Mating Markets

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1 Introduction

- The economic analysis of the “market for marriage” has a long tradition, marked by the seminal contributions of Becker (1973, 1974).

- Two more recent developments have made it the focus of renewed interest: new models of household behavior, and a class of tractable specifications for econometric work.

- These two threads have converged to generate richer predictions and empirical applications.

- The aim of the current survey is to provide an overview of these recent advances.

- We mostly concentrate on bipartite, one-to-one matching, e.g. on the traditional situation of marriage between one man and one woman.

2 Matching Markets: Theory

2.1 The Marital Surplus

- From a theoretical perspective, the analysis of marriage relies on the simple but fundamental intuition that marriage generates a **surplus**: when married, two individuals can both achieve a higher level of well-being than they would as singles.

- The exact nature of the surplus is complex; depending on the issues under consideration, it may be described in different ways.

- Non-monetary aspects, including what is usually called love, certainly play an important role.

2.1.1 Consumptions technology and domestic production

- **Public goods** A first gain generated by marriage (or cohabitation) stems from the existence of commodities that are publicly consumed within the household.

- The cost of providing such commodities is split between members, which generates economic gains.

- These can be illustrated by a simple example in a two-person framework; extending the argument to larger households is straightforward.

- Consider a two-person household consuming two commodities, one private (individual consumptions being denoted q^A; q^B) and one public (common consumption Q); utilities are Cobb-Douglas

$$u^i\left(q^i, Q\right) = q^i Q \quad \text{for} \quad i = A, B.$$

- Let x^A and x^B denote female and male income respectively, and let prices be normalized to 1.

- If single, spouses would each independently purchase (and privately consume) both commodities, leading to respective consumptions and utilities equal to

$$q^i = Q = \frac{x^i}{2} \text{ and } u_S^i = \frac{\left(x^i\right)^2}{4} \text{ for } X = A, B.$$

- If the couple reaches an efficient decision, its aggregate consumption of the private good will satisfy

$$q^A + q^B = Q = \frac{x^A + x^B}{2},$$

- resulting in utilities u_M^A and u_M^B that satisfy

$$u_M^A + u_M^B = \frac{\left(x^A + x^B\right)^2}{4}.$$

- The marital surplus is simply:

$$S = \left(u_M^A + u_M^B\right) - \left(u_S^A + u_S^B\right) = \frac{\left(x^A + x^B\right)^2}{4} - \frac{\left(x^A\right)^2}{4} - \frac{\left(x^B\right)^2}{4} = \frac{x^A x^B}{2}$$

- so that marriage has pushed up the utility possibilities frontier by $(x^A x^B)/2$ utils.

- **Economies of scale** Alternatively, marital gains may coexist with purely private individual consumptions when the family is a source of economies of scale.

- In Voena's model, for instance, individual consumptions (q^A, q^B) require total household expenditures equal to:

$$X = \left(\left(q^A\right)^\rho + \left(q^B\right)^\rho\right)^{1/\rho},$$

- where the price of the unique good has been normalized to 1.

- For $\rho > 1$, one can readily check that $X < q^A + q^B$ - the right hand side being the total cost faced by singles who would individually purchase the good.

- **Domestic Production and Specialization** Domestic production covers a large array of goods and services, from agricultural products to health care and food processing.

- Importantly, it also comprises investment in human capital-children's education being an obvious example.

- Domestic production can easily be discussed using a variant of the previous model.

- Assume that the public good is now produced from individual time, t^A and t^B respectively, according to the Cobb-Douglas production function:

$$Q = \left(0.1 + t^A\right)^{1/2} \left(0.1 + t^B\right)^{1/2}$$

- Moreover, the time not devoted to children is spent on the labor market; let w_A and w_B denote individual wages, and let us normalize the total available time to 1.

- Start with the behavior of a single parent, say A; we therefore assume that $t^B = 0$, and A's budget constraint is simply

$$q^A = w_A \left(1 - t^A\right)$$

- Then A optimally chooses

$$q_S^A = \frac{2.2}{3} w_A \text{ and } t_S^A = \frac{0.8}{3}.$$

- Considering now the household, aggregate budget constraint is:

$$q^A + q^B = w_A\left(1 - t^A\right) + w_B\left(1 - t^B\right)$$

- and efficient allocations satisfy

$$t^A = \min\left(\frac{0.7}{4} + \frac{1.1w_B}{4w_A}, 1\right),\ t^B = \min\left(\frac{0.7}{4} + \frac{1.1w_A}{4w_B}, 1\right).$$

- Individuals now specialize, as the time they each spend on domestic production depends the wage ratio w_B/w_A: the lower wage person spends more time on domestic production and less on salaried work.

- In particular, if the wage ratio is larger than 3 then $t^A = 1$: A leaves the labor market and exclusively specializes into the production of the public good.

- This specialization is a source of additional efficiency: the higher wage individual devotes more time to salaried work, while their spouse exploits their comparative advantage on domestic work.

2.1.2 Risk Sharing

- The household's ability to alleviate some market inefficiencies through bi- or multilateral agreements is another source of surplus.

- In the absence of complete insurance markets, individuals remain vulnerable to idiosyncratic shocks.

- Sharing the corresponding risk within the household potentially improves the (ex ante) welfare of all members.

- Assume for instance that household members consume a unique private good $q^i (i = A, B)$, and individual VNM utilities are CARA:

$$u^i\left(q^i\right) = -\exp\left(-s^i q^i\right)/s^i$$

- with $s^A, s^B > 0$ so that both partners are strictly risk averse.

- Each individual is endowed with a random income $\tilde{x}^i$.
- Once married, they can make ex ante efficient contracts, involving in particular risk sharing.
- For any particular realization $x = (x^A, x^B)$, of individual incomes, let $(\rho^A(x), \rho^B(x))$denote the individual consumptions.
- They are feasible if and only if

$$\rho^A(x) + \rho^B(x) = x^A + x^B. \tag{1}$$

- We call a feasible pair $(\rho^A(x), \rho^B(x))$ a *sharing rule*.

- If agents share risk efficiently, individual consumptions ρ^A *and* ρ^B only depend on total income $\bar{x} = x^A + x^B$:

Proposition 1 (Mutuality Principle) *If a sharing rule is efficient, then it only depends on the realization of total income:*

$$\rho^i(x) = \bar{\rho}^i\left(x^A + x^B\right) \quad i = A, B \tag{2}$$

for some functions $\bar{\rho}^i(\bar{x})$ *such that* $\bar{\rho}^A(\bar{x}) + \bar{\rho}^B(\bar{x}) \equiv \bar{x}$.

Proof. *Take any sharing rule* $\rho = \left(\rho^A, \rho^B\right)$ *and consider, for* $i = A, B$*:*

$$\bar{\rho}^i(\bar{x}) = \mathbb{E}\left[\rho^i\left(x^A, x^B\right) \mid x^A + x^B = \bar{x}\right].$$

$(\bar{\rho}^A(x^A + x^B), \bar{\rho}^B(x^A + x^B))$ *is clearly a sharing rule and for* $i = A, B$*:*

$$\begin{aligned} \mathbb{E}u^i\left(\bar{\rho}^i(\bar{x})\right) &= \mathbb{E}u^i\left[\mathbb{E}\left(\rho^i\left(x^A, x^B\right) \mid x^A + x^B = \bar{x}\right)\right] \\ &\geq \mathbb{E}\left[\mathbb{E}\left[u^i\left(\rho^i\left(x^A, x^B\right)\right) \mid x^A + x^B = \bar{x}\right]\right] \\ &= \mathbb{E}\left[u^i\left(\rho^i\left(x^A, x^B\right)\right)\right]. \end{aligned}$$

- Efficiency also requires that the mappings $(\bar{\rho}^A(\bar{x}), \bar{\rho}^B(\bar{x}) = \bar{x} - \bar{\rho}^A(\bar{x}))$ maximize a weighted sum of individual expected utilities:

$$\bar{\rho}^A(\bar{x}) \in \arg\max_{r(\cdot)} \left(\mathbb{E}u^A\left(r\left(\bar{x}\right)\right) \; + \mu \mathbb{E}u^B\left(\bar{x} - r\left(\bar{x}\right)\right)\right)$$

- for some $\mu > 0$.

- The first-order conditions give

$$\bar{\rho}^A\left(\bar{x}\right) = \frac{s^B \bar{x} - \ln \mu}{s^A + s^B} \quad \text{and} \quad \bar{\rho}^B\left(\bar{x}\right) = \frac{s^A \bar{x} + \ln \mu}{s^A + s^B},$$

- which results in individual expected utilities

$$\mathbb{E}u_M^A = -\frac{\mu^{s^A/(s^A+s^B)}}{s^A} \mathbb{E}\left[\exp(-s\bar{x})\right] \tag{3}$$

$$\mathbb{E}u_M^B = -\frac{\mu^{-s^B/(s^A+s^B)}}{s^B} \mathbb{E}\left[\exp(-s\bar{x})\right]$$

- where:

$$s = \frac{s^A s^B}{s^A + s^B} \Rightarrow \frac{1}{s} = \frac{1}{s^A} + \frac{1}{s^B}.$$

2.2 Mating Models: A Taxonomy

- While formal models of mating markets differ in many aspects, they all share a common feature: they consider individuals who are fundamentally heterogeneous.

- Following the standard approach of the hedonic literature, this heterogeneity can be described by a list of characteristics (or "traits").

- As a consequence, individuals typically have different valuations of the observable characteristics of potential mates.

- The fundamentals of marriage markets consist of two components: a description of the two populations, and an evaluation of the benefits that would be generated by the match of any two potential spouses.

- Any theoretical analysis of the market must answer two sets of questions:
  - Q1: the equilibrium matching patterns-who stays single, and who marries whom?
  - Q2: the equilibrium payoffs-how is the marital surplus distributed between the spouses?

- These questions have been analyzed within two different frameworks: frictionless matching theory and search models.

- The basic distinction between the two is related to the role given to frictions in the description of the market.

2.2.1 Searching and Frictionless Matching

- In search models, frictions are paramount.
- Typically, individuals each sequentially and randomly meet one person of the opposite gender; after such a meeting, they both must decide whether to settle for the current mate or to continue searching.
- The latter option involves various costs, from discounting to the risk of never finding a better partner.
- If both individuals agree to engage in a relationship, then a negotiation begins on how the surplus is to be shared.

- Matching models, on the contrary, assume a frictionless environment.
- In the matching process, each individual is assumed to have free access to the pool of all potential spouses, with perfect knowledge of the characteristics of each of them.
- Matching models thus disregard the cost of acquiring information about potential matches, as well as the role of meeting technologies of all sorts (from social media to dating sites to pure luck).

2.2.2 Utility Transfers

- Within the family of frictionless matching frameworks, a second and crucial distinction relies on the role of transfers: are partners in a match able to transfer utility to each other?

- Transfers make a fundamental difference: when available, they allow agents to “bid” for their preferred mate by offering to reduce their own gain from the match in order to increase the partner’s.

- The literature on matching has mostly focused on two polar extremes.

- In the so-called Non Transferable Utility (NTU) case, there is simply no technology enabling agents to transfer utility to any potential partner.

- When transfers are possible, the surplus created by a match must be allocated between partners.

- The answers to both questions Q1 and Q2 are inextricably linked in this framework.

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2.3 Matching Models under Transferable Utility

2.3.1. The Basic Framework

- We consider two compact sets $\mathcal{X} \subset \mathbb{R}^n$and $\mathcal{Y} \subset \mathbb{R}^m$, which respectively represent the space of female and male characteristics.

- The corresponding vectors of characteristics fully describe the agents; i.e., for any $x \in \mathcal{X}$, two women with the same vector of characteristics x are perfect substitutes as far as matching is concerned (and similarly for men).

- These spaces are endowed with measures F and G respectively; both $\mathrm{F}(\mathcal{X})$ and $\mathrm{G}(\mathcal{Y})$ are finite.

- In order to capture the case of persons remaining single within this framework, a standard trick is to "augment" the spaces by including an isolated point in each: a dummy partner ϕ_X for any unmatched man and a dummy partner ϕ_Y for any unmatched woman.

- Therefore, from now on we consider the spaces $X \coloneqq \mathcal{X} \cup \{\phi_X\}$ and $\mathrm{Y} \coloneqq \mathcal{Y} \cup \{\phi_Y\}$, where the point ϕ_X (resp. ϕ_Y) is endowed with a mass measure equal to the total measure of $\mathcal{Y}(\mathcal{X})$.

- In particular, a hypothetical matching in which all women remain single would be described by matching them all with ϕ_Y .

- To answer question Q1 ("Who marries whom?"), we define a measure h on $X \times \mathrm{Y}$; intuitively, one can think of $h(x, y)$ as the probability that x is matched to y in the matching h.

- Note that this definition allows for randomization.

- Randomization simplifies the problem by convexifying it; moreover, allowing for randomization is sometimes necessary.

- When each x has a unique match $y = \phi_X$, and conversely, the matching is said to be *pure*; it will be the case at equilibrium in many of the examples considered in this chapter.

- A matching h is feasible if its marginals on X and Y are F and G respectively; formally:

Definition 2 (Feasible Matching) *A measure h on $X \times Y$ is a feasible matching if and only if for all $x \in X$ and $y \in Y$,*

$$\int_{t \in Y} dh\,(x,t) = F(x) \text{ and } \int_{z \in X} dh\,(z,y) = G(y). \tag{4}$$

- Note that the feasibility constraints are linear in h, a point that will become important later on.

- The (perfectly) TU case relies on the additional assumption that, for a well chosen cardinalization of individual utilities, a potential match between x and y generates a joint surplus S (x; y) that is additively split into the individual surpluses of the two partners.

- The joint surplus, is then the differences between the sum of utilities that the spouses can reach when matched and the sum of their individual utilities if both stay single.

- In particular, the "surplus" generated by singlehood (i.e., a match with the dummy partner ϕ_X or ϕ_Y) is zero.

- This brings us to question Q2: how is the surplus split?

- Consider any feasible matching h. If x and y are matched with positive probability under h, we denote $u(x)$ and $v(y)$ their individual surpluses if they match, and we have:

$$h(x, y) > 0 \Rightarrow u(x) + v(y) = S(x, y). \tag{5}$$

- Condition (5) simply states that matched people share the resulting surplus.

- Note that if x stays single, then $u(x) = S(x, \phi_Y) = 0$.

- Like most of the literature, we model equilibrium by assuming stability (Gale and Shapley, 1962; Shapley and Shubik, 1972).

Definition 3 (Stable Matchings) *A matching is stable iff it is feasible and:*

(i) no matched individual would prefer being single, and

(ii) no pair of individuals would both *prefer being matched together (for a well-chosen distribution of the surplus) over their current situation.*

- Requirement (ii) implicitly incorporates a notion of “divorce at will”: whenever it is violated, if (one of) the corresponding individuals are currently matched they will each divorce their current spouse at no cost to form a new union.

- One can readily see that stability requires the following inequalities:

$$u(x) + v(y) \geq S(x, y) \quad \forall (x, y) \in X \times Y \qquad (6)$$

- Indeed, assume there exists a pair $(x, y) \in X \times Y$ such that $u(x) + v(y) < S(x, y)$.

- Then by (5), x and y are matched with zero probability; yet they could both strictly benefit from being matched together, since the surplus $S(x, y)$ they generate is sufficient to provide x with strictly more than $u(x)$ and y with strictly more than $v(y)$.

- But that would violate requirement (ii) of stability.

- An equivalent statement is the following: if a matching h is stable, the corresponding functions u and v, from X to R and from Y to R respectively, are such that:

$$u(x) = \max_{t \in Y} \{S(x,t) - v(t)\} \quad (7)$$

$$\text{and } v(y) = \max_{z \in X} \{S(z,y) - u(z)\}; \quad (8)$$

- and in each of these equalities, the maximum is reached for all potential spouses (possibly including the dummy one) to whom the individual is matched with positive probability under h.

- Note that (7) has a natural interpretation in hedonic terms: $v(y)$ is the "price" (in utility terms) that x would have to pay should she choose to marry y; then she would keep what is left of the surplus, namely $S(x,y) - v(y)$.

- Obviously, the same argument applies (*mutatis mutandis*) to (8).

2.3.2 Household behavior and TU

- For well chosen cardinalizations of individual preferences, the Pareto frontier generated by a given budget constraint is a straight line with slope - 1 for *all values of prices and incomes*. That is, its equation is simply:

$$u^A + u^B = \Phi \tag{9}$$

- for some function ϕ of prices and income.
- This, in turn, requires specific assumptions on individual preferences, that we now describe.
- We consider a two-person (A, B) household (the extension to any number of individuals is straightforward).
- The household consumes n private goods and N public goods; an allocation thus is a $(2n + N) - vector$

$$\mathcal{Q} = \left(q_1^A, \ldots, q_n^A, q_1^B, \ldots, q_n^B, Q_1, \ldots, Q_N\right)$$

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- We assume egoistic preferences of the form $u^i(q^i, Q)$ for $i = A, B$, and we define the *conditional indirect utility* of i by:

$$v^i(p, Q, \rho) = \max_q \left\{ u^i(q, Q) \mid p'q = \rho \right\}.$$

- In words, $v^i(p, Q, \rho)$ is the maximum utility that individual i can reach when consuming the vector Q and optimally choosing their private consumption subject to the budget constraint $p'q = \rho$.

- **A Basic Model** As is well known (see for instance Browning, Chiappori, and Weiss (2014)), any efficient allocation can be interpreted as the outcome of a two-stage decision process.

- In stage 1, members collectively choose the household demands for public goods Q and decide how the remaining income $x - P'Q$ is split between members.

- We denote p^i the income of member $i = A, B$, with $p^A + p^B = x - P'Q$.

- In stage 2, each member independently decides on their private consumption q^i under the budget constraint $p'q^i = \rho^i$, and achieves conditional indirect utility $v^i(p, Q, \rho^i)$.

- As a consequence, any efficient first stage choice solves:

$$\max_{Q,\rho^A,\rho^B} v^A\left(p, Q, \rho^A\right) + \mu v^B\left(p, Q, \rho^B\right)$$

- under the constraint

$$\rho^A + \rho^B = x - P'Q$$

- for some scalar $\mu > 0$.

- Chiappori and Gugl (2020) proved that TU holds for a pair of preferences is and only if they can be represented by conditional indirect utility functions that are affine in private expenditures and share the same slope.

Definition 4 (ACIU) *A utility function u^i satisfies the Affine Conditional Indirect Utility (ACIU) property if one can find a continuous scalar function $\alpha^i(Q,p)$ from $\mathbb{R}^{N+n}$ to $\mathbb{R}$ that is (-1)-homogeneous in p, and a continuous scalar function $\beta^i(Q,p)$ from $\mathbb{R}^{N+n}$ to $\mathbb{R}$ that is 0-homogeneous in p, such that the conditional indirect utility corresponding to u^i can be written as:*

$$v^i(p,Q,\rho) = \alpha^i(p,Q)\,\rho + \beta^i(p,Q) \quad \textit{for all } (p,Q,\rho). \tag{10}$$

Proposition 5 (Characterization of TU Preferences) *A pair of preferences satisfy the TU property if and only if one can find two representations (u^A, u^B) that both satisfy the ACIU property (10), with moreover*

$$\alpha^A(p,Q) = \alpha^B(p,Q). \tag{11}$$

Proof. *See Chiappori and Gugl (2020).* ■

- The property defined in Proposition 5, which can be called ISACIU (for Identical Shape Affine Conditional Indirect Utility), is thus necessary and sufficient.

- **Uncertainty: the one-dimensional case** The TU property can be characterized in more complex frameworks.

- Given a feasible sharing rule (ρ^A, ρ^B), the expected utility of agent i is $\mathbb{E}v^i(\rho^i(\tilde{x}^A, \tilde{x}^B))$, where v^i is i's (indirect) von Neumann-Morgenstern utility and the expectation is taken over the distribution of $(\tilde{x}^A, \tilde{x}^B)$.

- As always, ex ante efficiency requires that no alternative sharing rule could increase expected utility for both individuals.

- By the mutuality principle (Proposition 1), the efficiency sharing rule only depends on total income $\bar{x} = \tilde{x}^A + \tilde{x}^B$: for $i = A, B$, it is of the form $\rho^i(\bar{x})$.

- Mazzocco (2004) and Schulhofer-Wohl (2006) provide a characterization of vNM utilities that exhibit the TU property. As before, we start with a definition:

Definition 6 *A pair of vNM utility functions* (v^A, v^B) *belongs to the ISHARA class if the corresponding indices of absolute risk aversion are harmonic:*

$$-\frac{d^2 v^i/\partial\rho^2}{dv^i/\partial\rho} = \frac{1}{a^i + b^i\rho} \qquad (12)$$

for $i = A, B$, *and moreover* $b^A = b^B$.

- Condition (12) expresses that for each individual utility, the index of Absolute Risk Aversion is an Harmonic function of income; the shape coefficient is then b^i, and the Identical Shape requirement imposes $b^A = b^B$.

- For instance, any pair of CARA utility functions always belong to the ISHARA class (with $b^A = b^B = 0$), whereas two CRRA utilities are ISHARA if and only if they have the same coefficient of relative risk aversion b (then $a^A = a^B = 0$ and $b^A = b^B = b$).

Proposition 7 (ISHARA implies TU) *Consider a pair of vNM utility function* (v^A, v^B) *that belongs to the ISHARA class, and assume that individuals share their income risk efficiently. Then:*

1. *The sharing rule is an affine function of household income.*
2. *The household behaves as a single consumer, in the sense that all efficient sharing rules generate the same aggregate behavior; the latter maximizes expected utility for some representative vNM utility* U *that is also HARA.*
3. *The model is TU, in the sense that there exists two increasing mappings* f^A, f^B *from* $\mathbb{R}$ *to* $\mathbb{R}$ *such that, for any probability distribution of* $(\tilde{x}^A, \tilde{x}^B)$*, all efficient sharing rules* (ρ^A, ρ^B) *satisfy*

$$f^A\left(\mathbb{E}v^A\left(\rho^A(\bar{x})\right)\right) + f^B\left(\mathbb{E}v^B\left(\rho^B\left(\bar{x}\right)\right)\right) = K \tag{13}$$

where K *depends on the preferences and on the distribution of the total income* $\bar{x}$.

4. *In particular, if* (ρ^A, ρ^B) *is an efficient sharing rule, let* C_M^i *denote the certainty equivalent, for* i*, of the (random) allocation* $\rho^i(\tilde{x})$*. Then*

$$C_M^A + C_M^B = C$$

where C *is the certainty equivalent, for the representative consumer, of the random allocation* $\tilde{x}$*; in particular,* C *does not depend on the choice of the efficient sharing rule.*

Conversely, if the four previous properties are satisfied for all probability distributions $(\tilde{x}^A, \tilde{x}^B)$*, then the pair of vNM utility function* (v^A, v^B) *belongs to the ISHARA class.*

Proof. *See Mazzocco (2004) and Schulhofer-Wohl (2006).* ■

- As an illustration, consider the CARA utility functions, we had

$$-\log\left(-\mathbb{E}u_M^A\right) = \log s^A - \frac{s^A}{s^A + s^B}\log\mu - \log\mathbb{E}\exp(-s\bar{x})$$
$$-\log\left(-\mathbb{E}u_M^B\right) = \log s^B + \frac{s^B}{s^A + s^B}\log\mu - \log\mathbb{E}\exp(-s\bar{x}).$$

- This directly implies

$$-\frac{1}{s^A}\log\left(-\mathbb{E}u_M^A\right) - \frac{1}{s^B}\log\left(-\mathbb{E}u_M^B\right) = \frac{\log s^A}{s^A} + \frac{\log s^B}{s^B} - \frac{1}{s}\log\mathbb{E}\exp(-s\bar{x})$$

- which is of the form (13) for $f^i(t) = -\log(-t)/s^i$.

- In certainty equivalent terms:

$$C^A = L - \ln \left(\int \exp \left(- \frac{s^A s^B}{s^A + s^B} x \right) dF(x) \right) \quad \text{and}$$
$$C^B = -L - \ln \left(\int \exp \left(- \frac{s^A s^B}{s^A + s^B} x \right) dF(x) \right)$$

- where the constant L depends on the Pareto weight μ.

- For all values of μ, we have:

$$C^A + C^B = -\ln \left(\int \exp \left(- \frac{s^A s^B}{s^A + s^B} x \right) dF(x) \right) = C$$

- where C is the certainty equivalent of a representative consumer with CARA preferences defined by an index of Absolute Risk Aversion equal to $s = \frac{s^A s^B}{s^A + s^B}$.

- **Uncertainty: the general case** The previous result can readily be extended to the multiple-goods framework.

- Specifically, assume that (i) individual preferences satisfy the ISACIU property for some well-chosen cardinalization, and (ii) individual vNM utilities, considered as functions of individual private incomes, belong to the ISHARA class.

- Then the model is TU.

- To see why, start with the ISACIU property: for $i = A, B$,

$$v^i(p, Q, \rho) = \alpha(p, Q)\rho + \beta^i(p, Q). \qquad (14)$$

- Since ex ante efficient allocations are also ex post efficient, for any income realization the choice of the public consumption vector Q must maximize the sum of utilities using the cardinalization corresponding to the ACIU property.

- That is, Q solves:

$$\max_{Q}\left(\alpha\left(p,Q\right)\left(\bar{x}-P'Q\right)\ +\sum_{i}\beta^{i}\left(p,Q\right)\right).$$

- Let $\bar{Q}$ denote the solution; note that $\bar{Q}$ only depends on prices and on total household income x.

- Now assume that the vNM utility of $i = A, B$ is $\phi^i(v^i(p, Q, \rho))$,where the pair (ϕ^A, ϕ^B) belongs to the ISHARA class.

- Any ex ante efficient allocation must solve, for some $\mu > 0$,

$$\max_{\rho^A,\rho^B}\mathbb{E}\left[\phi^A\left(v^A\left(p,\bar{Q},\rho^A\right)\right)\right]+\mu\mathbb{E}\left[\phi^B\left(v^B\left(p,\bar{Q},\rho^B\right)\right)\right].$$

- Given the ISACIU property, this can be rewritten as

$$\max_{W^A, W^B} \mathbb{E}\left(\phi^A\left(W^A\right) + \mu\phi^B\left(W^B\right)\right),$$

- Where $W^i = v^i(p, \bar{Q}, \rho^i)$, under the constraint that

$$W^A + W^B = \alpha\left(p, \bar{Q}\right)\left(\bar{x} - P'\bar{Q}\right) + \beta^A\left(p, \bar{Q}\right) + \beta^B\left(p, \bar{Q}\right) \equiv \bar{W}.$$

- By Proposition 7, there exist (f^A, f^B) such that all ex ante efficient allocations solve:

$$f^A\left(\mathbb{E}\left[\phi^A\left(W^A\right)\right]\right) + f^B\left(\mathbb{E}\left[\phi^B\left(W^B\right)\right]\right) = K$$

- which is exactly TU.

2.3.3 Duality and Supermodularity

- Optimal transportation and duality A crucial property of matching models under TU is their intrinsic relationship with a class of linear maximization problems called “optimal transportation”.

- Consider the following question: Find a measure h on $X \times Y$, the marginals of which are F and G respectively, that maximizes the integral

$$\mathcal{S} = \int_{X \times Y} S(x, y)\, dh(x, y). \tag{15}$$

- In a TU framework, where individual utilities can all be measured in the same units, the natural measure of total welfare is the sum of all surpluses generated; that is exactly the meaning of the right-hand side integral in (15).

- As this problem is linear in h, its value coincides with that of its dual.
- The dual problem consists in finding two functions u and v, respectively defined on X and Y, that minimize the sum

$$\tilde{S} = \int_i u(x)dF(x) + \int_Y v(y)dG(y) \qquad (16)$$

- under the constraints:

$$u(x) + v(y) \geq S\left(x, y\right) \quad \forall \left(x, y\right) \in X \times Y$$

- Note that these constraints are simply the stability constraints of (6).

- **Supermodularity** The one-dimensional case $m = n = 1$ allows us to introduce an important notion: the supermodularity of the surplus.

Definition 8 (Supermodularity) *A function $f : \mathbb{R}^2 \to \mathbb{R}$ is supermodular if and only if for for all $x \leq x'$ and $y \leq y'$,*

$$f(x, y) + f(x', y') \geq f(x, y') + f(x', y) \qquad (17)$$

- If f is twice continuously differentiable, supermodularity is equivalent to the Spence-Mirrlees condition:

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) \geq 0 \quad \forall x, y.$$

- When the surplus S is strictly supermodular, the only stable matching must be *positively assortative*; for any two matched couples (x, y) and (x', y'), if $x < x'$ then $y \leq y'$.

- With continuous distributions, matching patterns follow a simple rule: x is matched to y if and only if the total mass of matched women above x equals the total mass of matched men above y, that is (assuming equal total numbers of men and women) $1 - F(x) = 1 - G(y)$.

- Formally, the matching is pure and can be described by a function

$$y = (G^{-1} \circ F)(x).$$

- In particular, all supermodular surplus functions generate exactly the same matching patterns.

- Lastly, if (17) holds with the opposite inequality, then the surplus function is submodular, and the stable matching is now negative assortative (larger x match with smaller y and conversely).

2.3.4 Multidimensional matching under TU

- The previous approach can be extended to multi-dimensional settings.

- **Index Models** In the so-called index model, the various characteristics of at least one partner only enter the surplus through some one-dimensional index:

$$S(x_1, \ldots, x_n, y) = \overline{S}(I(x_1, \ldots, x_n), y) \qquad (18)$$

- for some functions $\bar{S}$ and I. The index I serves as an aggregator of the vector of characteristics $x = (x_1, \ldots, x_n)$ that fully reflects her "attractiveness" on the marriage market: two women with different vectors $x; x'$ but the same index value $(\mathrm{I}(x) = I(x'))$ are perfect substitutes.

- Suppose for simplicity that all characteristics are continuous.
- In a multidimensional setting, there exist trade-offs between the various traits that characterize a woman.
- They are described by the ratio (formally equivalent to a marginal rate of substitution)

$$M_{ij}(x,y) = \frac{\partial S/\partial x_j}{\partial S/\partial x_i}(x,y)$$

- Many-to-one dimensional matching Another interesting situation obtains when dimensions m and n differ.
- Assume for instance that $m = 1$ but $n \geq 2$.
- Then a husband with a given characteristic y will marry with positive probability any of a continuum of different women x, thus defining "iso-husband" curves in the space of female characteristics.
- Note that these curves are (in principle) identifiable from data on matching patterns.

- Theory generates testable predictions relating the surplus function to the shape of iso-husband curves.

- To see how, let us consider the case n = 2. The stability condition:

$$v(y) = \max_{x_1, x_2 \in X} \{S(x_1, x_2, y) - u(x_1, x_2)\}$$

- gives by the envelope theorem:

$$v'(y) = \frac{\partial S}{\partial y}(x_1, x_2, y) \tag{19}$$

- which defines an iso-husband curve.

- In a general (non index) framework, however, the shape of the iso-husband curves also depend on the marginals.

- Equation (19) still yields the following:

Proposition 9 *Assume that the cross-derivatives* $\frac{\partial^2 S}{\partial x_1 \partial y}$ *and* $\frac{\partial^2 S}{\partial x_2 \partial y}$ *are positive. Then the iso-husband curves are decreasing in the* (x_1, x_2) *plane.*

Proof. From the implicit function theorem, (19) implies that the equation of the iso-husband curve can be written as:

$$x_2 = \phi(x_1, y)$$

with

$$\frac{\partial \phi}{\partial x_2}(x_1, y) = -\frac{\frac{\partial^2 S}{\partial x_2 \partial y}}{\frac{\partial^2 S}{\partial x_1 \partial y}}(x_1, x_2, y) < 0.$$

■

2.4 Other Matching models: Imperfectly Transferable Utility, search

2.4.1 Matching under Imperfectly Transferable Utility (ITU)

- TU models rely on a highly specific property: for a well-chosen cardinalization of individual preferences, the Pareto frontier is a hyperplane orthogonal to the *unitary vector for all price and incomes*.

- A more general utility possibility set can be defined by an equation of the form:

$$U \leq \Phi\left(x, y, V\right) \qquad (20)$$

- where $U(V)$ is her (his) utility and Φ is non-increasing in V.

- The TU case corresponds to $\Phi(x, y, V) = S(x, y) - V$ and NTU has fixed $U = U(x, y)$ and $V = V(x, y)$.

- A matching is still defined as a 3-uple (h, u, v) where the marginals of measure h are F and G respectively, and (20) is satisfied with equality whenever $h(x, y) > 0$. Stability requires, moreover, that:

$$u(x) \geq \Phi\left(x, y, v(y)\right) \quad \forall x, y \tag{21}$$

- with the same interpretation as in the TU case. In particular, $u(x)$ must be the value of the maximum over y of $\Phi(x, y, v(y))$, so that at the stable matching

$$u'(x) = \frac{\partial \Phi}{\partial x}\left(x, y, v(y)\right).$$

2.4.2 Search models

- Divorce is as exogenous as can be: matches are dissolved randomly with probability ρ.
- Let $W(x)$ be the value of an unmarried woman of characteristic x, and $M(y)$ be that an unmarried man of characteristic y.
- If these two individuals meet, they can obtain a flow marital surplus $s(x, y)$ until their match is dissolved.
- Suppose that they agree to divide it as $\mathrm{u}(x, y) + v(x, y) = s(x, y)$.
- Since the match is dissolved with probability ρ and its utility is discounted at rate r, the value $W(x|y)$ of the match for woman x is the value of $\mathrm{u}(x, y)$ in perpetuity, minus the expected value lost if the match is dissolved:

$$rW(x|y) = u(x, y) - \delta(W(x|y) - W(x)).$$

- The term $W(x|y) - W(x)$, and its analog $\mathrm{M}(y|x) - M(y)$ for man y, represent their shares of the surplus relative to their outside option (waiting for a new partner).

- Like most of the search literature, Shimer and Smith (2000) assume that these shares are equal:

$$W(x|y) - W(x) = M(y|x) - M(y). \tag{22}$$

- Since $u + v = s$, combining these equations shows that the common value in (22) is

$$\frac{s(x,y) - rW(x) - rM(y)}{2(r+\delta)}.$$

- Now consider the value of an unmarried woman.
- Since with probability ρ she will meet a partner y drawn randomly from the distribution f of unmarried men, $W(x)$ is given by

$$rW(x) = \frac{\rho}{2(r+\delta)} \int \left(s(x,y) - rW(x) - rM(y)\right) f(y)dy.$$

- Similarly,

$$rM(y) = \frac{\rho}{2(r+\delta)} \int \left(s(x,y) - rW(x) - rM(y)\right) g(x)dx$$

- if g is the pdf of the distribution of unmarried women.

- The densities f and g are equilibrium objects, however.
- Suppose that the pdf of the characteristics of all women (married or not) is n and that of all men is m.
- Then the pdf of the characteristics of married women is $\mathrm{n} - f$.
- Since their matches dissolve with probability δ, in steady-state the number of new matches must exactly compensate.
- With fully random meetings, an unmarried woman x will match with probability $\rho \int \alpha(x,y)g(y)dy$.
- Therefore we have the flow balance equations

$$\delta(n(x) - f(x)) = \rho \int \alpha(x,y)g(y)dy$$

$$\delta(m(y) - g(y)) = \rho \int \alpha(x,y)f(x)dx.$$

2.5 Dynamic aspects

2.5.1 Pre-marital investments

2.5.2 The commitment issue

2.5.3 Dynamic matching and divorce

- The decision to divorce or not will be ex post efficient.
- This argument can be summarized by the following figures, borrowed from Chiappori, Iyigun, and Weiss (2009a), where individual utilities are on the horizontal and vertical axes respectively.
- Point M (resp. D) denotes the current division of surplus if individuals remain married (resp. divorce).
- The red (resp. blue) line represents the Pareto frontier, i.e. the set of utility pairs that can be reached through transfers if spouses remain married (resp. divorce).

- Here:
  - in Figure 1a, point D belongs to the interior of the Pareto set when married (in red), while point M is located outside of the Pareto set after divorce (in blue). As a result, divorce is inefficient, and partners remain married, perhaps after renegotiating the existing agreement.
  - Figure 1b illustrates the opposite situation. Here point M belongs to the interior of the Pareto set when divorced, while D is outside the Pareto set if married. As a result, remaining married is inefficient, and individuals divorce; again, this may require a renegotiation of the post-divorce allocation.

Figure 1.

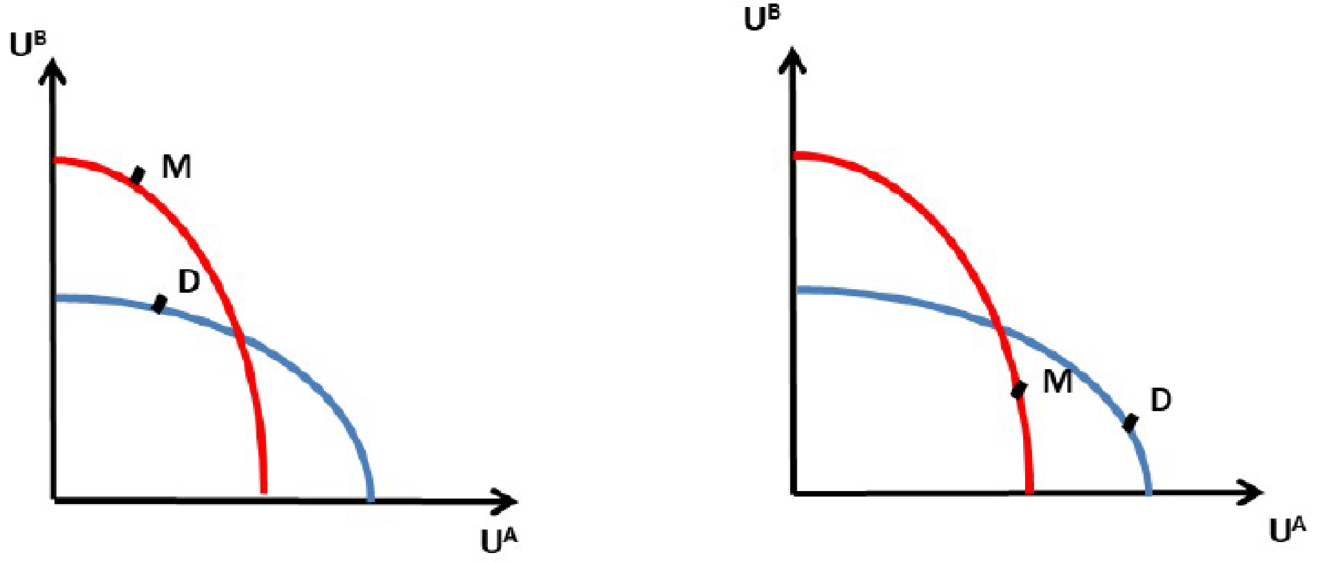

Figure 1a: spouses remain married

Figure 1b: spouses always divorce

Source: Chiappori, Iyigun, and Weiss (2009a).

- Agents divorce if and only if the shock is negative enough:

$$\theta + s(x,y) \leq U(x,y) + V(x,y)$$

- Or equivalently

$$\theta \leq \bar{\theta}(x,y)\,,\ \text{where}\ \bar{\theta}(x,y) = -s(x,y) + U(x,y) + V(x,y)$$

- In the first period, matching decisions depend on the ex ante expected surplus, which equals

$$\begin{aligned} ES(x,y) \equiv\ & s(x,y) + F_\theta\left[\bar{\theta}(x,y)\right]\bar{\theta}(x,y) \\ & + \left(1 - F_\theta\left[\bar{\theta}(x,y)\right]\right) E\left(\theta \mid \theta \geq \bar{\theta}(x,y)\right) \end{aligned}$$

- where F_θ denotes the cdf of θ and f_θ its density. If ES is supermodular, individuals match assortatively in the first period.

- The sign of $\partial^2 ES/\partial x \partial y$ depends on the sign of the cross derivatives of U and V, as well as on the signs of the first derivatives of $\bar{\theta}$.

2.5.4 Remarriage

3 Empirical Methods

3.1 The Separable Approach

- The marital surplus $\tilde{S}(\tilde{x}, \tilde{y})$ a priori may interact four groups of arguments: the observed characteristics x, y and the unobservable heterogeneities ε and η.

- Separability rules out any interaction between ε and η:

Assumption 10 (Separability) *The joint utility of a match between* $\tilde{x} = (x, \varepsilon)$ *and* $\tilde{y} = (y, \eta)$ *is*

$$\tilde{S}(\tilde{x}, \tilde{y}) = S_{xy} + \zeta_y(\tilde{x}) + \xi_x(\tilde{y}).$$

A single woman $\tilde{x}$ *has utility*

$$\tilde{S}(\tilde{x}, \emptyset_Y) = \zeta_0(\tilde{x})$$

and a single man $\tilde{y}$ *has utility*

$$\tilde{S}(\emptyset_X, \tilde{x}) = \xi_0(\tilde{y}).$$

- It is important to emphasize that separability does not rule out "matching on unobservables".

- The following result, due to Chiappori, Salanie, and Weiss (2017), describes its implications:

Theorem 11 (Splitting the Surplus under Separability) *Under Assumption 10, there exists a pair of matrices (U, V) such that at any stable matching (μ_{xy}):*

- *a woman of full type $\tilde{x} = (x, \varepsilon)$ will match with a man of an observable type y that maximizes $U_{xy} + \zeta_y(x, \varepsilon)$ over X*
- *a man of full type $\tilde{y} = (y, \eta)$ will match with a woman of an observable type x that maximizes $V_{xy} + \xi_x(y, \eta)$ over Y*
- $U_{x0} = V_{0y} = 0$
- $U_{xy} + V_{xy} \geq S_{xy}$, *with equality if* $\mu_{xy} > 0$.

Proof. We know from Section 2.3.1 that the utility of woman $\tilde{x}$ at a stable matching is

$$\tilde{u}(\tilde{x}) = \max_{\tilde{y}}(\tilde{S}(\tilde{x}, \tilde{y}) - \tilde{v}(\tilde{y})).$$

- Breaking down the maximization over y-then-η and using separability gives

$$\tilde{u}(\tilde{x}) = \max_y \left(S_{xy} + \zeta_y(\tilde{x}) + \max_\eta \left(\xi_x(y,\eta) - \tilde{v}(y,\eta) \right) \right)$$
$$= \max_y \left(S_{xy} + \zeta_y(\tilde{x}) - \min_\eta \left(\tilde{v}(y,\eta) - \xi_x(y,\eta) \right) \right).$$

- Denote $V_{xy} = \min_\eta(\tilde{v}(y,\eta) - \xi_x(y,\eta))$; then

$$\tilde{u}(\tilde{x}) = \max_y \left(S_{xy} - V_{xy} + \zeta_y(\tilde{x}) \right).$$

- Similarly, we can define $U_{xy} = min_\varepsilon(\tilde{u}(x,\varepsilon) - \zeta_y(x,\varepsilon))$.

- The stability constraints $\tilde{u}(\tilde{x}) + \tilde{v}(\tilde{y}) \geq \tilde{S}(\tilde{x},\tilde{y})$ imply that $U_{xy} + V_{xy} \geq 0$. If $\mu_{xy} > 0$, then there exist $(\tilde{x},\tilde{y})$ such that $\tilde{u}(\tilde{x}) + \tilde{v}(\tilde{y}) = \tilde{S}(\tilde{x},\tilde{y})$; then $U_{xy} + V_{xy} = S_{xy}$.

- Assuming separability greatly reduces the complexity of the matching problem: our unknown now is the matrix U, which is defined on the set of observable types rather than on the set of full types.

- With discrete x and y, the problem becomes finite-dimensional.

- Suppose that $\mu_{xy} > 0$ for all (x, y). Then given U, we can define $V = S - U$, and obtain the equilibrium utilities:

$$\tilde{u}(\tilde{x}) = \max_{y \in Y}(U_{xy} + \zeta_y(x, \varepsilon)) \tag{23}$$

and

$$\tilde{v}(\tilde{y}) = \max_{x \in X}(V_{xy} + \xi_x(y, \eta)). \tag{24}$$

- Moreover, the maxima in these simple, one-sided discrete choice problems are achieved by the stable matching partners.

3.2 Identification of Separable Models

- As explained in Section 2.3.3, the stable matching solves an optimal transportation problem whose objective function is the total joint utility generated by a matching:

$$\mathcal{W} = \int \tilde{u}(\tilde{x})\tilde{n}(d\tilde{x}) + \int \tilde{v}(\tilde{y})\tilde{m}(d\tilde{y}). \qquad (25)$$

- We will simply call it the social welfare from now on.

- The dual formulation of the matching problem states that W must be minimized under the stability constraints

$$\tilde{u}(\tilde{x}) + \tilde{v}(\tilde{y}) \geq \tilde{S}(\tilde{x}, \tilde{y}).$$

- Galichon and Salanie (2020) showed that in any separable model, the social welfare can be rewritten as follows:

$$\mathcal{W}(S) = \max_{\mu} \left(\sum_{x,y} \mu_{xy} S_{xy} + \mathcal{E}(\mu) \right) \qquad (26)$$

- where the *generalized entropy* $\mathcal{E}$ is a function whose shape only depends on the distributions $\mathbb{P}_x$ and $\mathcal{Q}_y$.

- The maximand in (26) consists of two terms.
- The first one is the value of social welfare if partners only matched on the basis of their observable types.
- Unobserved heterogeneity generates matching on unobservables, which adds another contribution to the social welfare W via the generalized entropy term.
- Taking the first-order conditions in this problem gives

$$S_{xy} = -\frac{\partial \mathcal{E}}{\partial \mu_{xy}}(\mu). \tag{27}$$

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- Taking the first-order conditions in this problem gives

$$S_{xy} = -\frac{\partial \mathcal{E}}{\partial \mu_{xy}}(\mu). \qquad (27)$$

3.3 The Logit Model

- The generalized entropy $\mathcal{E}$ is simply the standard entropy

$$\mathcal{E}(\mu; n, m) = -\sum_{x\neq 0, y\neq 0} \mu_{xy} \log \frac{\mu_{xy}^2}{n_x m_y} - \sum_{x\neq 0} \mu_{x0} \log \frac{\mu_{x0}}{n_x} - \sum_{y\neq 0} \mu_{0y} \log \frac{\mu_{0y}}{m_y}.$$

- and equation (27) gives the very simple Choo and Siow formula

$$S_{xy} = \log \frac{\mu_{xy}^2}{\mu_{x0}\mu_{0y}}. \qquad (28)$$

3.4 Estimation of Separable Models

- The data typically consists of a large sample of N households.
- Of those, $\hat{\mu}_{xy}$ are marriages between types x and y; $\hat{\mu}_{x0}$ are single women of type x, and $\hat{\mu}_{0y}$ are single men of type y.
- These natural estimates of the matching patterns generate margins

$$\hat{n}_x = \sum_y \hat{\mu}_{xy} + \hat{\mu}_{x0}$$

$$\hat{m}_y = \sum_x \hat{\mu}_{xy} + \hat{\mu}_{0y}.$$

- The estimators $\hat{\mu}$ are distributed as discrete count variables.
- If the N households are drawn with equal probabilities from an infinite population characterized by true matching patterns μ, then

$$\operatorname{cov}(\hat{\mu}_{xy}, \hat{\mu}_{zt}) = \frac{1}{N}\mu_{xy}(\mathbb{1}(x = z, y = t) - \mu_{zt}).$$

3.4.1 Nonparametric Estimation of the Surplus

3.4.2 Parametric Estimation

- **Maximum Likelihood Estimation** The most generally applicable way to estimate a parametric separable matching model is maximum likelihood.

- Suppose that we know how to compute the stable matching μ^{θ} for any given value of θ-we could use (25), but there are often much faster alternatives.

Note that this results in a number of households that typically differs from the observed N:

$$N^{\theta} = \sum_{x,y} \mu^{\theta}_{xy} + \sum_{x} \mu^{\theta}_{x0} + \sum_{y} \mu^{\theta}_{0y}.$$

The likelihood function of the sample is

$$\log L(\theta) = \sum_{x} \sum_{y} \hat{\mu}_{xy} \log \frac{\mu^{\theta}_{xy}}{N^{\theta}} + \sum_{x} \hat{\mu}_{x0} \log \frac{\mu^{\theta}_{x0}}{N^{\theta}} + \sum_{y} \hat{\mu}_{0y} \log \frac{\mu^{\theta}_{0y}}{N^{\theta}}.$$

- The estimator given by the maximization of log L has the usual properties: it is $\sqrt{N}$-consistent, asymptotically normal, and asymptotically efficient.

- **Moment matching and semilinear models** Suppose that the surplus function S is be linear in the unknown parameters:

$$S_{xy}^{\theta} = \sum_{k=1}^{K} \theta_k s_{xy}^k \tag{29}$$

where the s^k are known basis functions. In this model, it seems tempting to match the observed *comoments* $\hat{C}^k = \sum_{x,y} \hat{\mu}_{xy} s_{xy}^k$ with their simulated counterparts:

$$\hat{C}^k = \sum_{x,y} \hat{\mu}_{xy} s_{xy}^k = \sum_{x,y} \mu_{xy}^{\theta} s_{xy}^k.$$

- **Estimating the logit Model** In the logit model of Section 3.3, one can avoid having to compute the stable matching (or evaluating the social welfare W).

- Galichon and Salanie (2020) show that maximizing the function

$$\sum_{xy} S^{\theta}_{xy} - \sum_{x} \hat{n}_x (u_x + \exp(-u_x) - 1) - \sum_{y} \hat{m}_y (v_y + \exp(-v_y) - 1)$$
$$- 2 \sum_{x,y} \sqrt{\hat{n}_x \hat{m}_y} \exp\left(\frac{S^{\theta}_{xy} - u_x - v_y}{2} \right) \qquad (30)$$

- over (θ, u, v) yields a consistent estimator for the logit model, as well as the equilibrium utilities of all types.

- If the surplus S^{θ} is not too nonlinear in θ, then the objective function in (30) is globally concave over all of its arguments and therefore easy to maximize.

3.4.3 Continuous observed characteristics

- Dupuy and Galichon (2014) showed how the techniques described in previous subsections extend naturally to this continuous logit model.

- For instance, the objective function of (30) becomes

$$\int\int S^{\theta}(x,y)dxdy - 2\int\int \sqrt{\hat{n}(x)\hat{m}(y)}\exp\left(\frac{S^{\theta}(x,y)-u(x)-v(y)}{2}\right)dxdy$$
$$-\int \hat{n}(x)(u(x)+\exp(-u(x))-1)dx - \int_{y} \hat{m}(y)(v(y)+\exp(-v(y))-1)dy$$

- where $\hat{n}(x)$ and $\widehat{m}(y)$ are the estimated densities of the types.

3.5 Maximum-score methods

- Fox (2010) has developed an empirical approach to matching with transferable utility that relies on a selecting set of "matching inequalities."

- Suppose that $\tilde{x}$ marries y and $\tilde{x}'$ marries $\tilde{y}'$.

- If these two couples are part of a stable matching, then reshuffling partners cannot increase the sum of their surpluses:

$$\tilde{S}(\tilde{x}, \tilde{y}) + \tilde{S}(\tilde{x}', \tilde{y}') \geq \tilde{S}(\tilde{x}, \tilde{y}') + \tilde{S}(\tilde{x}', \tilde{y}).$$

- If we observe C couples $(\tilde{x}_i, \tilde{y}_i)$ and we assume that it belonged to a stable matching generated by a surplus $\tilde{S}^{\theta}(\tilde{x}_i, \tilde{y}_i) \equiv \tilde{S}_{ij}^{\theta}$, we could write

$$\sum_{i<j} (\tilde{S}_{ii}^{\theta} + \tilde{S}_{jj}^{\theta} - \tilde{S}_{ij}^{\theta} - \tilde{S}_{ji}^{\theta}) \geq 0.$$

- Under reasonable conditions, only a small set of values of would satisfy all of these inequalities.

- This is of course not a feasible approach in practice: we never observe matching between full types $\tilde{x}$ and $\tilde{y}$, only between types x and y.

- Now it is easy to see that in the logit model of Section 3.3, (28) implies that if we observe the couples (x, y) and (x', y'),

$$S_{xy} + S_{x'y'} - S_{xy'} - S_{x'y} = 2\left(\log \mu_{xy} + \log \mu_{x'y'} - \log \mu_{xy'} - \log \mu_{x'y}\right). \tag{31}$$

- Graham (2011, 2014) proved that if the unobserved heterogeneity terms ζ and ξ are independently and identically distributed, then the two sides of (31) must have the same sign.

- Now consider the function

$$F(\theta) \equiv \sum_{i<j} \mathbb{1}\left(S^{\theta}_{ii} + S^{\theta}_{jj} - S^{\theta}_{ij} - S^{\theta}_{ji} > 0\right)$$

- where i and j range over the set of observed matches.

4 Some empirical applications

4.1 Measuring homogamy

- Assume that an equal mass (normalized to 1) of men and women are distributed into two classes: Educated and Uneducated.

- Assuming away singles, matching patterns in this population are fully described by a 2×2 table: In Table 1, m and n are the proportions of Educated females and males, and r is the proportion of couples where both spouses are Educated.

- It is easy to define assortative matching here: a (m, n, r) table of this type exhibits Positive Assortative Matching (PAM) if the proportion of couples with equal education (the sum of the diagonal cells of the table) is larger than what would obtain under random matching; that is, if and only if $r \geq mn$.

Table 1: Matching by education

$_w\backslash^m$	Educated	Uneducated
Educated	r	$m - r$
Uneducated	$n - r$	$1 - m - n + r$

- The logit model described in Section 3.3 suggests an appealing criterion that satisfies the Chiappori, Costa-Dias, and Meghir (2020) axiom.

- It is easy to show that Table 1 is generated by any logit model such that

$$\frac{1}{2}\left(S_{EE}+S_{UU}-S_{EU}-S_{UE}\right) = \ln\left(\frac{r\left(1+r-m-n\right)}{\left(n-r\right)\left(m-r\right)}\right).$$

- The left-hand side of this equation is one-half of what Chiappori, Salanie,and Weiss (2017) called the supermodular core of the marital surplus, which is a direct measure of the preference for assortative matching on education.

- Define the ISEV assortativeness index as the right-hand side of this equation:

$$I_{SEV} = \ln\left(\frac{r\left(1+r-m-n\right)}{\left(n-r\right)\left(m-r\right)}\right)$$

- This index was originally proposed by Siow (2015).

4.2 Abortion law and marriage market outcomes

4.3 The marital college premium

- Define the “marital college premium” as the difference between the expected gains of college-educated individuals on the marriage market and those of less-educated individuals.

- Note that this marital premium comes over and above the labor market premium.

- Chiappori, Iyigun, and Weiss (2009b) show how the evolution of marital patterns over the period is compatible with a decrease (resp. increase) in the male (female) premium.

- The intuition is simple. When few women were educated, many uneducated women “married up” and not being educated did not hurt women's marital prospects much.

- As more and more women go to college (or beyond), those who do not face tougher competition on the marriage market.

- Symmetrically, less-educated men become more likely to marry a college-educated woman.

- This idea was taken to 30 years of data on the US marriage market by Chiappori, Salanie, and Weiss (2017).

- They start by fitting a logit model of the following form:

$$\tilde{S}(i,j) = S_{IJ} + \alpha_I^c + \beta_J^c + \zeta_J^c(i) + \xi_I^c(j)$$

- where woman i and man j belong to cohort c and have education levels I and J.

- This model allows for arbitrary changes in the marriage rates of the different types of men and women; but it restricts the supermodular core to be constant over these 30 cohorts.

- It is strongly rejected for the white population (although it is not for African-Americans).

- Next, they allow for a trend:

$$S^c_{IJ} = a_{IJ} + b_{IJ} \times c. \tag{32}$$

- The fit with actual patterns is considerably improved; moreover, the matrix $B = (b_{IJ})$ is supermodular, indicating stronger preferences for assortative matching over time

4.4 Household formation and dissolution

4.4.1 Divorce in a frictionless matching framework

4.4.2 Search models of divorce and (re-)marriage

4.4.3 Marital migrations

4.5 Personality traits and marriage

4.6 Same-sex marriage