

# Other Ways to Define Occupations

(Paretian Distributions and Income Maximizations)

Benoit Mandelbrot (1962)  
*Quarterly Journal of Economics*, 76(1): pp. 57–85.

Econ 350, Winter 2023

- Assume that each individual must choose *one* of  $N$  possible “occupations,”  $P_n(1 \leq n \leq N)$  with utility  $U_n$ .
- Model of multinomial choice.

- Within the total population, the distributions of the various  $U_n$  are random.

# Linear Factor Analysis of the Rental Price of an Undissoluble Bundle of Abilities

- $U_n(1 \leq n \leq N)$  is the rental price which the occupation  $P_n$  is ready to pay for the use of a man's abilities.
- Can be written as a nonhomogeneous *linear* form of  $F$  independent *factors*  $V_f(1 \leq f \leq F)$ , each of which is randomly distributed in the population, and “measures” one or several “abilities.”
- Then one can write:

$$U_n = \sum_{f=1}^F a_{nf} V_f + a_{n0} + a_n E_n.$$

## The Regions of Acceptance of the Different Offers

- The prospective employee's problem is to determine the  $n$  that maximizes the nonlinear function  $U_n$  of  $n$ .
- Let the random variable  $W_n$  designate the incomes *accepted* from the occupation  $P_n$ , and
- Let  $W$  designate all accepted incomes (observed income distributions).
- The region of acceptance  $R_n$  of the offer  $U_n$  will be the intersection of the  $N - 1$  half-hyperspaces defined by the equations:

$$U_n - U_i \geq 0 \quad (i \neq n).$$

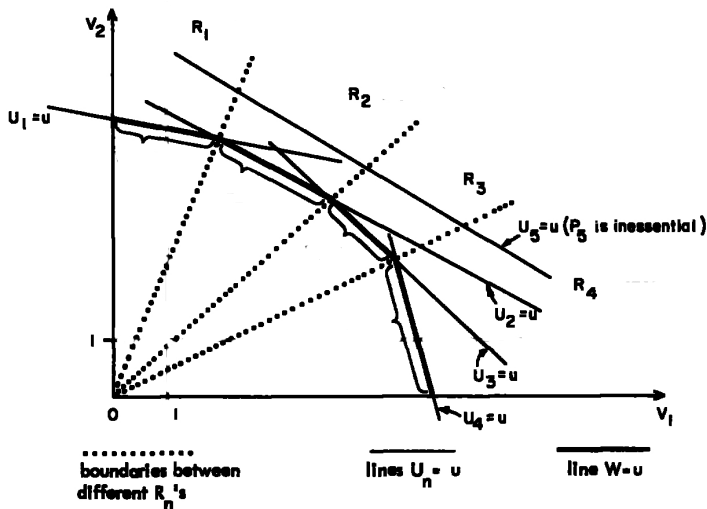
## Examples

### *Homogeneous offers.*

- The simplest case, from the viewpoint of  $R_n$  and of income iso-surfaces, occurs when the offers are homogeneous forms of the  $V_f$ .
- Then, the accepted  $P_n$  will obviously depend only upon the relative values of the factors, that is upon what we may call the “factor profile” of the individual.
- For the case  $F = 2$ , the typical shape of the regions  $R_n$  and of the surfaces  $W = u$ , is given in Figure 1.

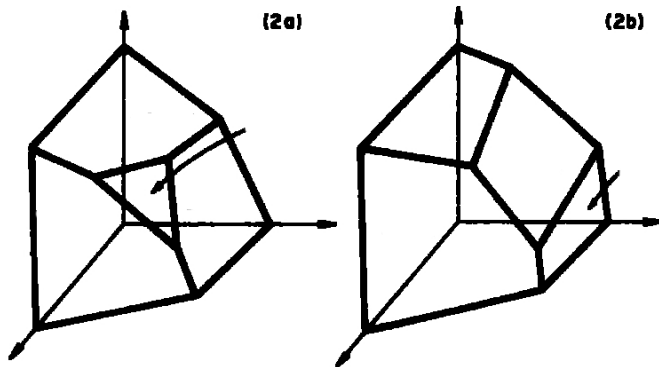


Figure 1: Example of Regions of Acceptance in the Case of Two Homogeneous Factors

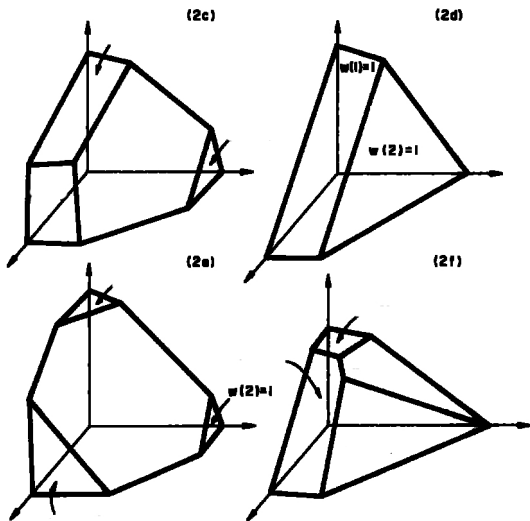


- Figures 2a to 2e are such that, if one factor is present in very large quantity, it fully determines the chosen occupation, so that changes in the values of other factors are irrelevant.
- But in Figure 2f, we see a case where the occupation is not determined by the value of the high factor  $V_1$  alone, and is greatly influenced by  $V_2$  and  $V_3$ , which are present in small quantities; small changes in those other factors can totally modify the value of  $n$  that will be chosen.

**Figure 2:** Six Miscellaneous Examples of the Shape of the Surface  $W = u$ , in the Case of Three Homogeneous Factors



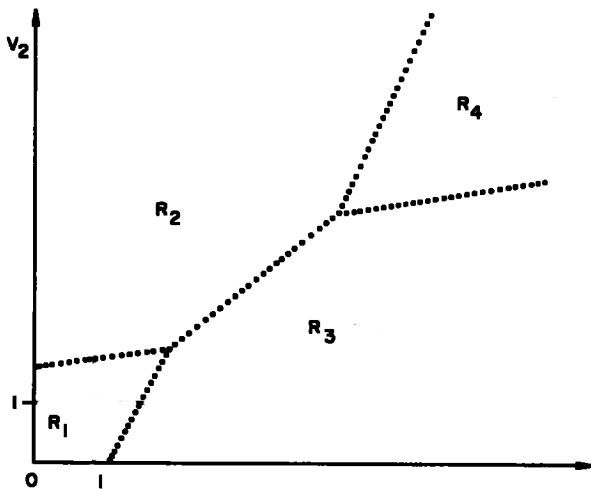
**Figure 2:** Six Miscellaneous Examples of the Shape of the Surface  $W = u$ , in the Case of Three Homogeneous Factors



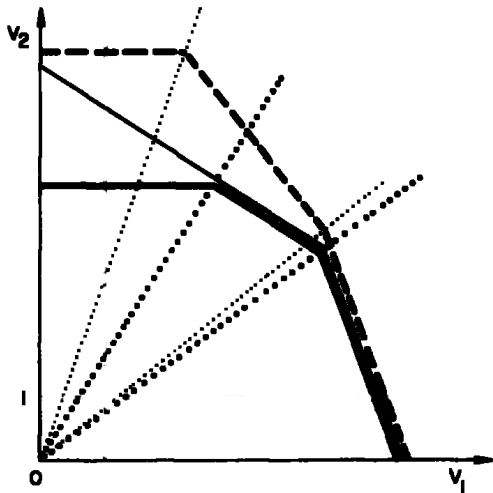
### *Linear and nonhomogeneous offers.*

- In this case, the regions of acceptance  $R_n$  need no longer be open cones.
- Some of these regions may be closed, leading to bounded incomes.
- The corresponding occupations will clearly be those which are not very sensitive to the values of the factors.
- On the contrary, fanning-out regions  $R_n$  are quite sensitive to the values of the factors.
- An example is given in Figure 3.

**Figure 3:** Example of Regions of Acceptance in the Case of Two Inhomogeneous Factors



**Figure 4:** Example of the Change in the Form of the Curve  $W = u$ , as Factor Loadings Are Modified



**solid bold line:**  
situation before  
the changes in the  
factor loadings

**solid thin line:**  
situation after a  
change in the fac-  
tor loadings of  $U_1$   
(which thereby be-  
comes inessential)

**bold dashed line:**  
situation after the  
changes in the fac-  
tor loadings of  
 $U_1$  and  $U_2$