Cost Benefit Analysis Using the MTE

Handbook of Econometrics

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- **Roy model**: agent selects into treatment if the benefits exceed the cost.
- Outcomes (Y_{1i}, Y_{0i}):

$$Y_{1i} = \mu_1(X) + U_1$$

$$Y_{0i} = \mu_0(X) + U_0$$

- Ψ is generated as: $\Psi_i = \varphi(W_i) + V_i$. This is **cost**.
- W_i is a (possibly vector-valued) observed random variable.
- V_i is an unobserved random variable.

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Assume:

$$D_z(\omega) = \mathbf{1}[\Delta \geq \Psi] \text{ where } \Psi = \varphi(W) + V$$

1. $\varphi(W)$ is a nondegenerate random variable, and $\varphi(W)$ is integrable.
2. V is absolutely continuous with respect to Lebesgue measure.
3. $Z = (W, X)$ is independent of (V, Y_0, Y_1),

Let $U \equiv V - (U_1 - U_0)$.

Let F denote the distribution function of U.

Let $\tilde{U} \equiv F(U)$.

Let $P(z) \equiv \Pr(D = 1|Z = z)$.

F is strictly increasing so that F will be invertible.

- Define the following treatment parameters corresponding to averages of the benefit received:

$$\begin{aligned}
\Delta^{ATE}(x) &\equiv E(Y_1 - Y_0 \mid X = x) \\
\Delta^{TT}(x, P(z), D = 1) &\equiv E(\Delta | X = x, P(Z) = P(z), D = 1) \\
\Delta^{TT}(x, D = 1) &\equiv E(\Delta | X = x, D = 1) \\
&= E(\Delta | X = x, \tilde{U} \leq P(Z)) \\
\Delta^{LATE}(x, P(z), P(z')) &\equiv \frac{E(Y|X = x, P(Z) = P(z)) - E(Y|X = x, P(Z) = P(z'))}{P(z) - P(z')} \\
\Delta^{MTE}(x, u) &\equiv E(\Delta | X = x, \tilde{U} = u)
\end{aligned} \tag{1}$$

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$$
\begin{aligned}
\Delta^{ATE}(x) &= \int E(\Delta | X = x, \tilde{U} = u) du \\
\Delta^{TT}(x, P(z), D = 1) &= \frac{1}{P(z)} \int_0^{P(z)} E(\Delta | X = x, \tilde{U} = u) du \\
\Delta^{TT}(x, D = 1) &= \int \Delta^{TT}(x, P(z), D = 1) dF_{P(Z)|X=x,D=1} \\
&= \int E(\Delta | X = x, \tilde{U} = u) g_x(u) du \\
\Delta^{LATE}(x, P(z), P(z')) &= \frac{1}{P(z) - P(z')} \int_{P(z')}^{P(z)} E(\Delta | X = x, \tilde{U} = u) du
\end{aligned} \tag{2}
$$

where

$$
g_x(u) = \frac{1 - F_{P(Z)|X=x}(u)}{\int (1 - F_{P(Z)|X=x}(t)) dt}
$$

Mirror Image Cost Parameters

- Roy model produces mirror-image set of treatment parameters, averages of the cost of treatment.
- Define

$$\Psi^{LIV}(w,u) \equiv E(\Psi | W = w, \tilde{U} = u).$$

$$
\begin{aligned}
\Psi^{ATE}(w) &\equiv E(\Psi|W=w) \\
&= \int E(\Psi|W=w,\tilde{U}=u)du \\
\Psi^{TT}(w,P(z),D=1) &\equiv E(\Psi|X=x,P(Z)=P(z),D=1) \\
&= \tfrac{1}{P(z)}\int_0^{P(z)} E(\Psi|X=x,\tilde{U}=u)du \\
\Psi^{TT}(w,D=1) &\equiv E(\Psi|W=w,D=1) \\
&= \int E(\Psi|X=x,\tilde{U}=u)g_w(u)du \\
\Psi^{LATE}(w,P(z),P(z')) &\equiv \tfrac{1}{P(z)-P(z')}\int_{P(z')}^{P(z)} E(\Psi|W=w,\tilde{U}=u)du
\end{aligned}
\tag{3}
$$

$$g_w(u) = \frac{1 - F_{P(Z)|W=w}(u)}{\int (1 - F_{P(Z)|W=w}(t))dt}$$

$F_{P(Z)|W=w}$ denotes the distribution of $P(Z)$, $W = w$.

$$\Delta^{LIV}(x, P(z))$$

$$\Delta^{LIV}(x, P(z)) \equiv \frac{\partial E(Y|X = x, P(Z) = P(z))}{\partial P(z)}. \tag{4}$$

$$\Delta^{LIV}(x, P(z)) = \Delta^{MTE}(x, P(z))$$

$$
\begin{aligned}
\Delta^{MTE}(x, P(z)) &= E\left(\Delta \middle| X = x, \tilde{U} = P(z)\right) \\
&= E\left(\Delta \middle| X = x, \Delta(x) = \Psi(w)\right) \\
&= E\left(\Delta(x) \middle| \Delta(x) = \Psi(w)\right)
\end{aligned}
$$

$$
\begin{aligned}
\Delta(x) &= \mu_1(x) - \mu_0(x) + U_1 - U_0 \\
\Psi(w) &= \varphi(w) + V
\end{aligned}
$$

$$
\begin{aligned}
E\left(\Delta(x)\middle|\Delta(x) \right. &= \left. \Psi(w)\right) = E\left(\Psi(w)\middle|\Delta(x) = \Psi(w)\right) \\
&= E\left(\Psi(w)\middle|W = w, \tilde{U} = P(z)\right) \\
&= \Psi^{MTE}(w, P(z))
\end{aligned}
$$

$$
\Delta^{LIV}(x, P(z)) = \Delta^{MTE}(x, P(z)) = \Delta^{LIV}(w, P(z)). \tag{5}
$$

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Benefit and Cost Parameters

- $\Delta^{LIV}(x, P(z)) = \dfrac{\partial E(Y|X = x, P(Z) = P(z))}{\partial P(z)}$.
- $\Delta^{LIV}(x, P(z))$ can be identified for any $(x, P(z))$.
- Follow the same arguments to use Ψ^{LIV} to identify or bound Ψ^{ATE} and Ψ^{TT}.

- Ψ^{LIV} identifies $\Psi^{ATE}(w)$ if the support of $P(Z)$ conditional on $W = w$ is the full unit interval.
- What information is available on the underlying benefit functions μ_0 and μ_1.

$$\Delta^{MTE}(x, P(z)) = E\left(\Delta \middle| X = x, \tilde{U} = P(z)\right) = \mu_1(x) - \mu_0(x) + \Upsilon(P(z)) \tag{6}$$

$$\Upsilon(P(z)) = E(U_1 - U_0 | \tilde{U} = P(z))$$

$$\Psi^{MTE}(w, P(z)) = E\left(\Psi \middle| W = w, U = P(z)\right) = \varphi(w) + \Gamma(P(z)). \tag{7}$$

$$\Gamma(P(z)) = E(V | \tilde{U} = P(z)).$$

$$\text{Let } \Delta^{LIV}(z) = \Delta^{LIV}(x, P(z)).$$

$$\Delta^{LIV}(z) = \Delta^{MTE}(x, P(z)) = \Psi^{MTE}(w, P(z))$$

$$\begin{aligned} \Delta^{LIV}(z) - \Delta^{LIV}(z') &= (\mu_1(x) - \mu_0(x)) - \left(\mu_1(x') - \mu_0(x')\right) \\ &= \varphi(w) - \varphi(w') \end{aligned}$$

Can identify $\varphi(w)$ up to support of W conditional on $P(Z) = P(z)$.

Shifting z while conditioning on $P(z)$ shifts $(\mu_1(x) - \mu_0(x))$ and $\varphi(w)$.

- If V is degenerate,

$$D = 1 \text{ if } Y_1 - Y_0 > \varphi(W).$$

$$\Delta^{LIV}(z) = \varphi(w).$$

- Deterministic cost function.

Summary

- Roy model structure can be exploited to identify cost parameters without direct information on the cost of treatment.