Life-Cycle Labour Supply with Human Capital: Econometric and Behavioural Implications

by Micheal P. Keane

James J. Heckman

THE UNIVERSITY OF CHICAGO

Econ 350, Winter 2023

1. A Simple Life-cycle Model with Human Capital

- In the MaCurdy (1981) formulation the period utility function is:

$$U_t = \frac{c_t^{1+\eta}}{1+\eta} - \beta\frac{h_t^{1+\gamma}}{1+\gamma} \quad t = 1, \ldots, T \quad \eta \leq 0, \gamma \geq 0. \qquad (1)$$

- Here, C_t and h_t are consumption and hours of labour supplied in period t, respectively.

- The parameter β captures tastes for leisure. It is typically assumed to be individual specific in empirical work (but that is not necessary to make the points I wish to make here).

- Given a discount factor of ρ, and assuming perfect-foresight, the present value of lifetime utility is given by:

$$V = \frac{c_1^{1+\eta}}{1+\eta} - \beta\frac{h_1^{1+\gamma}}{1+\gamma} + \sum_{t=2}^{T} \rho^{t-1}\left(\frac{c_t^{1+\eta}}{1+\eta} - \beta\frac{h_t^{1+\gamma}}{1+\gamma}\right). \qquad (2)$$

- Workers maximise (2) subject to the constraint that the present value of lifetime consumption equals the present value of lifetime earnings.

- Agents can borrow/lend across periods at interest rate r. In what follows I assume $\rho(1+r)=1$, in which case $C_t = C \forall$t: This simplifies the analysis, while not changing any of the key points.

- The constant level of consumption is:

$$C = \sum_{t=1}^{T} w_t(1-\tau_t)h_t(1+r)^{T-t} \Big/ \sum_{t=1}^{T} (1+r)^{T-t}. \quad (3)$$

- Here, w_t and τ_t are the wage and tax rates in period t, respectively.

- The final component of the model is the human capital production function.
- In order to obtain relatively simple and intuitive expressions for labour supply elasticities, I begin by assuming an extremely simple process:

$$w_t = w\left(1 + \alpha \sum_{s=1}^{t-1} h_s\right) \quad t = 2, \ldots, T; \quad w_1 = w. \tag{4}$$

- Here, w is the initial skill endowment.
- The term $\sum_{s=1}^{t-1} h_s$ is total work experience up until the start of period t.
- The parameter α maps work experience into human capital.
- This linear in experience specification has the analytically convenient feature that an extra unit of labour supply at time t raises the wage by αw in all future periods from $t+1$ to T.

- Given the model in (1)–(4), the first-order conditions of the worker's optimization problem imply the following:

$$\beta h_t^{\gamma} / C^{\eta} = w_t(1 - \tau_t) + \alpha w F_t \quad t = 1, \ldots, T, \tag{5a}$$

- where

$$F_t \equiv \sum_{s=t+1}^{T} \frac{h_s(1 - \tau_s)}{(1 + r)^{s-t}} \quad F_T = 0. \tag{5b}$$

- If $\alpha = 0$, then there is no human capital accumulation via returns to work experience and the standard model with exogenous wages is obtained.

- In that case, the following condition holds:

$$\beta h_t^{\gamma} / C^{\eta} = w_t(1 - \tau_t) \quad t = 1, \ldots, T. \tag{6}$$

- This is the familiar 'MRS' condition that equates the marginal rate of substitution between consumption and leisure to after-tax wage.

- But once I include human capital, as in (4), the opportunity cost of time (OCT) is the after-tax wage plus the effect of an extra hour of work at time t on the present value of after-tax earnings in all future periods ($\alpha w F_t$).

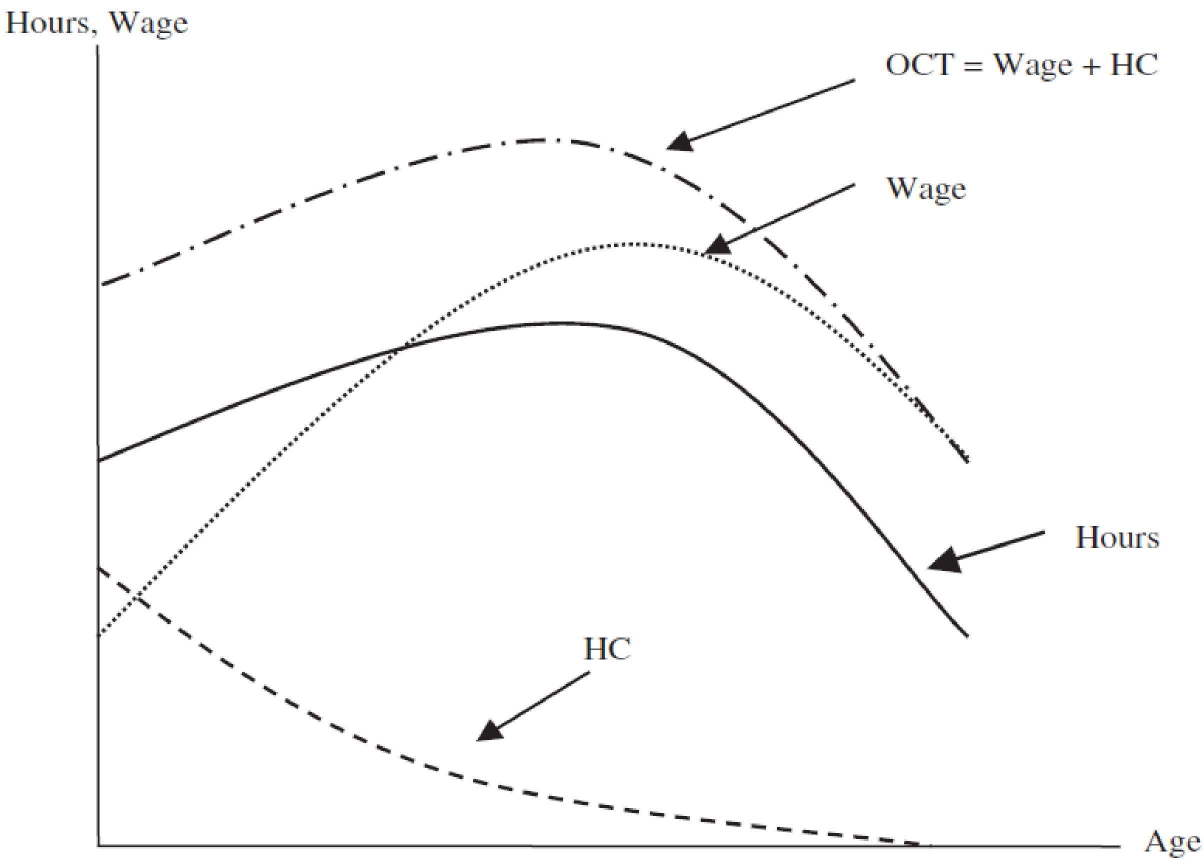
- An important econometric implication of $\alpha > 0$ becomes obvious if (5) is used to derive the familiar labour supply expression:

$$\ln h_t = \frac{1}{\gamma}\ln[w_t(1-\tau_t) + \alpha w F_t] + \frac{\eta}{\gamma}\ln C - \frac{1}{\gamma}\ln\beta. \qquad (7)$$

- Following MaCurdy (1981), a large literature estimates (7) under the assumption that $\alpha = 0$, often attempting to deal with endogenous wages using instrumental variables procedures.

- The classic papers by Browning et al. (1985), Altonji (1986) and Blundell and Walker (1986) mentioned in the introduction all take this approach.

Fig. 1. Hours, Wages and Price of Time over the Life-cycle



Notes. This Figure plots the components of the first-order condition for labour supply generated by the life-cycle model with human capital: $\frac{\beta h_t^{\gamma}}{C^{\eta}} = w_t(1-\tau_t) + \alpha w F_t$: Here, $Wage = w_t(1-\tau_t)$; the 'human capital term' $\mathrm{HC} = \alpha w F_t$; and the 'opportunity cost of time' $\mathrm{OCR} = \mathrm{Wage} + \mathrm{HC}$. Note that the term HC captures the return to an hour of work experience, in terms of increased present value of future wages.

- The reason this literature finds inelastic labour supply is illustrated in Figure 1.
- This Figure depicts typical paths of wages and hours over the life-cycle.
- Wages follow a familiar inverted U-shape (Mincer, 1958), rising quickly for young workers, peaking in middle age and declining as workers approach retirement.
- Hours of work also rise at young ages but, in sharp contrast to wages, they rise quite slowly.
- If our model is (7) with $\alpha = 0$, and it is observed that hours increase much more slowly than wages in the first half of the life cycle, then one is essentially forced to the conclusion that labour supply is quite inelastic.

1.1. The Frisch Elasticity

- Now I compare labour supply elasticities with respect to tax changes implied by the standard model vs. the human capital model.

- Consider first the Frisch elasticity. The Frisch is obtained from the total differential of (5) given a transitory tax change at time t:

$$\gamma \beta h_t^{\gamma-1} \times \mathrm{d}h_t = w_t C^\eta \times \mathrm{d}(1-\tau_t) + [w_t(1-\tau_t) + \alpha w F_t]\eta C^{\eta-1} \times \mathrm{d}C. \qquad (8)$$

- The Frisch holds the marginal utility of consumption fixed, which in the present case is equivalent to holding consumption itself fixed. That is, set $dC = 0$.

- Then from (8), the following is obtained:

$$e_{Ft} = \left.\frac{\mathrm{d}\ln h_t}{\mathrm{d}\ln(1-\tau_t)}\right|_{\mathrm{d}C=0} = \frac{1}{\gamma}\left[\frac{w_t(1-\tau_t)}{w_t(1-\tau_t)+\alpha\omega F_t}\right]. \qquad (9)$$

- If $\alpha = 0$ this reduces to the simple and familiar formula:

$$e_F = \left.\frac{\mathrm{d}\ln h_t}{\mathrm{d}\ln(1-\tau_t)}\right|_{\mathrm{d}C=0} = \frac{1}{\gamma}. \qquad (10)$$

- Alternatively, (9) and (10) can be obtained from partial differentiation of (7).

1.2. The Hicks Elasticity

- While the Frisch elasticity is useful for predicting effects of transitory tax changes, the Hicks and Marshall are relevant for predicting effects of permanent tax changes.

- In order to study permanent tax changes, it is convenient to assume that $\tau_t = \tau \forall t$ and rewrite (5) as:

$$\beta h_t^{\gamma} / C^{\eta} = w_t(1-\tau) + \alpha w \sum_{s=t+1}^{T} h_s(1-\tau)/(1+r)^{s-t} \quad t = 1, \ldots, T. \qquad (11)$$

- Taking the total differential (and redefining F_t appropriately), the following is obtained:

$$\gamma\beta h_t^{\gamma-1} \mathrm{d} h_t = \left(w_t C^{\eta} + \alpha w \frac{F_t}{1-\tau} C^{\eta} \right) \mathrm{d}(1-\tau) + [w_t(1-\tau) + \alpha w F_t] \eta C^{\eta-1} \mathrm{d} C. \qquad (12)$$

- The Hicks elasticity is obtained by setting dC so that a tax change does not alter a worker's consumption directly.
- That is, a compensating transfer is implemented on the right-hand side of the lifetime budget constrain (3), such that it continues to hold with equality at the initial hours vector fhtgT t¼1.
- Then, any change in consumption must be induced by a change in hours, and consumption is unchanged if hours remain at their initial level.
- Thus, the following is obtained:

$$\mathrm{d}C = \mathrm{d}C_H \equiv w_t(1-\tau) \times \mathrm{d}h_t + \alpha w F_t \times \mathrm{d}h_t. \tag{13}$$

- Using (12) and (13), the following is obtained:

$$e_{Ht} = \left.\frac{\mathrm{d}\ln h_t}{\mathrm{d}\ln(1-\tau_t)}\right|_{\mathrm{d}C=\mathrm{d}C_H} = \frac{1}{\gamma - \eta\left(C_t^*/C\right)}, \qquad (14a)$$

- where

$$C_t^* \equiv w_t(1-\tau)h_t + (\alpha w h_t)F_t. \qquad (14b)$$

- The variable C_t^* could be called 'effective' earnings at time t.

- It is actual earnings at t plus the present value of marginal future earnings obtained due to human capital investment at t.

- If $\alpha = 0$ then (14) reduces to:

$$e_H = \left.\frac{\mathrm{d}\ln h_t}{\mathrm{d}\ln(1-\tau_t)}\right|_{\mathrm{d}C=\mathrm{d}C_{H(\alpha=0)}} = \frac{1}{\gamma - \eta}, \qquad (15)$$

- which is, as noted by MaCurdy (1983), the Hicks elasticity in the standard model.

- Alternatively, (15) can be derived directly using the argument in Keane and Rogerson (2015).

- Equation (6) implies $\ln(h_t/h_1) = (1/\gamma)\ln(w_t/w_1)$.

- Thus, any policy that alters labour supply incentives but that holds the wage profile $\{w_t\}_{t=1,\ldots,T}$ fixed must alter hours proportionately at all ages.

- Furthermore, a proportional increase in hours at all ages, all else equal, increases C by exactly the same proportion.

- Thus $\frac{\partial lnC}{\partial lnh_t} = 1 \forall t$. The following is then obtained from (7):

$$\left.\frac{\mathrm{dln}h_t}{\mathrm{dln}(1-\tau)}\right|_{comp} = \frac{1}{\gamma} + \frac{\eta}{\gamma}\frac{\partial \ln C}{\partial \ln h_t}\left.\frac{\mathrm{dln}h_t}{\mathrm{dln}(1-\tau)}\right|_{comp} \quad \Rightarrow \quad \left.\frac{\mathrm{dln}h_t}{\mathrm{dln}(1-\tau)}\right|_{comp} = \frac{1}{\gamma-\eta}. \qquad (15')$$

- Comparing (9) and (14), the Hicks elasticity at age t exceeds the Frisch if:

$$\frac{1}{\gamma - \eta\left(C_t^*/C\right)} > \frac{1}{\gamma}\left[\frac{w_t(1-\tau_t)}{w_t(1-\tau_t) + \alpha w F_t}\right] \quad \Rightarrow \quad \alpha w F_t > \frac{(-\eta)}{\gamma}\frac{C_t^*}{C} w_t(1-\tau_t). \qquad (16)$$

- Strikingly, in the case of no income effects ($\eta = 0$) this inequality must hold, so the Hicks elasticity must exceed the Frisch.

- This is because permanent tax changes have a larger effect on the price of time than transitory and, hence, a larger pure substitution effect.

- But income effects work in the opposite direction.

- Thus, in general, (16) implies that the Hicks elasticity will exceed the Frisch if the return to human capital investment (awF_t) is sufficiently large relative to the product of the income effect and effective earnings ($-\eta C_t^*$).

1.3. The Marshallian Elasticity

- Finally, consider the Marshallian or uncompensated elasticity.
- This allows dC to incorporate both incremental earnings due to changes in hours and the income effect due to the change in the tax rate.
- In particular, the following is obtained:

$$\mathrm{d}C = \mathrm{d}C_M \equiv w_t(1-\tau)\times \mathrm{d}h_t + \alpha w F_t \times \mathrm{d}h_t + \frac{E_t}{1-\tau}\times \mathrm{d}(1-\tau), \qquad (17)$$

- where E_t denotes the present value of after-tax earnings:

$$E_t \equiv \sum_{s=t}^{T} \frac{w_s(1-\tau)h_s}{(1+r)^{s-t}}, \qquad (18)$$

- Using (12) and (17), the following is obtained:

$$e_{Mt} = \left.\frac{\mathrm{d}\ln h_t}{\mathrm{d}\ln(1-\tau_t)}\right|_{\mathrm{d}C=\mathrm{d}C_M} = \frac{1+\eta(E_t/C)}{\gamma-\eta\left(C_t^*/C\right)}, \qquad (19)$$

- Notice the denominator of the Marshall elasticity is identical to the Hicks.
- The income effect comes in through the additional $\eta(E_t/C)$ term in the numerator.
- If $\alpha = 0$ then (19) reduces to the following:

$$e_M = \frac{\mathrm{d}\ln h_t}{\mathrm{d}\ln(1-\tau_t)}\Big|_{\mathrm{d}C=\mathrm{d}C_{M(\alpha=0)}} = \frac{1+\eta}{\gamma-\eta}, \qquad (20)$$

- which is the Marshallian elasticity in the standard model.
- One can also derive (20) directly. Using (6) to substitute for ht in (3) gives:

$$\ln C = \frac{1+\eta}{\gamma-\eta}\ln(1-\tau) + \frac{\gamma}{\gamma-\eta}\ln k, \quad k \equiv \sum_{t=1}^{T} (w_t)^{\frac{1+\gamma}{\gamma}} \beta^{-\frac{1}{\gamma}} (1+r)^{T-t} \Big/ \sum_{t=1}^{T} (1+r)^{T-t}.$$

- Then (6) – or, equivalently, (7) with $\alpha = 0$ – implies that:

$$\ln h_t = \frac{1}{\gamma}\ln[w_t(1-\tau)] + \frac{\eta}{\gamma}\frac{1+\eta}{\gamma-\eta}\ln(1-\tau) + \frac{\eta}{\gamma}\frac{\gamma}{\gamma-\eta}\ln\mathrm{k} - \frac{1}{\gamma}\ln\beta,$$

- which gives:

$$\frac{\mathrm{d}\ln h_t}{\mathrm{d}\ln(1-\tau)} = \frac{1}{\gamma} + \frac{\eta}{\gamma}\frac{1+\eta}{\gamma-\eta} = \frac{1+\eta}{\gamma-\eta}. \qquad (20')$$

- Comparing (19) and (20), it is seen that the introduction of human capital again has two effects: (i) it makes the Marshallian elasticity a function of both preference and wage process parameters; and (ii) it makes the Marshallian elasticity a function of age.

- Strikingly, it is ambiguous whether the Marshallian elasticity is greater or less than the Frisch.

- Comparing (9) and (19), it is seen that the Marshall is greater than the Frisch if:

$$\alpha w F_t > \left[\frac{(-\eta)}{\gamma}\frac{C_t^*}{C} - \eta\frac{E_t}{C}\right] w_t(1-\tau_t)\frac{1}{1+\eta(E_t/C)} \quad \text{and} \quad 1+\eta(E_t/C) > 0. \qquad (21)$$

- If there are no income effects (g = 0), this condition reduces to $awF_t > 0$, so the Marshall must exceed the Frisch if there are any human capital effects.

- Again, this is because a permanent tax change has a larger effect on the OCT than a transitory one.

1.4. Quantifying the Bias From Ignoring Human Capital

- If $T = 2$ then the first-order conditions for hours are as follows:

$$\beta_1 h_1^{\gamma}/C^{\eta} = w_1(1-\tau_1) + \rho\alpha w h_2(1-\tau_2) \quad \beta_2 h_2^{\gamma}/C^{\eta} = w_2(1-\tau_2). \tag{22}$$

- Note that at $t = 1$ the OCT is augmented by the term $\rho\alpha w h_2(1-\tau_2)$, which captures the effect of an hour of work at $t = 1$ on the present value of earnings at $t = 2$.

- Here, I let tastes for work bt have an age subscript to create a source of stochastic variation in hours.

- Using (22) I obtain the following hours change equation:

$$\ln(h_2/h_1) = \frac{1}{\gamma}\ln\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1) + \rho\alpha w h_2(1-\tau_2)}\right] + \varepsilon, \tag{23}$$

- where $\varepsilon \equiv \left(\frac{1}{\gamma}\right)\ln(\frac{\beta_1}{\beta_2})$ is a shock to tastes for work (as in MaCurdy, 1981).

- A better sense of the magnitude of the problem is obtained by considering a regression of hours changes on wage changes:

$$\ln(h_2/h_1) = \Gamma \ln\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)}\right] + \upsilon. \qquad (24)$$

- Running the population regression (assuming data generated by (23)) gives:

$$\Gamma = \frac{1}{\gamma}\ln\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1) + \rho\alpha w h_2(1-\tau_2)}\right] \Big/ \ln\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)}\right]. \qquad (25)$$

- Thus, our estimate Γ of $(1/\gamma)$ is biased downwards by a factor equal to the ratio of the rate of growth in the effective wage rate to the rate of growth in the observed wage rate.

- As the Frisch, Hicks and Marshall elasticities all have the preference parameter c in their denominators, inferences about all three elasticities will be biased downwards.

- An important point is that instruments cannot be used to solve the problem of endogenous wage formation. To see this, rewrite (23) as follows:

$$\ln(h_2/h_1) = \frac{1}{\gamma}\ln\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)}\right] + \left\{\varepsilon + \frac{1}{\gamma}\ln\left[\frac{w_1(1-\tau_2)}{w_1(1-\tau_1)+\rho\alpha w h_2(1-\tau_2)}\right]\right\}. \qquad (26)$$

- The composite error term (in braces) includes the ratio of the first period wage to the opportunity cost of time.

- Any variable that affects the growth rate of after-tax wages will be correlated with this ratio as well.

- This invalidates any instrument that predicts wage growth.

2. Simulations of a Simple Two-period Model

2.1. Two-period Model Calibration

- Turning to the wage function, in contrast to the simple function assumed for analytical convenience in Section 1, (4), here we assume the more realistic:

$$w_2 = w_1 \exp\left(\alpha h_1 - \kappa \frac{h_1^2}{100} - \delta\right) \Rightarrow \ln w_2 = \ln w_1 + \alpha h_1 - \kappa \frac{h_1^2}{100} - \delta. \qquad (27)$$

- This is a Mincer-type earnings function, with w1 as the skill endowment, a quadratic in hours (experience), and a depreciation term δ, which causes earnings to fall if a person does not work sufficient hours in period one.

- Keane and Wolpin (1997) find depreciation is important.

- I calibrate the model so that: (i) a person must work at least part-time at $t = 1$ for the wage not to fall at $t = 2$; and (ii) the return to additional work falls to zero at $h = 200$.

- Given these constraints, the wage function reduces to:

$$w_2 = w_1 \exp\left(\alpha h_1 - \frac{\alpha}{4}\frac{h_1^2}{100} - \frac{175}{4}\alpha\right). \tag{28}$$

- Thus, the single parameter α determines how work experience maps into human capital.

2.2. Standard Model Without Human Capital

- Now I use the two-period model to simulate labour supply elasticities.
- Table 1 gives results for models with $\eta = -0.75$ (the Imai and Keane (2004) estimate).
- The first four rows show results for the standard model with no human capital (a = 0). The lower rows show results for progressively higher values of a.
- The columns correspond to different values of c. The left panel of Table 1 shows elasticities with respect to temporary tax changes at t = 1.
- The first two rows show uncompensated (total) and compensated (Hicks) elasticities.
- The next two rows show the Frisch elasticity calculated in the conventional way as hours growth divided by wage growth.
- The right panel of Table 1 shows elasticities with respect to permanent tax changes (i.e. changes that apply in both periods).

Table 1
Labour Supply Responses to Tax Changes, Case of $\eta = -0.75$

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	Elasticity	0	0.25	0.5	1	2	4	0	0.25	0.5	1	2	4
0	Total		1.570	0.835	0.445	0.235	0.122		0.249	0.199	0.142	0.090	0.052
	Compensated		2.059	1.222	0.721	0.410	0.223		0.990	0.792	0.566	0.361	0.209
	Frisch		4.060	2.010	1.000	0.499	0.249						
	Frisch (Δtax)		4.060	2.010	1.000	0.499	0.249						
0.001	Total	7.784	1.278	0.733	0.408	0.220	0.116	0.212	0.236	0.194	0.140	0.090	0.052
	Compensated	8.192	1.731	1.104	0.675	0.392	0.215	0.841	0.935	0.770	0.558	0.358	0.208
	Frisch	3.200	0.430	0.221	0.109	0.053	0.025						
	Frisch (Δtax)	−5.203	−0.839	−0.478	−0.263	−0.141	−0.074						
0.003	Total	2.267	0.891	0.572	0.341	0.192	0.103	0.223	0.220	0.186	0.137	0.089	0.052
	Compensated	2.663	1.297	0.917	0.596	0.357	0.200	0.883	0.874	0.739	0.546	0.354	0.207
	Frisch	1.404	0.390	0.208	0.099	0.045	0.020						
	Frisch (Δtax)	0.814	0.197	0.086	0.027	0.003	−0.002						
0.005	Total	1.185	0.645	0.450	0.285	0.166	0.091	0.231	0.213	0.181	0.135	0.088	0.052
	Compensated	1.571	1.020	0.773	0.528	0.326	0.185	0.913	0.843	0.719	0.538	0.352	0.206
	Frisch	1.019	0.359	0.195	0.090	0.038	0.015						
	Frisch (Δtax)	0.874	0.314	0.162	0.065	0.020	0.005						
0.007	Total	0.714	0.473	0.353	0.236	0.142	0.079	0.234	0.208	0.178	0.134	0.088	0.052
	Compensated	1.079	0.820	**0.657**	0.469	0.297	0.171	0.920	0.822	**0.705**	0.532	0.350	0.206
	Frisch	0.836	0.332	0.183	0.082	0.032	0.011						
	Frisch (Δtax)	0.817	0.350	0.190	0.079	0.024	0.005						

Notes. $\eta = -0.75$ is the Imai and Keane (2004) estimate. The 'Total' elasticity is the uncompensated. The 'Frisch' elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change, using the simulated data. In the rows labelled simply 'Frisch' the estimate is obtained from simulated data where the tax rate is equal in the two periods. In the rows labelled 'Frisch (tax)', the Frisch estimate is obtained using data that contain a tax cut at t = 1. The figures in bold are cases where, for my preferred value of c = 0.5, permanent tax effects exceed transitory tax effects.

Table 1
Labour Supply Responses to Tax Changes, Case of $\eta = -0.75$

(*Continued*)

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	Elasticity	0	0.25	0.5	1	2	4	0	0.25	0.5	1	2	4
0.008	Total	0.565	0.405	0.312	0.214	0.131	0.074	0.232	0.205	0.176	0.133	0.088	0.052
	Compensated	0.913	0.738	**0.606**	0.441	0.283	0.164	0.911	0.811	**0.698**	0.530	0.350	0.206
	Frisch	0.774	0.319	0.177	0.079	0.029	0.009						
	Frisch (Δtax)	0.791	0.358	0.198	0.083	0.025	0.004						
0.010	Total	0.358	0.295	0.241	0.174	0.111	0.064	0.221	0.198	0.173	0.132	0.088	0.052
	Compensated	0.663	0.597	**0.515**	0.391	0.257	0.151	0.865	0.783	**0.683**	0.525	0.349	0.206
	Frisch	0.682	0.296	0.165	0.072	0.024	0.006						
	Frisch (Δtax)	0.752	0.370	0.210	0.088	0.024	0.002						
0.012	Total	0.229	0.211	0.183	0.139	0.092	0.054	0.200	0.188	0.168	0.131	0.088	0.052
	Compensated	0.479	0.478	**0.434**	0.344	0.233	0.139	0.784	0.741	**0.663**	0.520	0.349	0.207
	Frisch	0.615	0.276	0.154	0.065	0.019	0.003						
	Frisch (Δtax)	0.727	0.380	0.220	0.091	0.023	−0.001						

Notes. $\eta = -0.75$ is the Imai and Keane (2004) estimate. The 'Total' elasticity is the uncompensated. The 'Frisch' elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change, using the simulated data. In the rows labelled simply 'Frisch' the estimate is obtained from simulated data where the tax rate is equal in the two periods. In the rows labelled 'Frisch (tax)', the Frisch estimate is obtained using data that contain a tax cut at t = 1. The figures in bold are cases where, for my preferred value of c = 0.5, permanent tax effects exceed transitory tax effects.

- Keane (2009) shows that, in the standard model, the uncompensated and compensated labour supply elasticities with respect to temporary tax changes at t = 1 are:

$$\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left(\frac{1+\eta}{\gamma-\eta}\right) - \left(\frac{\eta}{\gamma-\eta}\frac{1+\gamma}{\gamma}\frac{1}{2+r}\right) \quad \frac{\partial \ln h_1}{\partial \ln(1-\tau_1)|_{comp}} = \frac{1}{\gamma}\frac{1}{2+r} + \frac{1}{\gamma-\eta}\frac{1+r}{2+r}, \tag{29}$$

- in the two-period case.

- These equations give values of 0.84 and 1.228, which align closely with the values of 0.835 and 1.222 obtained in the simulation.

2.3. Models With a Small Human Capital Effect

2.4. Models With Plausible Human Capital Effects

2.5. Sensitivity of Results to Income Effects

- Table 2 reports results for a model with $\eta = -0.50$, the value estimated by Keane and Wolpin (2001).

- This implies weaker income effects than in the previous simulations.

- Focus again on the $\gamma = 0.50$ case. In the model without human capital ($\alpha = 0$), the uncompensated elasticity with respect to a temporary tax cut at t = 1 is, as expected, much greater than that with respect to a permanent tax cut (1.03 versus 0.50).

- But with plausible returns to work experience ($\alpha = 0.008$), the uncompensated elasticity with respect to a permanent tax cut is greater than that with respect to a temporary tax cut (0.445 *versus* 0.420).

Table 2
Labour Supply Responses to Tax Changes, Case of η=−0.50

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	Elasticity	0	0.25	0.5	1	2	4	0	0.25	0.5	1	2	4
0	Total		1.844	1.030	0.568	0.305	0.160		0.666	0.499	0.332	0.199	0.111
	Compensated		2.279	1.353	0.783	0.434	0.231		1.326	0.994	0.663	0.398	0.221
	Frisch		4.060	2.010	1.000	0.499	0.249						
	Frisch (Δtax)		4.060	2.010	1.000	0.499	0.249						
0.001	Total	7.854	1.518	0.915	0.526	0.289	0.153	0.650	0.634	0.487	0.329	0.198	0.110
	Compensated	8.265	1.923	1.226	0.735	0.415	0.223	1.289	1.262	0.971	0.656	0.396	0.220
	Frisch	3.865	0.480	0.238	0.114	0.054	0.026						
	Frisch (Δtax)	−3.734	−0.713	−0.431	−0.249	−0.138	−0.073						
0.003	Total	2.306	1.083	0.732	0.451	0.257	0.139	0.700	0.599	0.472	0.324	0.197	0.110
	Compensated	2.703	1.447	1.022	0.651	0.379	0.207	1.384	1.191	0.940	0.646	0.393	0.220
	Frisch	2.064	0.503	0.251	0.112	0.048	0.021						
	Frisch (Δtax)	1.502	0.346	0.147	0.046	0.009	−0.001						
0.005	Total	1.176	0.795	0.589	0.386	0.228	0.125	0.710	0.579	0.462	0.321	0.196	0.110
	Compensated	1.541	1.127	**0.861**	0.578	0.347	0.192	1.399	1.149	**0.920**	0.640	0.392	0.220
	Frisch	1.686	0.509	0.256	0.110	0.043	0.016						
	Frisch (Δtax)	1.438	0.493	0.242	0.092	0.028	0.006						
0.007	Total	0.654	0.583	0.472	0.329	0.202	0.113	0.641	0.552	0.452	0.319	0.196	0.110
	Compensated	0.945	0.878	**0.724**	0.513	0.316	0.177	1.259	1.095	**0.898**	0.635	0.392	0.220
	Frisch	1.536	0.507	0.257	0.107	0.038	0.013						
	Frisch (Δtax)	1.334	0.553	0.287	0.114	0.034	0.007						

Notes: η=−0.50 is the Keane and Wolpin (2001) estimate. The 'Total' elasticity is the uncompensated. The 'Frisch' elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change, using the simulated data. In the rows labelled simply 'Frisch', the estimate is obtained from simulated data where the tax rate is equal in the two periods. In the rows labelled 'Frisch (tax)' the Frisch estimate is obtained using data that contain a tax cut at t = 1. The figures in bold are cases where, for my preferred value of c = 0.5, permanent tax effects exceed transitory tax effects.

Table 2
Labour Supply Responses to Tax Changes, Case of η=−0.50

(Continued)

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	Elasticity	0	0.25	0.5	1	2	4	0	0.25	0.5	1	2	4
0.008	Total	0.489	0.495	**0.420**	0.303	0.189	0.107	0.578	0.532	**0.445**	0.318	0.197	0.110
	Compensated	0.732	0.768	**0.661**	0.482	0.302	0.171	1.134	1.054	**0.884**	0.633	0.392	0.220
	Frisch	1.500	0.505	0.256	0.105	0.036	0.011						
	Frisch (Δtax)	1.297	0.574	0.304	0.121	0.036	0.007						
0.010	Total	0.275	0.349	**0.327**	0.254	0.165	0.095	0.436	0.475	**0.424**	0.315	0.197	0.111
	Compensated	0.431	0.570	**0.541**	0.423	0.274	0.157	0.854	0.938	**0.841**	0.626	0.393	0.221
	Frisch	1.470	0.502	0.254	0.102	0.032	0.008						
	Frisch (Δtax)	1.246	0.610	0.334	0.134	0.038	0.005						
0.012	Total	0.162	0.240	**0.249**	0.210	0.143	0.084	0.315	0.400	**0.390**	0.309	0.197	0.111
	Compensated	0.255	0.407	**0.431**	0.367	0.248	0.144	0.618	0.789	**0.774**	0.614	0.393	0.222
	Frisch	1.470	0.501	0.251	0.098	0.029	0.005						
	Frisch (Δtax)	1.213	0.642	0.362	0.145	0.039	0.003						

Notes: η=−0.50 is the Keane and Wolpin (2001) estimate. The 'Total' elasticity is the uncompensated. The 'Frisch' elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change, using the simulated data. In the rows labelled simply 'Frisch', the estimate is obtained from simulated data where the tax rate is equal in the two periods. In the rows labelled 'Frisch (tax)' the Frisch estimate is obtained using data that contain a tax cut at t = 1. The figures in bold are cases where, for my preferred value of c = 0.5, permanent tax effects exceed transitory tax effects.

3. Quantitative Assessment of the Role of Human Capital

3.1. The Structure and Fit of the Imai–Keane Model

3.2. Simulations of the Model – Short-Run Labour Supply Elasticities

- Table 3 reports elasticities of current hours with respect to transitory and permanent tax changes.

- An unanticipated tax change has a (small) wealth effect but an anticipated change has no wealth effect, only a pure Frisch effect.

- As seen in Table 3, the Frisch elasticity increases substantially with age, from only 0.30 at age 20 to 1.96 at age 60. This agrees with the prediction of subsection 1.1: transitory taxes have small effects at young ages because they only affect a fraction of the OCT (i.e. the current after-tax wage, not returns to human capital investment).

- But, as workers age and the return to human capital investment falls, the Frisch elasticity increases.

Table 3
Short-run Labour Supply Responses to Taxes in the Imai–Keane Model

	Transitory		Permanent (unanticipated)	
Age	Unanticipated	Anticipated (Frisch)	Uncompensated (Marshall)	Compensated (Hicks)
20	**0.30**	**0.30**	0.14	**0.64**
25	**0.36**	**0.36**	0.12	**0.54**
30	**0.44**	**0.44**	0.12	**0.48**
35	0.52	0.52	0.10	0.46
40	0.64	0.66	0.14	0.46
45	0.76	0.84	0.20	0.56
50	0.94	1.06	0.46	0.84
55	1.24	1.44	1.06	1.44
60	**1.74**	**1.96**	1.88	**2.09**

Notes: All figures are elasticities of current hours with respect to tax changes. The 'transitory' increase only applies for one year at the indicated age. In the 'anticipated' case this has no wealth effect, so it is a pure Frisch effect. The 'permanent' tax increases take effect (unexpectedly) at the indicated age and last until age 65. In the 'compensated' case the proceeds of the tax (in each year) are distributed back to agents in lump sum form. Figures in bold are cases where permanent tax effects exceed transitory tax effects.

3.3. Simulations of the Model – Long-Run Tax Effects

- In Table 4, I use the Imai–Keane model to simulate the impact of a permanent change in the tax rate on labour earnings that starts at age 20 and lasts till age 65.
- Consider first the elasticity of lifetime hours from age 20 to 65.
- According to Table 4, the long-run compensated elasticity is a very substantial 1.3.
- Notably, the Hicks elasticity implied by the Imai–Keane parameter estimates in the standard model without human capital is, from (15), eq$e_H = \frac{1}{\gamma - \eta} = \frac{1}{0.262+0.736} \approx 1.0$
- Thus, the human capital mechanism amplifies the compensated elasticity of lifetime hours by 30% (from 1.0 to 1.3).

Table 4
Lifetime Effects of a Permanent Tax Increase on Labour Supply

Age	Uncompensated	Compensated
20	0.14	0.64
30	0.14	0.66
40	0.18	0.84
45	0.24	1.14
50	0.42	1.74
60	1.82	4.00
Lifetime hours (Ages 20–65)	0.40	1.32

Notes. This Table compares the baseline simulation of the Imai and Keane (2004) model with an alternative scenario where the tax rate on earnings is permanently higher. The increase is in effect from the first period (age 20) until the terminal period (age 65). The Table reports both the uncompensated case and the case where the proceeds of the tax (in each year) are distributed back to agents in lump sum form.

- Table 4 also reports elasticities of hours at selected ages.
- A permanent tax hike reduces hours much more at older ages than at young ages. The effect of the tax grows with age for two reasons:
- First, as workers get older the after-tax wage makes up a larger fraction of the opportunity cost of time.
- Second, a permanent tax rise slows the rate of human capital accumulation, which produces a ‘snowball’ effect on wages.

3. Quantitative Assessment of the Role of Human Capital

5. Conclusion