Item Response Theory

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- IRT entails three assumptions:
- A unidimensional trait denoted by θ ;
- Local independence of items;
- The response of a person to an item can be modeled by a mathematical *item response function* (IRF).

Three Parameter Logistic Model

- For example, in the **three parameter logistic model (3PL)**, the probability of a correct response to a dichotomous item i, usually a multiple-choice question, is:

$$p_i(\theta) = c_i + \frac{1 - c_i}{1 + e^{-a_i(\theta - b_i)}}$$

- θ indicates that the person's abilities are modeled as a sample from a normal distribution for the purpose of estimating the item parameters.

- a_i, b_i, and c_i are the item parameters.

- The item parameters can be interpreted as changing the shape of the standard logistic function:

$$P(t) = \frac{1}{1 + e^{-t}}.$$

- In brief, the parameters are interpreted as follows (dropping subscripts for legibility); b is most basic, hence listed first:

- b – difficulty, item location: $p(b) = (1 + c)/2$, the half-way point between c_i (min) and 1 (max), also where the slope is maximized.

- a – discrimination, scale, slope: the maximum slope $p'(b) = a \cdot (1 - c)/4$.

- c – pseudo-guessing, chance, asymptotic minimum $p\,(-\infty) = c$.

IRT Models

- Broadly speaking, IRT models can be divided into two families: unidimensional and multidimensional.
- Unidimensional models require a single trait (ability) dimension.
- Multidimensional IRT models model response data hypothesized to arise from multiple traits.
- However, because of the greatly increased complexity, the majority of IRT research and applications utilize a unidimensional model.

Logistic and Normal IRT Models

- An alternative formulation constructs IRFs based on the normal probability distribution; these are sometimes called *normal ogive models*.

- For example, the formula for a two-parameter normal-ogiveb IRF is:

$$p_i(\theta) = \mathbf{\Phi}(\frac{\theta - b_i}{\sigma_i})$$

where $\mathbf{\Phi}$ is the cumulative distribution function (CDF) of the standard normal distribution.