# Rising Wage Inequality and the Effectiveness of Tuition Subsidy Policies: Explorations with a Dynamic General Equilibrium Model of Labor Earnings

# Based on Heckman, Lochner, and Taber, *Review of Economic Dynamics* (1998)

Econ 350, Winter 2020



Heckman, Lochner, and Taber

# The Microeconomic Model The Problem of the Agent Demographics

- Overlapping Generations Model.
- Agents are endowed with cognitive ability stock  $\theta$ .
- $\Psi(\theta)$  is the cross-section distribution of  $\theta$ .
- Agents live for  $a_T$  years.
- Mandatory retirement  $a_R \leq a_T$ .



# The Microeconomic Model: Notation

- $H_{s,a}$  is the stock of type-s human capital at age a of an agent with schooling S.
- $K_a$  is the stock of physical capital of age *a* agent.
- *I<sub>a</sub>* time spent on post-schooling training of age *a* agent.
- $C_a$  is the consumption of age *a* agent.
- *R*<sub>s,t</sub> is the price at period *t* of a type-*s* unit of human capital.



Endowments:

- A<sub>s</sub> (θ) is how ability θ affects the productivity of post-schooling investment I<sub>a</sub>.
- $H_{s,1}$  is the initial stock of type-s human capital.
- $K_1$  is the initial stock of physical capital.



Agent Solves:

$$V_{a}(H_{s,a}, K_{a}, s, \theta, r_{t}, R_{s,t})$$

$$= \max\left\{\frac{C_{a}^{\gamma} - 1}{\gamma} + \delta V_{a+1}(H_{a+1}, K_{a+1}, s, \theta, r_{t+1}, R_{s,t+1})\right\}$$

subject to:

$$C_{a} + K_{a+1} = (1 - \tau) R_{s,t} H_{s,a} (1 - I_{a}) + (1 + (1 - \tau) r_{t}) K_{a}$$



The Euler equations are:

$$(C_a)^{\gamma-1} = \delta (1 + (1 - \tau) r_{t+1}) (C_{a+1})^{\gamma-1}$$

$$(C_{a})^{\gamma-1} R_{s,t}H_{s,a}$$

$$= \delta (C_{a+1})^{\gamma-1} R_{s,t+1} (1 - I_{a+1}) + \alpha_{s} A_{s} (\theta) (I_{a})^{\alpha_{s}-1} (H_{s,a})^{\beta_{s}}$$



Heckman, Lochner, and Taber

The initial conditions and Euler equations define solutions to the problem:

$$C_{a}^{*}=g_{a}^{C}\left(H_{s,a},K_{a},S, heta,r_{t},R_{s,t}
ight)$$

$$I_{a}^{*}=g_{a}^{I}\left(H_{s,a},K_{a},S,\theta,r_{t},R_{s,t}\right)$$

$$K_{a}^{*} = g_{a}^{K} \left( H_{s,a}, K_{a}, S, \theta, r_{t}, R_{s,t} \right)$$



Comments:

- The policy functions are age-specific because agents have a finite lifetime (OLG model).
- (a) The policy functions depend on price  $R_{s,t}$  and  $r_t$  because they vary over time (perfect foresight).

Now, use the policy functions to obtain:

$$V_{1}\left(H_{s,1},K_{1},s,\theta,t\right) = \frac{C_{a}^{*\gamma}-1}{\gamma} + \delta V_{2}\left(A_{s}\left(\theta\right)\left(I_{a}^{*}\right)^{\alpha_{s}}\left(H_{s,a}\right)^{\beta_{s}} + H_{s,a},K_{a}^{*},s,\theta,t+1\right)$$

The agent then decides schooling by solving:

$$s^* = \arg \max_{s} \left[ V_1 \left( H_{s,1}, K_1, s, \theta, t \right) - D_s - \varepsilon_s \right]$$



# The Problem of the Firm: Notation

- $\bar{H}_{s,t}$  is the total amount of type-s human capital demanded by the firm at period t.
- $K_t$  is the total amount of physical capital demanded by the firm at period *t*.
- $\sigma_k$  is the depreciation of physical capital.



The problem of the firm is:

$$\pi (R_{1,t}, R_{2,t}, r_t) = \max \left\{ F \left( \bar{H}_{1,t}, \bar{H}_{2,t}, K_t \right) - R_{1,t} \bar{H}_{1,t} - R_{2,t} \bar{H}_{2,t} - (r_t + \sigma_k) \bar{K}_t \right\}$$

The first-order conditions are:

$$R_{s,t} = \frac{\partial F\left(\bar{H}_{1,t}, \bar{H}_{2,t}, \bar{K}_{t}\right)}{\partial \bar{H}_{s,t}}, s = 1, 2$$
$$(r_{t} + \sigma_{k}) = \frac{\partial F\left(\bar{H}_{1,t}, \bar{H}_{2,t}, \bar{K}_{t}\right)}{\partial \bar{K}_{t}}$$



Heckman, Lochner, and Taber

The production function F is assumed to be:

$$\begin{split} & \mathsf{F}\left(\bar{H}_{1,t},\bar{H}_{2,t},\bar{K}_{t}\right) \\ &= \left\{\mathsf{a}_{2}\left(\bar{K}_{t}\right)^{\rho_{2}} + (1-\mathsf{a}_{2})\left[\mathsf{a}_{1}\left(\bar{H}_{1,t}\right)^{\rho_{1}} + (1-\mathsf{a}_{1})\left(\bar{H}_{2,t}\right)^{\rho_{1}}\right]^{\frac{\rho_{2}}{\rho_{1}}}\right\}^{\frac{1}{\rho_{2}}} \end{split}$$



# Aggregation

- $t_c$  is the year of birth of cohort c.
- $a = t t_c$  is the age of cohort c at year t.
- P<sub>tc</sub> = {r<sub>i</sub>, R<sub>1,i</sub>, R<sub>2,i</sub>}<sup>tc+aR</sup><sub>i=tc</sub> is the sequence of prices cohort *c* will face during working life.
- $N_s(\theta, t_c)$  is the number of agents of type.
- $\theta$  in cohort *c* and schooling level *s*.



- *H*<sub>s,a</sub> (θ, *P*<sub>tc</sub>) is the stock of type-s human capital at age a of an agent of cohort c.
- *K<sub>s,a</sub>*(θ, *P<sub>tc</sub>*) is the stock of type-*s* human capital at age *a* of an agent of cohort *c*.
   Therefore:

$$\hat{H}_{s,t} = \sum_{t_c=t-a_R}^{t-1} \int N_s(\theta, t_c) H_{s,t-t_c}(\theta, P_{t_c})$$
$$(1 - I_{t-t_c}(s, \theta, P_{t_c})) d\Psi(\theta)$$

$$\hat{K}_{t} = \sum_{t_{c}=t-a_{R}}^{t-1} \sum_{s=1}^{2} \int N_{s}\left(\theta, t_{c}\right) K_{t-t_{c}}\left(s, \theta, P_{t_{c}}\right) d\Psi\left(\theta\right)$$



• Earnings of a person age *a* at time *t* of cohort *c* :

$$W(a, t, H_{a,s}(\theta, P_{t_c})) = R_{s,t}H_{a,s}(\theta, P_{t_c})(1 - I_a(s, \theta, P_{t_c}))$$

• Suppose that for two consecutive ages a and a + 1,  $I_a(s, \theta, P_{t_c}) = I_{a+1}(s, \theta, P_{t_c}) = 0$ .

$$\frac{W(a+1, t+1, H_{a+1,s}(\theta, P_{t_c}))}{W(a, t, H_{a,s}(\theta, P_{t_c}))} = \frac{R_{s,t+1}H_{a+1,s}(\theta, P_{t_c})}{R_{s,t}H_{a,s}(\theta, P_{t_c})} = \frac{R_{s,t+1}(1-\sigma_s)H_{a,s}(\theta, P_{t_c})}{R_{s,t}H_{a,s}(\theta, P_{t_c})} = \frac{R_{s,t+1}(1-\sigma)}{R_{s,t}}$$

- We can get the ratio of  $\frac{R_{s,t+1}}{R_{s,t}}$  up to a constant. Next step, we show how to get  $\sigma$ .
- We can get σ from microestimates of human capital production function.

• Consider the firm's wage bill of schooling level *s* at period *t*:

$$WB_{s,t} = R_{s,t}\overline{H}_{s,t}$$

• Rearranging terms:

$$\frac{WB_{s,t}}{\left(1-\sigma\right)^{t}R_{s,t}} = \frac{\bar{H}_{s,t}}{\left(1-\sigma\right)^{t}}$$

Thus:

$$egin{aligned} & ilde{\mathcal{R}}_{s,t} = (1-\sigma)^t \, \mathcal{R}_{s,t} \ & ilde{\mathcal{H}}_{s,t} = rac{ar{\mathcal{H}}_{s,t}}{(1-\sigma)^t} \end{aligned}$$



# Digression:

Identifying the Parameters of Interest: Identifying  $\sigma$ 

Let

$$Q_t = (a_1 (H_{t,1})^{\rho_1} + (1 - a_1) (H_{t,2})^{\rho_1})^{\frac{1}{\rho_1}} .$$
 (1)

•  $\frac{1}{1-\rho_1}$  = elasticity of substitution

• The price  $R_t^Q$  of one unit of the basket  $Q_t$  is the solution to

$$R_t^Q = \min_{H_{t,1}, H_{t,2}} R_{t,1} H_{t,1} + R_{t,2} H_{t,2}$$

subject to

$$\left( \mathsf{a}_1 \left( \mathsf{H}_{t,1} 
ight)^{
ho_1} + \left( 1 - \mathsf{a}_1 
ight) \left( \mathsf{H}_{t,2} 
ight)^{
ho_1} 
ight)^{rac{1}{
ho_1}} = 1$$
 .



• The first-order conditions are:

$$R_{t,1} = \lambda \left( a_1 \left( H_{t,1} \right)^{\rho_1} + (1 - a_1) \left( H_{t,2} \right)^{\rho_1} \right)^{\frac{1 - \rho_1}{\rho_1}} a_1 \left( H_{t,1} \right)^{\rho_1 - 1}$$
(2)  
$$R_{t,2} = \lambda \left( a_1 \left( H_{t,1} \right)^{\rho_1} + (1 - a_1) \left( H_{t,2} \right)^{\rho_1} \right)^{\frac{1 - \rho_1}{\rho_1}} (1 - a_1) \left( H_{t,2} \right)^{\rho_1 - 1}$$

• The solution to this problem is well-known:

$$R_{t}^{Q} = \left[ (a_{1})^{\frac{1}{1-\rho_{1}}} (R_{t,1})^{\frac{\rho_{1}}{\rho_{1}-1}} + (1-a_{1})^{\frac{1}{1-\rho_{1}}} (R_{t,2})^{\frac{\rho_{1}}{\rho_{1}-1}} \right]^{\frac{\rho_{1}-1}{\rho_{1}}}$$
(4)

• The problem of the firm can be recast as:

$$\pi \left( R_{t}^{Q}, r_{t} \right) = \max \left\{ \left[ a_{2} Q_{t}^{\rho_{2}} + \left( 1 - a_{2} \right) K_{t}^{\rho_{2}} \right]^{\frac{1}{\rho_{2}}} - R_{t}^{Q} Q_{t} - r_{t} K_{t} \right\}$$



(3)

The first-order conditions are

$$\left[a_{2}\left(Q_{t}\right)^{\rho_{2}}+\left(1-a_{2}\right)\left(K_{t}\right)^{\rho_{2}}\right]^{\frac{1-\rho_{2}}{\rho_{2}}}a_{2}\left(Q_{t}\right)^{\rho_{2}-1}=R_{t}^{Q}\qquad(5)$$

$$\left[a_2 Q_t^{\rho_2} + (1 - a_2) \,\mathcal{K}_t^{\rho_2}\right]^{\frac{1 - \rho_2}{\rho_2}} (1 - a_2) \,(\mathcal{K}_t)^{\rho_2 - 1} = r_t \qquad (6)$$

• Taking ratios of (5) and (6) and applying logs it follows that:

$$\log \frac{R_t^Q}{r_t} = \log \left(\frac{a_2}{1-a_2}\right) + (\rho_2 - 1) \log \left(\frac{Q_t}{K_t}\right)$$
(7)

• But note that from (1) and (4) are defined in terms of  $H_{s,t}$  and  $R_{s,t}$ .



However, we only observe H
<sub>s,t</sub> and R
<sub>s,t</sub>. Let Q
<sub>t</sub> and R
<sub>t</sub><sup>Q</sup> be defined as in (1) and (4) but based on observables H
<sub>s,t</sub> and R
<sub>s,t</sub>:

$$ilde{Q}_t = \left( extsf{a}_1 \left( ilde{H}_{t,1} 
ight)^{
ho_1} + \left( 1 - extsf{a}_1 
ight) \left( ilde{H}_{t,2} 
ight)^{
ho_1} 
ight)^{rac{1}{
ho_1}}$$

$$ilde{R}_{t}^{Q} = \left[ (a_{1})^{rac{1}{1-
ho_{1}}} \left( ilde{R}_{t,1} 
ight)^{rac{
ho_{1}}{
ho_{1}-1}} + (1-a_{1})^{rac{1}{1-
ho_{1}}} \left( ilde{R}_{t,2} 
ight)^{rac{
ho_{1}}{
ho_{1}-1}} 
ight]^{rac{
ho_{1}-1}{
ho_{1}}}$$

• It is easy to show that:

$$Q_t = (1 - \sigma)^t \tilde{Q}_t$$

$$R_t^Q = (1 - \sigma)^{-t} \tilde{R}_t^Q$$
(8)
(9)



• Plugging (8) and (9) into (7) it follows that:

$$\log \frac{\tilde{R}_t^Q}{r_t} = \log \left(\frac{a_2}{1-a_2}\right) + \rho_2 \log \left(1-\sigma\right) t + (\rho_2 - 1) \log \left(\frac{\tilde{Q}_t}{K_t}\right)$$

• Suppose we run a regression:

$$\log \frac{\tilde{R}_t^Q}{r_t} = \beta_0 + \beta_1 t + \beta_2 \log \left(\frac{\tilde{Q}_t}{K_t}\right) + \varepsilon_t$$

• Then we can identify:

$$a_2 = rac{e^{eta_0}}{1+e^{eta_0}} ~~
ho_2 = 1+eta_2 ~~\sigma = 1-e^{rac{eta_1}{1+eta_2}}$$

• This assumes no technical progress and is a bad assumption.

# End of Digression

(10)

 To identify the other parameters of interest, consider the log of the ratio of (3) to (2):

$$\log \frac{R_{t,2}}{R_{t,1}} = \log \left(\frac{1-a_1}{a_1}\right) + (\rho_1 - 1) \log \left(\frac{H_{t,2}}{H_{t,1}}\right)$$
(11)

- Again, note that we do not observe either  $R_{t,s}$  or  $H_{t,s}$ , but only  $\tilde{R}_{t,s}$  and  $\tilde{H}_{t,s}$ .
- Therefore

$$\log \frac{\frac{\tilde{R}_{t,2}}{(1-\sigma)^t}}{\frac{\tilde{R}_{t,1}}{(1-\sigma)^t}} = \log \left(\frac{1-a_1}{a_1}\right) + (\rho_1 - 1) \log \left(\frac{(1-\sigma)^t \tilde{H}_{t,2}}{(1-\sigma)^t \tilde{H}_{t,1}}\right) \Rightarrow$$
$$\Rightarrow \log \frac{\tilde{R}_{t,2}}{\tilde{R}_{t,1}} = \log \left(\frac{1-a_1}{a_1}\right) + (\rho_1 - 1) \log \left(\frac{\tilde{H}_{t,2}}{\tilde{H}_{t,1}}\right). \tag{12}$$

• An OLS regression on (12) can identify  $\rho_1$  and  $a_1$ . CHICAGO

- This assumes that  $a_1$  is not time varying or, if it is,  $\ln\left(\frac{1-a_1}{a_1}\right)$  is not collinear with  $\widetilde{H}_{t,2}/\widetilde{H}_{t,1}$ .
- But we can get  $\sigma$  from the production function of human capital.
- We bring this to the macro data.



# **Estimating the Human Capital Production Function**

- We use wage and schooling data on white males from the NLSY.
- We assume that there are four observable  $\theta$  types which we define according to AFQT quartile.
- We assume that the interest rate is fixed at r = 0.05 and that rental rates are fixed and normalized to one.



 For any given (a, θ, S) and any set of parameters π we can calculate the optimal wage

$$w(a, \theta, S; \pi)$$
.

• We assume that these wages are measured with error and we estimate the parameters,  $\pi$ , using nonlinear least squares, minimizing

$$\sum_{i=1}^{N}\sum_{a}\left(w_{i,a}^{*}-w(a,\theta,S;\pi)\right)^{2},$$

where  $w_{i,a}^*$  is the observed wage.



- Given these estimated parameters, we can obtain the present value of earnings for each type as college graduates or high school graduates,  $\widehat{V_{\theta}^{S}}$ .
- We assume that the nonpecuniary tastes for college are normally distributed, so

$$\Pr\left(\mathsf{Coll} \mid D^{\mathcal{S}}, \theta\right) = \Phi\left(\frac{(1-\tau)\left(V_{\theta}^{2} - V_{\theta}^{1}\right) - D^{\mathcal{S}} + \mu_{\theta}}{\sigma_{\varepsilon}}\right)$$

Using data on state tuition we estimate this model as a probit.



• We take

$$au = 0.15$$
  $\delta = 0.96$   $\gamma = 0.10$ 

 We calibrate the model to "look like" the NLSY in the original steady state:

$$(1 - \tau) r = 0.05$$
  $R^1 = 2.00$   $R^2 = 2.00$ 

- In order to match the capital-output ratio, we need a transfer from old cohorts to young.
- We take an exogenous transfer from a cohort as it retires and give it to a new cohort as it is born.
- This transfer is approximately \$30,000.



• We estimate a nested CES production function allowing for a linear time trend

$$a_3 \left(a_2 \left(a_1 (ar{H}^1_t)^{
ho_1} + (1-a_1) (ar{H}^2_t)^{
ho_1}
ight)^{
ho_2/
ho_1} + (1-a_2) ar{\mathcal{K}}^{
ho_2}_t
ight)^{1/
ho_2}$$

- We estimate  $\rho_1 = 1 \frac{1}{\sigma} \doteq \frac{1}{3}$  and  $\rho_2 = 0$  based on those estimates.
- We calibrate  $(a_1, a_2, a_3)$  and the transfer to yield prices  $(r, R^1, R^2)$  and a capital-output ratio of 4 in the initial steady state.



# **Skill-Biased Technical Change**

- Unexpected shock resulting in a constant decline in *a*<sub>1</sub> for 30 years.
- The total decline in the share of low skilled labor is 30% (matching the rate of decline in the data).
- Perfect foresight.
- Transition period of 200 years.



# Skill-Biased Technical Change: The Effects of Skill-Biased Technology Change

- Movements in measured wages are different from movements in skill prices, especially for young workers.
- Without intervention, economy converges to a new steady state with lower wage inequality than before the technology change.
- In the long run, society is richer and all types are better off. In the short run, low ability/low skilled workers caught in the transition are worse off.



- In the new steady state, there are more high skilled workers, but human capital per skilled worker is lower.
- During transition periods, cross-section estimates of "returns" to skill are substantially different from the actual returns faced by cohorts making educational decisions.



# **Tuition Subsidy**

- Partial equilibrium analysis ignores the effects of changes in skill quantities on the price of skill.
- As individuals acquire more skill in response to policy change, the returns to skill decline.
- This lowers the proportion of individuals taking advantage of the policy.
- The increase in aggregate skill also affects the earnings of individuals who do not take advantage of the new policy.



- Partial equilibrium analysis fails for two reasons:
  - 1 Overstates the effect of the program on participants.
  - Ø Misses the effect of the program on non-participants
- Accounting for these effects in evaluating policy requires a general equilibrium, structural model of skill formation.



# **Tuition Subsidy: Example**

- \$500 tuition subsidy.
- Balance the budget in the steady states.
- Perfect foresight.
- Transition period of 200 years.



# Main Findings

- Estimates of college enrolment responses based on cross-section variations in tuition are substantially overstated.
- Individuals who do not change their schooling decision are affected.



# Summary

- We develop an empirically-grounded dynamic overlapping generations general-equilibrium model of skill formation with heterogeneous human capital.
- Model roughly consistent with changing wage structure.
- Partial equilibrium program evaluation can be very misleading.
- We distinguish between effects measured in a cross-section and the effects on different cohorts.



# Extensions

- Additional tax and subsidy policies.
- Closer link between macro and micro models.
- Relax perfect foresight assumption.
- Incorporate a separate sector for schooling-education requires high skilled labor inputs.



# **Tables and Figures**



Heckman, Lochner, and Taber

Figure 1: Estimated parameters for human capital production function and schooling decision (standard errors in parentheses)

Human Capital Production $H_{a+1}^{S} = A^{S}(\theta) I_{a}^{\alpha_{S}} H_{a}^{\beta_{S}} + (1 - \sigma) H_{a}^{S}$ S = 1, 2						
	High School $(S = 1)$	College $(S = 2)$				
α	0.945(0.017)	0.939(0.026)				
eta	0.832(0.253)	0.871(0.343)				
A(1)	0.081(0.045)	0.081(0.072)				
$H_{a_R}(1)$	9.530(0.309)	13.622(0.977)				
A(2)	0.085(0.053)	0.082(0.074)				
$H_{a_R}(2)$	12.074(0.403)	14.759(0.931)				
A(3)	0.087(0.056)	0.082(0.077)				
$H_{a_R}(2)$	13.525(0.477)	15.614(0.909)				
A(4)	0.086(0.054)	0.084(0.083)				
$H_{a_R}(4)$	12.650(0.534)	18.429(1.095)				

Figure 2: Estimated parameters for human capital production function and schooling decision (standard errors in parentheses)

	Probit	Average	
	Parameters	Derivatives	
λ	0.166(0.062)	-0.0655(0.025)	
$\alpha(1)$	-1.058(0.097)	-	
$\alpha(2)$	-0.423(0.087)	0.249(0.037)	
$\alpha(3)$	0.282(0.089)	0.490(0.029)	
<i>α</i> (4)	1.272(0.101)	0.715(0.018)	
Sample Size:			
Persons	869	1069	
Person Years	7996	11626	

(1)  $D^2$  is the discounted tuition cost of attending college.

(2)  $\alpha(\theta)$  is the nonparametric estimate of  $(1 - \tau)[V^2(\theta) - V^1(\theta)]$ , the monetary value of the gross discounted returns to attending college.

(3)  $\delta^2 = 1$  if attend college;  $\delta^2 = 0$  otherwise. A is the unit normal cdf.

Figure 3: Derived parameters for human capital production function and schooling decision (units are thousands of dollars)

Human Capital Production						
	High School $(S = 1)$	College $(S = 2)$				
$H^{S}(1)$	8.042(0.0.094)	11.117(0.424)				
$H^{S}(2)$	10.0634(0.118)	12.271(0.325)				
$H^{S}(3)$	11.1273(0.155)	12.960(0.272)				
$H^{S}(4)$	10.361(0.234)	15.095(0.323)				
Present Value Earnings 1	260.304(3.939)	289.618(12.539)				
Present Value Earnings 2	325.966(5.075)	319.302(10.510)				
Present Value Earnings 3	360.717(6.352)	337.260(9.510)				
Present Value Earnings 4	335.977(8.453)	393.138(11.442)				



Figure 4: Derived parameters for human capital production function and schooling decision (units are thousands of dollars)

College Decision: Attend College if $(1-\tau)V^2(\theta) - D^2 + \varepsilon_i \ge (1-\tau)V^1(\theta)$					
$\varepsilon_{ heta} \sim N(\mu_{ heta}, \sigma_{m{arepsilon}})$					
$\sigma_{\varepsilon}$ (Std. deviation of $\varepsilon$ )	22.407(8.425)				
Nonpecuniary costs by ability level					
	-53.0190(16.770)				
	-2.8173(12.760)				
$\mu_3$ (Third Ability Quartile)	29.7712(11.540)				
$     \mu_4 $ (Highest Ability Quartile)	-28.6494(16.966)				

(1)  $V^{i}(\theta)$  is the monetary value of going to schooling level *i* for a person of AFQT quartile  $\theta$ .

i = 1 for high school; i = 2 for college. We assume  $\tau_r = \tau_h = \tau$ .

(2)  $\varepsilon_{\theta}$  is the nonpecuniary benefit of attending college for a person of ability quartile  $\theta$ .

(3)  $D^2$  is the discounted tuition cost of attending college

Figure 5: Estimates of aggregate production function estimated from factor demand equations (III-1) and (III-2), 1965–1990, allowing for technical progress through a linear trend (standard errors in parentheses)

Instruments	$\rho_1$	Implied Elasticity of Substitution $(\sigma_1)$	Time Trend	ρ2	Implied Elasticity of Substitution $(\sigma_2)$	Time Trend
OLS (Base Model)	$\begin{array}{c} 0.306 \\ (0.089) \end{array}$	1.441 (0.185)	$\begin{array}{c} 0.036 \\ (0.004) \end{array}$	-0.034 (0.200)	0.967 ( 0.187)	-0.004 (0.007)
Percent Working Pop. $<30$ & Defense Percent of GNP	$\begin{array}{c} 0.209 \\ (0.134) \end{array}$	1.264 (0.215)	$\begin{array}{c} 0.039 \\ (0.005) \end{array}$	-0.036 (0.200)	$0.965 \\ (0.187)$	-0.004 (0.007)
Defense Percent of GNP	$0.157 \\ (0.125)$	$1.186 \\ (0.175)$	$\begin{array}{c} 0.041 \\ (0.004) \end{array}$	-0.171 (0.815)	$   \begin{array}{c}     0.854 \\     (0.594)   \end{array} $	-0.008 (0.024)
Percent Working Pop. $< 30$	$\begin{array}{c} 0.326 \\ (0.182) \end{array}$	1.484 (0.400)	$0.036 \\ (0.006)$	$0.364 \\ (1.150)$	1.572 (2.842)	$\begin{array}{c} 0.007 \\ (0.034) \end{array}$



Figure 6: Simulated changes in wages and wage inequality from 1960–1990. Includes the estimated trend in technology and entrance of baby boom cohorts from 1965–80 (multiplied by 100)

Years	Coll HS Log Wage Diff.	Mean HS Age 25	Log Wage Age 50	Mean Co Age 25	ll. Log Wage Age 50	Std. D HS	eviation of College	Log Wages All
1960-70	6.66	-26.98	-9.17	19.41	-2.2	0.06	0.67	2.49
1970-80	-5.33	3.51	-2.32	-8.72	-5.11	2.06	-0.84	0.14
1980-90	11.74	-4.94	-1.74	11.22	-2.72	10.68	-7.87	8.12
1960-90	13.07	-28.4	-13.22	21.91	-10.03	12.8	-8.03	10.75



Figure 7: Predicted vs. actual hourly wages (in 1992 dollars) by AFQT quartile (high school category)



Figure 8: Predicted vs. actual hourly wages (in 1992 dollars) by AFQT quartile (college category)



Figure 9: Comparison of Mincer vs. estimated investment profiles (high school)



Figure 10: Comparison of Mincer vs. estimated investment profiles (college)



Figure 11: Labor and capital shares over time



Note: The breakdown of labor's share is based on wages and excludes other forms of compensation.



### Figure 12: Estimated trend in $\alpha_1$ for 30 years



### Figure 13: Estimated trend in $\alpha_1$ for 30 years



### Figure 14: Estimated trend in $\alpha_1$ for 30 years



### Figure 15: Estimated trend in $\alpha_1$ for 30 years



### Figure 16: Estimated trend in $\alpha_1$ for 30 years.



# Figure 17: Estimated trend in $\alpha_1$ for 30 years



### Figure 18: Estimated trend in $\alpha_1$ for 30 years



Figure 19: Figure 11B: Estimated trend in  $\alpha_1$  for 30 years



### Figure 20: Estimated trend in $\alpha_1$ for 30 years



### Figure 21: Estimated trend in $\alpha_1$ for 30 years



### Figure 22: Estimated trend in $\alpha_1$ for 30 years



# Figure 23: Estimated trend in $\alpha_1$ for 30 years



### Figure 24: Estimated trend in $\alpha_1$ for 30 years



### Figure 25: Estimated trend in $\alpha_1$ for 30 years



### Figure 26: Estimated trend in $\alpha_1$ for 30 years



Heckman, Lochner, and Taber

### Figure 27: Estimated trend in $\alpha_1$ for 30 years



# Figure 28: Estimated trend in $\alpha_1$ for 30 years



### Figure 29: Estimated trend in $\alpha_1$ for 30 years



### Figure 30: Estimated trend in $\alpha_1$ for 30 years



Figure 31: Estimated trend in  $\alpha_1$  for 30 years. Baby boom (expansion of cohort size by 32%) between years 1965–80



# Figure 32: Estimated trend in $\alpha_1$ for 30 years. Baby boom (expansion of cohort size by 32%) between years 1965–80

