A Theory of Intergenerational Mobility

Becker, Kominers, Murphy, and Spenkuch (*JPE*, 2018)

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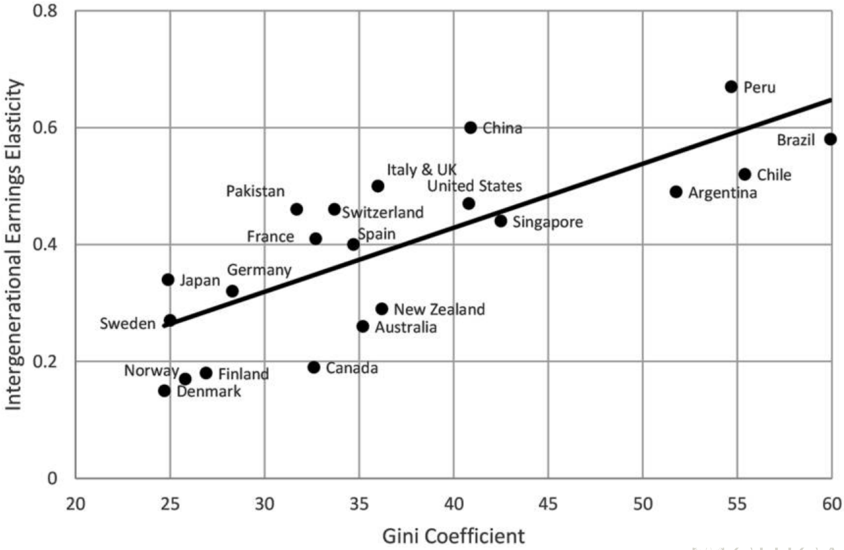
I. Introduction

Figure 1: The Great Gatsby Curve



II. A Model of Intergenerational Mobility

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- Goal: To understand how the persistence of economic status depends on where people originate in the distribution of human capital and income.
- Build on Cunha et al. (2007) to relax previous assumption in Becker-Tomes that all parents are equally good in investing in children.
- Doing so creates a richer model of intergenerational mobility more concordant with facts.
- Model preserves or accentuates status across generations through complementarities (parents' human capital affects production of investments in child human capital).
- Complementarity may $\Rightarrow$ convexity in impact of family influence on child outcomes.

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- Can lead to separation of classes if high levels of human capital $\Rightarrow$ disproportionate returns in the market and/or in nonmarket production.
- Shows how inequality $\uparrow \Rightarrow$ IGE $\uparrow$.
- Related to Durlauf and Benabou neighborhood models.
- An alternative explanation (both may be at work).
- Poverty traps may persist even with credit constraints.

- Parental preferences depend on parents' own consumption, z and on the well-being of their children

$$\underbrace{V(I_p)}_{\substack{\text{parental}\\ \text{utility}}} = \underbrace{u(z)}_{\substack{\text{utility}\\ \text{of parent}}} + \delta \underbrace{U_c(\bar{I}_c)}_{\substack{\text{child}\\ \text{utility}}} . \tag{1}$$

- Intergenerational discount factor $\delta \in (0, 1)$: parents' degree of altruism toward their children
- I_p: parental monetary resources
- $\bar{I}_c$: expected resources of children

- Assume
  $u' > 0$, $u'' < 0$, $U_c' > 0$, $U_c'' < 0$, and $\lim_{\bar{I}_c \to 0} U_c' = \infty$.
- Thus, all parents want to invest at least a little bit in the human capital of their children as long as $\delta > 0$.

- Earnings: assume an isoelastic relationship with human capital, H:

$$E = rH^{\sigma}\varepsilon. \tag{2}$$

- r: Price of human capital in society.
- σ: Elasticity of earnings with respect to human capital.
- ε: Luck; $\varepsilon \perp\!\!\!\perp H$, $E(\varepsilon) = 1$.
- ε: Unknown to the parent at time investments in children made.
- Will discuss right skew of earnings (see, e.g., Sattinger, 1979).

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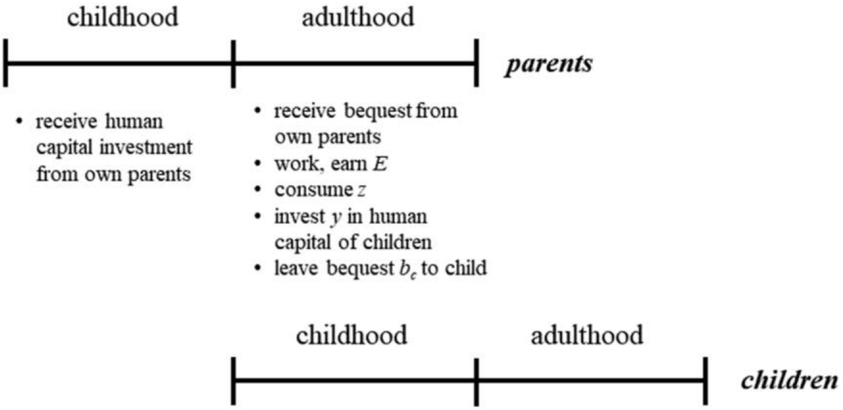
Figure 2: Timing of Actions



- Follow Cunha and Heckman (2007) and Cunha et al. (2010).
- General function for the production of children's human capital:

$$H_c = F(y, G, A_c, H_p, \nu_c). \tag{3}$$

- H_c and H_p are the human capital of children and parents, respectively.
- y denotes parental investments in children;
- G denotes government spending on education.
- A_c stands for the abilities of children.
- ν_c records other influences (assumed fixed genetically).

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- Considerable evidence suggests that parental human capital and investments in children are complements (see, e.g., Heckman and Mosso, 2014).
- To make analysis tractable, specialize (3) to a Cobb-Douglas production function of only A_c, y, and H_p:

$$H_c = A_c y^{\alpha} H_p^{\beta}. \tag{4}$$

- Becker-Tomes: $A_c = a + bA_p + U_c, U_c \perp\!\!\!\perp A_p$.
- This paper: $A_c \equiv 1$ (shuts down genetic link).
- Perfect capital market: People lend or borrow at R_k.
- Allows for negative bequests.

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- b_c: bequests given to children.
- b_p: bequests adults get from parents.
- R_y: return on investment in children.
- R_k: return on capital.
- Parents choose consumption level z, investments y, and bequests b_c in order to maximize V subject to the production function of human capital in (4), the determinants of earnings in (2), and the lifetime budget constraint

$$z + \frac{b_c}{R_k} + y = I_p \equiv E_p + b_p. \tag{5}$$

- Combining the first-order conditions for y and b_c, efficient investment in children's human capital:

$$R_y \equiv \frac{d\bar{I}_c}{dy} = r\alpha\sigma y^{\alpha\sigma - 1} H_p^{\beta\sigma} = R_k. \tag{6}$$

- Thus, if capital markets are perfect, parents invest in their children's human capital until the marginal return on these investments equals the exogenously given return on capital.
- Use (6) to solve for the optimal investment:

$$y^* = \left(\frac{r\alpha\sigma}{R_k}\right)^{1/(1-\alpha\sigma)} H_p^{\beta\sigma/(1-\alpha\sigma)}. \tag{7}$$

- When $\beta = 0$, no impact of parental income (IGE=0).
- When $\beta > 0$, $\sigma \uparrow \Rightarrow$ greater dependence of child income on family income.

- By choosing optimal investments that depend positively on parental human capital, parents affect the equilibrium mapping between their own human capital and that of their children.
- Use equation (7) to eliminate y from the production function for H_c.
- The result differs from the production function in (4):

$$H_c = \left(\frac{r\alpha\sigma}{R_k}\right)^{\alpha/(1-\alpha\sigma)} H_p^{\beta/(1-\alpha\sigma)}. \tag{8}$$

- The influence that the family has on the earnings of children.
- Combine equations (2) and (8) to obtain:

$$\log(E_c) = \frac{1}{1-\alpha\sigma_c}\log(r_c) + \frac{\alpha\sigma_c}{1-\alpha\sigma_c}\log\left(\frac{\alpha\sigma_c}{R_k}\right) + \frac{\beta\sigma_c}{1-\alpha\sigma_c}\log(H_p) + \log(\varepsilon_c). \quad (9)$$

- r_c: Reward per unit human capital for child.
- Subscripts indicate the respective generation.

- Aside from σ_c, the elasticity between human capital and earnings in the children's generation, the coefficients in equation (9) are all determined by parameters in the production function for H_c and by the way these parameters affect parental investments in children through equation (7).

- Use (2) to substitute for H_p, the above relationship can be transformed into an equation that describes the intergenerational transmission of earnings:

$$\log(E_c) = \mu + \frac{\beta}{1-\alpha\sigma_c}\frac{\sigma_c}{\sigma_p}\log(E_p) + \tilde{\varepsilon}, \tag{10}$$

where

$$\mu \equiv \frac{1}{1-\alpha\sigma_c}\log(r_c) - \frac{\beta}{1-\alpha\sigma_c}\frac{\sigma_c}{\sigma_p}\log(r_p) + \frac{\alpha\sigma_c}{1-\alpha\sigma_c}\log\left(\frac{\alpha\sigma_c}{R_k}\right)$$

and

$$\tilde{\varepsilon} \equiv \log(\varepsilon_c) - \frac{\beta}{1-\alpha\sigma_c}\frac{\sigma_c}{\sigma_p}\log(\varepsilon_p).$$

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- From equations (8) and (10), in the steady state (when $\sigma_c = \sigma_p$) the IGE equals the intergenerational human capital elasticity:

$$\frac{d \log E_c}{d \log E_p} = \frac{d \log H_c}{d \log H_p} = \frac{\beta}{1 - \alpha\sigma}. \tag{11}$$

- **Question**: *Compare this result with that in Becker-Tomes (1986). Discuss the role of imperfect capital markets and heritability in that model compared to this model.*

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III. How Changes in the Marketplace Affect Intergenerational Mobility

- Equation (11) shows that the IGE depends positively on the production function parameters α and β, as well as the elasticity of earnings with respect to human capital (σ).
- It does not, however, depend on r, the economywide "base price" of human capital.
- As a result, the model predicts that changes in the marketplace that simply stretch the income distribution do not affect the IGE, that is,

$$\frac{d}{dr}\left(\frac{d \log E_c}{d \log E_p}\right) = 0. \tag{12}$$

- However,
$$\frac{d}{d\sigma}\left(\frac{d\log E_c}{d\log E_p}\right) > 0.$$
- Complementarities affect the upper tail (rising return to skill raises IGE):
- $$\sigma_c \uparrow \Rightarrow \text{ IGE } \uparrow$$
$$\frac{d}{d\alpha}\left(\frac{d\log E_c}{d\log E_p}\right) > 0$$
$$\alpha \uparrow \Rightarrow \text{ IGE } \uparrow$$
- Greater the productivity parameter of parental investment.

IV. Intergenerational Dynamics and the Long-Run Evolution of Dynasties

- Suppose $H_c = k + \tilde{\beta} H_p + \nu_c$. $\tilde{B} < 1$. This is a traditional specification.
- $\Rightarrow$ Convergence.

Figure 3: Intergenerational Dynamics in Linear Models (Convergence)

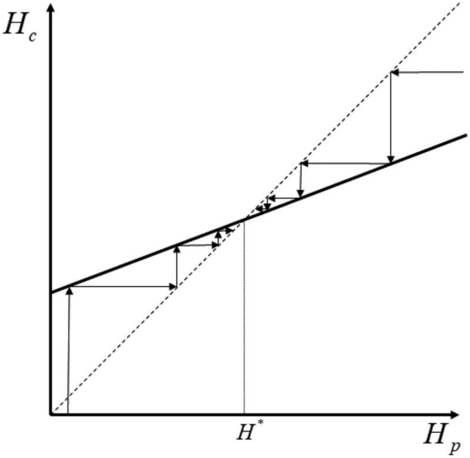

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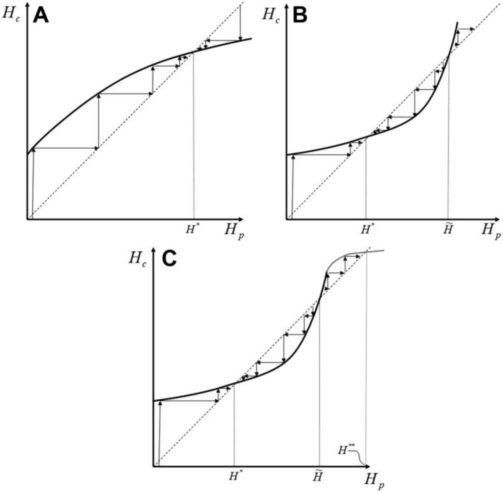
Figure 4: Intergenerational Dynamics in Becker et al.'s 2018 Model



- Examine the A, B, & C components of Figure 4.
  - A $\Rightarrow$ convergence.
  - Above $\tilde{H}$ in B $\Rightarrow$ divergence.
  - B: $\ln H_c = K + \frac{\beta}{1-\alpha\sigma} H_p$
  - C: Two stable classes. Formation of classes.
- See Durlauf (1996) and Durlauf & Sheshadri neighborhood sorting is another reason for bifurcated equilibrium.

- Both β, σ, and α contribute to bifurcated equilibria.

Summary

- IGE: Income Inequality produced by family factors.

Appendix

A Model Without Credit Market
Solon Model (2004)

Link to the Solon Model

- No lending or borrowing
- No bequests
- First, the budget constraint assumes families must allocate all after-tax lifetime income to either parental consumption (z) or investment in the child (y):

$$(1-\tau)E_p = z + y \tag{13}$$

- E_p is money income of family (no lending or borrowing)
- τ is the tax rate

A Model Without Credit Market

- Human capital of the child (θ_1) is produced by a semi-log production function:

$$\underbrace{H_c}_{\text{human capital of child}} = \underbrace{\psi}_{\text{productivity of the transmission process}} \log(y + \underbrace{G}_{\text{governmental investment}}) + \underbrace{\nu_c}_{\text{child initial endowment}} \tag{14}$$

- Observe y and G are perfect substitutes. (A property of many models.)
- $E_p = r_p H_p + \underbrace{L_p}_{\text{Luck}}.$
- Abstracts from "Luck" $L_p \perp\!\!\!\perp H_p$.

- Child endowments follow AR(1) process:

$$\nu_c = \kappa + \lambda \nu_p + \eta_c;$$
$$\eta_c \perp\!\!\!\perp \nu_p \tag{15}$$

λ is between 0 and 1 and η_c is white noise (Becker-Tomes, 1986).

- Earnings equation:

$$\log(E_c) = \mu + r_c H_c \tag{16}$$

- r_c is the return to a unit of human capital for child.

- The family maximizes $V = (1-\delta)\log(z) + \delta\log(E_c)$.
- δ measures the degree of altruism towards the child.
- Solon (2004) models provision of governmental goods: $G/[(1-\tau)E_p] = \varphi - \gamma\log(E_p)$.
- $\gamma > 0$ ratio of government investment to after-tax income is decreasing in income.
- γ: a measure of the progressivity of government spending on children.
- By maximizing the utility function with respect to parental investment and collecting terms, one arrives at

$$\log(E_c) = \mu^* + [(1-\gamma)\psi r]\log(E_p) + r\nu_c \tag{17}$$

  The form of the standard IGE regression.
- ν_c correlated with $\ln(E_p)$ through common shock ν_p.
- $\nu_c \not\perp\!\!\!\perp E_p$.

- Substitute for ν_c using (15)
- In steady state, $\text{Var}(\nu_C) = \text{Var}(\nu_P)$

$$\text{IGE } \eta = \frac{(1-\gamma)\psi r + \lambda}{1 + (1-\gamma)\psi r \lambda} \quad \uparrow \text{ as } \lambda \uparrow, \psi \uparrow, r \uparrow, \gamma \downarrow . \qquad (18)$$

- Estimated IGE (and intergenerational correlation) greater if
  1. the heritability coefficient λ is higher so ability is more highly correlated across generations,
  2. ψ is higher so that the human capital accumulation process is more productive,
  3. earnings returns to human capital are higher so r is larger, or
  4. governmental investment in human capital is less progressive so γ is smaller.

- Cross section variance of log E (steady state)

$$\text{Var}(\ln E) = \frac{[1 + (1-\gamma)\psi r \lambda] r^2 \,\text{Var}(\nu)}{[1 - (1-\gamma)\psi r \lambda](1-\lambda^2)[1 - (1-\gamma)\psi r]^2}$$

- $\text{Var}(\nu)$ is variance in heritability of endowments.
- $\text{Var}(\ln E)$

$$\uparrow \text{ in } \lambda, \psi, r, 1-\gamma$$

- New term not in β is $\text{Var}(\nu)$.
- Can show that out of steady state as income inequality $\uparrow$, $\beta \uparrow$.
- Note crucial role for r in Solon.
- Absence of any important role in Becker et al.