leq -\bar{L}^s_t(e_{t+1}, k_{t+1}, \boldsymbol{\theta})$ & $\lambda_{s,t} > 0)$ at Age 40, Cont'd

(c) Fraction Constrained *vs* θ_n



(d) Fraction Constrained *vs* $\log F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$

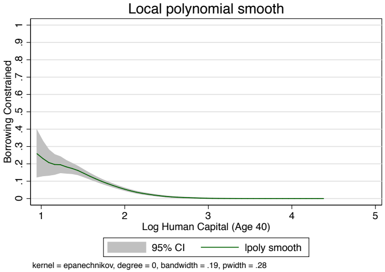
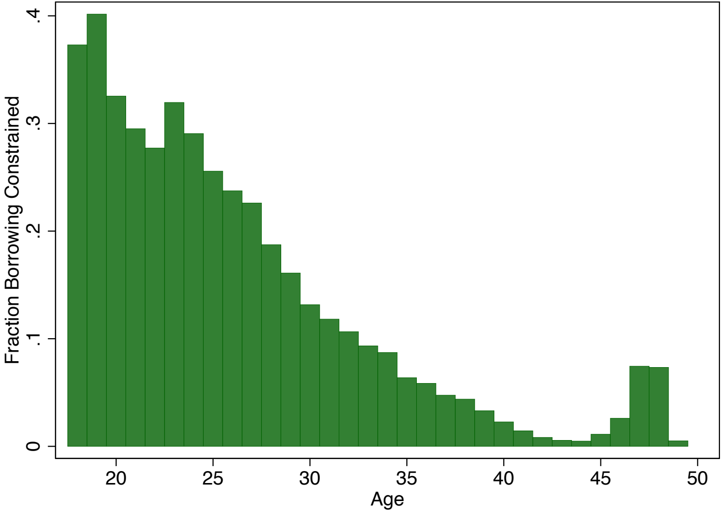
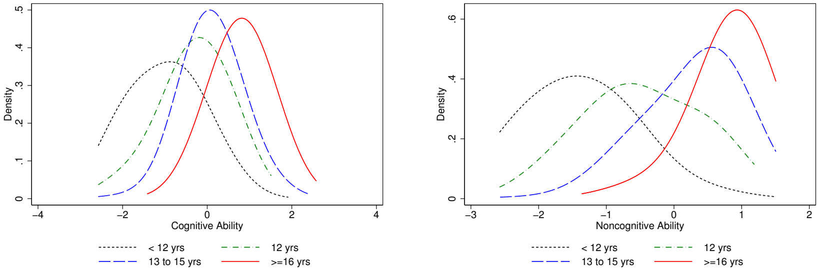


Figure 27: % Borrowing Constrained by Age



6.5 Sorting into Education by Ability

Figure 28: Density of Initial Factors Conditional on Age-30 Education



7. Counterfactual Exercises

7.1 Equalizing Initial Endowments

Table 7: Inequality in Education, Wages, and Consumption (Age 30)

	Inequality (Var of log)			Changes in Inequality (%)		
	Educ	Wage	C	Educ	Wage	C
Benchmark	0.0395	0.3313	0.1002	N.A.	N.A.	N.A.
	Counterfactual Experiments					
Equalizing Cognitive Ability	0.0255	0.2924	0.0714	-35.53	-11.75	-28.73
Equalizing Noncognitive Ability	0.0222	0.3012	0.0775	-43.88	-9.08	-22.69
Equalizing Parental Factors	0.0363	0.3292	0.0963	-8.14	-0.64	-3.92

7.2 Two Policy Experiments:

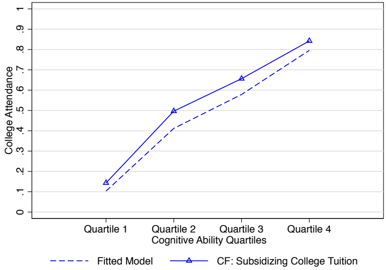
1. Subsidizing College Tuition
2. Increasing Student Loan Limits

Table 8: Years of Schooling and College Attendance and Graduation under Different Experiments

	Years of Schooling at Age 30	College Attendance at Age 21 (%)	4-Year College Graduation at Age 25 (%)	% Students who Work Part-Time
Benchmark	13.76	47.25	27.76	63.45
Counterfactual Experiments	Changes Relative to Benchmark			
(1) Subsidizing College Tuition	0.18	6.22	3.55	-5.72
(2) Increasing Student Loan Limits	0.04	3.95	1.09	1.28

Figure 29: Effects of College Tuition Subsidy

(a) College Attendance & Cognitive Ability (Age 21)



(b) College Attendance & Noncognitive Ability (Age 21)

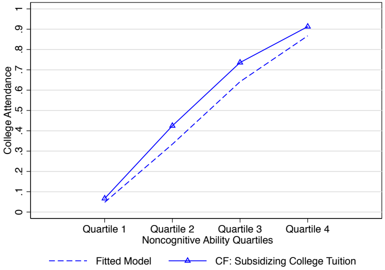
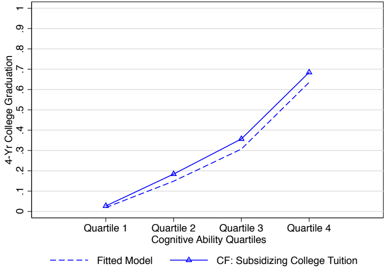


Figure 29: Effects of College Tuition Subsidy, Cont'd

(c) 4-Year College & Cognitive Ability (Age 25)



(d) 4-Year College & Noncognitive Ability (Age 25)

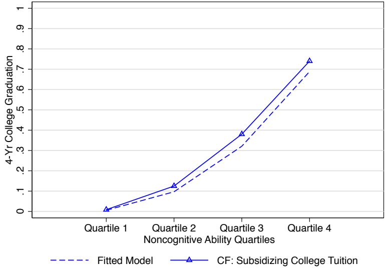
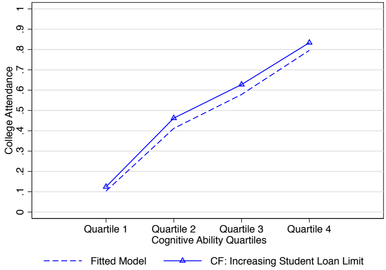


Figure 30: Effects of Increasing Student Loan Limits

(a) College Attendance & Cognitive Ability (Age 21)



(b) College Attendance & Noncognitive Ability (Age 21)

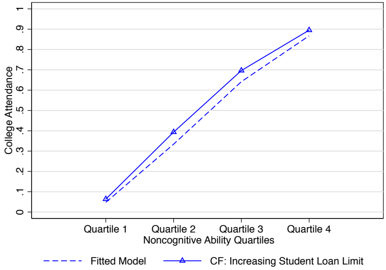


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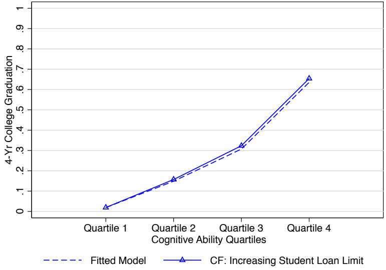
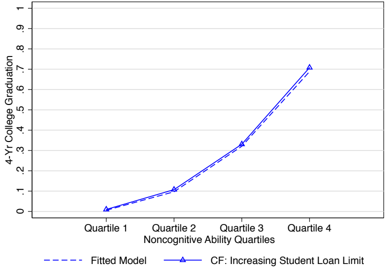
Figure 30: Effects of Increasing Student Loan Limits, Cont'd

(c) 4-Year College & Cognitive Ability (Age 25)



(d) 4-Year College & Noncognitive Ability (Age 25)



Link to Comparison with and Alternative Model

9. Summary and Conclusion

- This paper estimates a life cycle model of human capital and work experience with parental transfers in the presence of endogenous borrowing limits and precautionary savings motives.
- Individuals are subject to uninsured human capital risks and choose to invest in education, accumulate work experience and assets, and smooth consumption.
- Borrowing is permitted up to an endogenously determined limit.
- The limit is explicitly derived.

- Model extends Aiyagari's (1994) analysis by considering the effects of labor supply and human capital accumulation on the supply of available credit.
    - Account for the private lending market and government student loan programs.
    - Use our estimated model to investigate the determinants of human capital inequality and to examine the relationship between educational attainment, cognitive and noncognitive abilities, and parental education and wealth.
    - Analyze the effects of tuition subsidies and enhanced student loan limits on educational attainment and human capital inequality.
    - Do not introduce arbitrary free parameters into our analysis of credit constraints, nor do we impose *ad hoc* borrowing constraints.

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- Substantial evidence of life cycle credit constraints that affect human capital accumulation and inequality.
- Constrained individuals fall into two groups.
- A large proportion of the chronically poor with low initial endowments and abilities who acquire little skill over their lifetimes are constrained.
- There is also a smaller portion of initially well-endowed persons with high levels of acquired skills who are constrained early in their life cycles.
    - The first group has flat life cycle wage profiles. Most remain constrained over their lifetimes.
    - The second group has rising life cycle wage profiles. They are constrained only early on in life because they cannot immediately access their future earnings. As they age, their constraints are relaxed as they access their future earnings.

- Equalizing cognitive and noncognitive ability (separately and together) has dramatic effects on reducing inequality in education.
- Equalizing parental backgrounds has a much weaker effect on inequality in earnings and consumption.
- Reducing tuition has substantial impacts on schooling, but has only minor effects on our measures of inequality (even lifetime utility).
- Enhancing student loan limits has minor effects on all outcomes studied.

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Appendix: Comparison with an Alternative Model with Fixed Credit Limits by Education

- Follow **?** and consider the following specification of credit limits:

$$\bar{L}_t^s(e) = \begin{cases} \$75,000 & \text{if} \quad e \geq 16 \\ \$25,000 & \text{if} \quad e < 16 \ \& \ e \geq 12 \\ \$15,000 & \text{if} \quad e < 12 \end{cases} \tag{35}$$

- if $t \geq 22$ and $\bar{L}_t^s(e) = 0$ if $t < 22$.

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8.1 Estimation Results

Figure 31: Density of Estimated Discount Factors (Fixed Credit Limits by Education): $\exp(-\rho(\theta_c, \theta_n))$

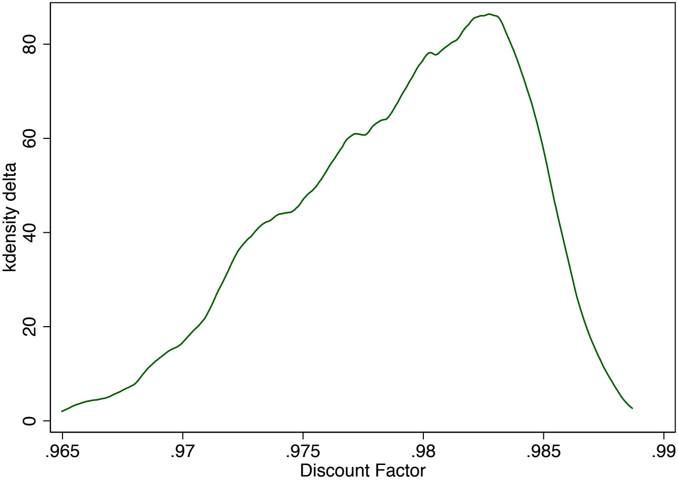
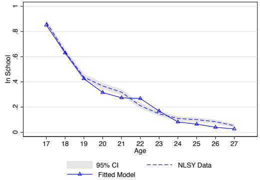
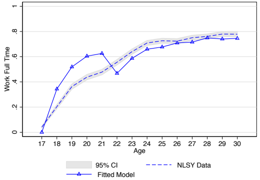


Figure 32: Goodness of Fit over the Lifecycle (Alternative Model)

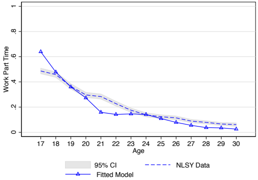
(a) In School



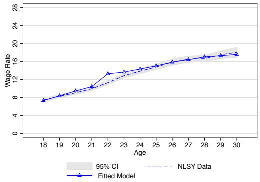
(b) Working Full-time



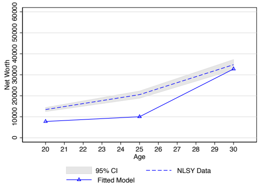
(c) Working Part-time



(d) Hourly Wages over Age



(e) Net Worth by Age



(f) Negative Net Worth by Age

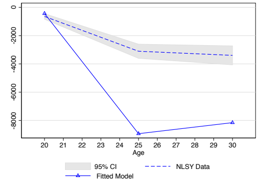


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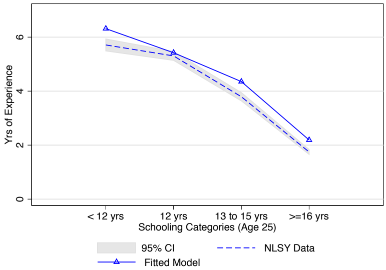
Figure 33: Goodness of Fit by Education (Alternative Model)

(a) Yrs Worked by Education



(b) Hourly Wages by Education

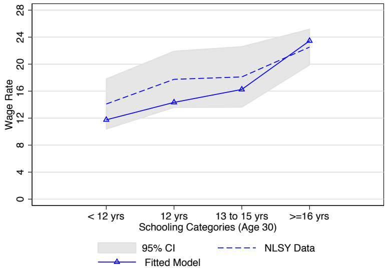


Figure 34: Density of Initial Factors Conditional on Age-30 Education (Alternative Model)

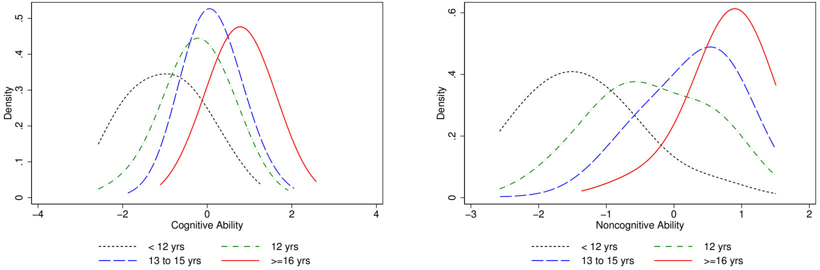
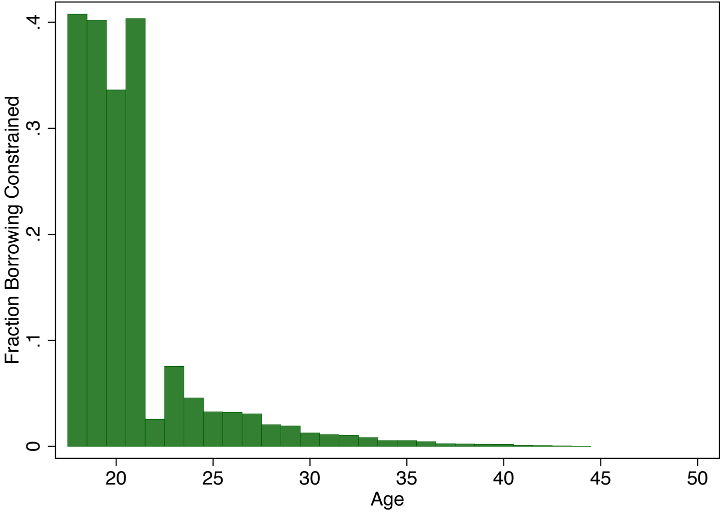


Figure 35: % Borrowing Constrained by Age for Alternative Model



8.2 Counterfactual Experiments

Table 9: Inequality in Education, Wages, and Consumption at Age 30 (Fixed Credit Limits by Education)

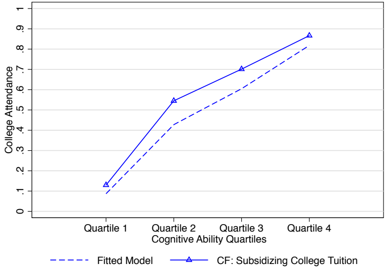
	Inequality (Var of log)			Changes in Inequality (%)		
	Educ	Wages	C	Educ	Wages	C
Benchmark	0.0398	0.3825	0.1168	N.A.	N.A.	N.A.
	Counterfactual Experiments					
Equalizing Cognitive Ability	0.0253	0.3360	0.0704	-36.37	-12.17	-39.75
Equalizing Noncognitive Ability	0.0248	0.3471	0.0784	-37.76	-9.27	-32.89
Equalizing Parental Factors	0.0375	0.3782	0.1112	-5.75	-1.14	-4.83

Table 10: Years of Schooling and College Attendance and Graduation under Different Experiments (Fixed Credit Limits by Education)

	Years of Schooling at Age 30	College Attendance at Age 21 (%)	4-Year College Graduation at Age 25 (%)	% Students who Work Part-Time
Benchmark	13.52	48.44	34.53	64.83
Counterfactual Experiments	Changes Relative to Benchmark			
(1) Subsidizing College Tuition	0.29	7.63	5.78	-5.11
(2) Increasing Student Loan Limits	0.12	4.60	2.34	0.60

Figure 36: Effects of College Tuition Subsidy on Attendance and Graduation (Fixed Credit Limits by Education)

(a) College Attendance & Cognitive Ability (Age 21)



(b) College Attendance & Noncognitive Ability (Age 21)

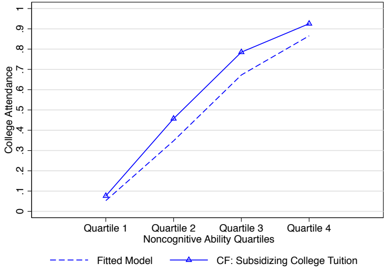
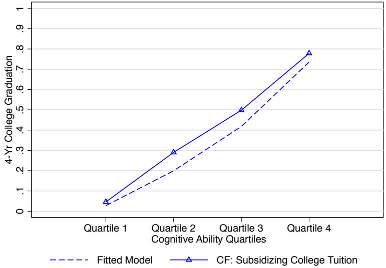


Figure 36: Effects of College Tuition Subsidy on Attendance and Graduation (Fixed Credit Limits by Education), Cont'd

(c) 4-Year College & Cognitive Ability (Age 25)



(d) 4-Year College & Noncognitive Ability (Age 25)

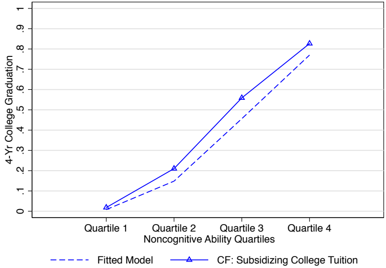


Figure 37: Effects of Increasing Student Loan Limits on Attendance and Graduation (Fixed Credit Limits by Education)

(a) College Attendance & Cognitive Ability (Age 21)

(b) College Attendance & Noncognitive Ability (Age 21)

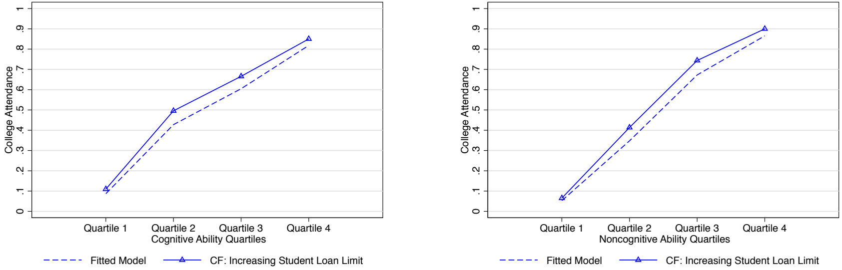
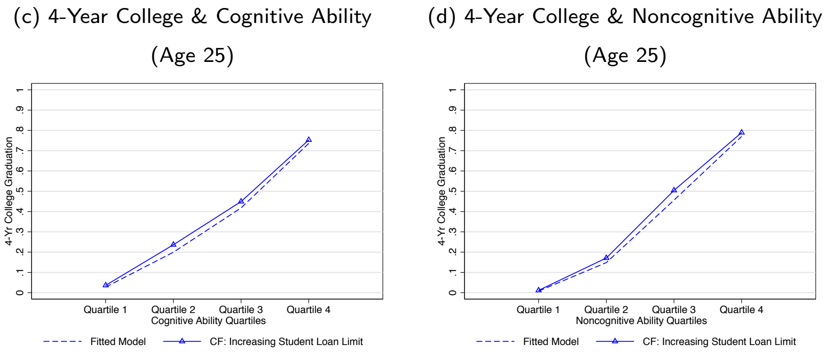


Figure 37: Effects of Increasing Student Loan Limits on Attendance and Graduation (Fixed Credit Limits by Education), Cont'd



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